An email from Dalton ......

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From:    Dalton Schnack <schnack@wisc.edu>
Sent:    Wednesday, July 25, 2007 5:28 pm
To:   Nimrod Developer announcements <nimrod-devel@nimrodteam.org>
Cc:   Steve Jardin <jardin@pppl.gov>
Subject:   [Nimrod-devel] GV benchmarking with 3.2.4
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Colleagues,

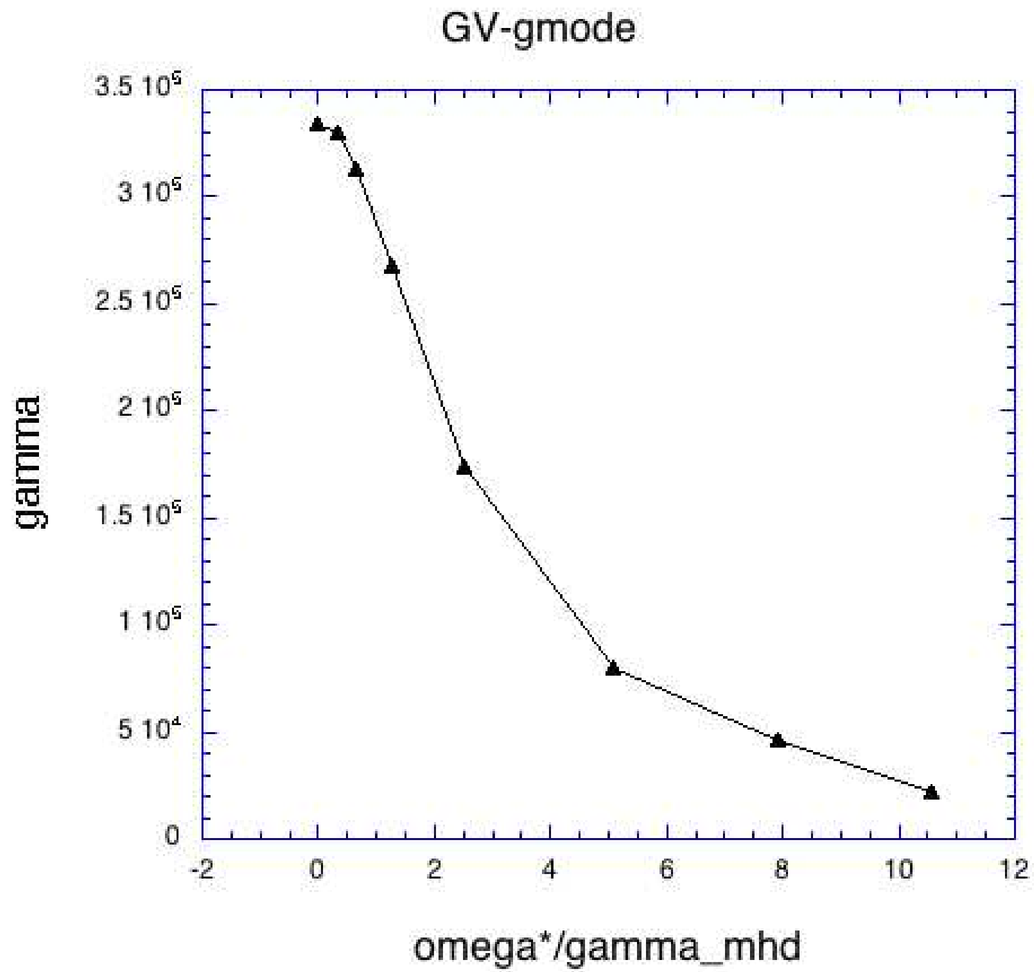
As a result of my recent visit to Boulder and collaboration with Scott K., I have concluded that my previous validation tests on the g- mode with gyro-viscosity (NOT Hall) were buggy and should be completely discarded. Scott and I found errors in the equilibrium specification for these cases. This has been fixed and the cases have been repeated with nimrod3.2.4. The results are appended. Previously stabilization occurred at $\omega_*/\gamma_{\mathrm{MHD}} \sim 1.67$. Now, as you can see, the mode is never completely stabilized with GV alone. I think Scott confirmed that identical results for a single case were obtained with the latest version of nimuw. As part of our debugging, Scott and I went over the GV coding with a fine tooth comb and, to be best of our knowledge, it is coded correctly.

Dalton

Growth rate remains nonzero when ω_* well above $2\gamma_{\rm MHD}$

(Schnack and Kruger [07])

$$B = 6.0$$
$$g = 10^{12}$$
$$n = 2.0 \times 10^{20}$$
$$\beta = 2\mu_0 p/B^2 = 1.0$$
$$p = 1.4323944 \times 10^6$$
$$c_s = 5.974138 \times 10^6$$
$$\Omega_i = 2.87507603 \times 10^8$$
$$k_{\rm 2fl} = 353$$
$$k_{\rm gyr} = 203$$
$$k_y = 2094$$
$$k_y L_n = 20943$$
$$k_y d_i = 30$$
$$d_i/L = 1.47 \times 10^{-3}$$
$$\gamma_{\rm mhd} = 3.35761 \times 10^5$$
$$V_A = 6.544340 \times 10^6$$



FLR Stabilization of Interchange Mode in Extended MHD

Ping Zhu

University of Wisconsin-Madison

In collaboration with

D. D. Schnack, F. Ebrahimi, E. G. Zweibel, M. Suzuki, C. C. Hegna, and C. R. Sovinec

Textbook Theory

Extended MHD models the dominant finite Larmor radius (FLR) effects.

FLR fully stabilizes g-mode when $\omega_* \geq 2\gamma_{\mathrm{MHD}}$, where $\omega_* \propto k_\perp$ (Roberts and Taylor [62]).

FLR stabilization of high-n ballooning is cruicial for ELM simulations in extended MHD.

Benchmark Question

Which is correct, theory [RT62] or simulation [SK07]?

A Revisit to g-mode Dispersion in Extended MHD

- Extended MHD: gyroviscosity $\boldsymbol{\pi}$ and 2-fluid Ohm's law:

(1) $$\frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{J}\times\mathbf{B} + \rho\mathbf{g} - \nabla\cdot\boldsymbol{\pi}_i$$

(2) $$\mathbf{E} + \mathbf{u}\times\mathbf{B} = \frac{1}{ne}\left(\mathbf{J}\times\mathbf{B} - \nabla p_e\right)$$

(3) $$(\pi_i)_{xx} = -(\pi_i)_{yy} = -\frac{p_i}{2\Omega}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)$$

(4) $$(\pi_i)_{xy} = (\pi_i)_{yx} = \frac{p_i}{2\Omega}\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y}\right)$$

- Equilibrium: $d[p(x) + B(x)^2/2]/dx = \rho(x)g$
- Pure interchange perturbation: $\mathbf{u} = [u_x(x)\mathbf{e}_x + u_y(x)\mathbf{e}_y]e^{ik_y y - i\omega t}$
- Local approximation orderings: $k_y L_x \sim \epsilon$, $k_y d_i \sim 1$, $u_y \sim \epsilon u_x$, $\epsilon \ll 1$, where $L_x = (d\ln/dx)^{-1}$, $d_i = v_{Ti}/\Omega$.

FLR stabilization due to gyroviscosity only

(5) $$\omega^2 + \omega_* \omega + \gamma_{\mathrm{GYR}}^2 = 0$$

where

(6) $$\omega_* = \frac{\dfrac{k_y \delta}{\Omega}\left[(1+\beta)\dfrac{p'}{\rho} - \dfrac{2+\gamma\beta}{1+\gamma\beta} g\beta\right]}{1 + \dfrac{k_y^2 \delta^2}{4\Omega^2}\dfrac{p}{\rho}\dfrac{\beta}{1+\gamma\beta}}$$

(7) $$\gamma_{\mathrm{GYR}}^2 = \frac{\gamma_{\mathrm{MHD}}^2}{1 + \dfrac{k_y^2 \delta^2}{4\Omega^2}\dfrac{p}{\rho}\dfrac{\beta}{1+\gamma\beta}}$$

(8) $$\gamma_{\mathrm{MHD}}^2 = \frac{g^2}{u_A^2(1+\gamma\beta)} - \frac{\rho'}{\rho} g.$$

Here, $\Omega = eB/m_i$, $\beta = \mu_0 p/B^2$, $u_A^2 = B^2/\mu_0 \rho$, γ is the adiabatic index, and $\delta = p_i/p$. Reduce to [RT62] when $\beta \to 0$.

FLR stabilization could be absent in certain high β regime

In the case of constant magnetic fi eld B, $dp/dx = \rho g$, so that

$$\omega_* = \frac{\frac{k_y \delta g}{\Omega}\left(1 - \frac{\beta}{1+\gamma\beta}\right)}{1 + \frac{k_y^2\delta^2}{4\Omega^2}\frac{p}{\rho}\frac{\beta}{1+\gamma\beta}}$$

FLR stabilization requires

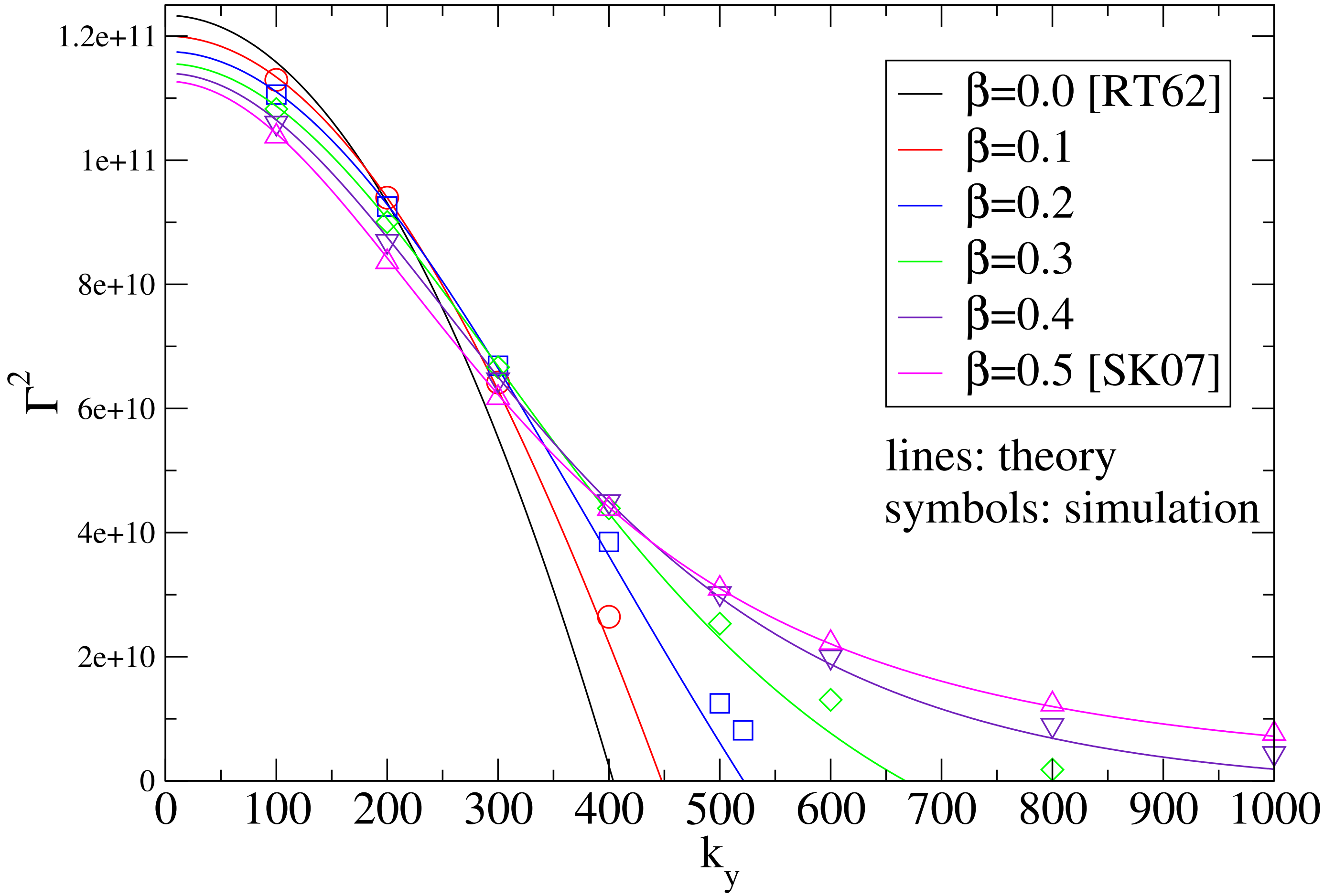
$$\omega_*^2 > 4\gamma_{\mathrm{GYR}}^2, \tag{9}$$

or

$$\frac{k_y^2\delta^2}{\Omega^2} \geq \frac{4\gamma_{\mathrm{MHD}}^2}{\left[g^2\left(1 - \frac{\beta}{1+\gamma\beta}\right)^2 - \frac{p}{\rho}\frac{\beta}{1+\gamma\beta}\gamma_{\mathrm{MHD}}^2\right]} \tag{10}$$

As it turns out, in the case studied by Dalton and Scott in NIMROD simulation, the stabilization criterion can not be satisfi ed for any real k_y when $\beta \geq 0.445857$. The equilibrium in that simulation has a $\beta \sim 0.5$.

Comparison between NIMROD simulation and theory



FLR stabilization due to 2-fluid Ohm's law only

(11) $$\omega(\omega^2 + \omega_* \omega + \gamma_{\mathrm{MHD}}^2) + D = 0$$

where

(12) $$\omega_* = -\frac{k_y \lambda}{\Omega} \frac{1}{1+\gamma\beta} \left[g - \tau \frac{p}{\rho} \left(\ln \frac{p}{\rho^\gamma} \right)' \right]$$

(13) $$D = -\frac{k_y \lambda}{\Omega} \frac{\frac{\rho'}{\rho} g}{1+\gamma\beta} \tau \frac{p}{\rho} \left(\ln \frac{p}{\rho^\gamma} \right)'$$

and $c_s^2 = \gamma p/\rho$, $\tau = p_i/p$, and λ is a tracer multiplier. Reduce to [RT62] in isentropic case when $d \ln (p/\rho^\gamma)/dx = 0$.

When $D \neq 0$, there are 3 eigenmodes. When D is not small, there are situations when there are 2 complex conjugate roots so that there's alway one growing mode for any k_y. In that case, FLR stabilization could be lost.

FLR stabilization due to both gyroviscosity and 2-fluid effects

(14) $$\omega(\omega^2 + \omega_* \omega + \gamma_{\mathrm{FLR}}^2) + D = 0,$$ where

$$\omega_* = \frac{k_y}{\Omega} \frac{\delta \left[(1+\gamma\beta)(1+\beta)\frac{p'}{\rho} - (2+\gamma\beta) g\beta \right] - \lambda \left[g - \tau \frac{p}{\rho} \left(\ln \frac{p}{\rho^\gamma} \right)' + \frac{k_y^2 \delta^2}{4\Omega^2} \frac{p^2}{\rho^2} \rho \right.}{(1+\gamma\beta)\left(1 + \frac{k_y^2 \delta^2}{4\Omega^2} \frac{p}{\rho} \frac{\beta}{1+\gamma\beta} \right)}$$

$$\gamma_{\mathrm{FLR}}^2 = \gamma_{\mathrm{GYR}}^2 + \frac{k_y^2 \lambda \delta}{\Omega^2} \frac{p}{\rho} \frac{(1+\beta)\left(\tau \frac{p'}{\rho} - g \right) \frac{p'}{p} + \left[(1+\gamma\beta\tau) g - (1+\beta)\gamma\tau \frac{p'}{\rho} \right] \frac{\rho'}{\rho} + \left(\frac{\rho g}{p} - \right.}{(1+\gamma\beta)\left(1 + \frac{k_y^2 \delta^2}{4\Omega^2} \frac{p}{\rho} \frac{\beta}{1+\gamma\beta} \right)}$$

$$D = -\frac{k_y \lambda}{\Omega} \frac{\frac{\rho'}{\rho} g \tau \frac{p}{\rho} \left(\ln \frac{p}{\rho^\gamma} \right)'}{(1+\gamma\beta)\left(1 + \frac{k_y^2 \delta^2}{4\Omega^2} \frac{p}{\rho} \frac{\beta}{1+\gamma\beta} \right)}$$

Summary

- New theory for pure interchange g-mode dispersion reaches agreement with NIMROD simulations [SK07].
- Previous textbook theory on complete FLR stabilization of pure interchange g-mode [RT62] by gyroviscosity or 2-fluid effects, strictly applies only in low β or isentropic regime.
- In high β or non-isentropic regime, full FLR stabilization of pure interchange g-mode may not be attainable by gyroviscosity or 2-fluid effects alone, respectively.
- Finite-β effects on FLR stabilization may not negligible either for other interchange type of modes, such as ballooning in ELMs.