Spectral Elements in the M3D extended MHD code

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PPPL

outline

- Advantages of SEL
- Structure of M3D
- SEL library
- Comparison of FEM / SEL computations
  - MHD equilibrium
  - Linear stability
  - nonlinear

Advantages of Spectral Elements

- Benefits of spectral elements
  - Exponential decrease of error with mesh size
  - Mesh can adapt, align with boundary, flux surfaces
- Efficient implementation
  - Tensor product of 1-D elements (quads, hex)
    - Much less computational work than 2-D FEM or 2-D SEL
  - Static decomposition for elliptic solves
    - Element Interior values known from element boundary values
    - Solve much smaller matrix of element boundary values
  - diagonal mass matrix
    - Nodal points = GL quadrature points
    - Higher order FEM has non diagonal mass matrix, many inversions
  - Same code for arbitrary order
    - Have good results for 12^{th} order

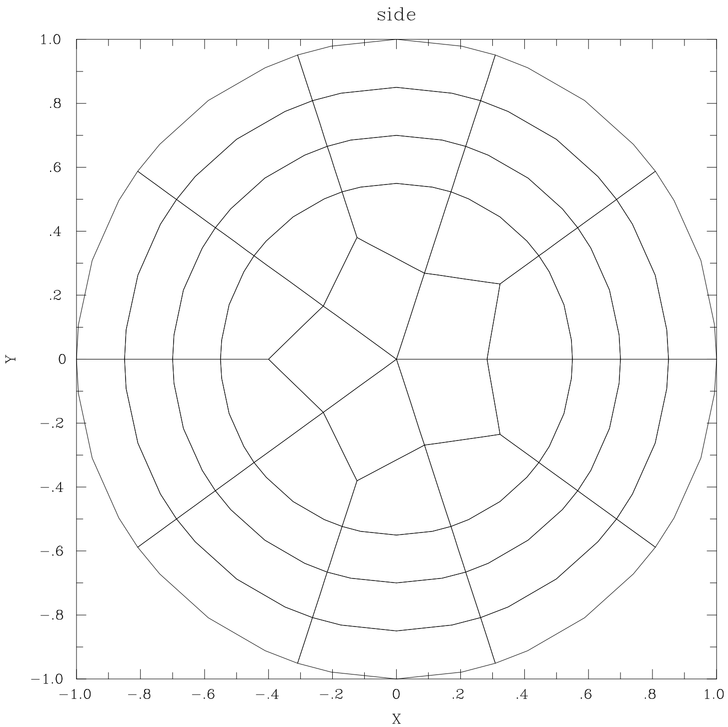
M3D code

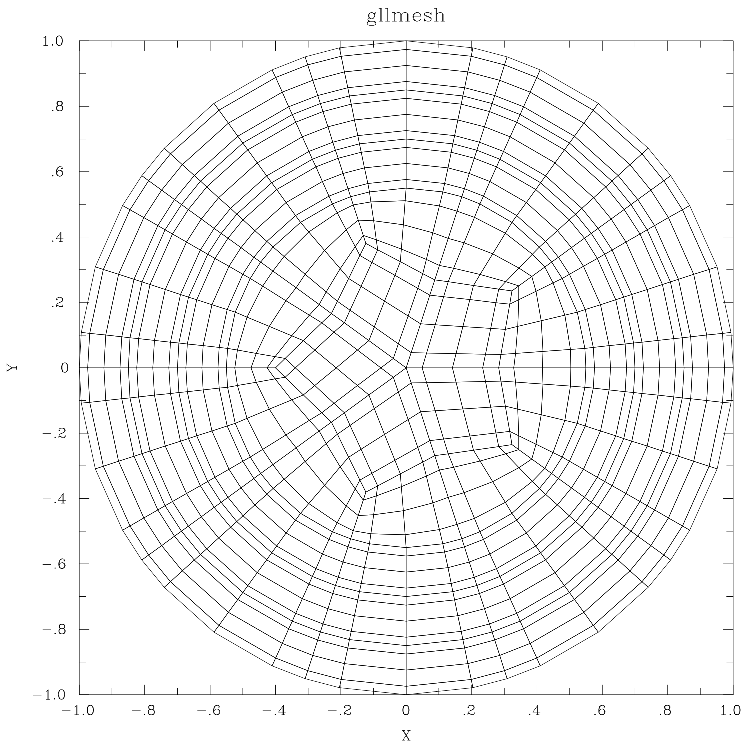
- Extended MHD (MHD + two fluid effects)
- Partly explicit (shear Alfven, sound waves), partly implicit (compressional Alfven waves) time advance
- Highly modular driver code (Fortran)
    - Forms right hand sides and does time advance
    - No explicit reference to grid
    - 3 discretization implementations
        - Finite differences + 2D spectral discretization
        - Triangular 2D finite elements + 1D pseudospectral
        - 2D quadrilateral spectral elements + 1D pseudospectral

Discretization library

- Discretization library (Fortran, C)
  - Galerkin discretization
  - C0 basis functions
- Called by driver code
  - Implements differentiation, integration, elliptic solves
  - Mesh generation initiated in driver code (skeleton mesh)
    - Full mesh generated in library
  - Interface maps data representation
    - Driver: “flat” Fortran array
    - Library: data organized by elements
- SEL library implemented for M3D
  - B. Hientzsch, (no domain decomp) Openmp parallelization
  - J. Chen, MPI / Petsci version in progress

Mesh generation





Skeleton mesh
Curvilinear, GLL points
on element vertices
and edges

SEL mesh including GLL
Interior element points
Interior coordinates are blended
From skeleton mesh

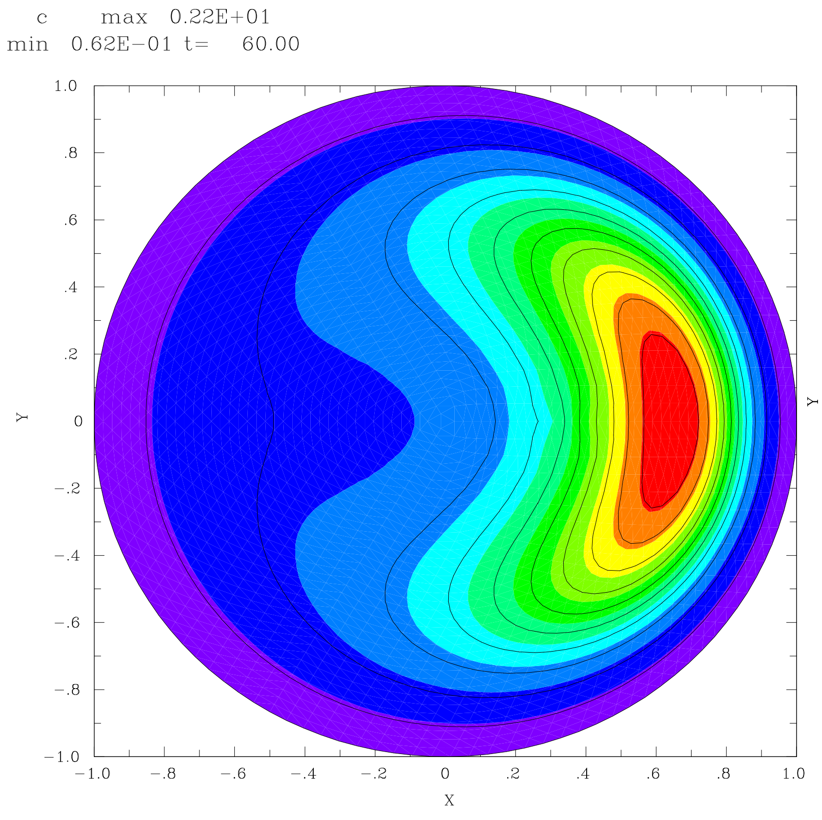
Comparison of FEM and SEL

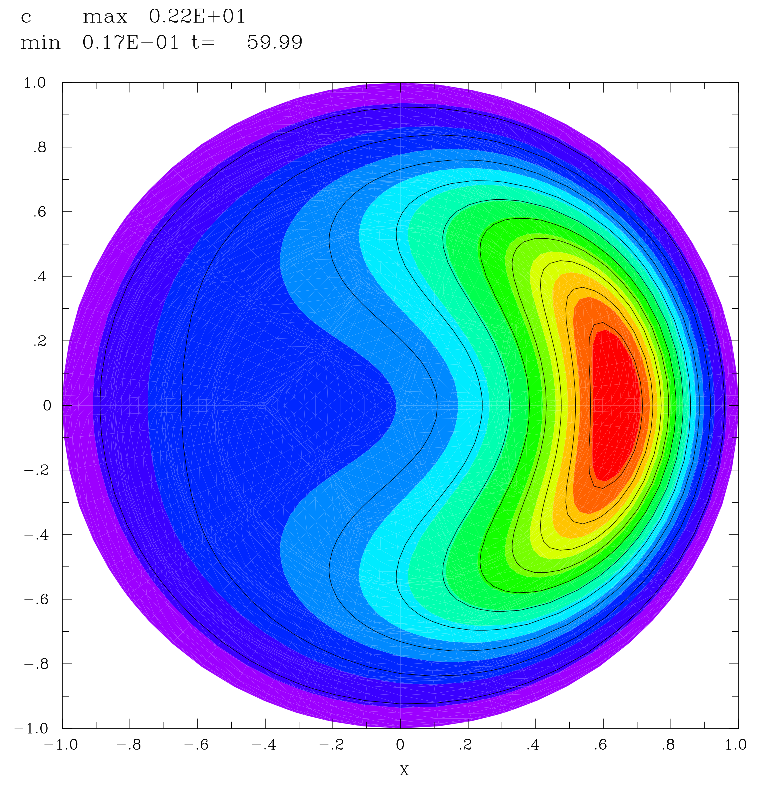
- Same M3D code
  - Only mesh and discretization is different
  - Can compare the effect of discretization
  - Linear FEM, 2 – 12 order SEL
  - Resistive MHD
- Same initialization
  - Same parameters: resistivity, viscosity, time step
  - Same initial magnetic field and pressure
- Comparisons
  - Equilibrium
    - Evolve initial non equilibrium to a steady state equilibrium
  - Linear stability
    - Perturb equilibrium and time advance, mode grows exponentially
  - Nonlinear evolution
    - Initialize with equilibrium and linear eigenmode

Equilibria: FEM / SEL

toroidal current density, R/a = 3

(current is more sensitive than flux surface quantities like pressure)

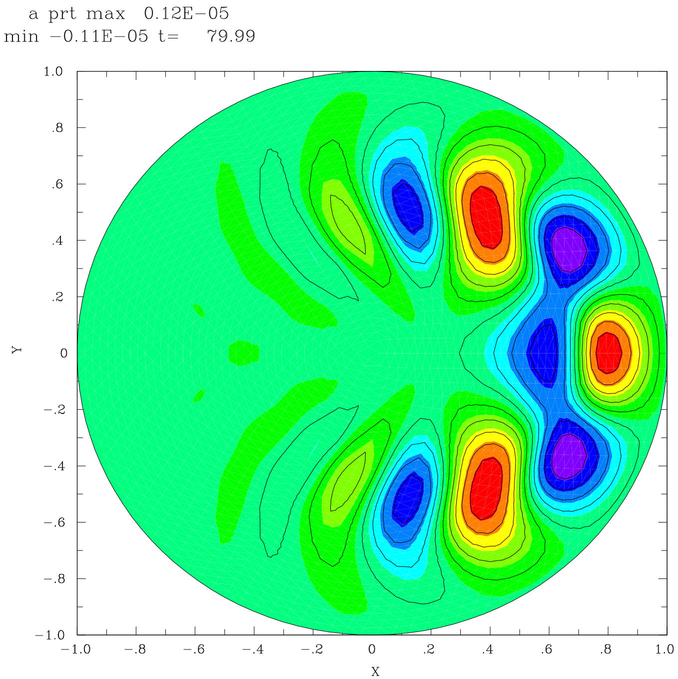


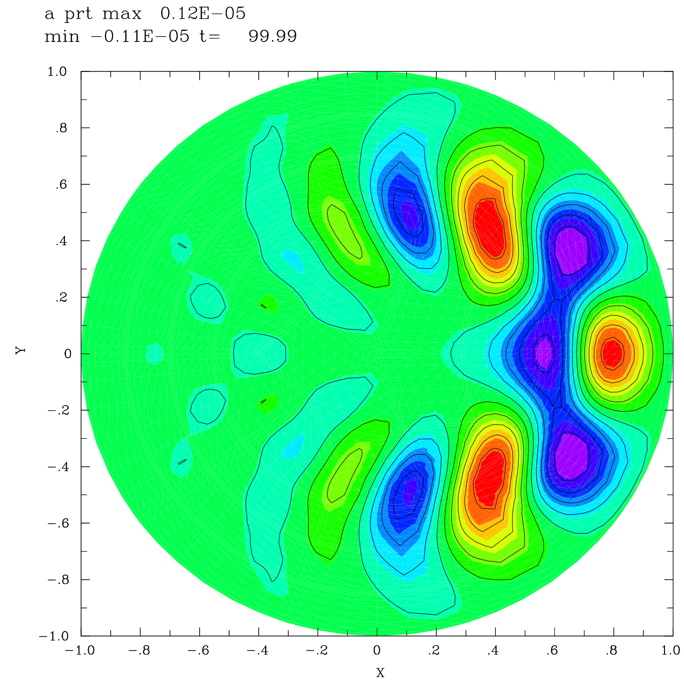


Linear FEM equilibrium
Initial non equilibrium state
Relaxed to equilibrium

8th order SEL equilibrium
Same initial state, same method
Slight differences

Linearly unstable n=3 perturbed poloidal magnetic flux function (ballooning)

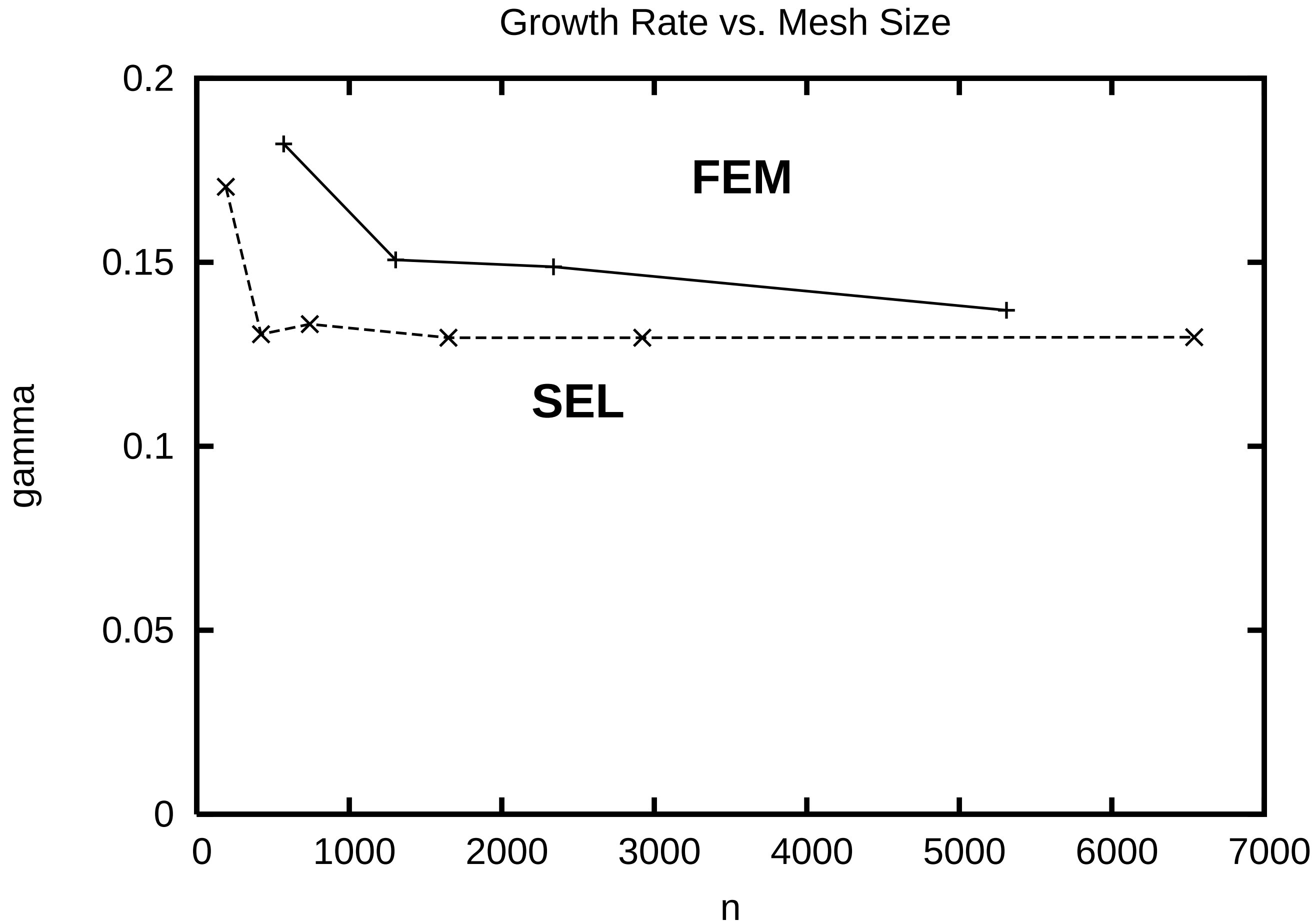




FEM: equilibrium perturbed and time advanced until Dominated by a mode Growing exponentially in time (2340 mesh points)

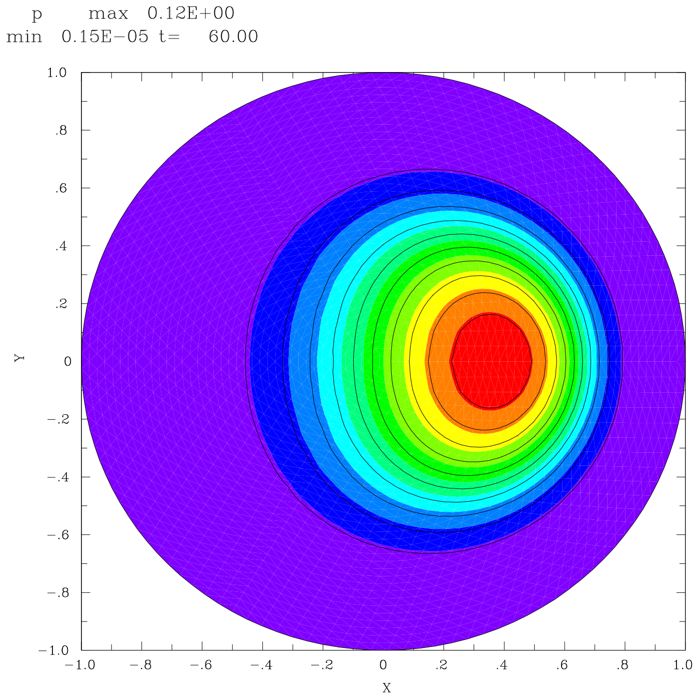
SEL (8^{th} order): same method Slight differences in mode Structure (2921 mesh points)

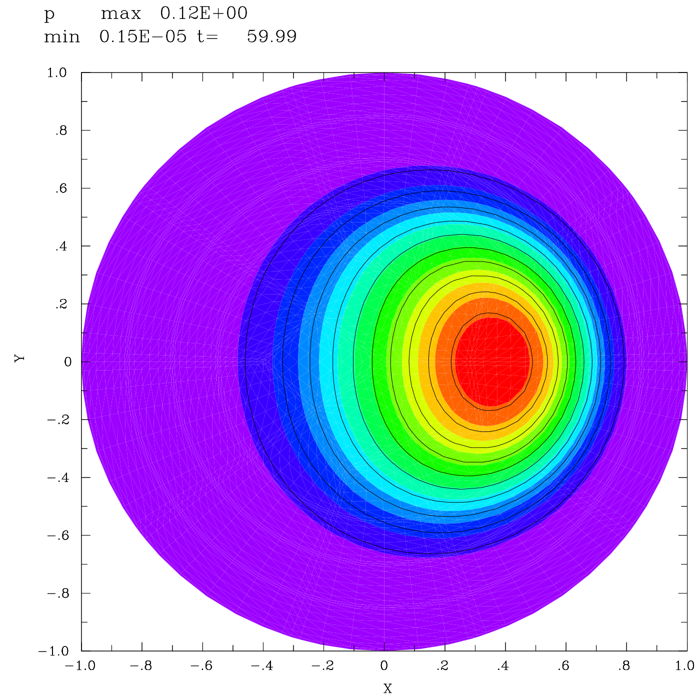
Convergence of linear growth rate with mesh size



As expected ,SEL converges to growth rate at smaller mesh size
Same skeleton mesh, degree varied from 2 to 12

Nonlinear evolution – initial pressure



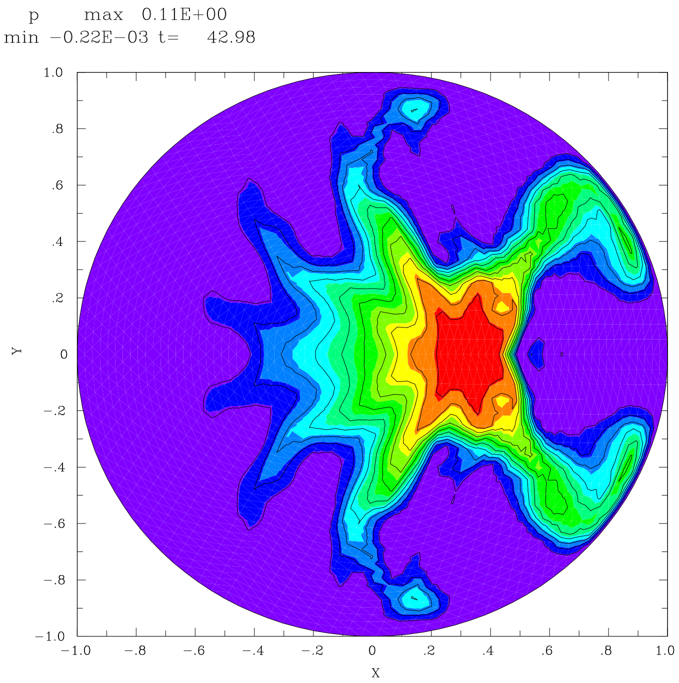


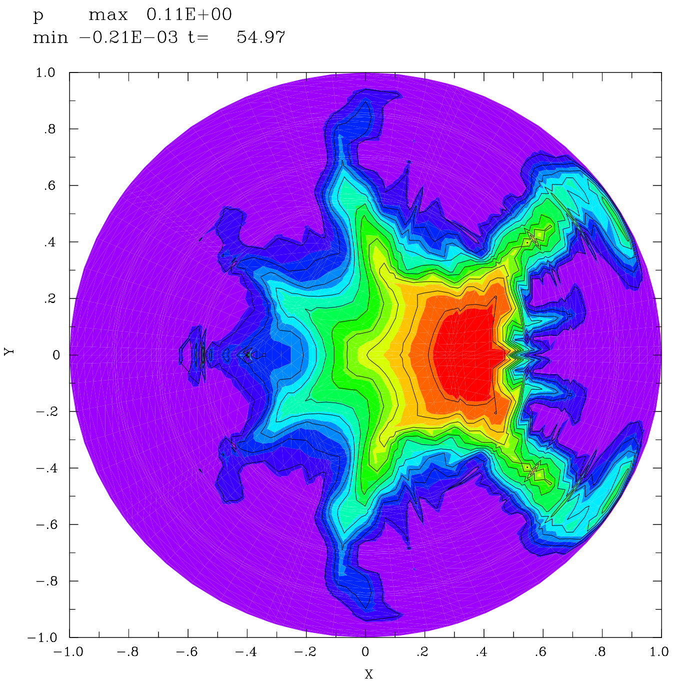
FEM

SEL

Both FEM and SEL initialized with same equilibrium and Linear perturbations

Nonlinear evolution - pressure

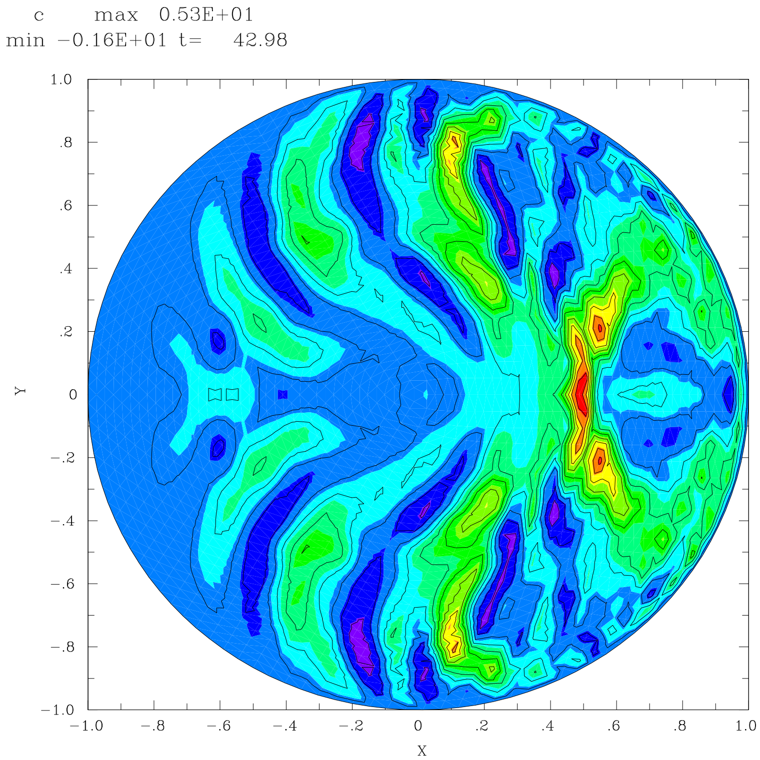




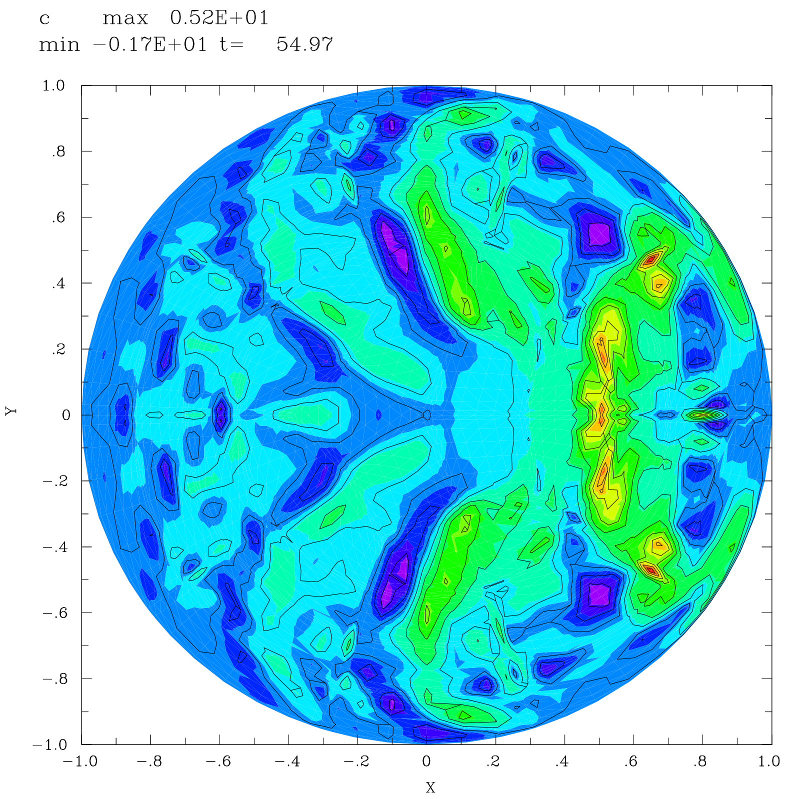
FEM: equilibrium perturbed
With linear mode, solution
is smoother

SEL (8th order): same method
More differences in nonlinear
evolution
Less smooth than FEM – at
element boundaries

Nonlinear evolution – toroidal current



FEM



SEL

Corresponding to previous pressure plots, FEM is smoother

Summary and future work

- SEL library implemented for M3D
  - Library by B. Hientzsch, Openmp parallelization
  - MPI / Petsci version by J. Chen, in progress
- Same driver code
  - Permits direct comparison of FEM and SEL
- SEL / FEM performance
  - SEL has improved convergence, linear, smooth problems
  - SEL can be noisier for highly nonlinear problems – filtering
  - FEM compares favorably with SEL
- Future work
  - Develop MPI/Petsci implementation
  - More complicated skeleton mesh geometry (stellarator)
  - Compare two fluid effects in FEM and SEL