Short Mean-Free Path Drift-Ordered Two-Fluid Plasma Closure

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The Bottom Line

- Braginskii's equations are **usually not appropriate** for modeling (collisional) tokamak plasma
- **Reason:** they neglect effects of heat fluxes on plasma viscosity
- Effects of heat fluxes on plasma viscosity are important for plasma of arbitrary collisionality
- For collisional plasma should use **corrected Mikhailovskii and Tsypin's** equations

Heat Flux Viscosity

- Assume that hot (fast) ions move in the positive direction and cold (slow) ions move in the negative direction on some flux surface
- Choose the number of hots and colds in such a way that the net particle flow is zero (fewer hots than colds)
- Then there is a net heat flow in the positive direction
- Colds are more collisional and random walk (across the flux surface) faster than hots: $D \sim \rho_i^2 \nu_i \propto T_i^{-1/2}$
- There exists a net perpendicular flux of parallel momentum

Outline

- Brief overview of short mean-free path closure assumptions
- Mikhailovskii and Tsypin's (drift) closure
- Catto and Simakov's corrections of Mikhailovskii and Tsypin's closure
- A step towards closures for plasmas of arbitrary collisionality: drift kinetic equation

Motivation

- Collisional plasma can be rigorously described by a closed system of fluid equations for regimes of both sonic and subsonic plasma flows
- The case of plasma with subsonic flows is more complicated but also of more interest
- Until recently only ion, but not electron, description existed, however the collisional pieces of ion viscous stress tensor (parallel and perpendicular viscosities) contained errors
- Needed to correct the ion description and obtain the electron one

Closures

- To close the system of fluid equations have to know $\overleftrightarrow{\pi}_j$ (viscous stress tensor), $\boldsymbol{q}_j$ (heat flux), $\boldsymbol{R}_j$ (collisional momentum exchange) and W_j (collisional energy exchange) in terms of n_j, $\boldsymbol{V}_j$ and T_j
- The better known closures were obtained by Braginskii (1957) and by Mikhailovskii & Tsypin (1971)
- Braginskii's closure assumes sonic flows (MHD-like ordering) and is usually not appropriate for tokamaks
- Mikhailovskii & Tsypin's closure assumes subsonic flows on the order of diamagnetic heat fluxes over plasma pressure (drift ordering)
- The latter closure is appropriate for tokamaks and contains all Braginskii's results as a special case

Closure Assumptions
(Braginskii, Mikhailovskii & Tsypin)

$$\frac{\partial}{\partial t},\ \omega_{dj} \ll \nu_j \tag{1}$$

with ω_{dj}, ν_j drift and collision frequencies of species j;

$$\delta_j \equiv \frac{\rho_j}{L_\perp} \ll 1, \quad \Delta_j \equiv \frac{\lambda_j}{L_\parallel} \ll 1 \tag{2}$$

with ρ_j, λ_j species j gyroradius and mean-free path, $L_\perp$ and $L_\parallel$ length scales $\perp$ and $\parallel$ to magnetic field;

$$\text{Braginskii} \implies v_{Ti} \equiv \sqrt{2T_i/m_i} \sim V_j \gg q_j/p_j \tag{3}$$

$$\text{Mikhailovskii-Tsypin} \implies v_{Ti} \equiv \sqrt{2T_i/m_i} \gg V_j \sim q_j/p_j$$

Mikhailovskii & Tsypin's Procedure

- Assumed that distribution function is close to a Maxwellian f_{Mj} : $f_j = f_{Mj}(1 + \Phi_j)$
- Wrote the function Φ_j as

$$\Phi_j = \sum_{l=2}^{\infty} a_j^l L_l^{(1/2)}(x_j) + \boldsymbol{w}_j \cdot \sum_{l=1}^{\infty} \boldsymbol{b}_j^l L_l^{(3/2)}(x_j) + \left(\boldsymbol{w}_j \boldsymbol{w}_j - \frac{w_j^2}{3} \overleftrightarrow{I} \right) : \sum_{l=0}^{\infty} \overleftrightarrow{c}_j^{\,l} L_l^{(5/2)}(x_j) + \cdots$$

with $\boldsymbol{w}_j \equiv \boldsymbol{v}_j - \boldsymbol{V}_j$ the random velocity vector, $x_j \equiv m_j w_j^2 / 2T_j$, and $L_l^{(i+1/2)}$, $i = 0, 1, 2, \ldots$, Laguerre polynomials

- Coefficients a_j^l, $\boldsymbol{b}_j^l$, $\overleftrightarrow{c}_j^{\,l}$ are moments of f_j: $\boldsymbol{b}_j^1 = -(2m_j / 5\, p_j\, T_j)\boldsymbol{q}_j$, $\overleftrightarrow{c}_j^{\,0} = (m_j / 2\, p_j\, T_j) \overleftrightarrow{\pi}_j$ and so on
- Obtained moment equations for the coefficients and solved them

Mikhailovskii & Tsypin's Results

- Expressions for $\boldsymbol{q}_j$, $\boldsymbol{R}_j$, W_j are equivalent to those of Braginskii
- Expressions for $\overleftrightarrow{\pi}_j = \overleftrightarrow{\pi}_{||j} + \overleftrightarrow{\pi}_{gj} + \overleftrightarrow{\pi}_{\perp j}$ are more complicated, where $\overleftrightarrow{\pi}_{||j}$ is the parallel viscosity (pressure anisotropy), $\overleftrightarrow{\pi}_{gj}$ is the gyro-viscosity, and $\overleftrightarrow{\pi}_{\perp j}$ is the perpendicular (or collisional) viscosity
- Example: gyro-viscosity

$$\overleftrightarrow{\pi}_{gj} = \frac{p_j}{4\Omega_j}\left\{\hat{\boldsymbol{b}} \times [\overleftrightarrow{\alpha}_j + (\overleftrightarrow{\alpha}_j)^{\mathrm{T}}] \cdot (\overleftrightarrow{\boldsymbol{I}} + 3\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}) - (\overleftrightarrow{\boldsymbol{I}} + 3\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}) \cdot [\overleftrightarrow{\alpha}_j + (\overleftrightarrow{\alpha}_j)^{\mathrm{T}}] \times \hat{\boldsymbol{b}}\right\}$$

with Ω_j the gyro-frequency and $\hat{\boldsymbol{b}}$ the unit vector along magnetic field

$$\text{Braginskii} \Longrightarrow \overleftrightarrow{\alpha}_j = \boldsymbol{\nabla}\boldsymbol{V}_j$$

$$\text{Mikhailovskii-Tsypin} \Longrightarrow \overleftrightarrow{\alpha}_j = \boldsymbol{\nabla}\boldsymbol{V}_j + \frac{2}{5p_j}\boldsymbol{\nabla}\boldsymbol{q}_j$$

Mikhailovskii & Tsypin's Errors

- Use of Mikhailovskii & Tsypin's technique makes it difficult to predict where to truncate the polynomial expansion of f_j (i.e. of Φ_j)
- Indeed, they truncated the polynomial expansion too soon to retain the gyro-phase dependent portion of f_j to the accuracy required
- They neglected the contribution from the non-linear piece of like-particle collision operator $C_{jj}(f_{1j}, f_{1j})$ in the second order of like-particle collision operator $C_{jj2}(f_j) \equiv C_{jj}(f_{2j}, f_{0j}) + C_{jj}(f_{0j}, f_{2j}) + C_{jj}(f_{1j}, f_{1j})$
- As a result, some terms are omitted in their expressions for $\overset{\leftrightarrow}{\pi}_{||j}$ and $\overset{\leftrightarrow}{\pi}_{\perp j}$ and some numerical factors are wrong in the expression for $\overset{\leftrightarrow}{\pi}_{\perp j}$

Catto & Simakov's Corrections

- Assumed $\delta_i \sim \Delta_i$; more general than neoclassical ordering $\delta_i \ll \Delta_i$
- Can retain all short mean-free path turbulent and neoclassical effects of interest to tokamaks
- Solved ion kinetic equation order by order:

$$\Omega_i \boldsymbol{w}_i \times \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla}_{w_i} f_{0i} = C_0,$$

$$\Omega_i \boldsymbol{w}_i \times \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla}_{w_i} f_{1i} = C_1 + [\boldsymbol{w}_i \cdot \boldsymbol{\nabla} f_{0i} + (m_i n_i)^{-1} \boldsymbol{\nabla} p_i \cdot \boldsymbol{\nabla}_{w_i} f_{0i}],$$

$$\Omega_i \boldsymbol{w}_i \times \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla}_{w_i} f_{2i} = C_2 + [\boldsymbol{w}_i \cdot \boldsymbol{\nabla} f_{1i} + (m_i n_i)^{-1} \boldsymbol{\nabla} p_i \cdot \boldsymbol{\nabla}_{w_i} f_{1i}]$$
$$+ \frac{\partial f_{0i}}{\partial t} + \boldsymbol{V}_i \cdot \boldsymbol{\nabla} f_{0i} - \boldsymbol{w}_i \cdot \boldsymbol{\nabla} \boldsymbol{V}_i \cdot \boldsymbol{\nabla}_{w_i} f_{0i}$$

- Gyrophase dependent portion of f_i through second order is evaluated exactly, gyrophase independent - variationally

Ion Viscosity

- Knowing f_i to second order can evaluate parallel and gyro- viscosities directly
- Can evaluate perpendicular (and gyro-) viscosity using the moment approach
- Application: results were used to evaluate the neoclassical radial electric field in a collisional tokamak (Pfirsch-Schlüter regime)
- See the poster **CP1.049** on Monday afternoon

Catto & Simakov vs. Mikhailovskii & Tsypin

- The ion heat fluxes are the same as those of M&T (and Braginskii)
- The ion gyro-viscosity is the same as that of M&T (Braginskii missed heat flux contributions)
- The ion parallel viscosity differs from that of M&T by q_i^2 terms from the $C_{ii}(f_{1i}, f_{1i})$ contribution that they neglected
- The ion perpendicular viscosity differs from that of M&T because their gyrophase dependent portion of f_{2i} is incomplete and they ignored $C_{ii}(f_{1i}, f_{1i})$ contributions
- We also evaluated electron parallel and gyro- viscosities (M&T only considered ions)

A Step Towards Less-Collisional Closures: Drift Kinetic Equation

- Less collisional plasmas can only be **rigorously** described by using either purely kinetic or perhaps combined kinetic/fluid approaches
- Can use drift kinetic (as opposed to gyro-kinetic) equation for plasmas of arbitrary collisionality if $\rho_i / L_\perp \ll 1$ can be assumed
- The most popular is drift kinetic equation by R.D. Hazeltine (1973)
- We have recently discovered that this equation does not correctly account for effects of order $(\rho_i / L_\perp)^2$ (such as gyro-viscosity)
- Consequently at the very least this equation cannot be used for a purely kinetic description of plasmas
- **Need a better drift kinetic equation**
- We have derived the appropriate corrections to Hazeltine's drift kinetic equation
- See the poster **FP1.142** on Tuesday afternoon

Gyro-Viscous Stress Tensor for Plasma of Arbitrary Collisionality

- Using the more precise expression for $\tilde{f}_j$ (as compared with that of Hazeltine) can evaluate gyro-viscosity for plasmas of arbitrary collisionality
- The answer is in terms of $q_{1j} \equiv m_j \int \mathrm{d}^3v\, v_{||}\, \mu B\, \bar{f}_j$ and $q_{2j} = (m_j/2) \int \mathrm{d}^3v\, v_{||}^3\, \bar{f}_j$
- Here, $\tilde{f}_j$ and $\bar{f}_j$ are gyro-phase dependent and independent portions of f_j, respectively
- $\mu \equiv v_\perp^2/(2B)$ is the magnetic moment
- In the collisional limit the Mikhailovskii and Tsypin's answer is recovered

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Conclusions

- Short mean-free path two-fluid plasma closures have been discussed
- The most famous Braginskii's (MHD-like) closure assumes sonic plasma flows and is usually not appropriate for tokamaks
- Mikhailovskii and Tsypin's (drift) closure is for subsonic plasma flows and is appropriate for tokamaks
  - contains all Braginskii's results as a special case
  - keeps extra heat flux terms in viscous stress tensor
- Mikhailovskii and Tsypin made several errors in their derivation, which resulted in errors in parallel and perpendicular viscosity
- They only treated ions, not electrons
- Catto and Simakov used an alternative technique to correct the errors in the ion description and obtained for the first time the electron description

Conclusions: Continued

- As a step towards obtaining closures for plasmas of arbitrary collisionality Hazeltine’s drift kinetic equation (1973) has been reexamined
- It was found to be exact only through the first order in the small gyro-radius expansion ⬇ does not account for effects of gyro-viscosity, Reynolds’ stress tensor etc.
- Simakov and Catto have obtained corrections to make it exact through the second order
- The formalism developed has been employed to evaluate gyro-viscosity for plasmas of arbitrary collisionality
- The general expression for gyro-viscosity recovers the Mikhailovskii and Tsypin’s answer in the collisional plasma limit

Conclusions: Continued

- We would like to be made aware of any fluid or kinetic code which employs a complete and rigorous self-consistent closure in any regime of collisionality