Status, Plans, and Issues in 2-Fluid drift-MHD Equations

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CEMM Meeting

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SEAVIEW C

Long Beach Hyatt

Important References

L. E. Sugiyama and W. Park, "A nonlinear two-fluid model for toroidal plasmas", Phys. Plasmas **7**: 4644-4658 (2000)

E.V. Belova, "Nonlinear gyroviscous force in a collisionless plasma", PPPL-3569, 2001, Phys. Plasmas, 2001

R.D. Hazeltine and J.D. Meiss, "Shear-Alfven Dynamics of Toroidally Confined Plasmas", Physics Reports **121**: 1-164 (1985) "Plasma Confinement", Addison-Wesley, 1992

Z. Chang and J.D. Callen, "Generalized gyroviscous force and its effect on the momentum balance equation", Phys. Fluids B **4**: 1766-1771 (1992)

A.I. Smolyakov, "Gyroviscous forces in a collisionless plasma with temperature gradients", Can. J. Phys. **76**: 321-331 (1998)

L. C. Steinhauer and A. Ishida, "Relaxation of a two-species magnetofluid and application to finite-β flowing plasmas", Phys. Plasmas **5**: 2609-2622 (1998)

Consider the Ion and Electron Momentum equations in the

Drift Model: (HM, SP)

$$\delta \equiv \frac{\rho}{L}, \quad V \sim \delta V_{th}, \quad \frac{\partial}{\partial t} \sim \frac{\delta V_{th}}{L}$$

Gyrokinetic Model: (EB)

$$V \sim \delta V_{th}, \quad \frac{\partial}{\partial t} \sim \frac{V_{th}}{L}$$

$$\boxed{\begin{aligned} & n_i m_i \left(\frac{\partial \vec{V}_i}{\partial t} + \left(\vec{V}_i \bullet \nabla \right) \vec{V}_i \right) + \nabla P = \vec{J} \times \vec{B} - \nabla \bullet \overleftrightarrow{\Pi}_i^{gv} + N_i \vec{B} \\ & \vec{E} + \vec{V}_e \times \vec{B} = \eta \vec{J}^* - \frac{1}{ne} \nabla p_e - \frac{0.71}{e} \nabla_{\|} k_B T_e - N_e \vec{B} \end{aligned}}$$

Here,

$$\eta \vec{J}^* \equiv \eta_{\|} \vec{J}_{\|} + \eta_{\perp} \left(\vec{J}_{\perp} - \frac{3}{2} n \frac{\vec{B} \times \nabla T_e}{B^2} \right)$$

Braginskii, NRL,
HM, SP,…

Need $\nabla \bullet \overleftrightarrow{\Pi}_i^{gv}, N_i, N_e$

Consider the ion momentum equation in the *Drift Model*:

$$\boxed{n_i m_i \left(\frac{\partial \vec{V}_i}{\partial t} + \left(\vec{V}_i \bullet \nabla \right) \vec{V}_i \right) + \nabla P = \vec{J} \times \vec{B} - \nabla \bullet \overleftrightarrow{\Pi}_i^{gv} + N_i \vec{B}}$$

$$\nabla \bullet \overleftrightarrow{\Pi}_i^{gv} = \left(\nabla \bullet \overleftrightarrow{\Pi}_i^{gv} \right)_\perp + \left(\nabla \bullet \overleftrightarrow{\Pi}_i^{gv} \right)_{||} \qquad \text{(EB)}$$

$$= -n_i m_i \left(\frac{\partial \vec{V}_*}{\partial t} + \left(\vec{V}_i \bullet \nabla \right) \vec{V}_* \right) - (n_i m_i \vec{V}_* \bullet \nabla) \vec{V}_{||i} + \underline{\nabla_\perp \tilde{\chi} + \upsilon_{gv||}}$$

$$= -n_i m_i (\vec{V}_* \bullet \nabla) \vec{V}_i + \underline{\hat{b} \times \nabla \left(\nabla_{||} \left\| U \mu \right\| \right) + \nabla_\perp \tilde{\chi} + \upsilon_{gv||}}$$

$$\vec{V}_* \equiv \vec{B} \times \nabla p_i / enB^2$$

$$\tilde{\chi} \equiv -\frac{p_\perp}{2\Omega_i} \hat{b} \bullet \nabla \times V_\perp - \frac{1}{4\Omega_i} \nabla \times \vec{q}_\perp$$

$$\upsilon_{gv||} \equiv \frac{1}{B^2} \nabla_\perp \left\{ \nabla_\perp \varphi, \left\| \mu U \right\| \right\}$$

$$\left\| U \mu \right\| B = p_\perp V_{||} + q_{||}^\perp$$

Note: HM &CC agree with EB in limit ∇T=0.

Also, HM argue underlined terms can sometime be dropped

The 2 forms in (EB) lead to 2 equivalent expressions of the ion momentum equation in the *Drift Model*:

A. $$n_i m_i \left(\frac{\partial \vec{V}_i}{\partial t} + \left((\vec{V}_i - \vec{V}_*) \bullet \nabla \right) \vec{V}_i \right) + \underline{\upsilon_{gv\perp} + \nabla_\perp \tilde{\chi} + \upsilon_{gv\parallel}} + \nabla P = \vec{J} \times \vec{B}$$

B. $$n_i m_i \left(\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \bullet \nabla \right) \vec{V} + (\vec{V}_* \bullet \nabla) \vec{V}_\perp \right) + \underline{\nabla_\perp \tilde{\chi} + \upsilon_{gv\parallel}} + \nabla P = \vec{J} \times \vec{B}$$

$$\vec{V}_* \equiv \vec{B} \times \nabla p_i / enB^2$$

$$\vec{V}_{di} \equiv \vec{J}_\perp / en - \vec{B} \times \nabla p_e / enB^2$$

$$\vec{V} \equiv \vec{V}_i - \vec{V}_*$$

$$\vec{V}_{SP} \equiv \vec{V}_i - \vec{V}_{di}$$

$$\upsilon_{gv\perp} \equiv \hat{b} \times \nabla \left(\nabla_\parallel \| U \mu \| \right)$$

- HM uses form B. and neglect some underlined terms involving heat flux,
- SP approximate $\vec{V} \to \vec{V}_{SP}$

And $\vec{V}_* \to \vec{V}_{di}$ in form B and use a slightly modified form for the viscous terms. These approximations neglect terms higher order in their ordering parameter δ

Equivalent expressions of the electron equation:

A. $$\vec{E}+\vec{V}\times\vec{B}=\eta\vec{J}^{*}+\frac{1}{ne}\left(\vec{J}\times\vec{B}-\nabla p_e-\nabla_{\perp}p_i\right)-\frac{0.71}{e}\nabla_{\parallel}k_BT_e-N_e\vec{B}$$

B. $$\vec{E}+\vec{V}_{SP}\times\vec{B}=\eta\vec{J}^{*}-\frac{1}{ne}\nabla_{\parallel}p_e-\frac{0.71}{e}\nabla_{\parallel}k_BT_e-N_e\vec{B}$$

C. $$\vec{E}+\vec{V}_i\times\vec{B}=\eta\vec{J}^{*}+\frac{1}{ne}\left(\vec{J}\times\vec{B}-\nabla p_e\right)-\frac{0.71}{e}\nabla_{\parallel}k_BT_e-N_e\vec{B}$$

$$\vec{V}_* \equiv \vec{B}\times\nabla p_i / enB^2$$

$$\vec{V}_{di} = \vec{J}_{\perp}/en-\vec{B}\times\nabla p_e / enB^2$$

$$\vec{V}\equiv\vec{V}_i-\vec{V}_*$$

$$\vec{V}_{SP}\equiv\vec{V}_i-\vec{V}_{di}$$

$$\vec{T}\equiv n_im_i\left(\frac{\partial\vec{V}_i}{\partial t}+\left(\vec{V}_i\bullet\nabla\right)\vec{V}_i\right)+\nabla\bullet\overleftrightarrow{\Pi}_i^{gv}$$

- HM use form C.
- SP use form B. This is consistent with their substitution in the momentum equation:

$$\vec{V}\rightarrow\vec{V}_{SP},\quad or\quad \vec{V}_*\rightarrow\vec{V}_{di}$$

This approximation is equivalent to neglecting the (higher order) polarization drift, since:

$$\vec{V}-\vec{V}_{SP}=\vec{V}_{di}-\vec{V}_*=\vec{B}\times\vec{T}/enB^2$$

It also removes the whistler wave from the equations.

Alternate form in terms of the generalized vorticity (the curl of the canonical momentum):

Note that if we define: $\vec{B}^* \equiv \nabla\times(\vec{A}+\frac{m_i}{e}\vec{V})$ The HM form of the electron and ion momentum equations become:

$$\frac{\partial \vec{B}^*}{\partial t} = \nabla\times\left[\vec{V}\times\vec{B}^* - \frac{m_i}{e}(\vec{V}_*\bullet\nabla)\vec{V}_\perp + \frac{0.71}{e}\nabla_\parallel k_B T_e + N_e\vec{B} - \eta\vec{J}^*\right]$$

$$n_i m_i\left(\frac{\partial\vec{V}}{\partial t} + \left(\vec{V}\bullet\nabla\right)\vec{V} + (\vec{V}_*\bullet\nabla)\vec{V}_\perp\right) + \nabla P =$$

$$\left[\nabla\times\left(\vec{B}^* - \frac{m_i}{e}\nabla\times\vec{V}\right)\right]\times\left(\vec{B}^* - \frac{m_i}{e}\nabla\times\vec{V}\right)$$

$$\nabla\times\vec{B}=\nabla P=\vec{V}=0$$

2-fluid zero-pressure dispersion relation for HM equations:

$$\left[\frac{\omega^2}{V_A^2}-(k_x^2+k_z^2)\right]\left[\frac{\omega^2}{V_A^2}-k_z^2\right]-\frac{\omega^2}{V_A^2}\left(\frac{V_A^2}{\Omega^2}\right)k_z^2(k_x^2+k_z^2)=0$$

$$\vec{B}_0=(0,0,B),$$
$$\vec{k}=(k_x,0,k_z)$$

the Hall modified fast wave (+) and shear Alfven wave (-) are given by:

$$\omega^2/V_A^2=\frac{1}{2}\left[k_x^2+2k_z^2+\frac{V_A^2}{\Omega^2}k_z^2\left(k_x^2+k_z^2\right)\right]$$
$$\pm\frac{1}{2}\left[k_x^4+2\frac{V_A^2}{\Omega^2}\left(k_x^2+2k_z^2\right)k_z^2\left(k_x^2+k_z^2\right)+\frac{V_A^4}{\Omega^4}k_z^4\left(k_x^2+k_z^2\right)^2\right]^{1/2}$$

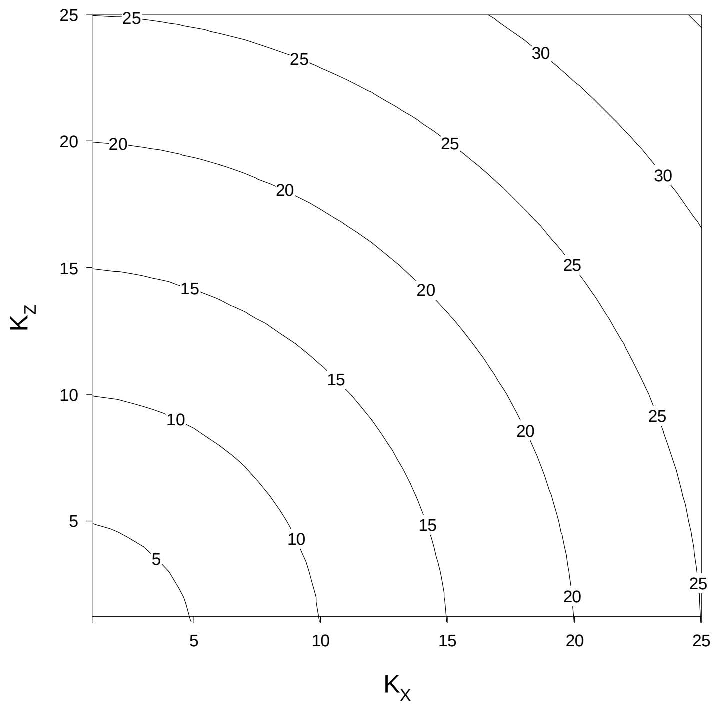
large k limit:

$$k^2\gg\left(\frac{V_A^2}{\Omega^2}\right)^{-1}$$

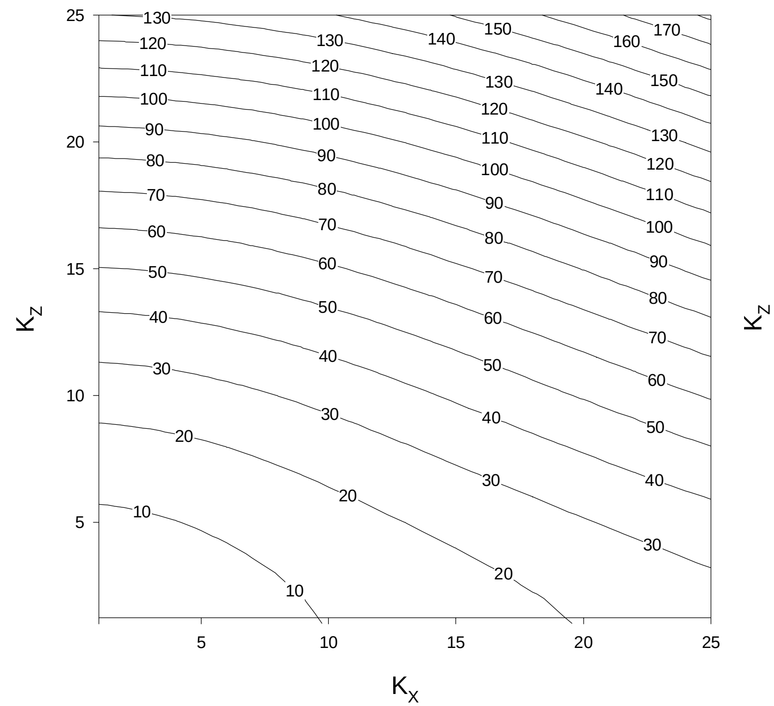
$$\frac{\omega^2}{V_A^2}\sim\left(1+\frac{V_A^2}{\Omega^2}k_z^2\right)\left(k_x^2+k_z^2\right)+\ldots$$ Fast wave

$$\frac{\omega^2}{V_A^2}\sim\left(\frac{V_A^2}{\Omega^2}\right)^{-1}-\left(\frac{V_A^2}{\Omega^2}\right)^{-2}\frac{\left(k_x^2+2k_z^2\right)}{k_z^2\left(k_x^2+k_z^2\right)}+\ldots$$ Shear Alfven

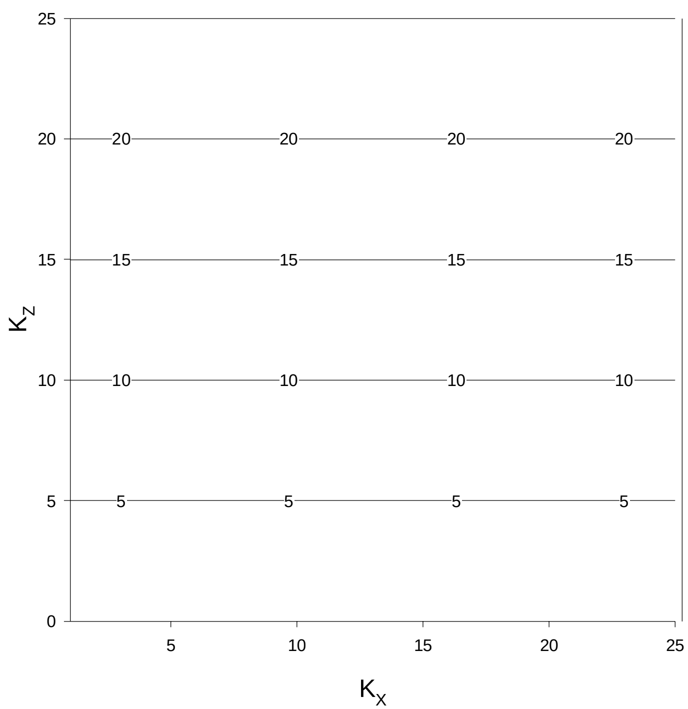
ω^2/V_A^2 for Fast Wave with $(V_A^2/\Omega^2=0)$



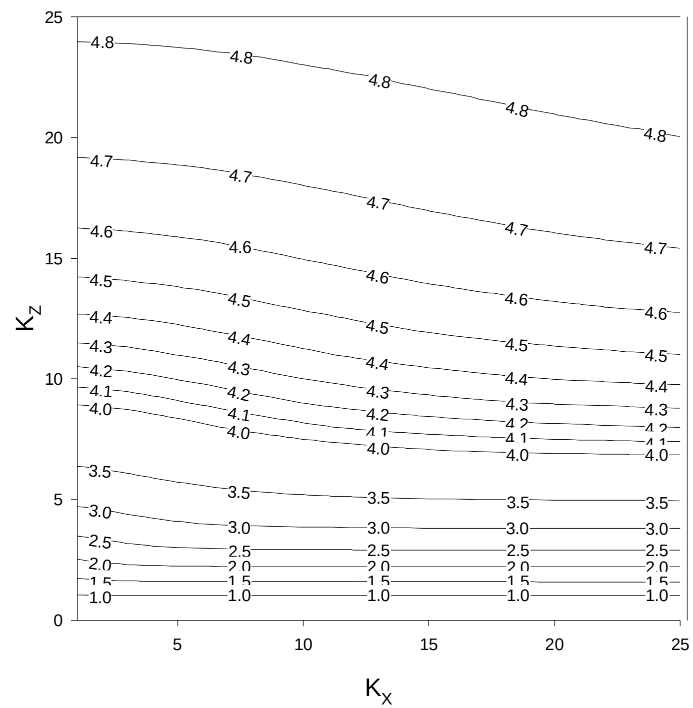
ω^2/V_A^2 for Hall modified Fast Wave with $(V_A^2/\Omega^2=0.04)$



ω^2/V_A^2 for Alfven Wave with $(V_A^2/\Omega^2=0)$



ω^2/V_A^2 for Hall modified Alfven Wave with $(V_A^2/\Omega^2=,04)$



Continuity and energy equations:

$$\frac{\partial n}{\partial t} + \nabla \bullet \left(n\vec{V}_i \right) = 0$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \nabla \bullet \left(\frac{3}{2} p_e \vec{V}_i \right) = -p_e \nabla \bullet \vec{V}_i + \frac{\vec{J}}{ne} \bullet \left[\frac{3}{2} \nabla p_e - \frac{5}{2} \frac{p_e}{n} \nabla n + \vec{R} \right] - \nabla \bullet \vec{q}_e - Q_\Delta$$

$$\frac{3}{2}\frac{\partial p_i}{\partial t} + \nabla \bullet \left(\frac{3}{2} p_i \vec{V}_i \right) = -p_i \nabla \bullet \vec{V}_i + \Pi_i : \nabla V_i - \nabla \bullet \vec{q}_i + Q_\Delta$$

$$\vec{R} = -0.71\, n\, \nabla_{\parallel} k_B T_e + ne \left[\eta_{\parallel} \vec{J}_{\parallel} + \eta_{\perp} \left(\vec{J}_{\perp} - \frac{3}{2} \frac{n\vec{B} \times \nabla T_e}{B^2} \right) \right] - neN_e \vec{B}$$

This form conserves energy, but needs an expression for $\Pi : \nabla V_i$

Two-fluid CEMM Activity:

A. m = 1 mode
   1. Cylinder (SP, RN)
      i. Testbed for comparison of 2 different formulations and effects of neglected terms
   2. Torus
      i. Identification of toroidal effects
      ii. Sawtooth mechanism

B. m > 1 modes
   1. Cylinder (SP)
      i. Testbed for comparison of 2 different formulations and effects of neglected terms
   2. Torus (SP)
      i. Magnetic Island rotation
      ii. Neoclassical tearing
      iii. Nonlinear island coupling

C. Integrated Effects
   1. Sawtooth trigger for NTM
   2. Mechanisms for onset of disruptions

Summary and recommendations:

- Belova paper demonstrates equivalence of the 2-forms of the gyroviscous cancellation allowing formulation either in terms of guiding center velocity V or ion velocity V_i
- Nature of approximation in neglecting gradient terms in ion-momentum equation (HM) should be clarified
- SP neglect of higher-order polarization drift terms ($V_* \rightarrow V_{di}$) removes Hall term in Ohm's law and hence remove whistler waves. Effect on applications should be clarified.
- Energy conservation is an outstanding problem since there is no agreed upon expression for $\Pi : \nabla V$
- Need conservative expression for $\nabla \bullet \Pi$ including gradients and curvature terms in B: (for both GV and N_e and N_i parts)
- CEMM goal should be to develop and document "standard" sets of equations: compare and contrast