NIMROD Two-Fluid, Algorithm and Calculations

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University of Colorado at Boulder

Center for Integrated Plasma Studies

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Outline

- Two-fluid MHD
- Time-implicit method – a stability theorem
- NIMROD-2F implementation and dispersion tests
- FRC application
- Conclusions

Single/Two-Fluid "MHD" (1F/2F)

$$Mn\frac{\partial \mathbf{u}}{\partial t} = (\nabla \times \mathbf{B})/\mu_0 \times \mathbf{B} - \nabla(P_e + P_i) - Mn\mathbf{u}\bullet\nabla\mathbf{u}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left\{ \mathbf{u}\times\mathbf{B} - \frac{1}{e}\left[\boxed{\frac{(\nabla\times\mathbf{B})\times\mathbf{B}/\mu_0 - \nabla P_e}{n}} \right.\right.$$

$$\left.\left. \boxed{- m_e\left(\frac{\partial}{\partial t} + \mathbf{u}\bullet\nabla - \frac{\nabla\times\mathbf{B}/\mu_0}{en}\bullet\nabla\right)\left(\mathbf{u} - \frac{\nabla\times\mathbf{B}/\mu_0}{en}\right)} \right]\right\}$$

$$\frac{\partial n}{\partial t} = -\nabla\bullet n\mathbf{u}$$

$$\frac{\partial P_e}{\partial t} = -\mathbf{u}\bullet\nabla P_e - \Gamma_e P_e \nabla\bullet\mathbf{u} + \boxed{\frac{\nabla\times\mathbf{B}/\mu_0}{en}\bullet\nabla P_e + \Gamma_e P_e \nabla\bullet\frac{\nabla\times\mathbf{B}/\mu_0}{en}}$$

$$\frac{\partial P_i}{\partial t} = -\mathbf{u}\bullet\nabla P_i - \Gamma_i P_i \nabla\bullet\mathbf{u}$$

Can Rewrite Hall Term

$$Mn\frac{\partial \mathbf{u}}{\partial t} = \left(\nabla\times\mathbf{B}\right)/\mu_0\times\mathbf{B} - \nabla\left(P_e+P_i\right) - Mn\mathbf{u}\bullet\nabla\mathbf{u}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left\{\mathbf{u}\times\mathbf{B} - \frac{1}{e}\left[M\frac{\partial \mathbf{u}}{\partial t} + M\mathbf{u}\bullet\nabla\mathbf{u} + \frac{\nabla P_i}{n}\right.\right.$$

$$\left.\left. - m_e\left(\frac{\partial}{\partial t} + \mathbf{u}\bullet\nabla - \frac{\nabla\times\mathbf{B}/\mu_0}{en}\bullet\nabla\right)\left(\mathbf{u} - \frac{\nabla\times\mathbf{B}/\mu_0}{en}\right)\right]\right\}$$

$$\frac{\partial n}{\partial t} = -\nabla\bullet n\mathbf{u}$$

$$\frac{\partial P_e}{\partial t} = -\mathbf{u}\bullet\nabla P_e - \Gamma_e P_e\nabla\bullet\mathbf{u} + \frac{\nabla\times\mathbf{B}/\mu_0}{en}\bullet\nabla P_e + \Gamma_e P_e\nabla\bullet\frac{\nabla\times\mathbf{B}/\mu_0}{en}$$

$$\frac{\partial P_i}{\partial t} = -\mathbf{u}\bullet\nabla P_i - \Gamma_i P_i\nabla\bullet\mathbf{u}$$

1F→2F Plasma Features

- Waves (uniform, unbounded plasma)
  - Whistler
  - Kinetic Alfven
  - Low frequency electrostatic
- Drift waves
- ω* stabilization of MHD modes
- Electron-Ion decoupling

Physics of HMHD

- Full (warm 2 fluid) dispersion relation has 3 waves (cubic)
- This was given by Stringer (1963) and is also discussed by Swanson

$$w = \omega^2$$

$$w^3 - A w^2 + B w - C = 0$$

$$A = k_{\|}^2 + \left(1 + \hat{\beta}\right) k^2 + H^2 k_{\|}^2 k^2$$

$$B = k_{\|}^2 k^2 \left(1 + 2\hat{\beta} + \hat{\beta} H^2 k^2\right)$$

$$C = \hat{\beta} k^2 k_{\|}^4$$

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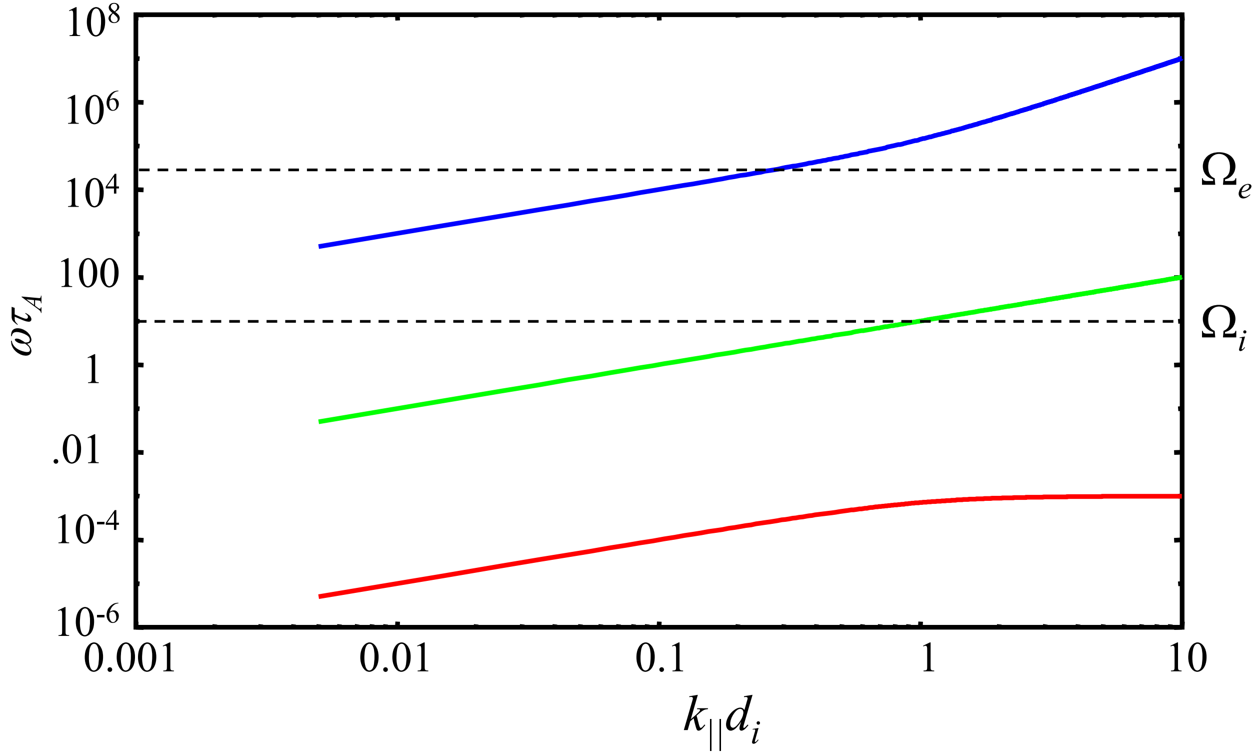
Modest Parameters Give Extreme Stiffness



E.G. $\hat{\beta} = 10^{-8}$, $k_{||}/k_{\perp} = 10^{-4}$, $d_i/L_x = 0.1 \Rightarrow$ 12 orders of magnitude in ω

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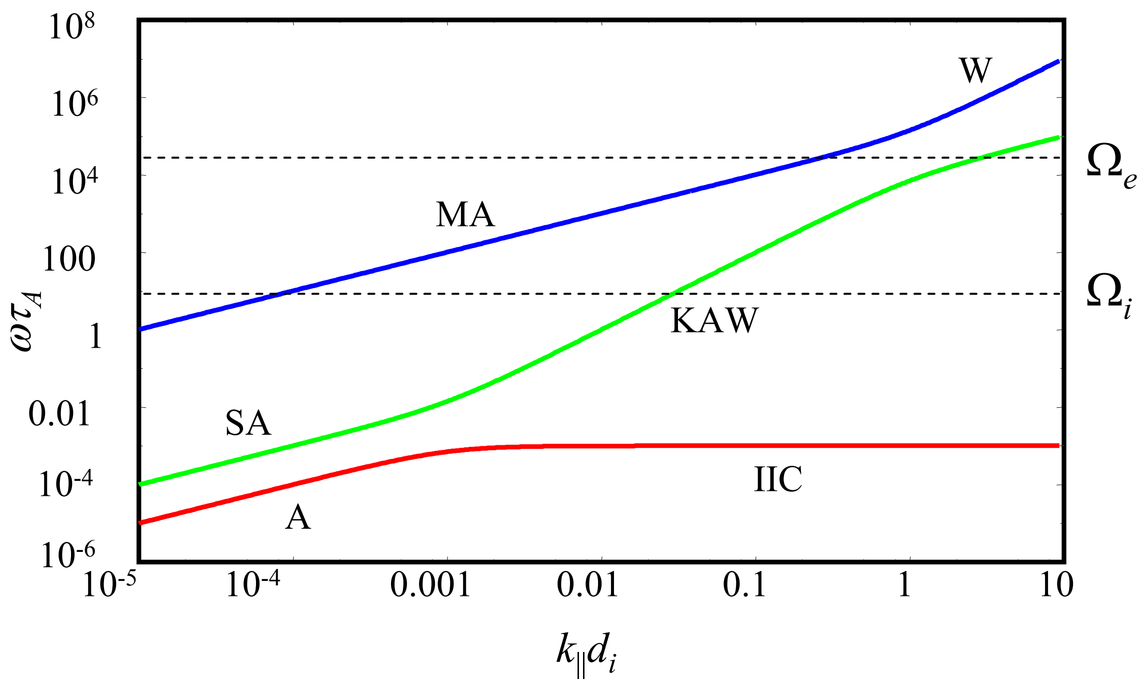

E.G. $\hat{\beta} = 10^{-2}$, $k_{\|}/k_{\perp} = 10^{-4}$, $d_i/L_x = 0.1 \Rightarrow$ 12 orders of magnitude in ω

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Extension of 1F Method to 2F Challenging

- 1F of form

$$\dot{\mathbf{B}} = \dot{\mathbf{B}}(\mathbf{u}), \dot{\mathbf{u}} = \dot{\mathbf{u}}(\mathbf{B}) \Rightarrow$$

$$\mathbf{B}^{n+1} - \mathbf{B}^{n} = \Delta t \dot{\mathbf{B}}\left(\frac{\mathbf{u}^{n+1} + \mathbf{u}^{n}}{2}\right), \mathbf{u}^{n+1} - \mathbf{u}^{n} = \Delta t \dot{\mathbf{u}}\left(\frac{\mathbf{B}^{n+1} + \mathbf{B}^{n}}{2}\right) \Rightarrow$$

$$\mathbf{B}^{n+1/2} - \mathbf{B}^{n-1/2} = \Delta t \dot{\mathbf{B}}\left(\mathbf{u}^{n}\right), L\left(\mathbf{u}^{n+1} - \mathbf{u}^{n}\right) = \Delta t \dot{\mathbf{u}}\left(\mathbf{B}^{n+1/2}\right)$$

- 2F is not!
- Challenge is to obtain method for low dissipation
  - Real frequency physical modes should give real frequency numerical modes

A Useful Stability Theorem

- To simulate low dissipation cases, need "spectral fidelity"
  - If physical system has only real frequency modes, numerical system will have only real frequency modes (or controlled damping would also work)
- One case of sufficient conditions

$$\dot{U} = F(U, \nabla) \Rightarrow \frac{U^{n+1} - U^{n}}{\Delta t} = F\left(U^{n+\sigma}, D\right)$$

$U^{n+\sigma}$ is linear combination of $U^{n+1}, U^{n}, U^{n-1}, \ldots$, centered at $\sigma \geq 1/2$, D is difference operator $\ni : iD$ is Hermitian

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Some Possible Schemes

$$U^{n+1/2} = \frac{U^{n+1} + U^n}{2} : (1L) \Rightarrow Z - 1 = -i\tilde{\omega}\Delta t \frac{Z+1}{2}$$

$$U^{n+1/2} = \lambda \frac{U^{n+1} + U^n}{2} + (1-\lambda)\frac{3U^{n+1} + U^{n-1}}{4} : (2L) \Rightarrow$$

$$\left[i\tilde{\omega}\Delta t(3-\lambda) + 4\right]Z^2 + (2i\tilde{\omega}\Delta t - 4)Z + i\tilde{\omega}\Delta t(1-\lambda) = 0$$

Both have vanishing damping for low frequencies. 1L has no damping for high frequencies as well, while 2L gives strongest damping when $\lambda = 3/4$ ($|Z| = 1/3$).

Using NIMROD Machine for This Form

- Use Predictor/Corrector method
- Advance all equations (slave) except momentum (master) with trial $\mathbf{u}^* \sim \mathbf{u}^{n+1}$
- Error in momentum gives correction
- Linear operator from linearizing change in slave variables in change in u
- P/C required because not possible to incorporate exact linear change of slave variables
  - $\nabla\nabla \neq \nabla^2$ (use compact stencil for 2nd order operator)
  - Slave equations contain operator inversion

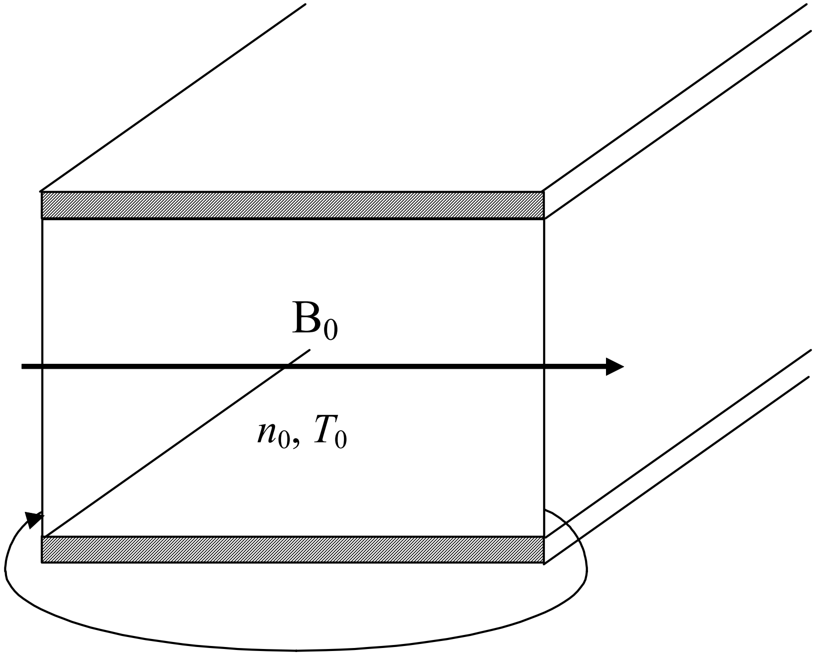
Some Numerical Details

- Care in integration by parts because of different BC
- Correct BC is to require *only* $u_n = 0$

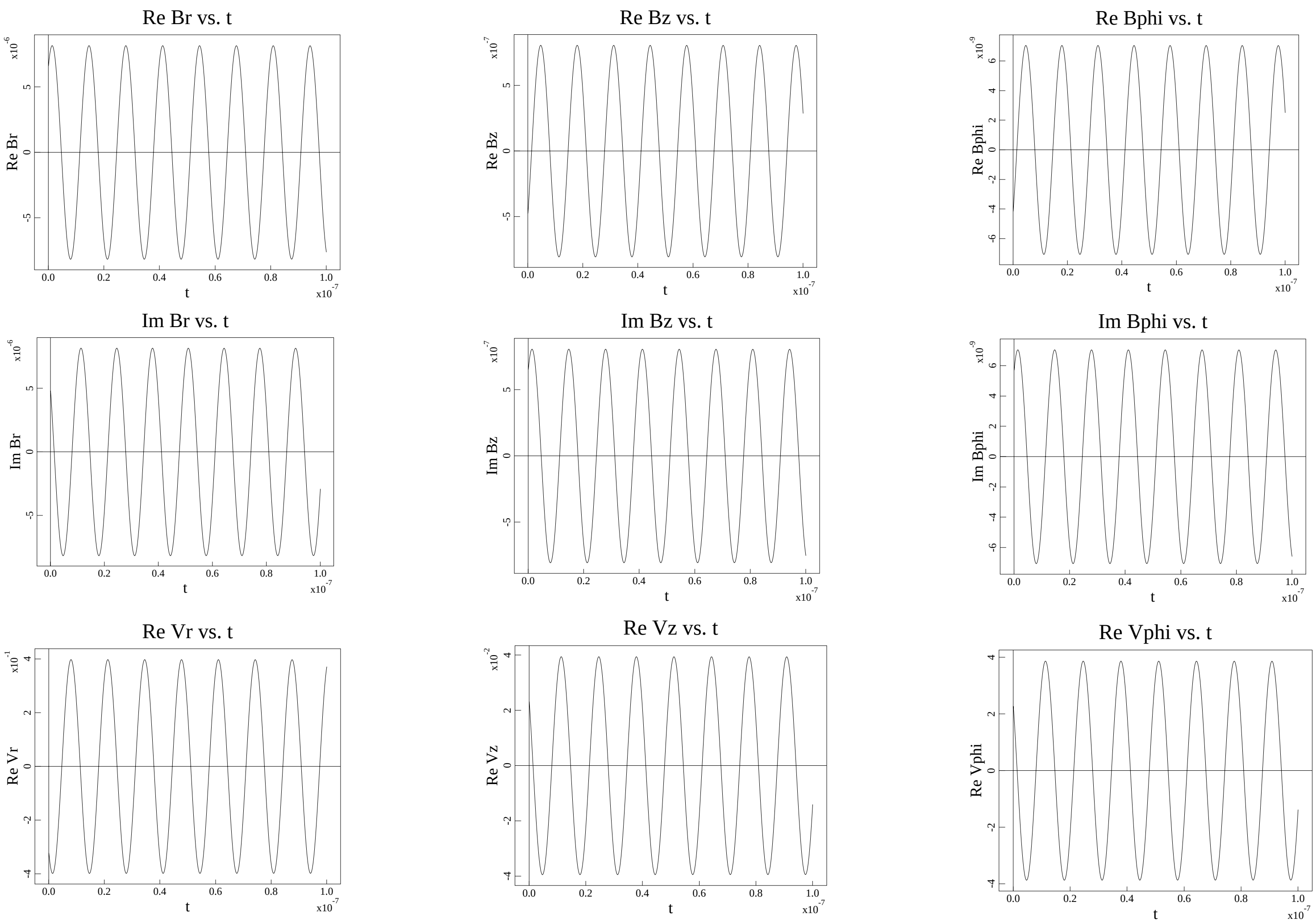
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{u} \times \mathbf{B} - \frac{M}{e} \nabla \times \frac{\partial \mathbf{u}}{\partial t} + \ldots$$

$$\int d\mathbf{r}\,\boldsymbol{\xi}^* \bullet \frac{\partial \mathbf{B}}{\partial t} \neq \int d\mathbf{r}\,\nabla \times \boldsymbol{\xi}^* \bullet \left(\mathbf{u} \times \mathbf{B} - \frac{M}{e}\frac{\partial \mathbf{u}}{\partial t} \right)$$

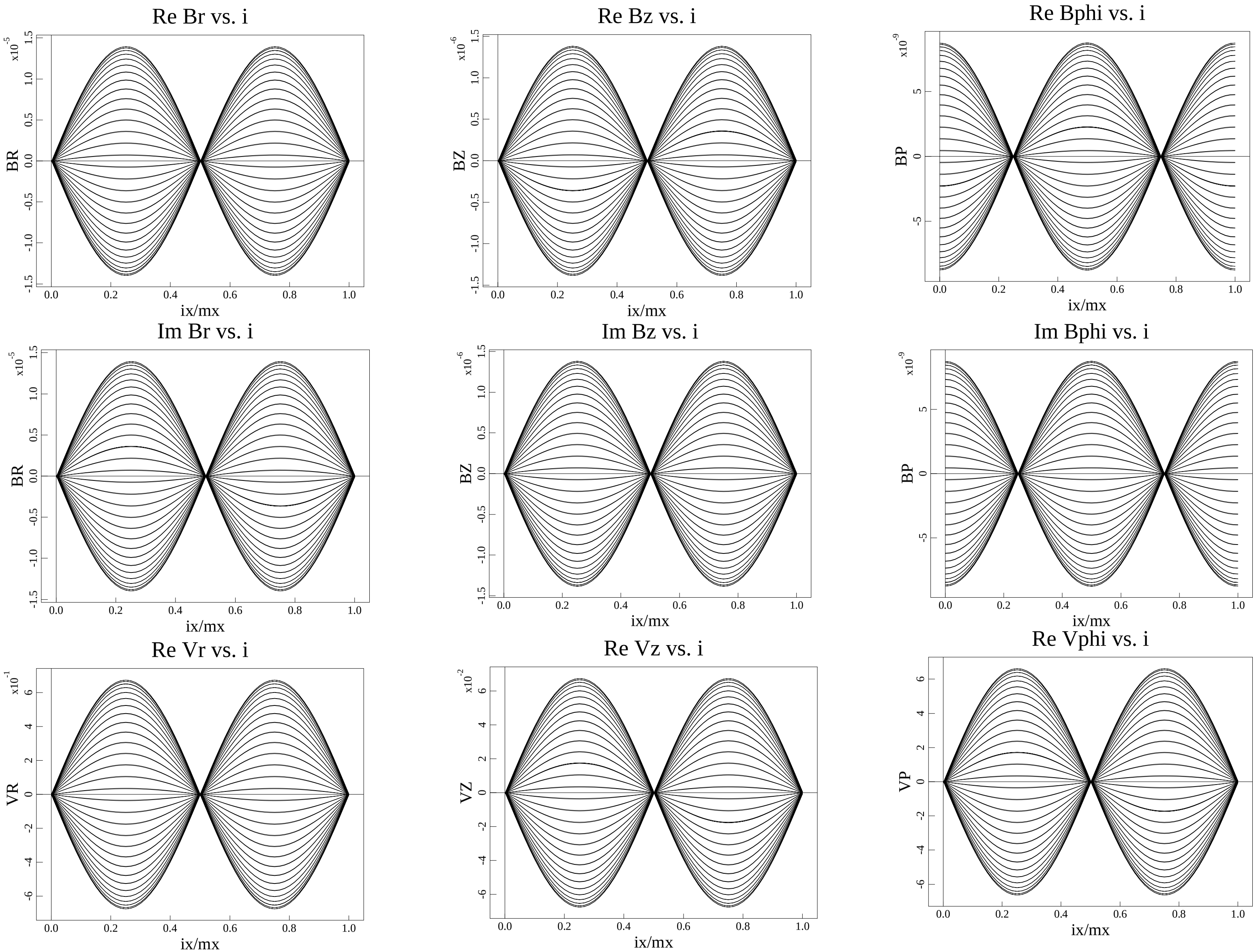
Waves in a Box Tests



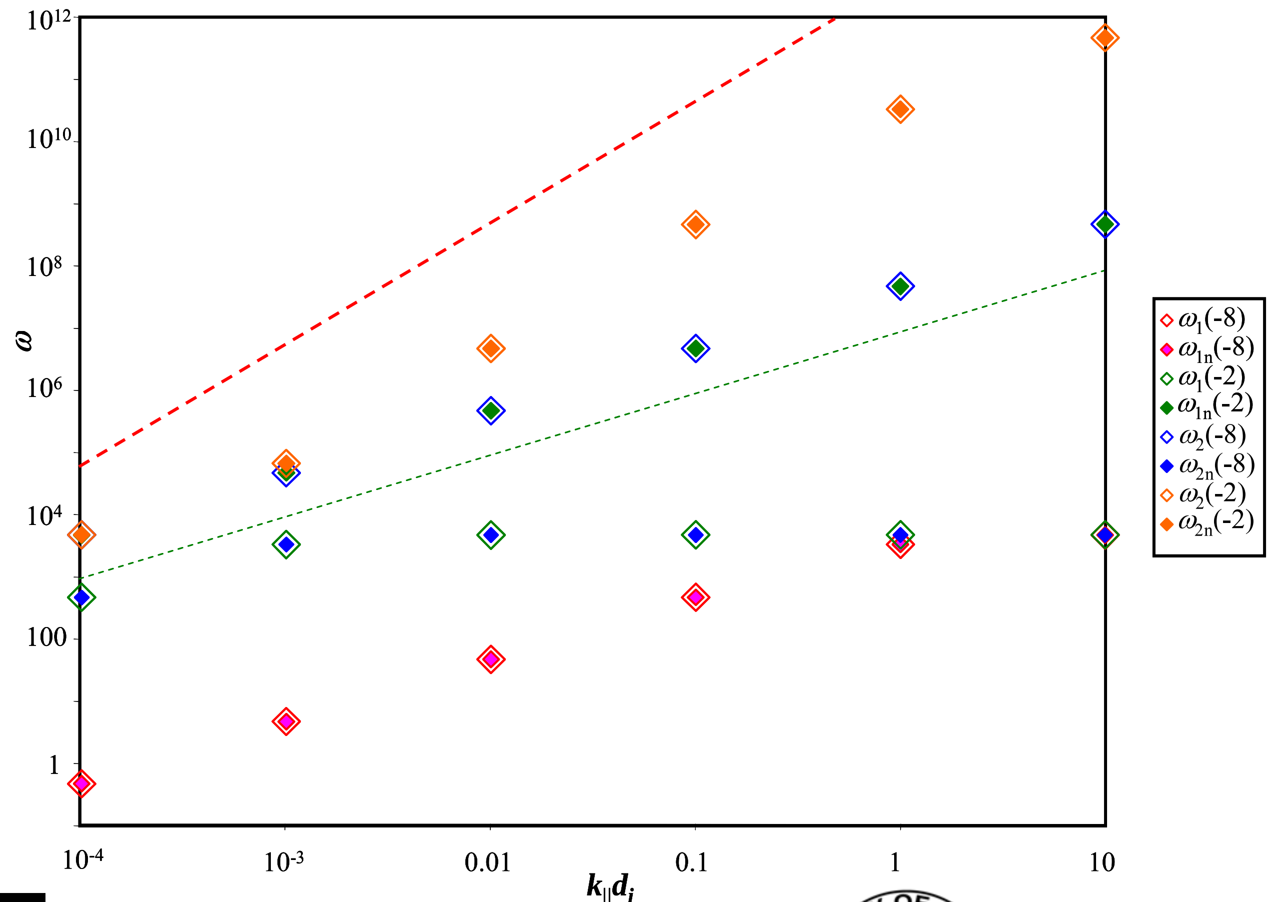
History for KAW Calculation



Profile for KAW Calculation



Numerical Results show good Dispersion



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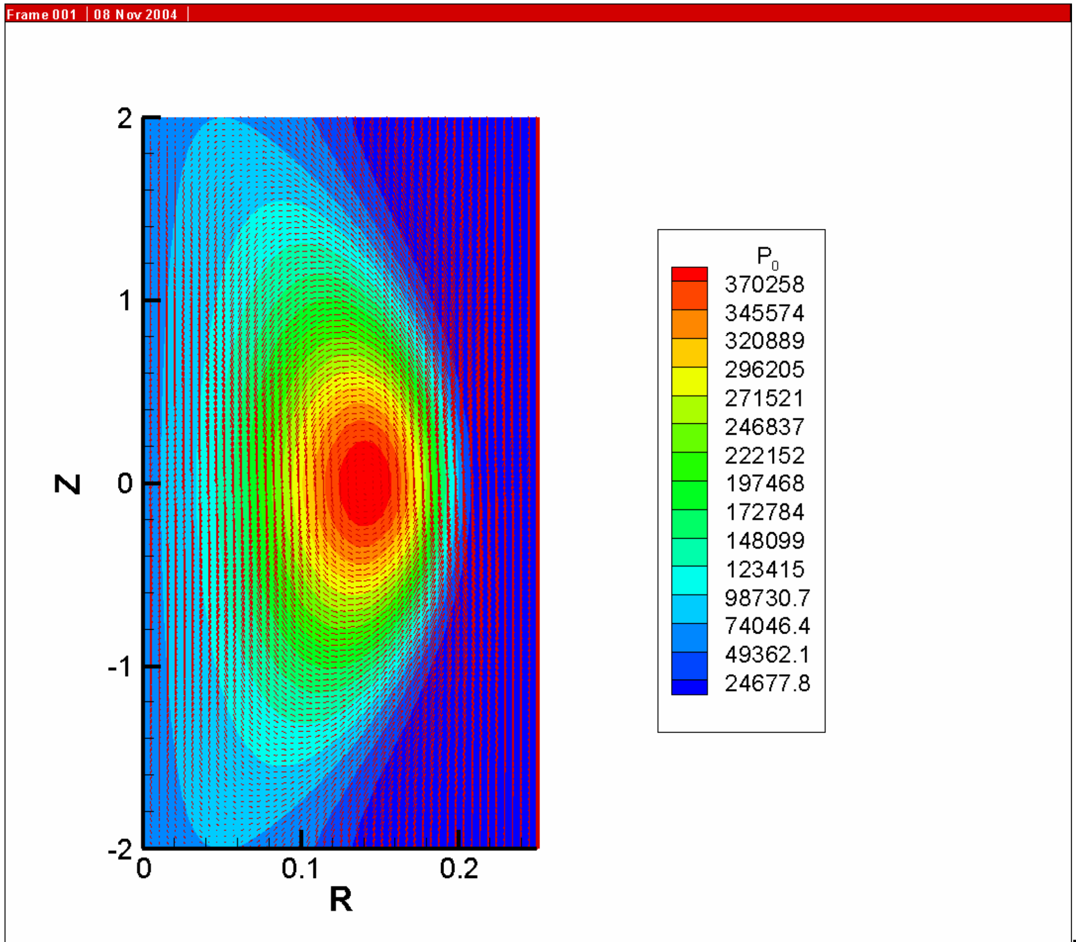
Convergence Acceleration of P/C

- Can consider present scheme as preconditioner
- Apply GMRES to iteration
- Some success with partial GMRES (1 lag level)
- For larger problems, can use direct solve per block to get Schwarz preconditioner

Extension of Linear Solve

- Instead of master/slave scheme, solve all equations simultaneously (8 unknowns/ node instead of 3)
- No P/C required
- NIMROD machinery supports this (almost)
- Almost same as scheme(s) of Chacón and Glasser

NIMROD Initialized with FRC Equilibrium



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Conclusion

- NIMROD 2F well under way
  - Waves in box test (nearly) passed
  - Early version into CVS (update soon)
  - All thermoelectric terms in (nonlinear?)
- FRC application beginning
  - Need better separatrix conditions
- Algorithm improvements continue in parallel with applications
  - Finite m_e
  - GMRES, Block direct + Schwarz
  - Alternative: fully couple all equations