NIMROD: a computational laboratory for studying nonlinear fusion magnetohydrodynamics

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Theme: NIMROD’s numerical capabilities create a virtual laboratory for fusion MHD research.

OUTLINE

I. Algorithm features
II. Tokamak and ST applications
III. Magnetic relaxation
IV. Conclusions

The project has been a multi-institutional effort since 1996.

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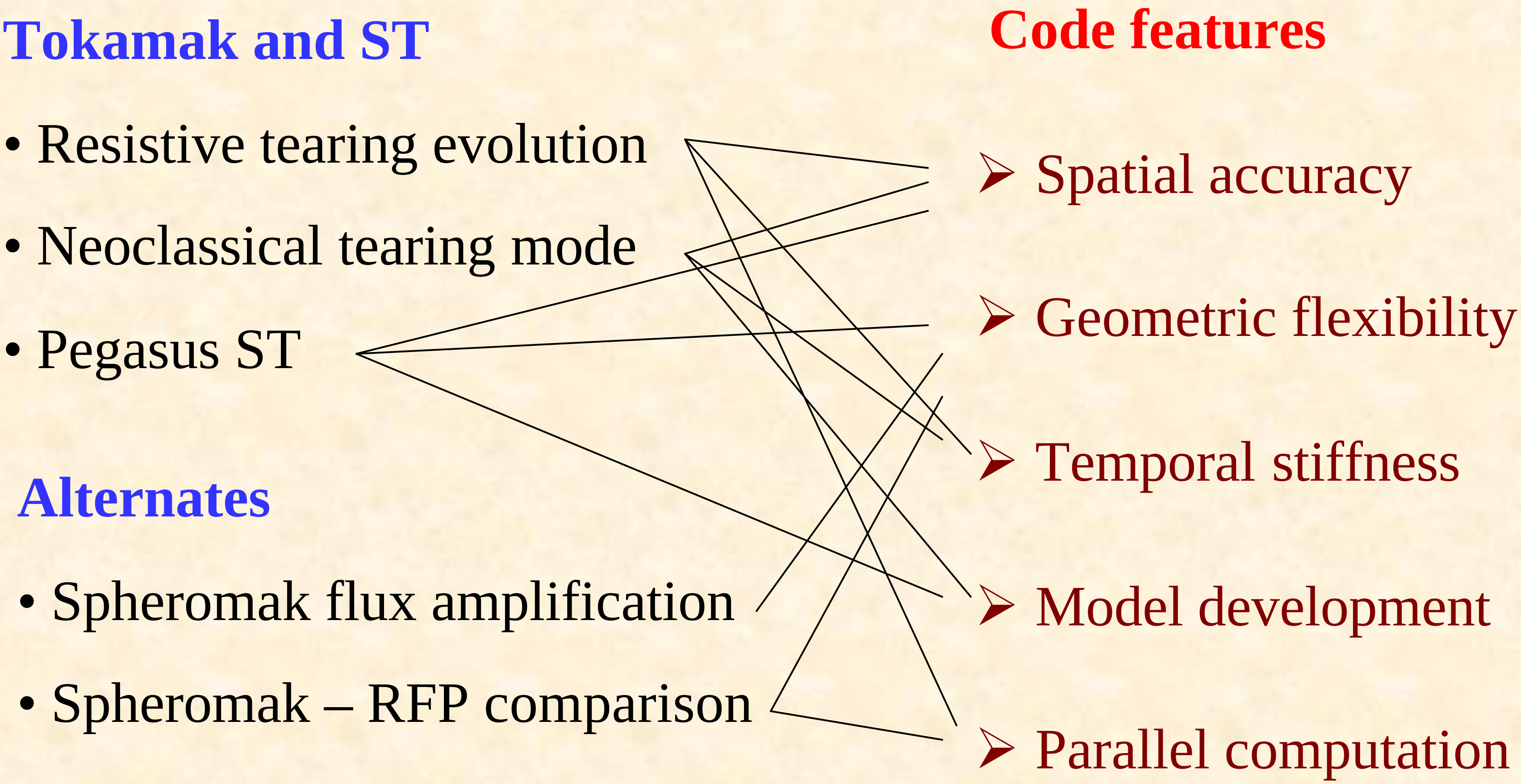
Presently, there is non-team-member use of the NIMROD code at LLNL, IFS, Univ. of WA, UCLA, and AIST-Japan.

The NIMROD code has a unique combination of advanced numerical methods for high-temperature simulation:

- High-order finite element representation of the poloidal plane:
    - accuracy for MHD and transport anisotropy at realistic parameters: $S>10^6$, $c_{\|}/c_{\mathrm{perp}}>10^9$
    - flexible spatial representation
- Temporal advance with semi-implicit and implicit methods:
    - multiple time-scale physics from ideal MHD (*m*s) to transport (10-100 ms)
- Coding modularity for physics model development
- Large-scale parallel computing

Selected applications highlight code features.



Tokamak and ST

- Resistive tearing evolution
- Neoclassical tearing mode
- Pegasus ST

Alternates

- Spheromak flux amplification
- Spheromak – RFP comparison

Code features

- ➢ Spatial accuracy
- ➢ Geometric flexibility
- ➢ Temporal stiffness
- ➢ Model development
- ➢ Parallel computation

Simulation of a classical tearing-mode demonstrates application in nonlinear low-dissipation conditions.

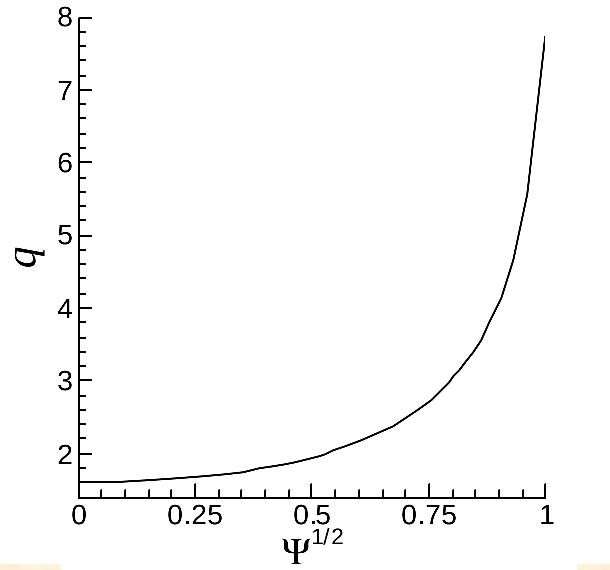
NIMROD Simulation

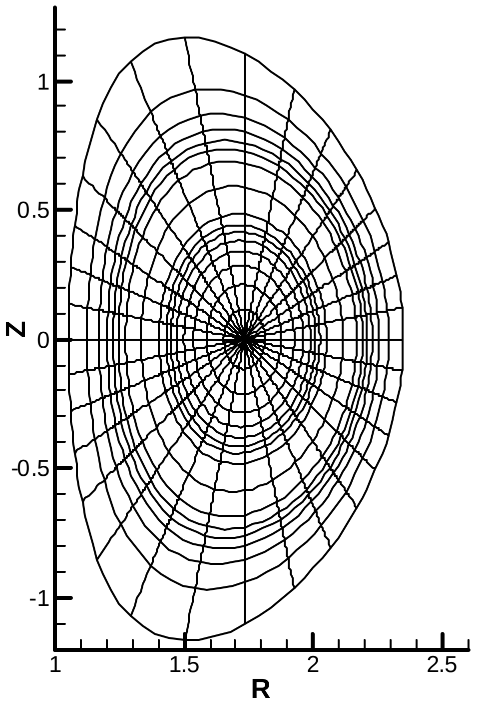
- $S=10^6$
- $Pm=t_R/t_n=0.1$
- $t_A=1ms$
- *b* <<1% to avoid GGJ stabilization

DIII-D L-mode Startup Plasma

[R. LaHaye, Snowmass Report]

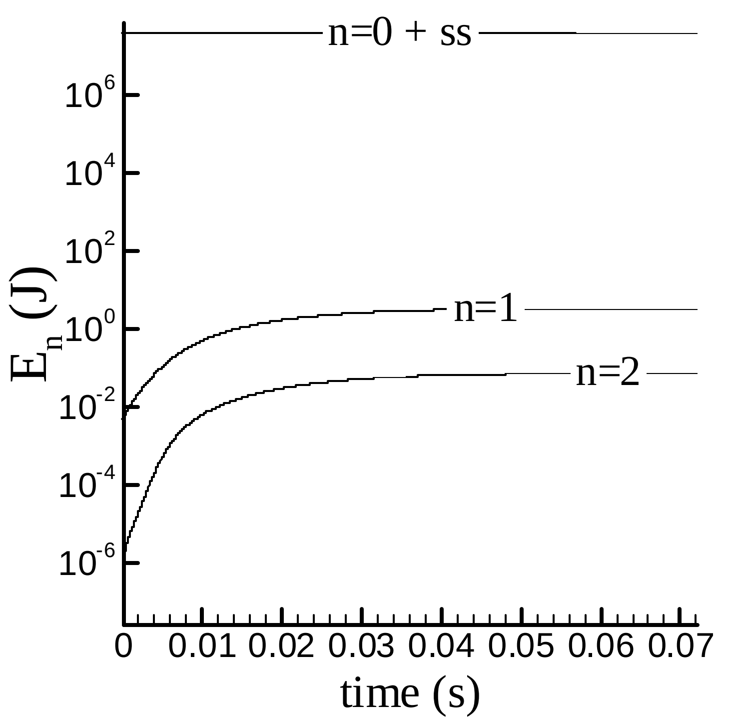
- $S=1.6x10^6$
- $Pm=4.5$
- $t_A=0.34ms$
- $t_E=0.03s$



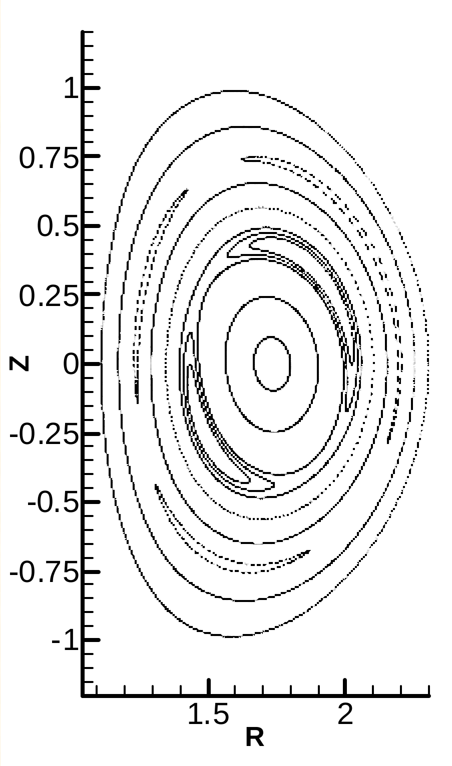


Small Δ' (linear gt_A=5x10^{-4}) leads to nonlinear evolution over the energy confinement time-scale.

Magnetic Energy vs. Time



Saturation of Coupled Island Chains



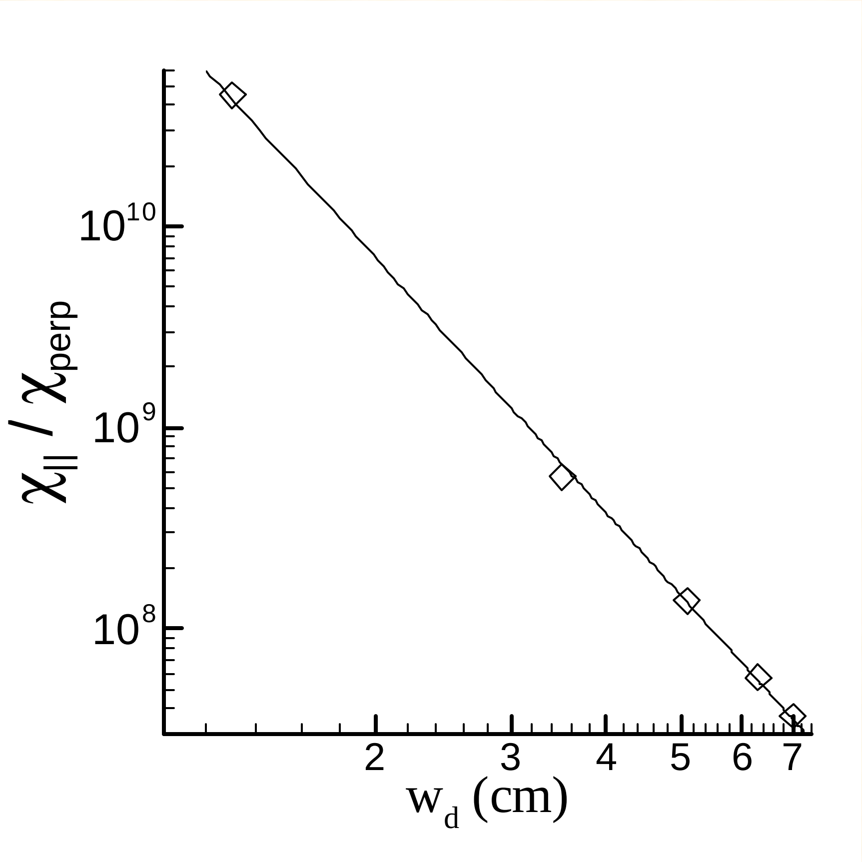
- 5th-order accurate biquartic finite elements resolve anisotropies.
- 20,000 semi-implicit time-steps evolve solution for times > t_E.
- Explicit computation is impossible→2x10^8 time-steps.

Testing anisotropic thermal conduction at various times reproduces the w_d^{-4} scaling. [Fitzpatrick, PoP 2, 825 (1995)]

- Conductivity ratio is scaled until an inflection in *T* within (2,1) island is achieved.

- Power-law fit is $c_{\|}/c_{\text{perp}}=3.0\text{x}10^{3}\,(w_d/a)^{-4.2}$.

- Result is for toroidal geometry.



- High-order spatial convergence is required for realistic anisotropy.
- Implicit thermal conduction is required for stiffness.

The time-scales arising with nonlinear resistive MHD evolution indicate the need for drift and transport effects.

- Flux-surface average closures derived for transport calculations [Hinton and Hazeltine, Rev. Mod. Phys. **48**, 239 (1976), for example] are not suitable for 3D simulation.
- Gianakon, Kruger, and Hegna assessed heuristic local closures for simulating NTM physics with NIMROD [PoP **9**, 536 (2002)]. Local poloidal flow damping (*a*=*i*,*e*),

$$\nabla \cdot \Pi_a = r_a m_a \left\langle B^2 \right\rangle \frac{\mathbf{V}_a \cdot \hat{\mathbf{e}}_q}{\left(\mathbf{B} \cdot \hat{\mathbf{e}}_q\right)^2} \hat{\mathbf{e}}_q$$

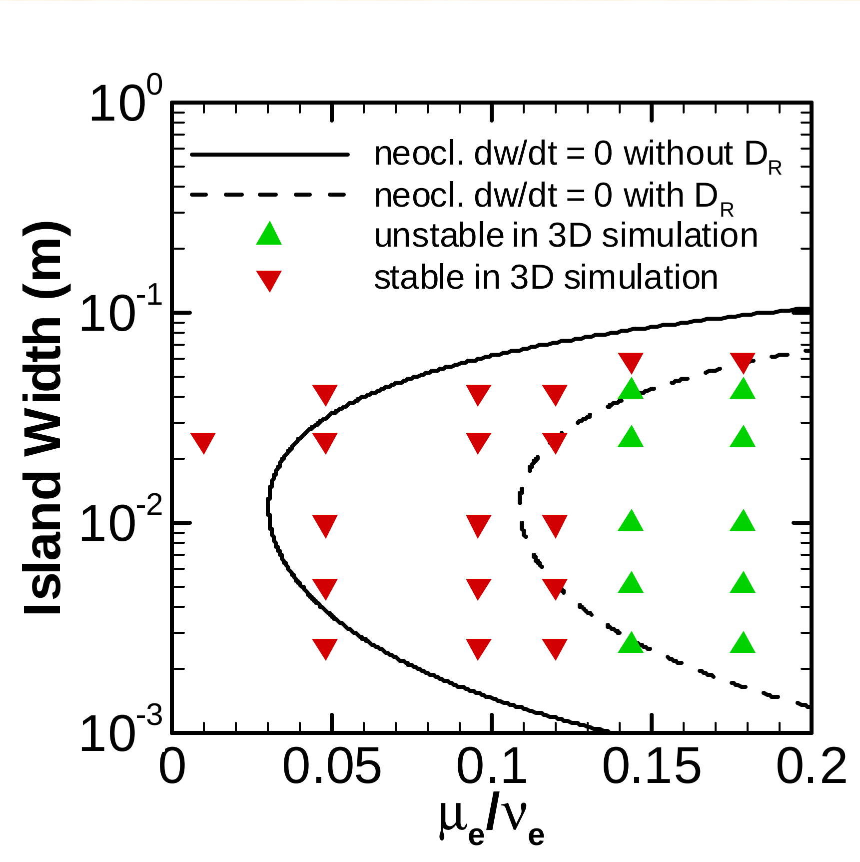
captures the important effects found in kinetic analysis that are needed for fluid moment evolution. [Alternatively, the electron force can be re-expressed using a resistive ordering:]

$$\nabla \cdot \Pi_e = -\frac{r_e m_e}{ne} \left\langle B^2 \right\rangle \frac{\mathbf{B}_0 \times \nabla p \cdot \hat{\mathbf{e}}_q}{B^2 \left(\mathbf{B} \cdot \hat{\mathbf{e}}_q\right)^2} \hat{\mathbf{e}}_q$$

NIMROD NTM simulations with (diamagnetic) poloidal flow damping reproduce the threshold found through small-island-width analysis including D_R.

- Calculations use an EFIT equilibrium for DIII-D discharge 86144.
- Simulations scan initial island width, $w(t=0)$, and viscous damping rate, m_e, to create a nonlinear stability diagram.

- NIMROD's high-order spatial accuracy and semi-implicit advance meet the need for realistic parameters, $c_{\|}/c_{perp}=10^{10}$ and $S=2.7\times10^{6}$.



NIMROD simulation results for NTM anisotropic conduction threshold [Gianakon, et al. PoP **9**, 536 (2001) and Hegna PoP **6**, 3980 (1999) for dw/dt].

For detailed modeling of nonlocal kinetic effects, we have developed an approach that solves drift kinetic equations to determine closures. [Held, Callen, Hegna, Sovinec, PoP **8**, 1171 (2001)]

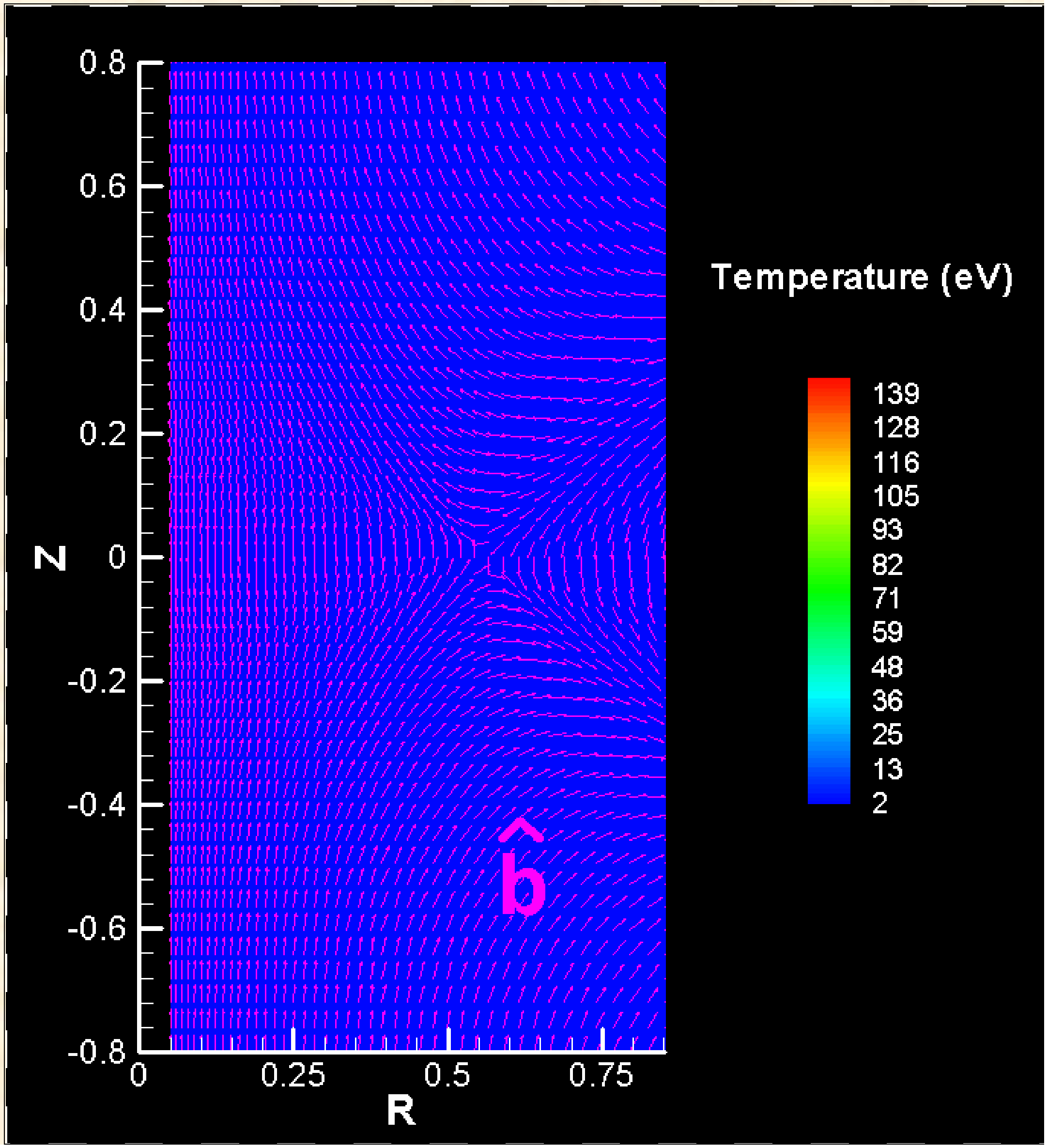
- Drift kinetic equations are solved allowing for maximal ordering of collisional and free-streaming terms (no assumptions on n wrt $\left|\mathbf{v}_{\|} \cdot \nabla\right|$)
- Kinetic fluxes are determined via integration along characteristics in a 3D solution.
  - Nonlocal fluxes interact self-consistently with fluid moments.
  - A semi-implicit advance allows large time-step.
- This approach is being applied for computing heat flux in RFP and NTM simulations.
- Electron stress computations will lead to a more fundamental description of NTM evolution.

Pegasus simulations apply anisotropic conduction and *h*(*T*) to reproduce nonlinear free-boundary evolution.

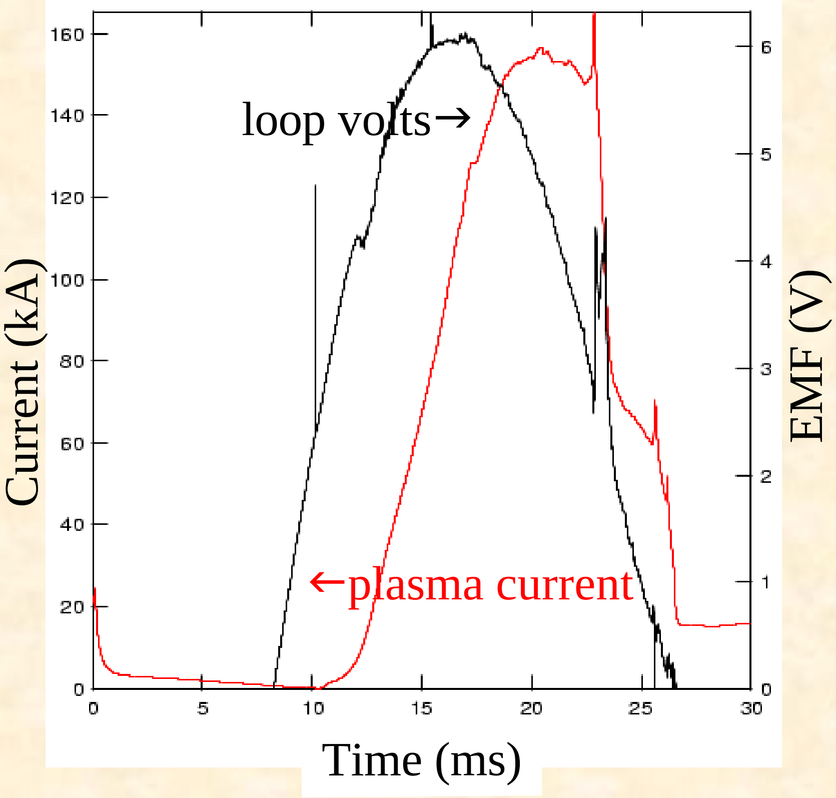
- Plasma current and separatrix evolve self-consistently with applied loop voltage and vertical-field ramp.

- Transport has a strong influence on dynamics.

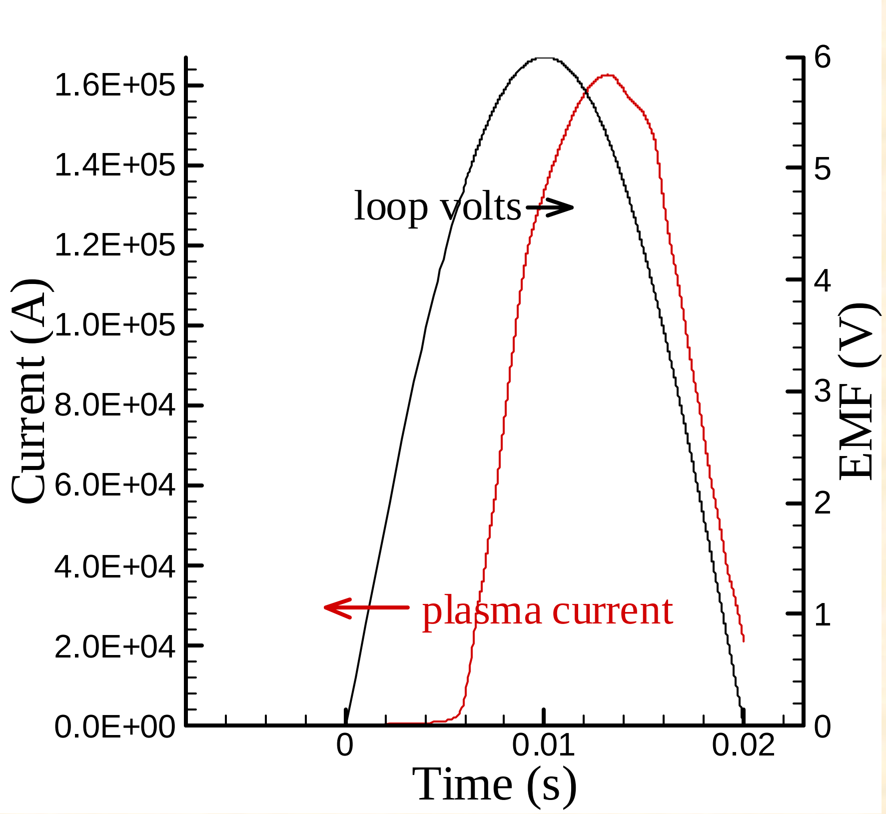
- High-order spatial accuracy is essential for distinguishing closed flux and open flux through modeled transport effects.



This study integrates MHD and transport effects with realistic geometry and experimental parameters.



Pegasus data courtesy of A. Sontag.

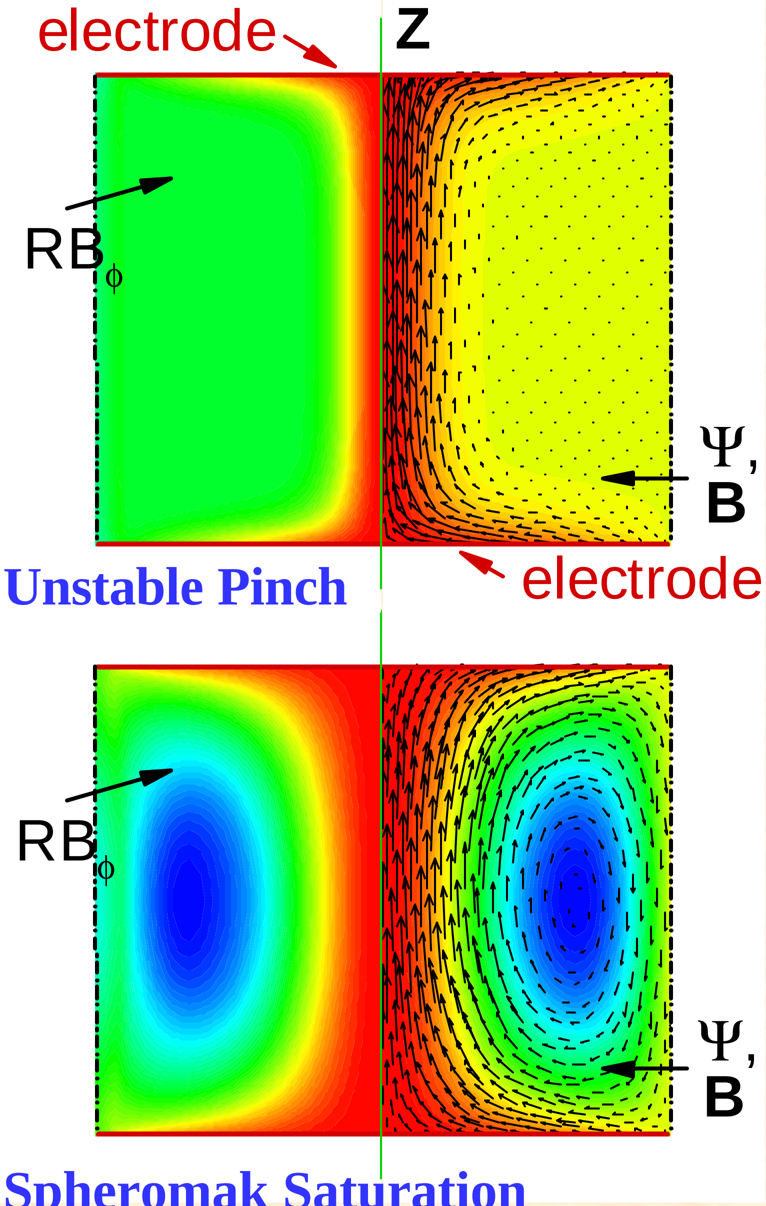


Axisymmetric simulation results.

- Study has focused on 2D evolution, but 3D tearing-mode simulation is a straightforward extension for NIMROD.
- Results emphasize interaction between MHD, transport effects, and overall performance.

A study of spheromak helicity injection and sustainment found flux amplification as a result of pinch current instability.

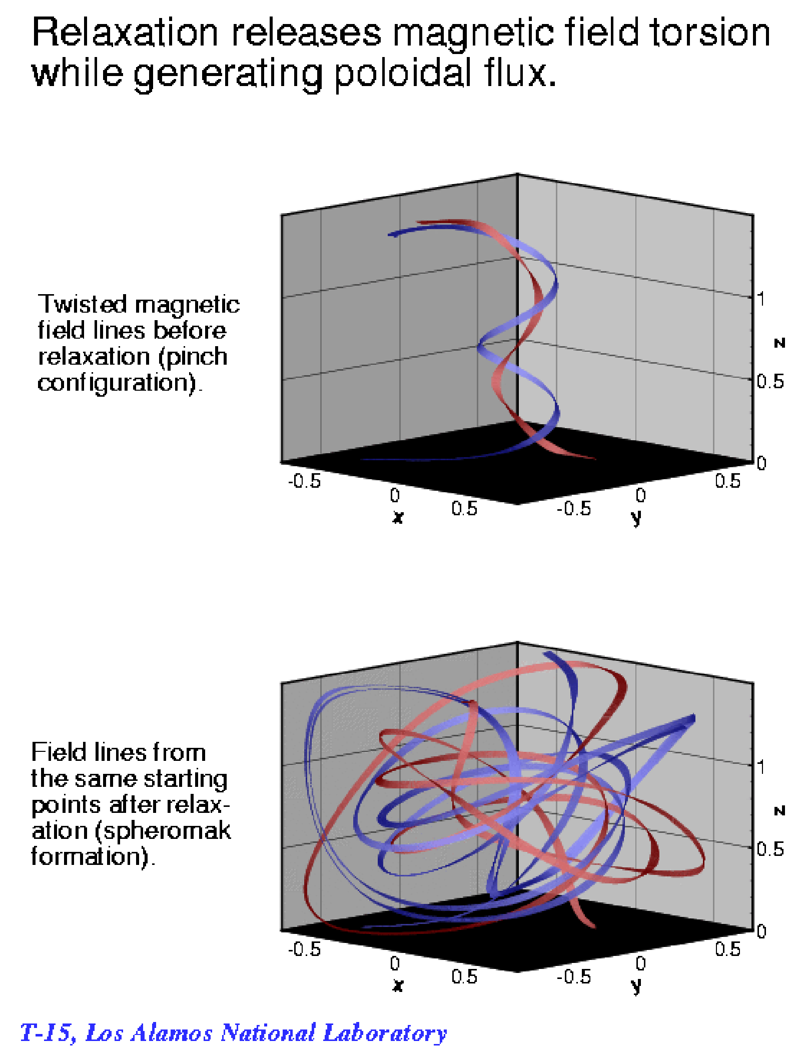
- Steady 2D solution to the resistive MHD equations is a pinch that is unstable to *n*=1.
- Full 3D evolution finds the conversion of toroidal flux to poloidal flux as part of the MHD saturation ($S=10^3$-10^4).
- Simply connected geometry is required.
- Geometric flexibility also allows study of gun-driven configurations.



Unstable Pinch

Spheromak Saturation

Flux amplification is a direct result of the resistive n=1 pinch mode and the azimuthal average of the resulting 3D field. [Sovinec, Finn, del-Castillo-Negrete, PoP **8**, 475 (2001).]



Relaxation releases magnetic field torsion while generating poloidal flux.

Twisted magnetic field lines before relaxation (pinch configuration).

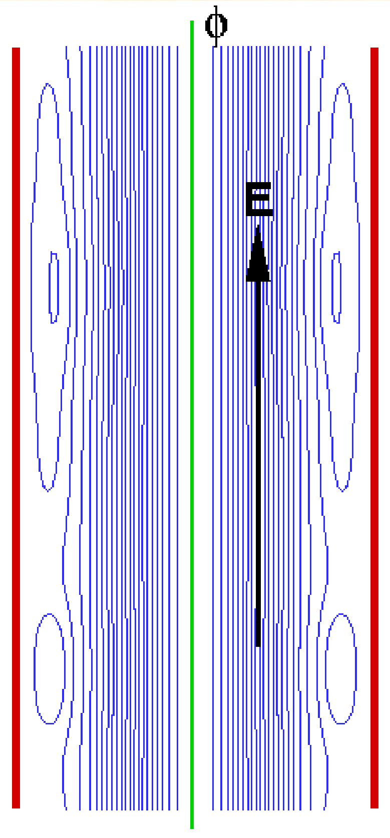
Field lines from the same starting points after relaxation (spheromak formation).

T-15, Los Alamos National Laboratory

- Formation includes magnetic reconnection.
- With sufficient drive, the final state exhibits limit-cycle behavior.
- Removing drive allows flux-surface formation as perturbations decay preferentially, and toroidal current is driven by induction.
- Detailed studies of confinement during transients are in progress at LLNL and Univ. of WI.

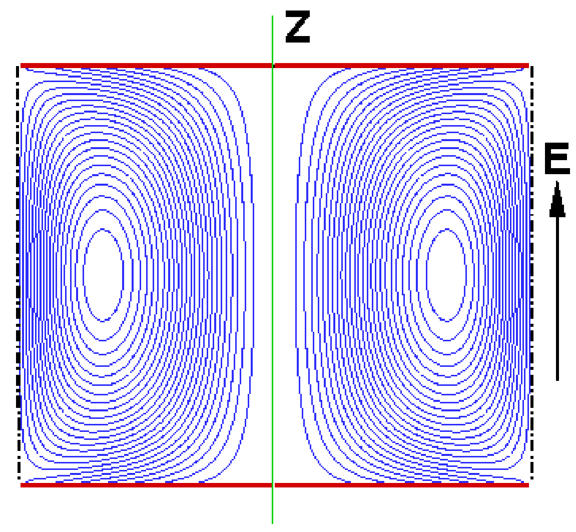
A comparison of spheromak and RFP simulations finds a detailed analogy when orientation is made with respect to the applied electric field.

Contours of Magnetic Flux



Cylindrical RFP

YES

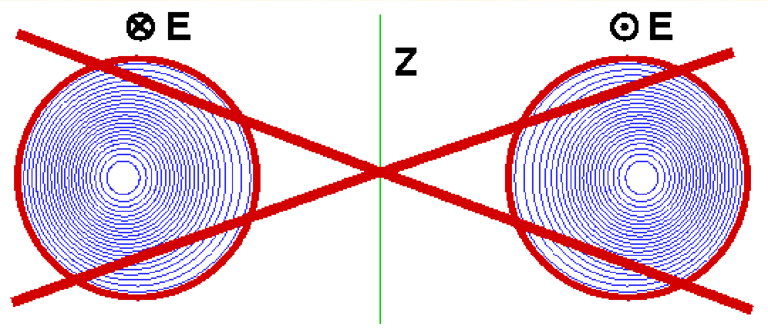


Flux-core Spheromak

- Geometric flexibility!!

NO

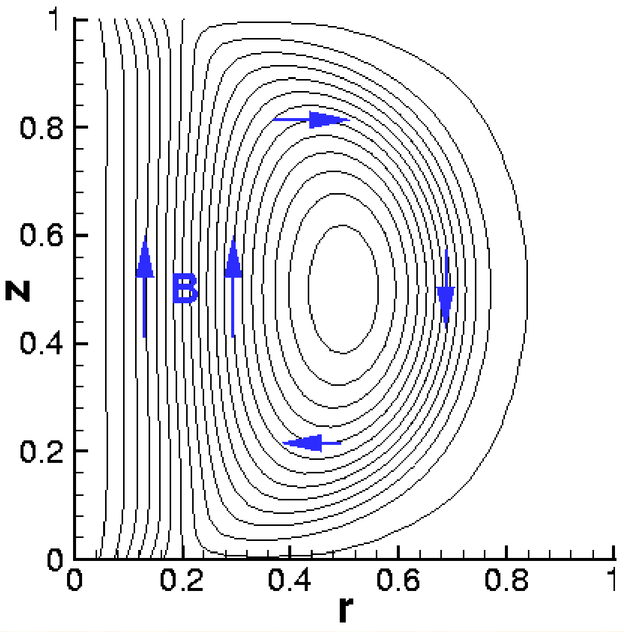
- Pinch drives fluctuations.
- Spheromak poloidal flux is analogous to RFP toroidal flux.



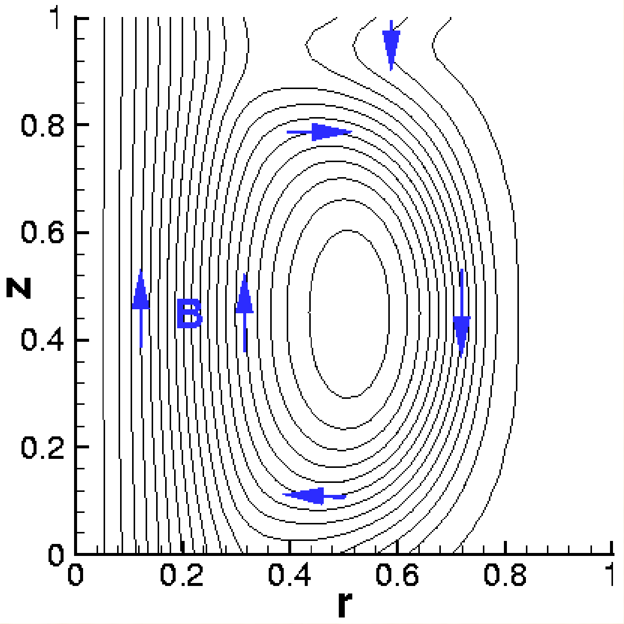
Toroidal RFP

A short periodic cylinder (~RFP) produces nearly the same ‘flux amplification’ as a line-tied (spheromak) configuration with the same applied **E**.

- Spheromak *n*=1 is analogous to the RFP *m*=1 ‘dynamo modes.’
- Flux amplification is an extreme version of RFP reversal.
- Physical dimensions influence the spectrum.

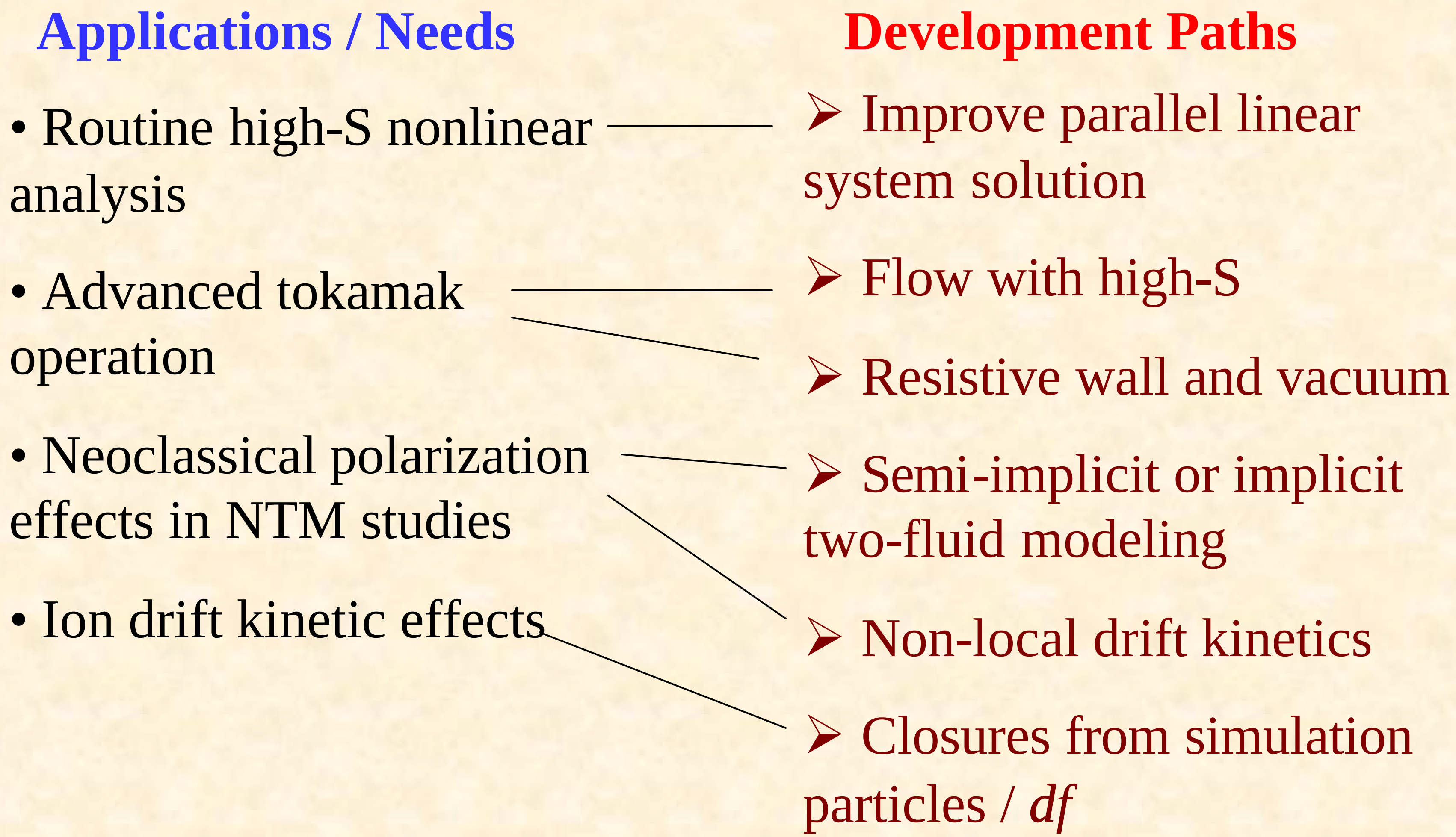


Line-tied Configuration



Periodic Configuration

Present and planned code development will provide ‘facility’ upgrades to extend application.



Applications / Needs

- Routine high-S nonlinear analysis
- Advanced tokamak operation
- Neoclassical polarization effects in NTM studies
- Ion drift kinetic effects

Development Paths

- ➢ Improve parallel linear system solution
- ➢ Flow with high-S
- ➢ Resistive wall and vacuum
- ➢ Semi-implicit or implicit two-fluid modeling
- ➢ Non-local drift kinetics
- ➢ Closures from simulation particles / *df*

Conclusions

- The unique combination of numerical capabilities in NIMROD enable a wide range of fusion MHD studies.
  - flexible and accurate spatial representation
  - semi-implicit advance for stiffness
- Application to realistic configurations
  - nonlinear analysis for experiments
  - extends and confirms theoretical predictions
  - motivates model development
- The productive synergy throughout the NIMROD effort anticipates future integrated modeling.

A study of the disruption precursor in D3D discharge 87009 (with NCS) is an example of application to ideal MHD physics in tokamaks.

- Analytical calculation [Callen, et al. PoP 6, 2963 (1999)] finds faster-than-exponential growth of and ideal interchange-like mode,

$$\exp[(t/t)^{3/2}]$$

where

$$t \equiv (3/2)^{2/3}\hat{g}^{-2/3}g_h^{-1/3} \quad \text{and} \quad g_h << \hat{g}$$

due to heating that changes the instantaneous growth rate

$$-w^2 = \hat{g}^2[b(t)/b_c - 1]$$

- The growth-rate scaling compares well with laboratory measurements, but the analytic treatment is very approximate.

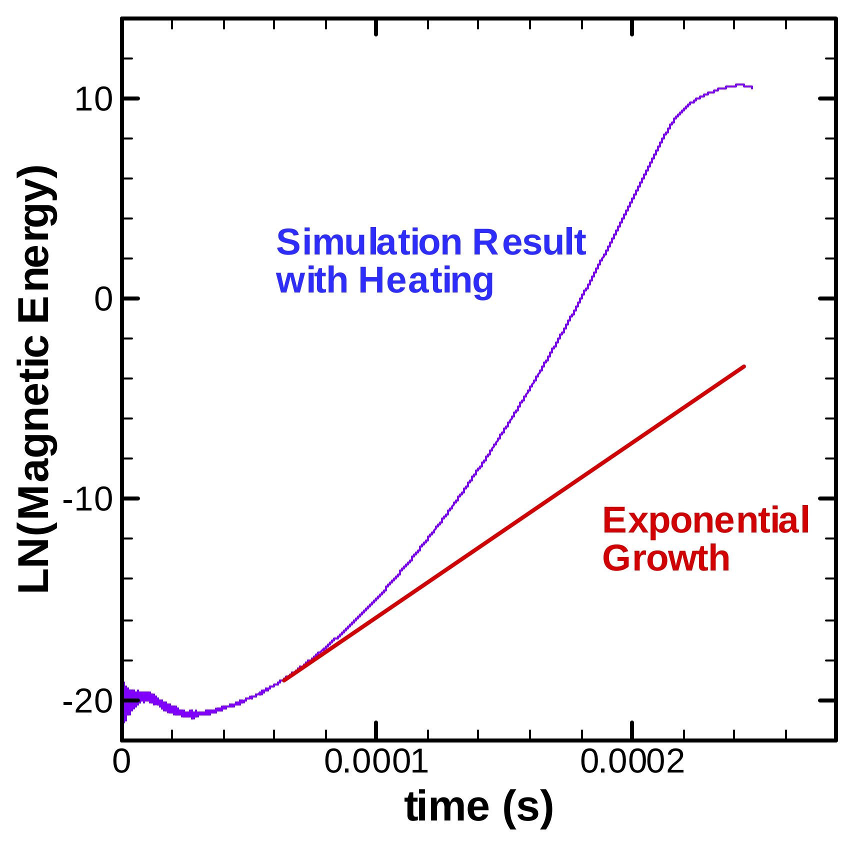
NIMROD simulations (with *b* increased for fixed boundary computation) confirm the predicted scaling in toroidal geometry with the interchange-like mode.

[Kruger, Schnack, Brennan, Gianakon, Sovinec, NF—submitted]

- Heating is applied proportional to the initial pressure distribution.

- Fitting the time-dependence for two heating rates produces the simulation-determined scaling

$$t \sim \hat{g}^{-0.72} g_h^{-0.28}$$



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