M3D-C^1 Linear Stability Benchmarking

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Features of M3D-C^1

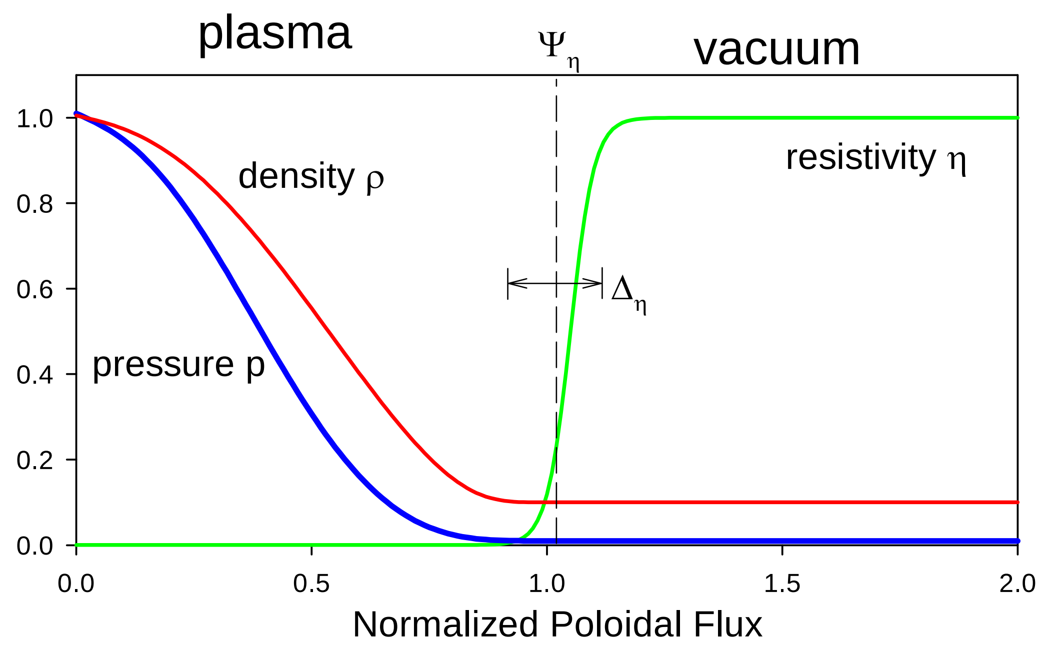
- Two-Fluid MHD with Hall physics and Gyroviscosity
- Efficient scalable implicit time-stepping with time step independent of zone size
- High-Order (5^{th}) finite elements (in plane) with continuous first derivatives (C^1 continuity)
- Fully unstructured mesh with adaptive capability
- Can solve subsets of 2-field or 4-field reduced MHD instead of full (8 field) equations

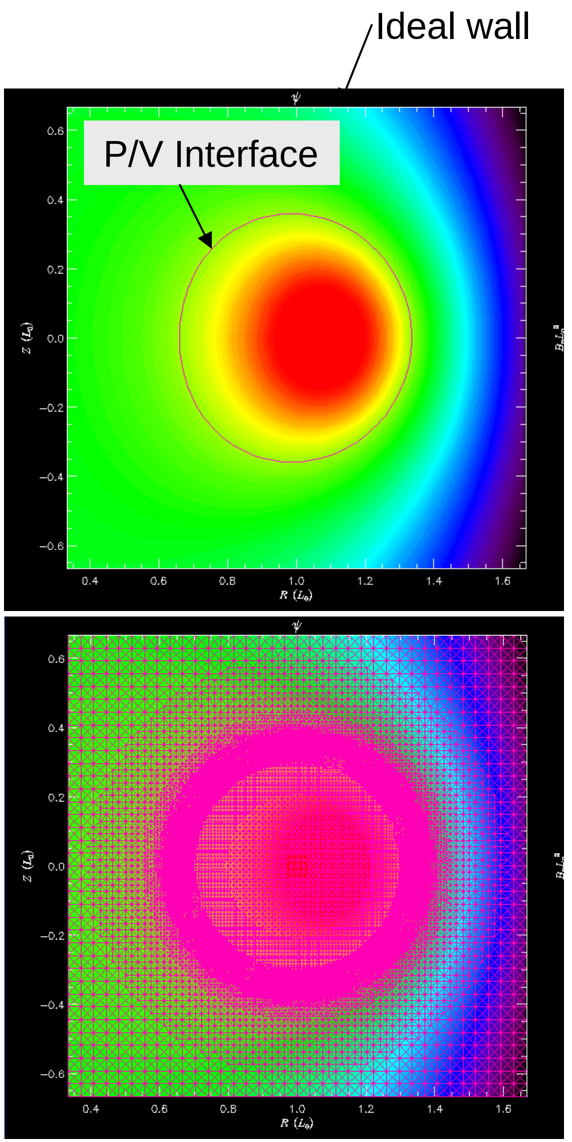
Linear Stability Studies with M3D-C^1

Intermediate step to the full nonlinear calculations for code verification and to explore new physics:

- Can linearize about two-fluid equilibrium with flow
- Two-fluid or resistive MHD evolution equations
- (high resistivity) vacuum region separating plasma and (resistive) wall
- Mesh packing and adaptation

Initial benchmark is to reproduce ideal MHD limit in a free boundary plasma (surrounded by vacuum)

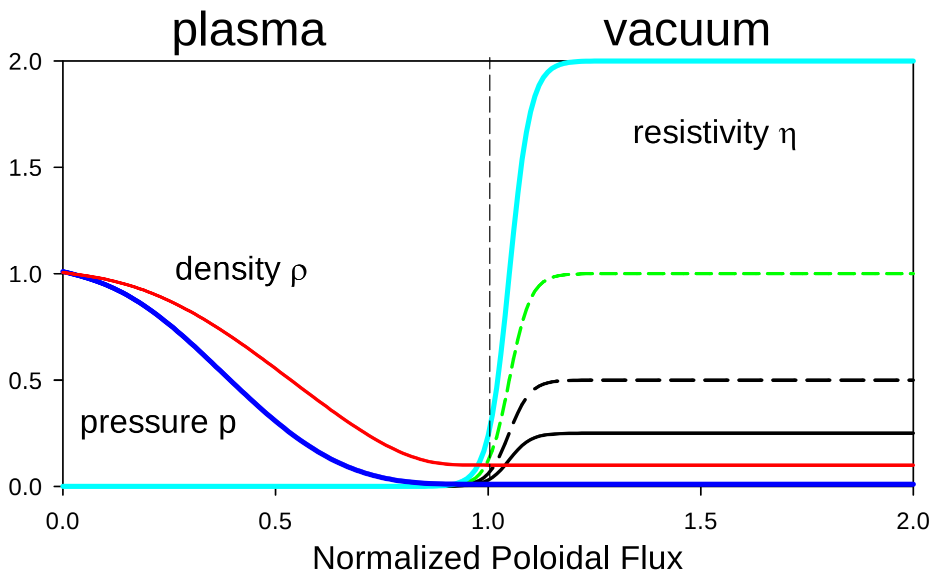




Should be able to approach ideal limit as:

- resistivity transition width $\Delta_\eta \rightarrow 0$
- resistivity transition location $\Psi_\eta \rightarrow 1$ (P/V interface),
- plasma resistivity $\eta_P \rightarrow 0$,
- vacuum resistivity $\eta_V \rightarrow \infty$

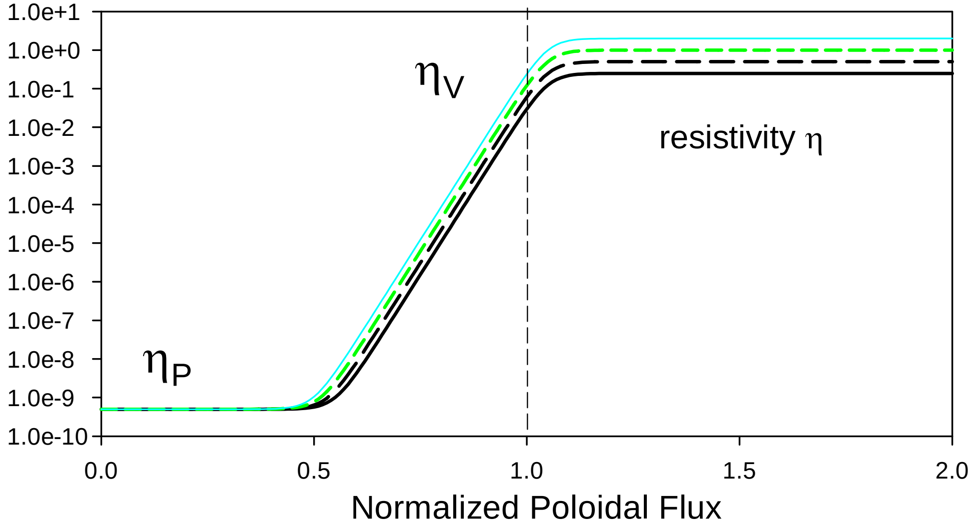
Initial convergence study varied η_V and η_P



$$\eta = \eta_P + \eta_V \left\{ \frac{1}{2} + \frac{1}{2}\tanh\left[\frac{\psi - (\psi_{\lim} + \varepsilon_{off}\Delta\psi)}{\varepsilon_\delta \Delta\psi} \right] \right\}$$

$$\Delta\psi \equiv \psi_{\lim} - \psi_0$$

$$\varepsilon_{off} = \varepsilon_\delta = .05$$



Growth rates for n=1 mode for equilibrium with q_a = 3.2, β_N = 4.92

η_P	η_V = 0.5	η_V = 1.0	η_V = 2.5
10^{-7}	.69876	.7086	.71877
10^{-8}	.69875	.7085	.71875
10^{-9}	.69875	.7085	.71875

Small dependence on η_V not entirely satisfactory!

Variables are initialized with a certain up/down parity, and this is preserved if equilibrium is symmetric

n=1 mode, ideal limit

$\eta_{VACUUM} = 0.5$

$\eta_{PLASMA} = 5.\times10^{-8}$

Odd parity in *U*

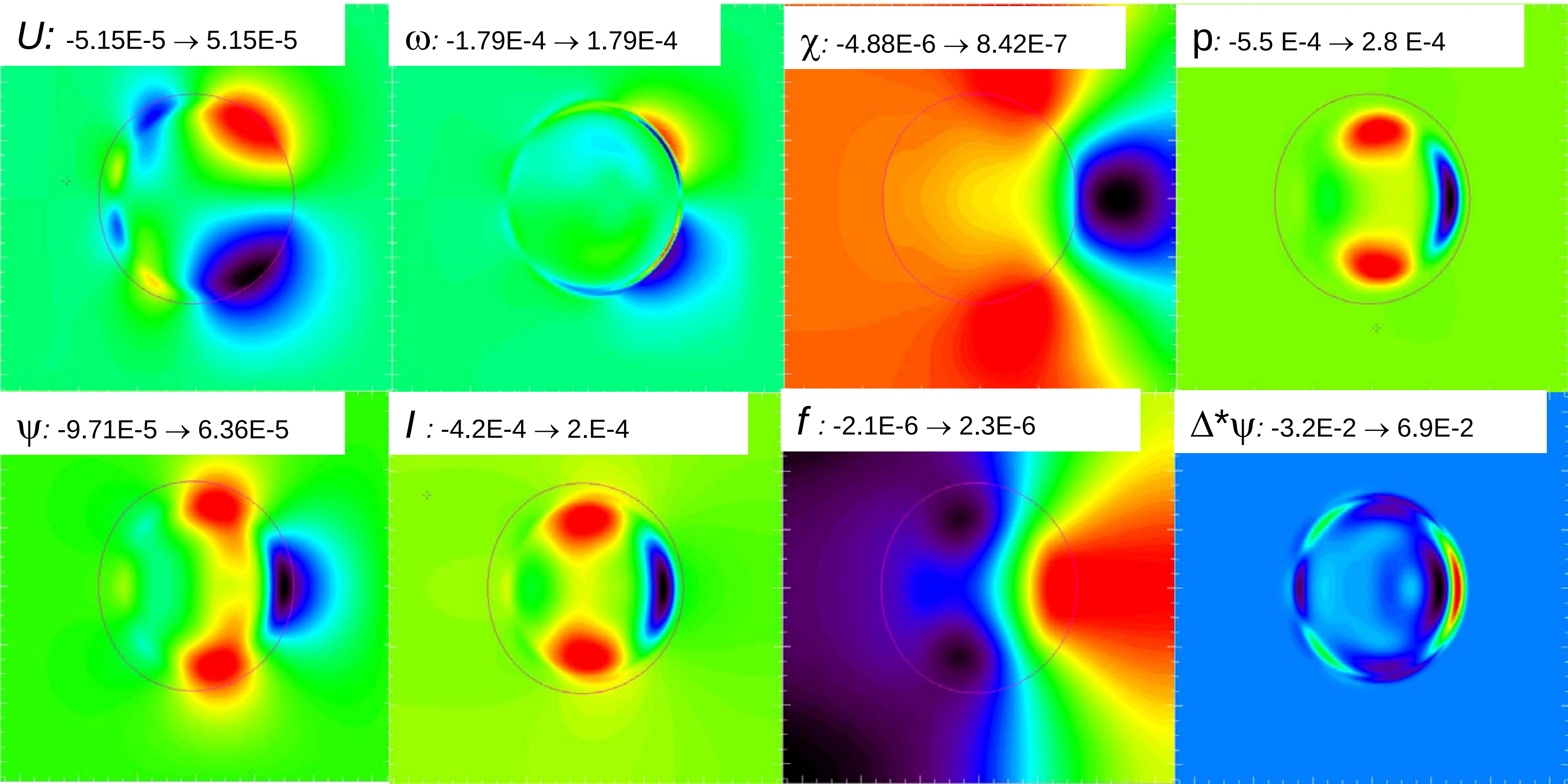
$A = 3$

$1.01 < q < 3.32$

$\beta_N = 4.92$

$$\mathbf{V} = R^2\nabla U \times \nabla\varphi + \omega R^2 \nabla\varphi + R^{-2}\nabla_\perp \chi$$

$$\mathbf{B} = \nabla\psi \times \nabla\varphi - \nabla f_\varphi + I\nabla\varphi$$



Variables are initialized with a certain up/down parity, and this is preserved if equilibrium is symmetric

n=1 mode, ideal limit

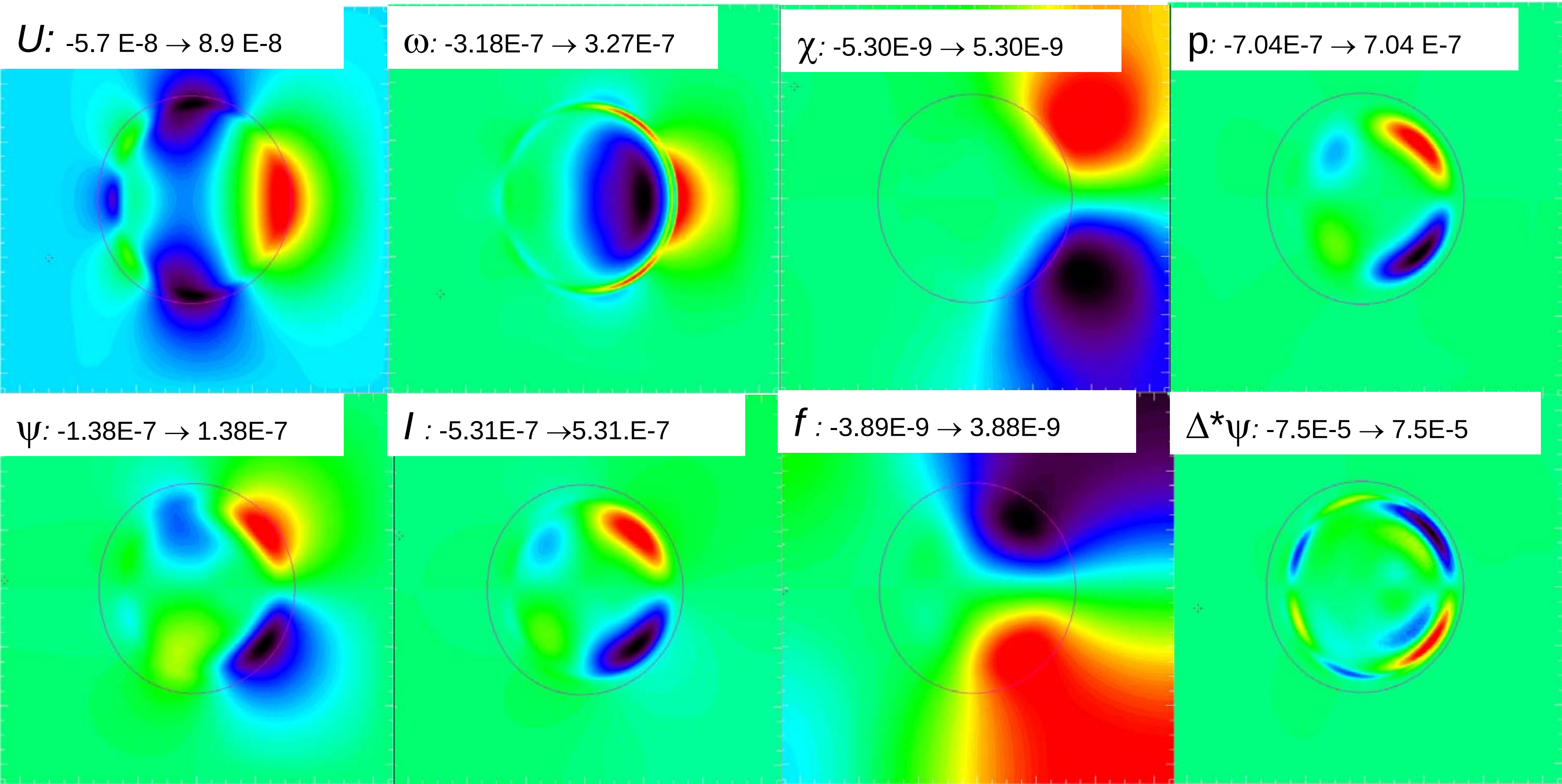
$\eta_{VACUUM} = 0.5$ $\quad$ A = 3

$\eta_{PLASMA} = 5.\times 10^{-8}$ $\quad$ $1.01 < q < 3.32$

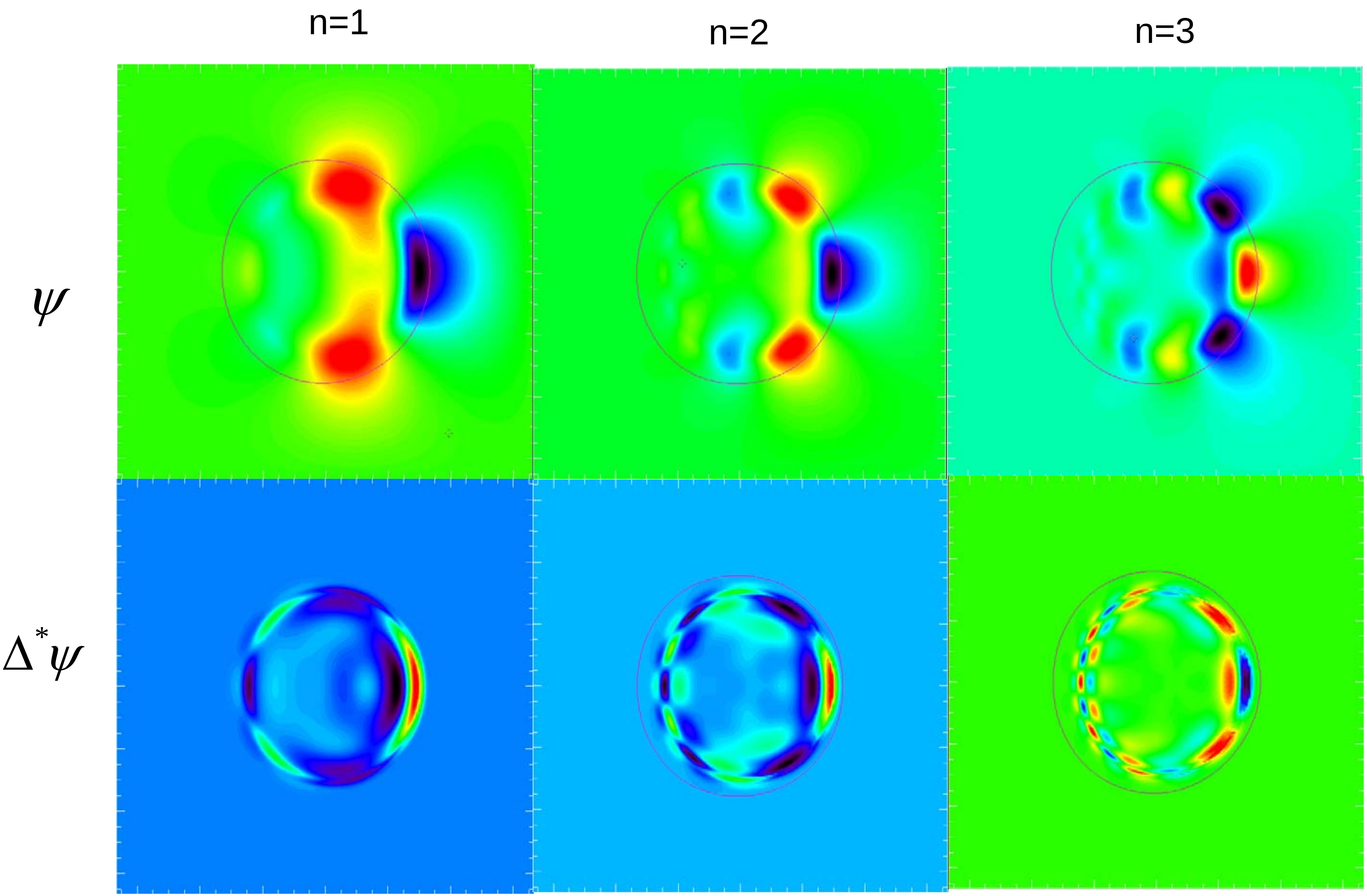
Even parity in U $\quad$ $\beta_N = 4.92$

$$\mathbf{V} = R^2 \nabla U \times \nabla \varphi + \omega R^2 \nabla \varphi + R^{-2} \nabla_\perp \chi$$

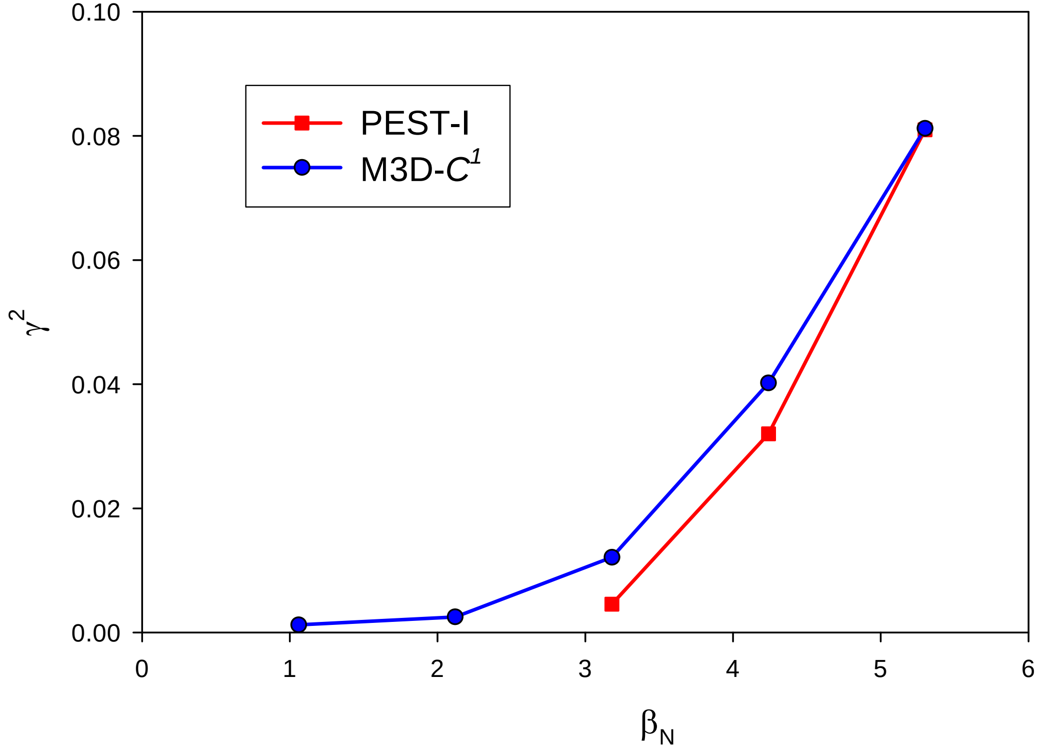
$$\mathbf{B} = \nabla \psi \times \nabla \varphi - \nabla f_\varphi + I \nabla \varphi$$



Perturbed field extends into vacuum but perturbed currents do not



Comparison of Growth Rate with PEST-I

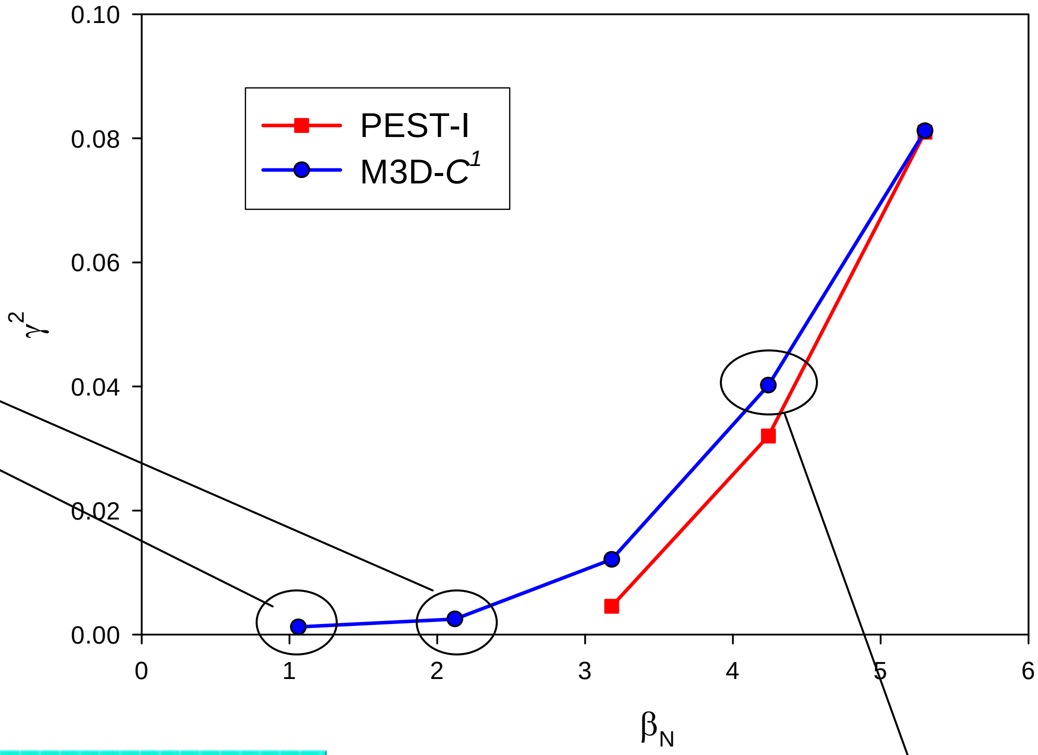


$S_V = 0.5$

$S_P = 10^8$

$\mu = 10^{-9}$

Comparison of Growth Rate with PEST-I



$S_V = 0.5$

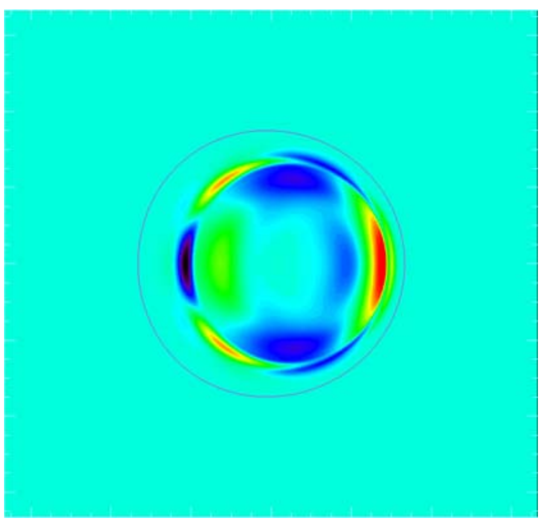
$S_P = 10^8$

$\mu = 10^{-9}$

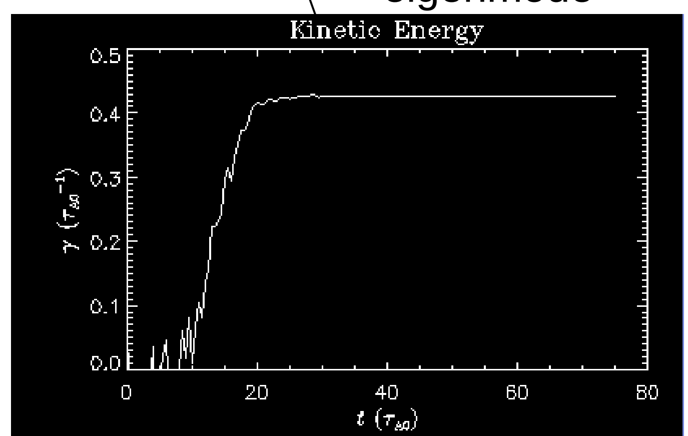
Residual modes ?

$\Delta t \sim 1\tau_A$

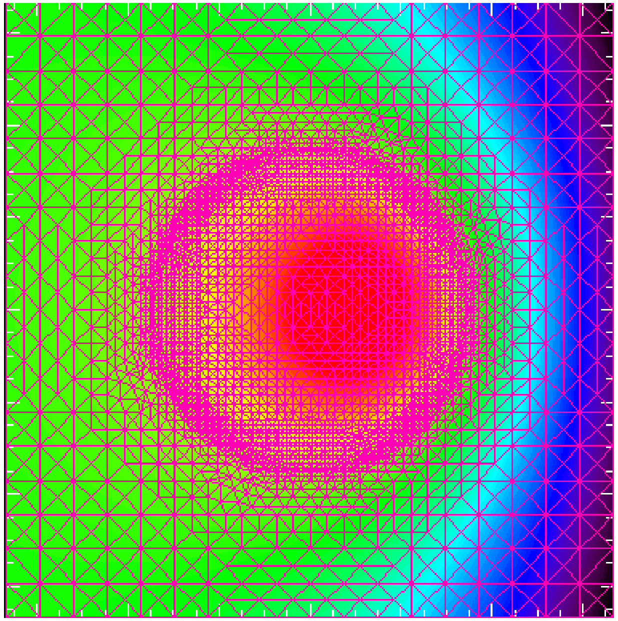
eigenmode

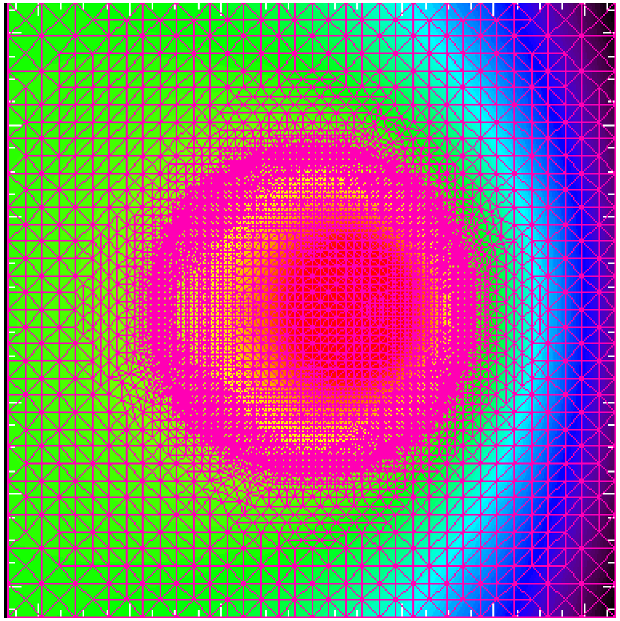


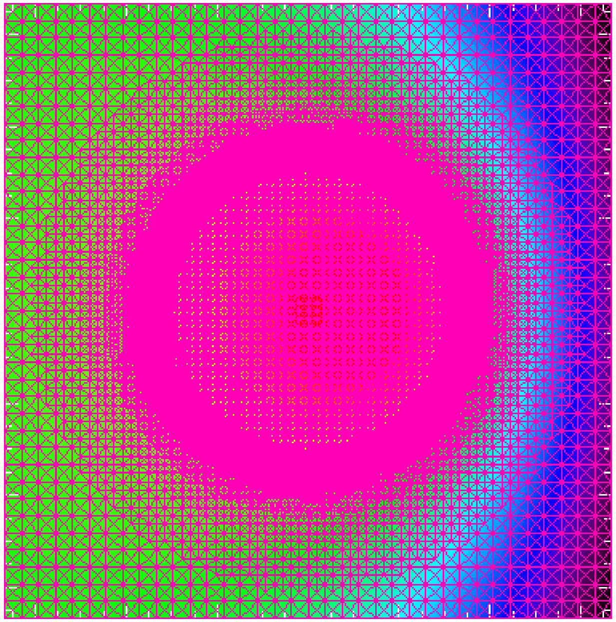
eigenfunction



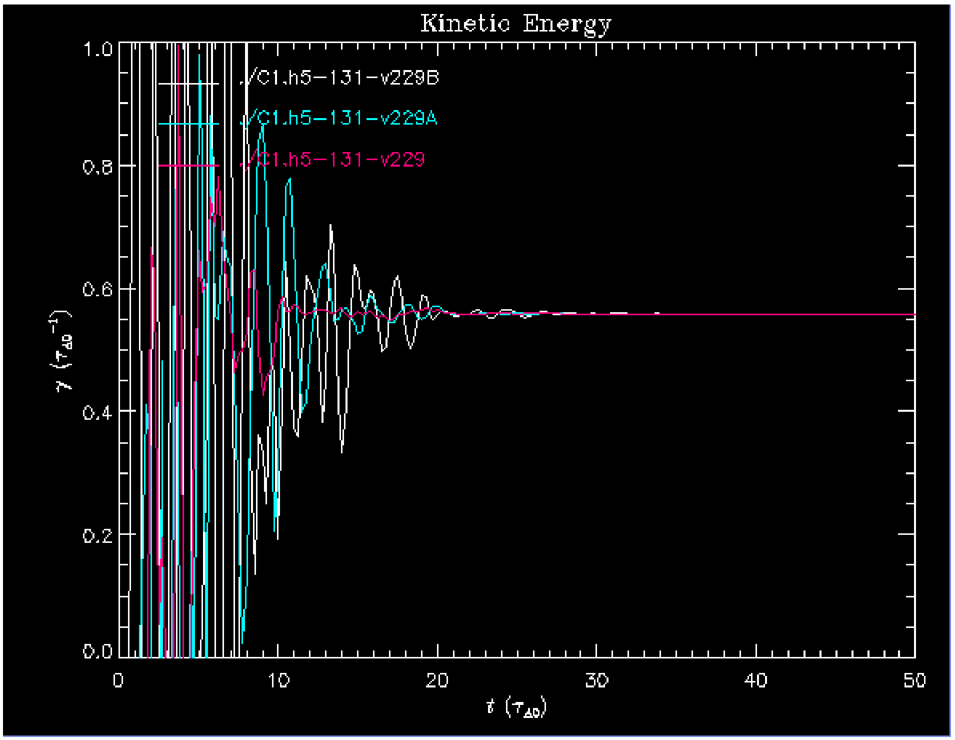
Spatial convergence study used 3 mesh resolutions

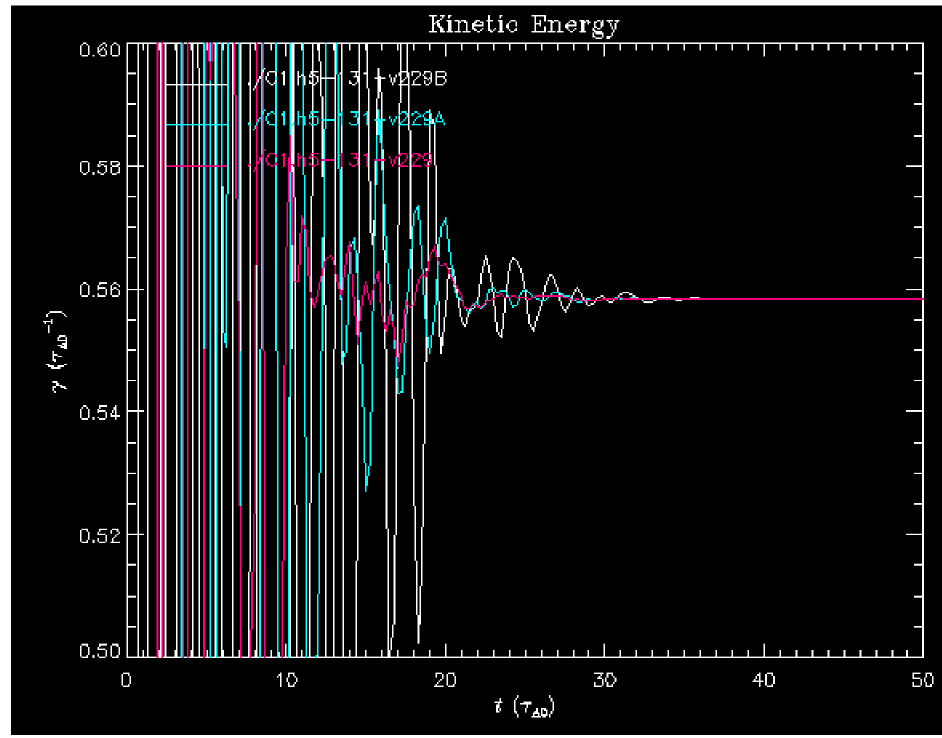

3557 nodes


6185 nodes


24697 nodes

Same converged growth rate for 3 mesh resolutions

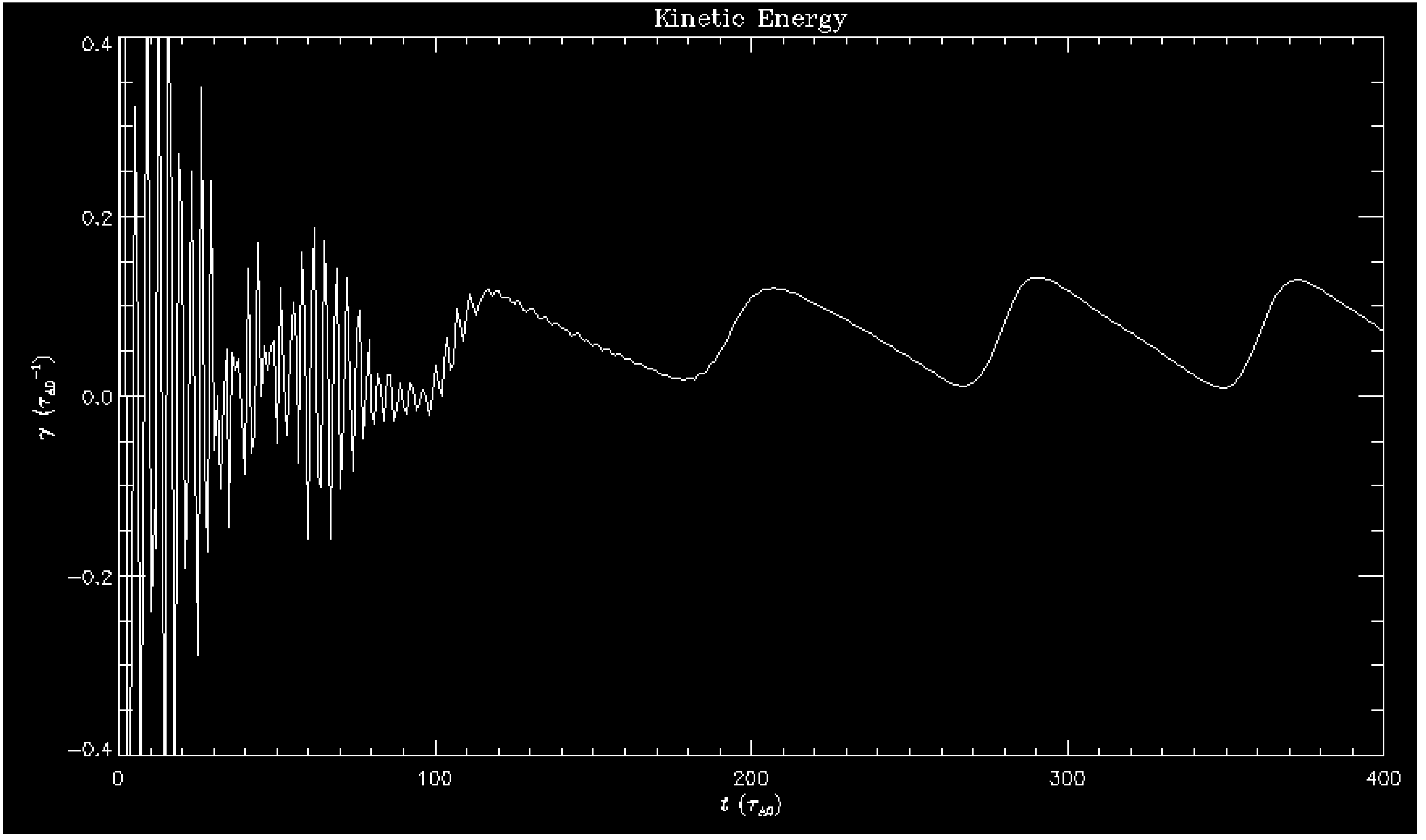




Vertical scale (γ): 0 - 1

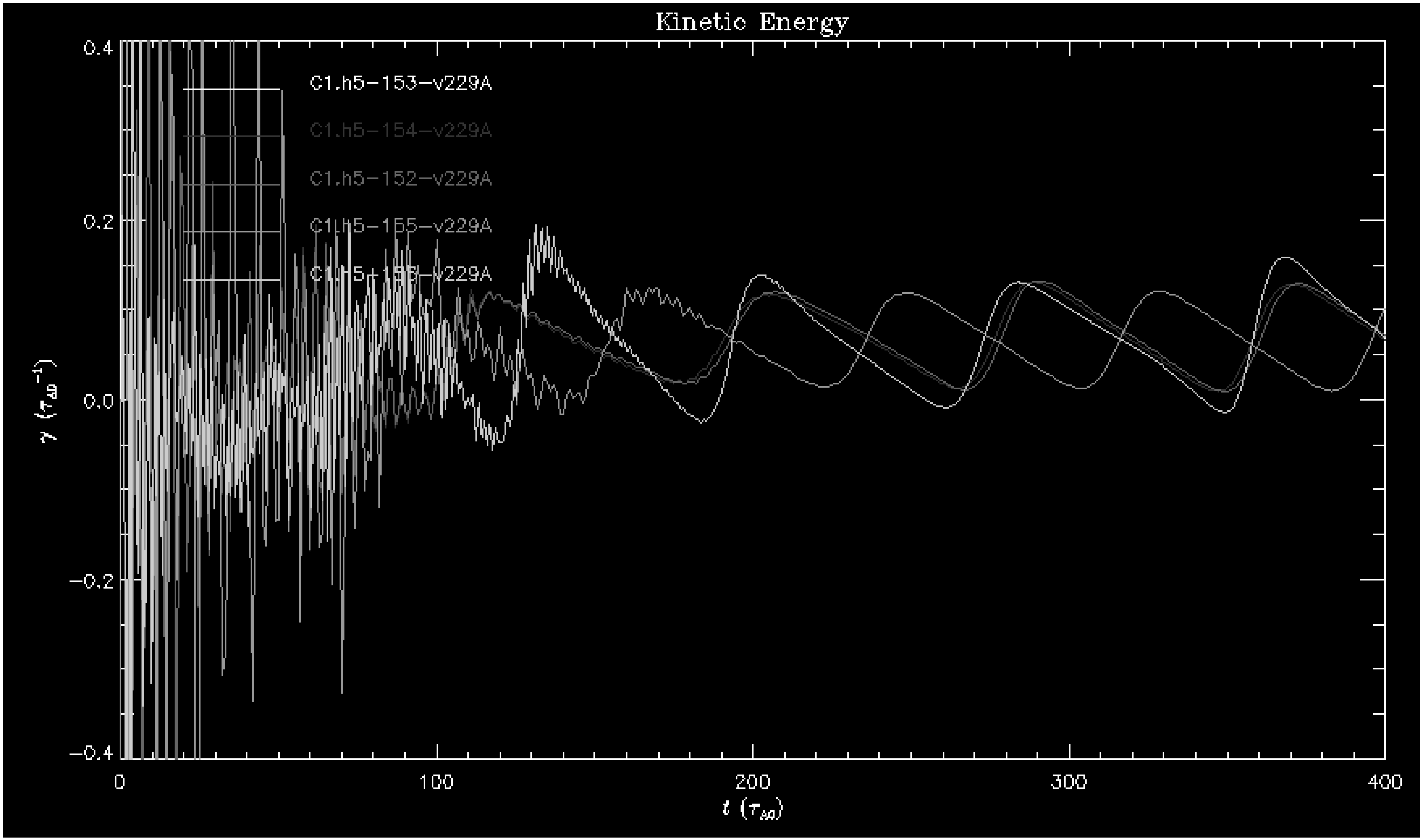
Vertical scale (γ): 0.5 – 0.6

What is the residual mode?



Appears to be two modes with same growth rate rotating in opposite directions.

Mean growth rate and frequency not sensitive to numerical parameters.



ν: $10^{-6} \rightarrow 10^{-7}$

S_v: $0.5 \rightarrow 5$

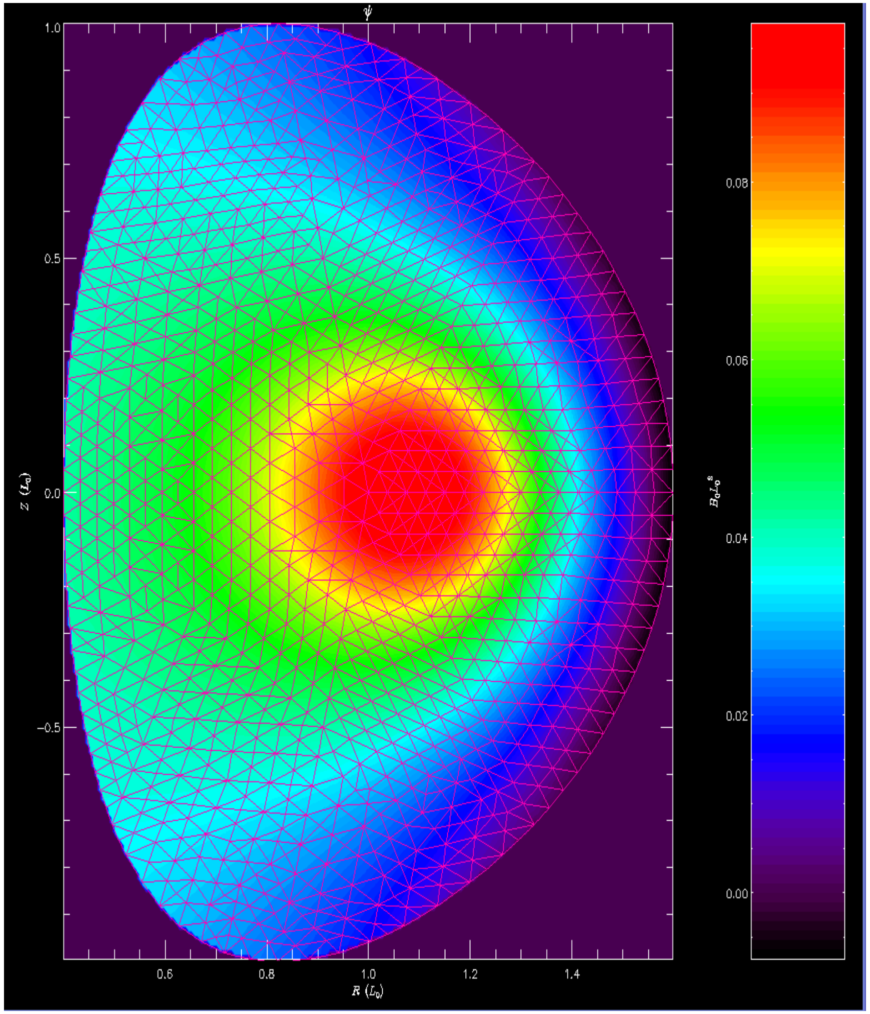
Zone size doubled

S_P: $10^{6} \rightarrow 10^{7}$

Δt: $0.5 \rightarrow 1.0$

New features now being implemented in M3D-C^1

- Arbitrarily shaped domain boundary (axisymmetric vacuum vessel shape)
- Zone packing by specifying anisotropic mesh size function as arbitrary function of the equilibrium flux function.
- resistive wall



Will soon transition into fully nonlinear 3D 2F MHD code.