Modeling of RF/MHD coupling using NIMROD and GENRAY

Physics results and code coupling issues

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Goal of SWIM Slow MHD campaign – numerically simulate ECCD stabilization of NTM's

•Experimental efforts to stabilize neoclassical tearing modes via electron cyclotron current drive have been very successful [La Haye, Phys. Plasmas **13**, 055501 (2006)]. Want to develop a self-consistent model for simulating this physics.

•For ECCD, the RF-induced current is relatively small (of the same order as the electric field) – small expansion parameter

•Add an RF term to the kinetic equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}} = C(f_\alpha) + Q(f_\alpha)$$

$$Q(f_\alpha) \equiv \frac{\partial}{\partial \mathbf{v}}\cdot\mathcal{D}\cdot\frac{\partial}{\partial \mathbf{v}}$$

Gyrophase-averaged Fokker-Planck Coulomb collision operator

Quasilinear diffusion tensor from RF source

RF terms appear in the fluid equations

- Taking fluid moments in the conventional manner yields

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0 \qquad \text{(RF produces no particles)}$$

$$m_\alpha n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha \right) = n_\alpha q_\alpha (\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) - \nabla p_\alpha - \nabla \cdot \pi_\alpha + \mathbf{R}_\alpha + \mathbf{F}_{\alpha 0}^{rf}$$

$$\mathbf{F}_{\alpha 0}^{rf} \equiv \int m_\alpha \mathbf{v} Q(f_\alpha) d\mathbf{v} \qquad \text{(additional momentum imparted by RF waves)}$$

$$\frac{3}{2} n_\alpha \left(\frac{\partial T_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) T_\alpha \right) + n_\alpha T_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - \pi_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha + S_{\alpha 0}^{rf}$$

$$S_{\alpha 0}^{rf} \equiv \int \frac{1}{2} m_\alpha v^2 Q(f_\alpha) d\mathbf{v} \qquad \text{(additional energy imparted by RF waves)}$$

- Closure calculations for $\mathbf{q}_\alpha$, π_α are also affected by RF.

Can approximate f_α as a local Maxwellian, here – RF perturbations are small

Much of the basic physics is independent of the problem details

•Ultimately, the closure scheme and the small parameter expansion yield a self-consistent set of fluid equations for ECCD-influenced MHD.

•While work proceeds on that front, we can study simpler models (e.g. resistive tearing modes, rather than NTM's) to gain physical insight. (Neoclassical effects enter the Rutherford equation additively.)

•Consider the electron momentum equation, which gives rise to the MHD Ohm's law. RF-induced momentum yields an additional term,

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J} - \frac{\mathbf{F}_{\mathrm{rf}}}{e}$$

•A physically reasonable form for the lowest-order effect of the RF is

$$\frac{\mathbf{F}_{\mathrm{rf}}}{e} = \frac{\eta \lambda_{rf} \mathbf{B} f(\mathbf{x}, t)}{\mu_0}$$

λ_{rf} = RF amplitude

$f(\mathbf{x},t)$ = space/time dependence

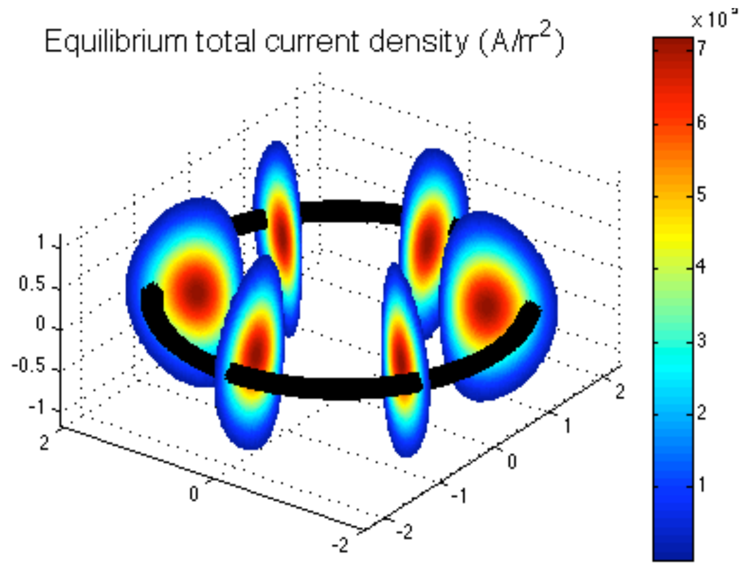
Simple RF models can yield sound physics results, even without self-consistency

- For these simulations, model the RF as a Gaussian function in the poloidal plane – neglect toroidal variation for the moment.

- Ramp up on some intermediate timescale (slow compared to Alfvén timescale, fast compared to resistive timescale), neglect closure problem.

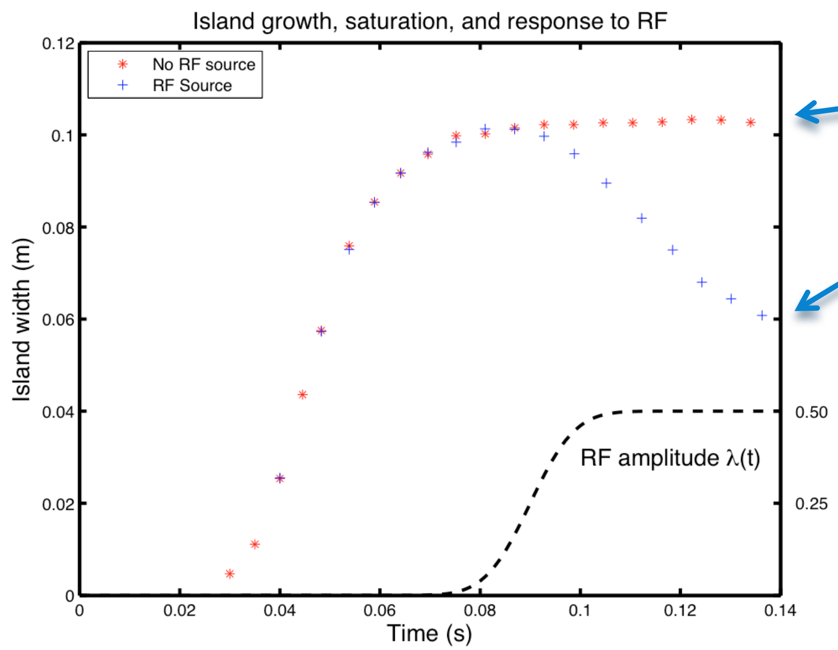
$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J} - \frac{\eta \lambda_{rf} \mathbf{B}}{\mu_0} \exp\left(-\frac{(R - R_{rf})^2 + (Z - Z_{rf}^2)}{w_{rf}^2} \right) \mathrm{erf}\left(\frac{t - t_{\text{offset}}}{t_{\text{build}}} \right)$$

- Parameters:
(R_{rf},Z_{rf}) = position of RF peak
w_{rf} = half-width
λ_{rf} = amplitude

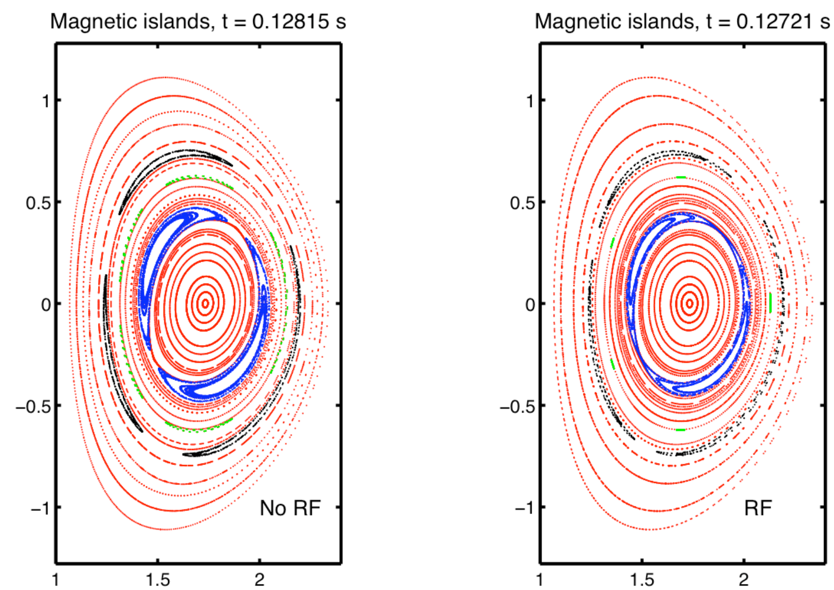


- What physics results arise from the inclusion of this term in the MHD model?

RF effects modify the saturated island width of resistive tearing modes



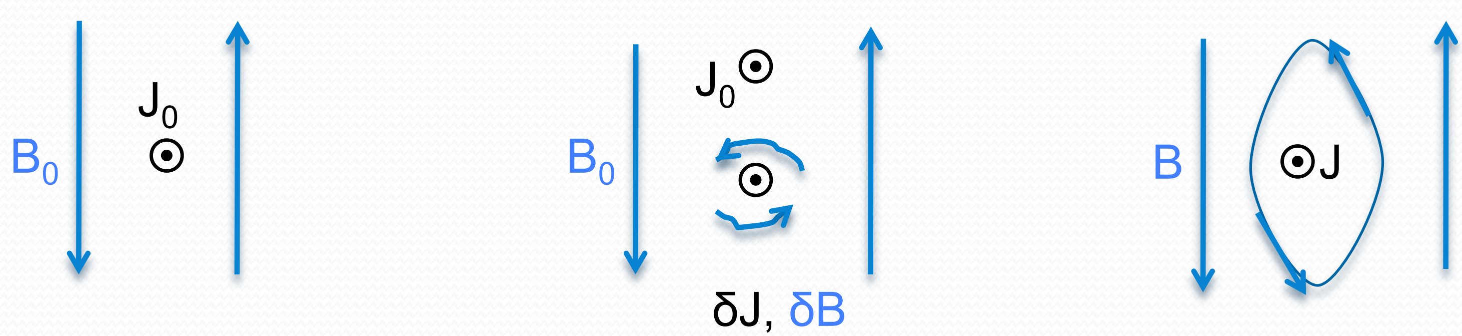
•Marked reduction in the size of saturated islands can be achieved – proof of principle.



•Island width data obtained from field line traces – expensive.

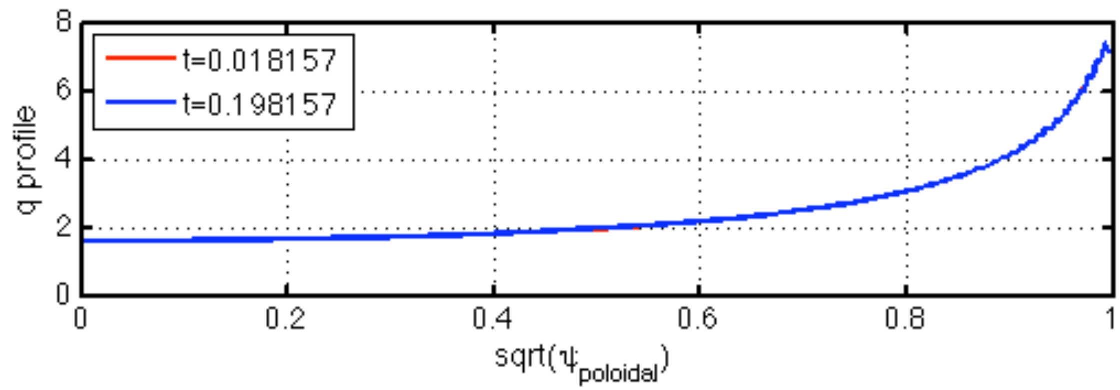
What is the effect of the RF on the tearing modes?

- Tearing mode can be influenced by the RF in two major ways:
  - Modification of tearing parameter Δ'
    -will affect linear growth rate and saturated island width
    -easy to diagnose – proportional to linear growth rate
  - Modification of helical current profile $\mu = \mu_0 (J \bullet B)/(B \bullet B)$
    -becomes important as mode saturates nonlinearly
    -not so easy to diagnose. Conceptual picture:

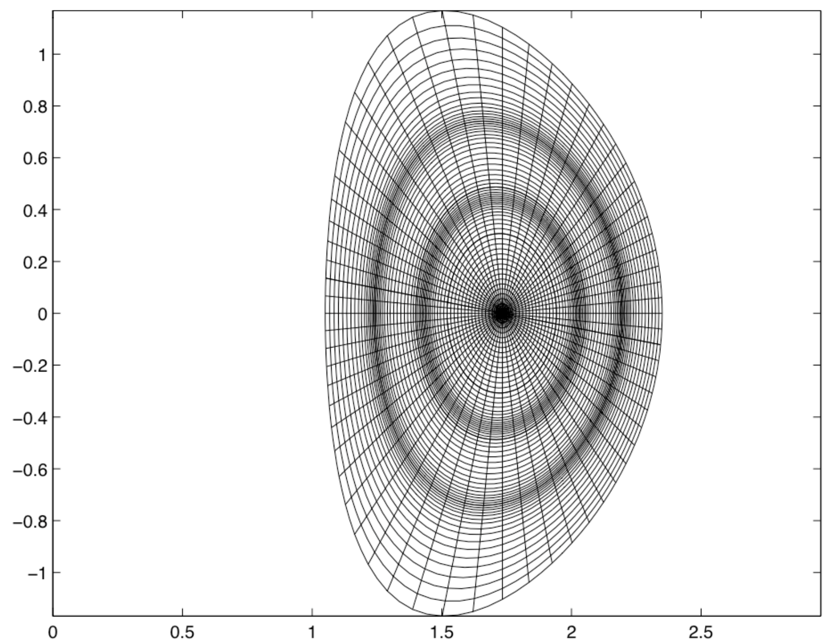


Study Δ’ and current profile modification in DIII-D-like geometry

•Begin with equilibrium where the dominant instability is the (2,1) resistive tearing mode

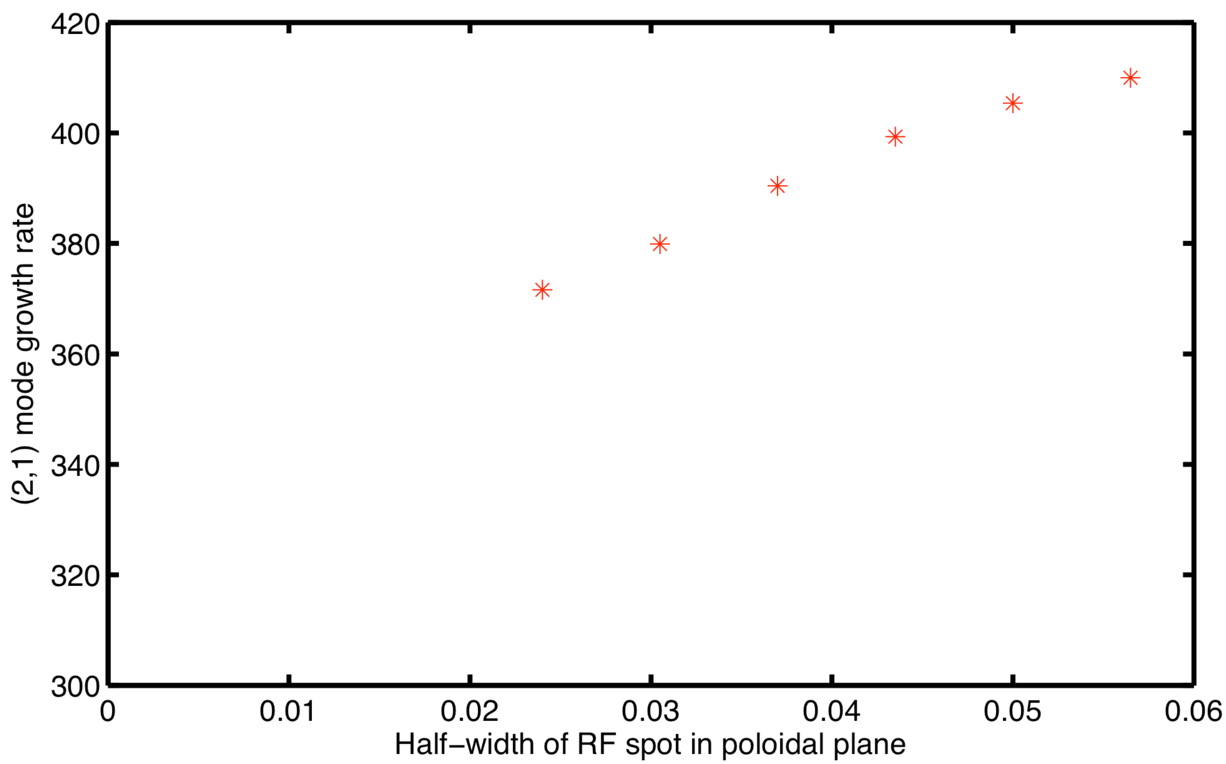


•Pack grid around (2,1) and (3,1) rational surfaces



To study Δ' modification, find growth rate after RF influence on background comes to steady-state

- Allow only toroidally symmetric (n=0) modes in simulation, ramp RF up to steady state – yields new RF-modified 'equilibrium'
- Then allow n=1 (and higher) modes into simulation, check linear growth



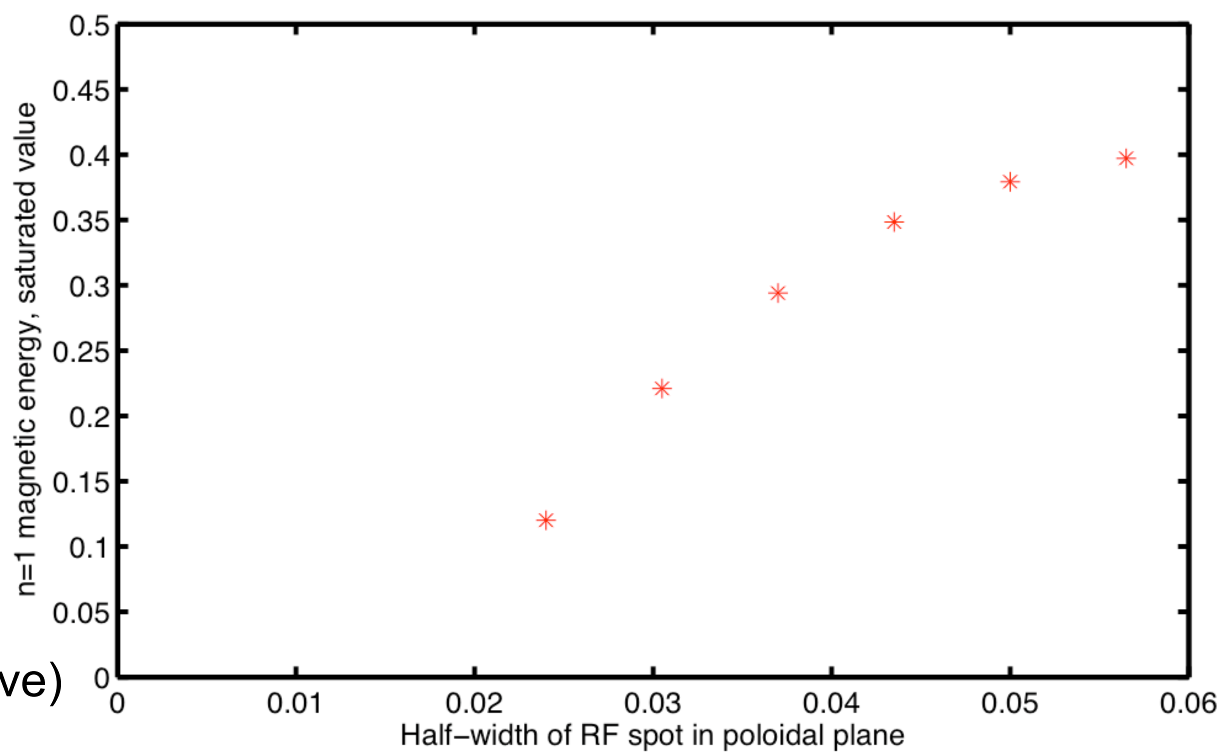
- For fixed RF current (centered on (2,1) rational surface), the growth rate (and thus Δ') is reduced slightly as the poloidal cross-section of the RF spot is reduced.
- In agreement with results of Pletzer/Perkins [PoP **6**, 1589 (1999)].

Δ' modification - saturated island width is decreased as RF cross-section is reduced

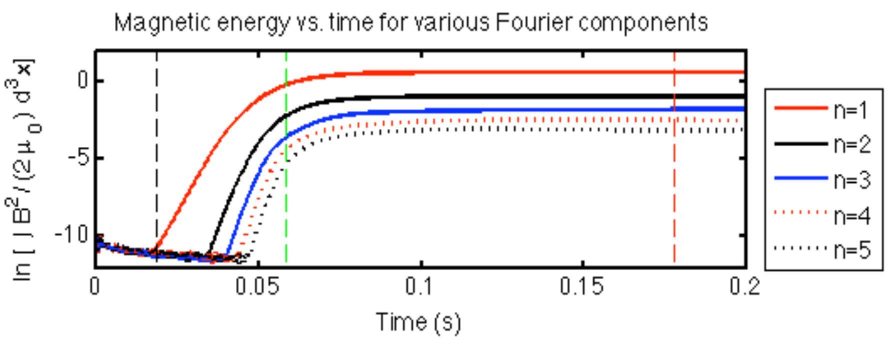
•For fixed RF current (centered on (2,1) rational surface), the saturated island width is also reduced for smaller RF poloidal cross-sections. (Δ' modification is not the dominant effect on island size here).

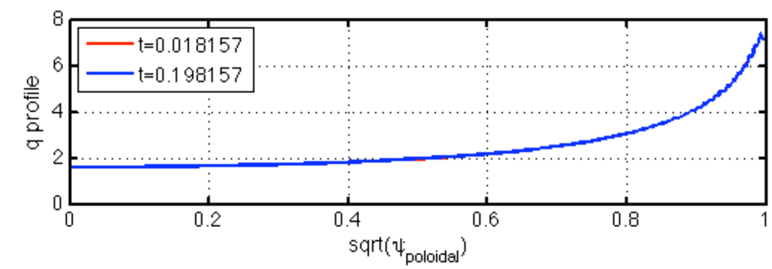
•In agreement with general conclusions of Hegna/Callen [PoP **4**, 2940 (1997)].

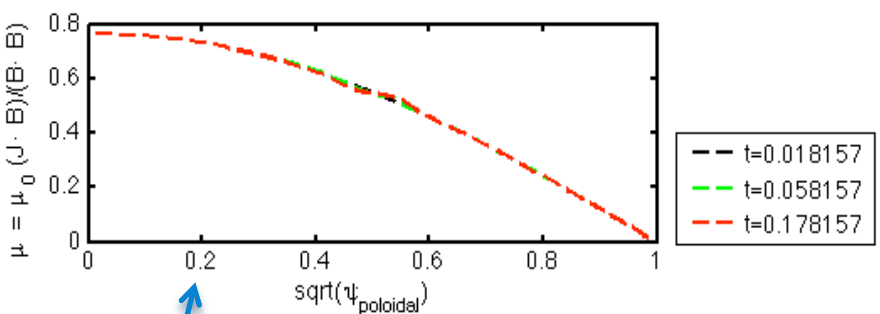
•Pletzer/Perkins find that for $I_{RF}/I_0 < 4\%$, offset from rational surface is destabilizing – simulations in progress to verify this effect (somewhat counterintuitive)

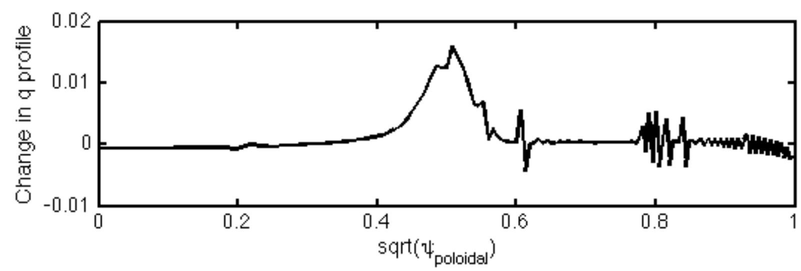


Current profile modification – quasilinear flattening occurs at the rational surface even in the absence of RF



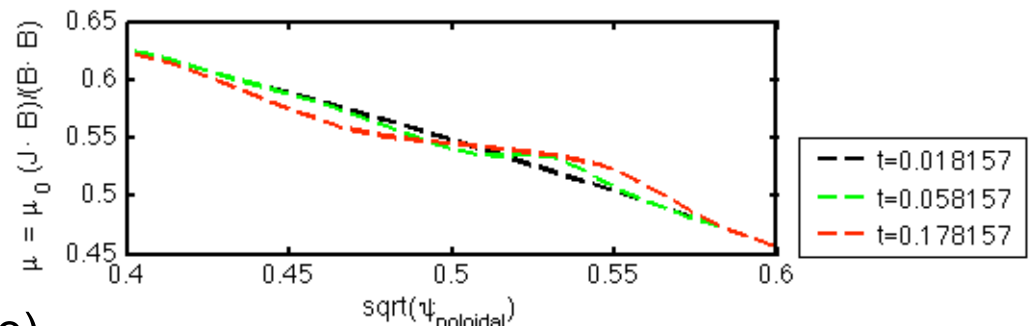




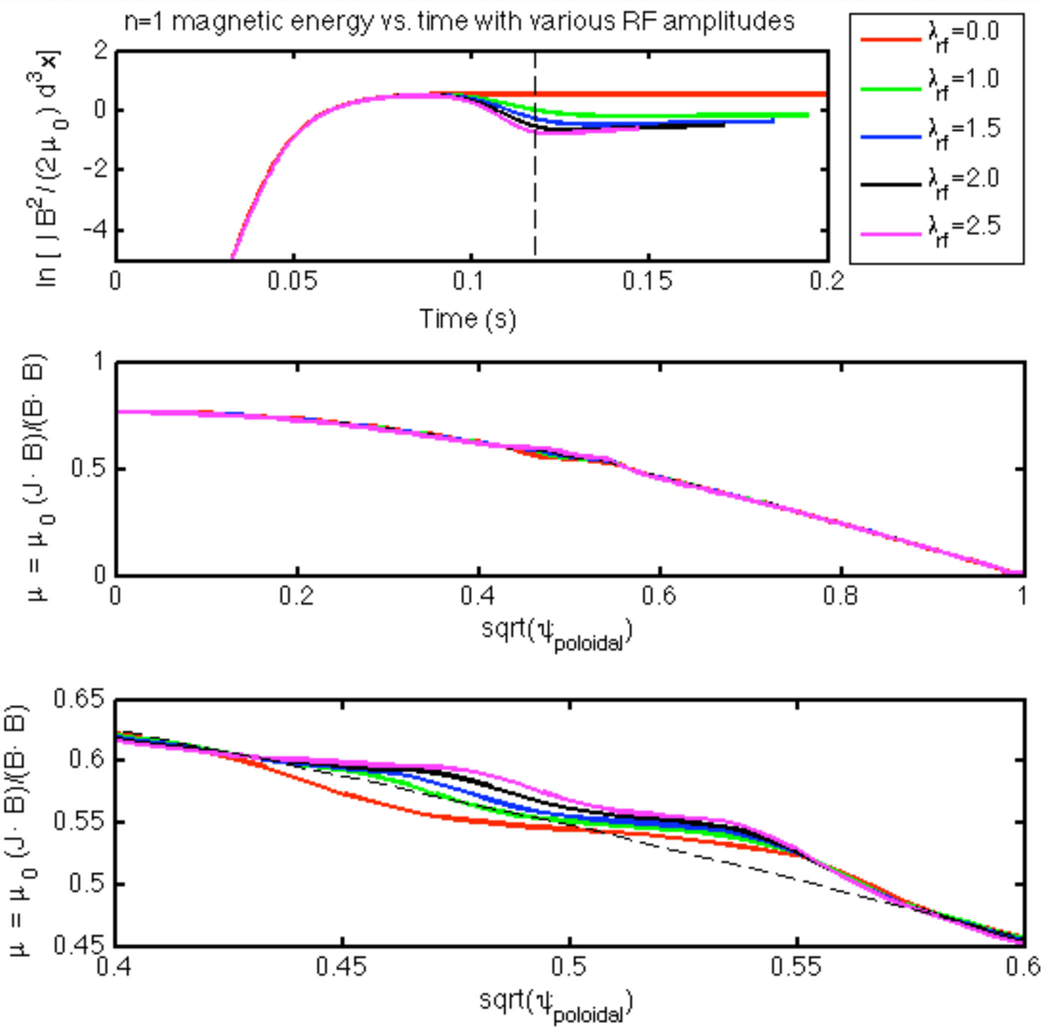


- Toroidally averaged μ profile and closeup. Relaxes to a quasilinearly flattened equilibrium at long times.

(see mu_without_RF_1D.avi movie)



Current profile modification – same simulation, but with RF added at (2,1) rational surface

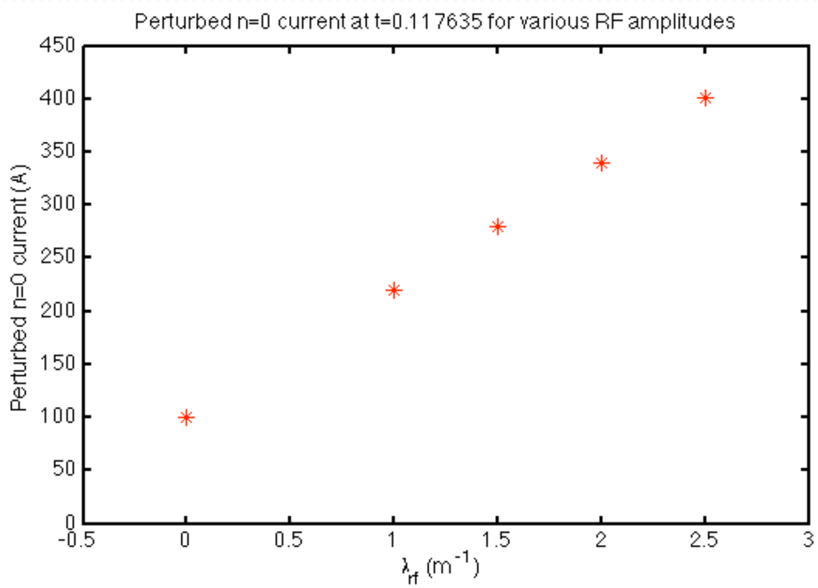


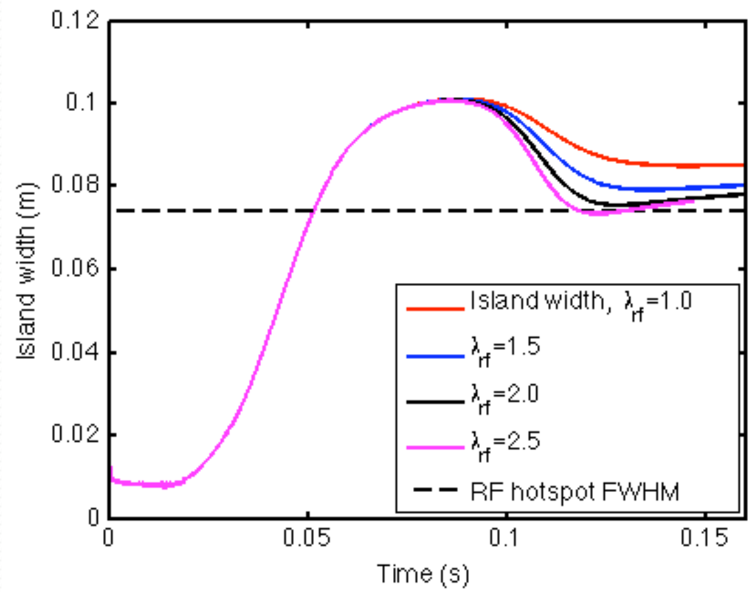
- To study current profile modification, turn on RF after resistive tearing mode has saturated; examine the island width reduction and μ profile (here $I_{RF(n=0)}/I_{0(n=0)}$ ~ 1%).

- Without the RF, quasilinear flattening leads to increased net current on outward side of rational surface; decreased net current on inward side (red).

- RF adds current on both sides of rational surface – destabilizing inside, but stabilizing outside (a net stabilizing effect). (See mu_with_RF_1D.avi movie)

The additional current is linear in λ_{RF}, but island reduction is limited by the RF cross-section





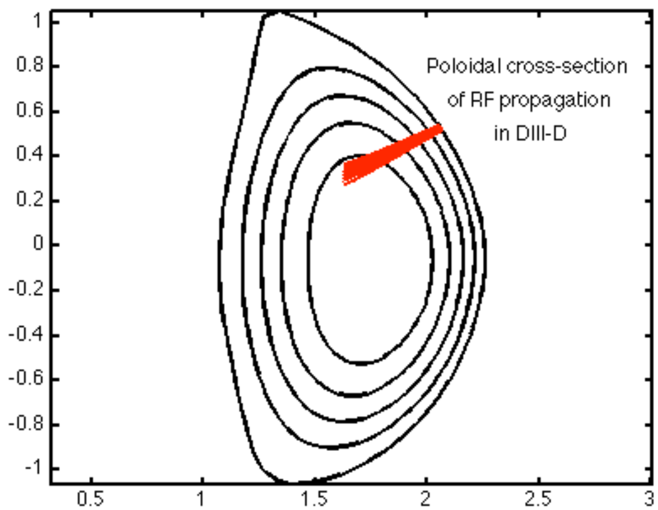
- The island cannot shrink appreciably below the width of the poloidal cross-section of the RF drive. In agreement with Hegna/Callen result

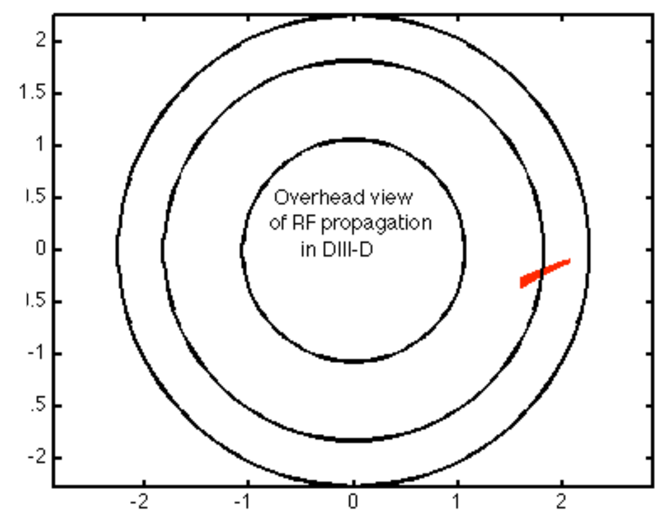
- When RF is offset from rational surface, investigating Δ' and current profile modifications relative to non-offset case; compare with Pletzer/Perkins conclusions (in progress). Size of RF cross-section relative to saturated island size also important.

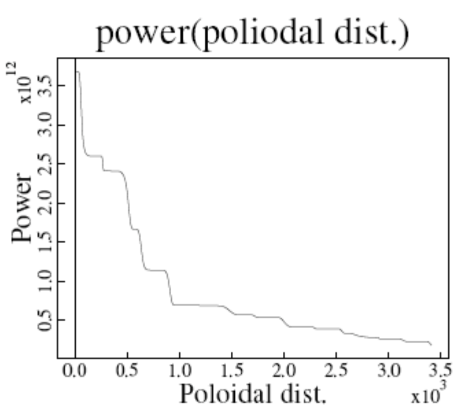
Replace ad hoc RF term with realistic RF physics using GENRAY

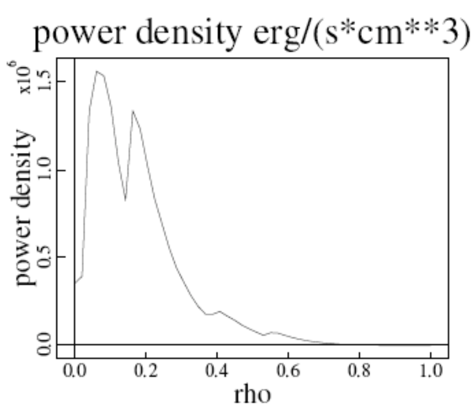
- GENRAY calculates ray trajectories and power deposition as a function of flux surface, given the magnetic geometry and wave parameters

- One difficulty has been the translation of NIMROD magnetic data into a form GENRAY can process.





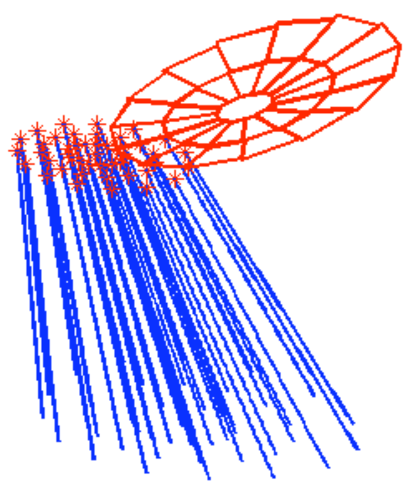




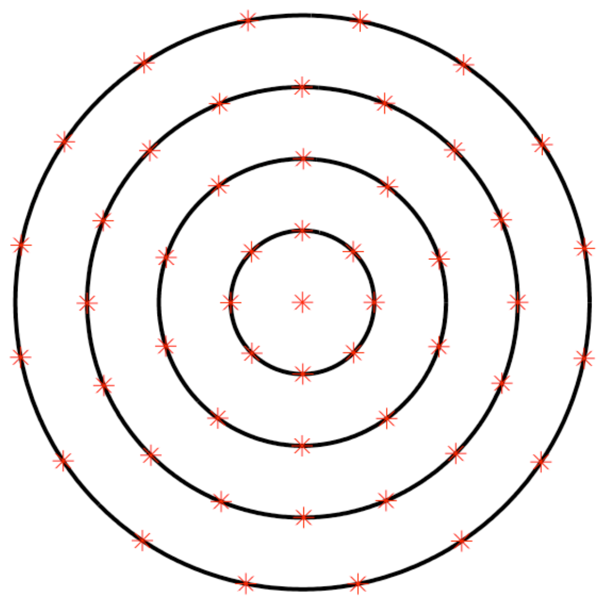
Ray data from GENRAY is passed to NIMROD in a CDF file

•Rays are launched in a cone of r concentric rings around a central ray, with s_r equally spaced rings on the rth ring with a relative phase shift of p_r. GENRAY writes this metadata and the ray data to a CDF file.

•GENRAY calculates flux-surface-averaged quantities. A nontrivial geometric calculation determines volume elements for 'undoing' this averaging; alternatively, can pass the full quasilinear operator to NIMROD.



$s = 4$, $s_r=[8,10,16,20]$,
$p_r=[0,\pi/10,0,\pi/20]$



After discrete GENRAY data has been passed to NIMROD, it needs to be mapped to continuous finite elements

- Some regions of the NIMROD grid will have more contributions from discrete points than there are basis functions (overdetermined); others will have only a few contributions (underdetermined).

- Eric Held – working on importance-sampling techniques (similar to some PIC methods) – a single point of ray data can be more important to some finite element basis functions than others.

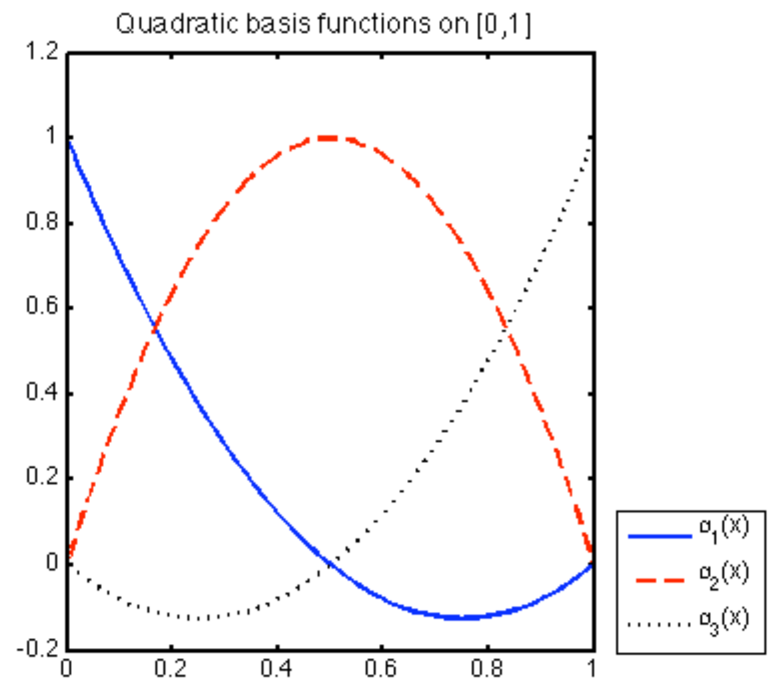
- Ultimately, we'll import the full quasilinear diffusion tensor from GENRAY into NIMROD.

$$Q(f_\alpha) \equiv \frac{\partial}{\partial \mathbf{v}} \cdot \mathcal{D} \cdot \frac{\partial}{\partial \mathbf{v}}$$

Importance sampling – a simple example

•Consider a set of quadratic basis functions on the interval [0,1].

$$\begin{aligned}\alpha_1(x) &= (2x-1)(x-1)\\ \alpha_2(x) &= 4x(1-x)\\ \alpha_3(x) &= x(2x-1)\end{aligned}$$



•An arbitrary (unknown) function F(x) can be approximated as

$$F(x) = \sum_{j=1}^{3} A_j \alpha_j(x)$$

What are these coefficients?

•To find the coefficients A_j, multiply by a trial basis function and integrate over the interval;

$$\int_0^1 F(x)\alpha_k(x)\,dx = \sum_{j=1}^{3} A_j \int_0^1 \alpha_j(x)\alpha_k(x)\,dx$$

Nonorthogonal basis functions yield a matrix equation for the A_i's, and integrals over the unknown F(x)

$$\begin{bmatrix} \int_0^1 F(x)\alpha_1(x)\,dx \\ \int_0^1 F(x)\alpha_2(x)\,dx \\ \int_0^1 F(x)\alpha_3(x)\,dx \end{bmatrix} = \begin{bmatrix} \int_0^1 \alpha_1(x)\alpha_1(x)\,dx & \int_0^1 \alpha_2(x)\alpha_1(x)\,dx & \int_0^1 \alpha_3(x)\alpha_1(x)\,dx \\ \int_0^1 \alpha_1(x)\alpha_2(x)\,dx & \int_0^1 \alpha_2(x)\alpha_2(x)\,dx & \int_0^1 \alpha_3(x)\alpha_2(x)\,dx \\ \int_0^1 \alpha_1(x)\alpha_3(x)\,dx & \int_0^1 \alpha_2(x)\alpha_3(x)\,dx & \int_0^1 \alpha_3(x)\alpha_3(x)\,dx \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

- Basis function integrals can be evaluated (explicitly here, or by Gaussian quadrature in general) to yield

$$\begin{bmatrix} \int_0^1 F(x)\alpha_1(x)\,dx \\ \int_0^1 F(x)\alpha_2(x)\,dx \\ \int_0^1 F(x)\alpha_3(x)\,dx \end{bmatrix} = \frac{1}{30} \begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

- Matrix inversion yields

$$\frac{3}{4} \begin{bmatrix} 12 & -2 & 4 \\ -2 & 3 & -2 \\ 4 & -2 & 12 \end{bmatrix} \begin{bmatrix} \int_0^1 F(x)\alpha_1(x)\,dx \\ \int_0^1 F(x)\alpha_2(x)\,dx \\ \int_0^1 F(x)\alpha_3(x)\,dx \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

How are these integrals done? F(x) is not known, it's what we want to find.

Integrals can be performed by Monte Carlo methods

•Suppose F(x) represents the effect of the RF fields – we don't know F(x), but we know F(x_i) at a set of N discrete ray points x_j. Perform the integrals

$$I_k = \int_0^1 F(x)\alpha_k(x)\,dx$$

by defining a normalized PDF *ψ(x)* on the interval [0,1]:

$$\int_0^1 \psi(x)\,dx = 1 \quad ; \quad I_k = \int_0^1 \psi(x)\left[\frac{F(x)\alpha_k(x)}{\psi(x)}\right]\,dx$$

What is *ψ(x)*? How do we know its relationship to the x_j's?

•Then one can use Monte Carlo integration:

$$I_k = \int_0^1 \psi(x)\left[\frac{F(x)\alpha_k(x)}{\psi(x)}\right]\,dx \approx \frac{1}{N}\sum_{j=1}^{N}\frac{F(x_j)\alpha_k(x_j)}{\psi(x_j)}$$

•Assumes that the individual x_j's are realizations of *ψ(x)*.

What is the optimal form of $\psi(x)$?

- Suppose the discrete data is uniformly distributed on the interval

$$\psi(x) = 1 \Rightarrow x_j\text{'s uniformly distributed; } I_k \approx \frac{1}{N}\sum_{j=1}^{N} F(x_j)\alpha_k(x_j)$$

- Data may vary widely – a single large value of F(x_j) may introduce large errors in the calculation.

- To prevent this, we want the square-bracketed quantity to be of order one in

$$I_k = \int_0^1 \psi(x)\left[\frac{F(x)\alpha_k(x)}{\psi(x)}\right]\,dx \approx \frac{1}{N}\sum_{j=1}^{N}\frac{F(x_j)\alpha_k(x_j)}{\psi(x_j)}$$

- Choose a reasonable guess for the (unknown) F(x): $F(x) \approx \bar{F}(x)$

- Since the basis functions can be negative, renormalize:

$$\psi(x) = \psi_k(x) = \frac{\bar{F}(x)\alpha_k(x) + |\min[\bar{F}(x)\alpha_k(x)]|}{\int_0^1 \left(\bar{F}(x)\alpha_k(x) + |\min[\bar{F}(x)\alpha_k(x)]|\right)\,dx}$$

Possible forms of *ψ(x)*

Add a constant to the numerator so that it is nonnegative

$$\psi(x) = \psi_k(x) = \frac{\bar{F}(x)\alpha_k(x) + |\min[\bar{F}(x)\alpha_k(x)]|}{\int_0^1 \left(\bar{F}(x)\alpha_k(x) + |\min[\bar{F}(x)\alpha_k(x)]|\right)\, dx}$$

Denominator normalizes the numerator on [0,1]

Approximate $\bar{F}(x)$ can incorporate any known information (e.g. spatial localization of deposited RF power) to improve accuracy

- In the absence of useful information, can also set $\bar{F}(x) = 1$; then ψ(x) is largest where basis functions are largest or "most important".

- A single ray point can contribute more to some basis function integrals than others – "importance sampling" concept

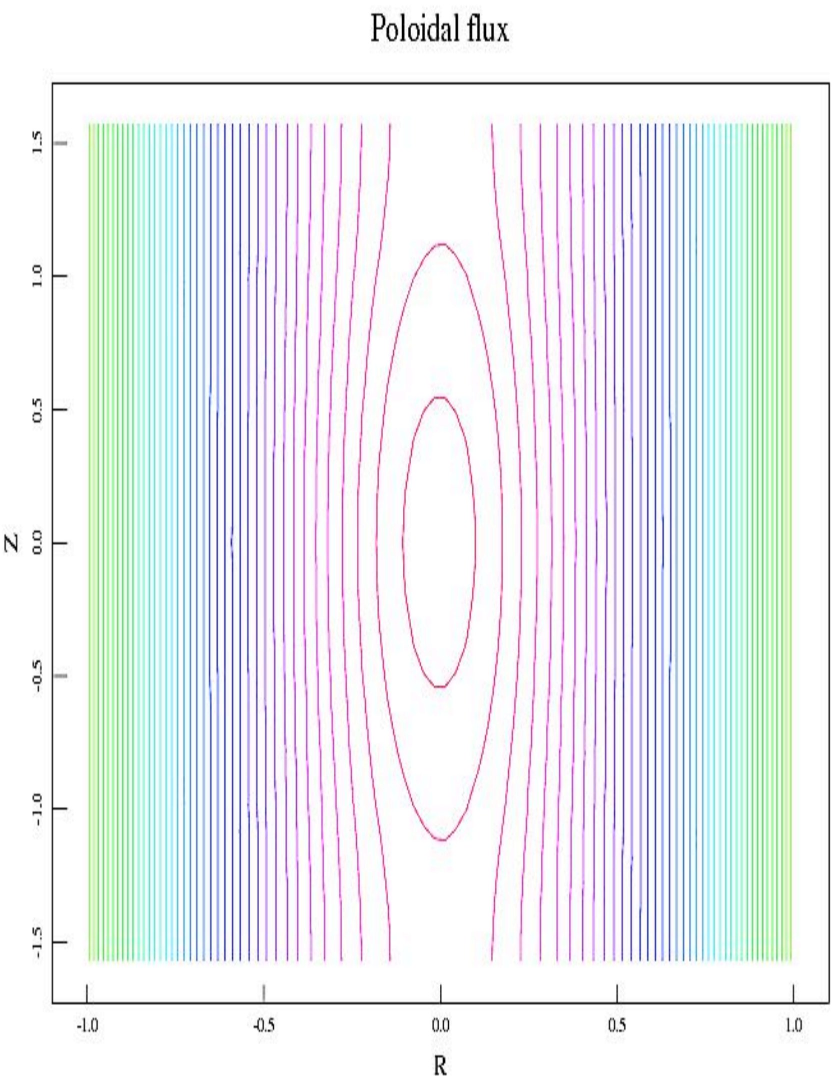
How well does it work?

- More generally, we want to represent the effect of the RF fields by this method – $J_{RF}(R,Z,\varphi)$ in NIMROD

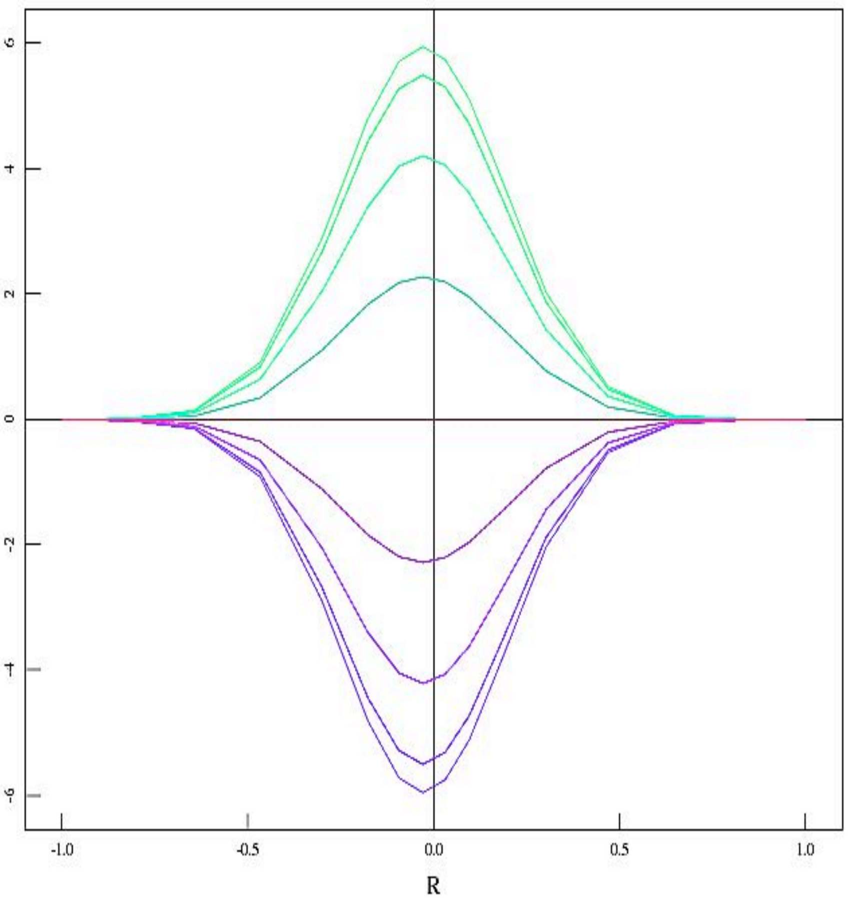
- Works for bilinear basis functions in the poloidal plane – need to generalize to arbitrary-order polynomial basis functions (not too hard) and toroidal asymmetry

- Can also find I_k by bivariate interpolation of ray data onto NIMROD's Gaussian quadrature points… possibly inaccurate.

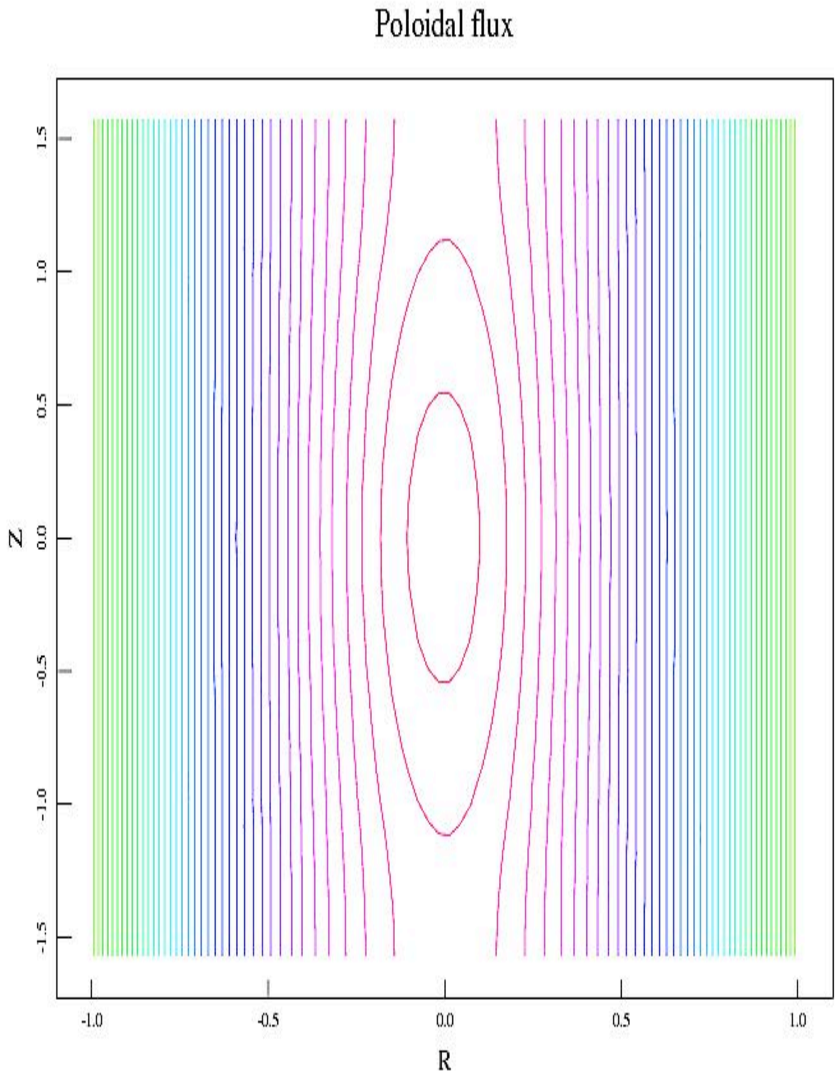
- Work in progress…



flux surfaces for n=0 slab magnetic island, heat flows in at left boundary and out at right



profiles of Braginskii parallel heat flow closure at t=0 for linear T profile.



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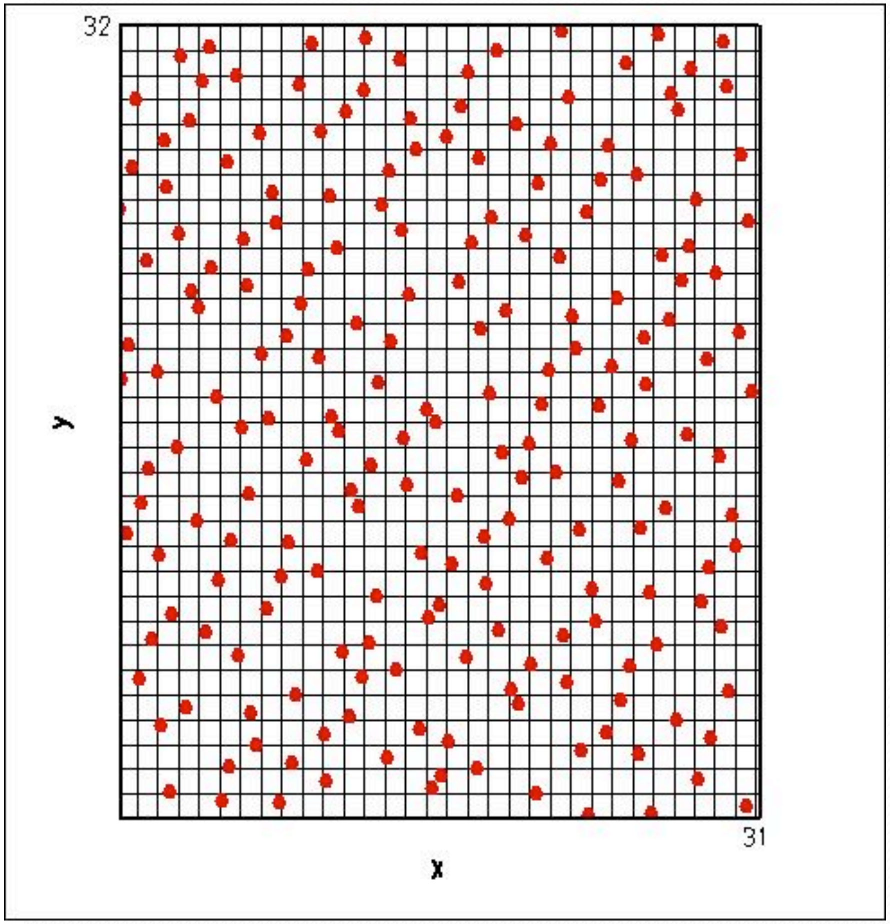
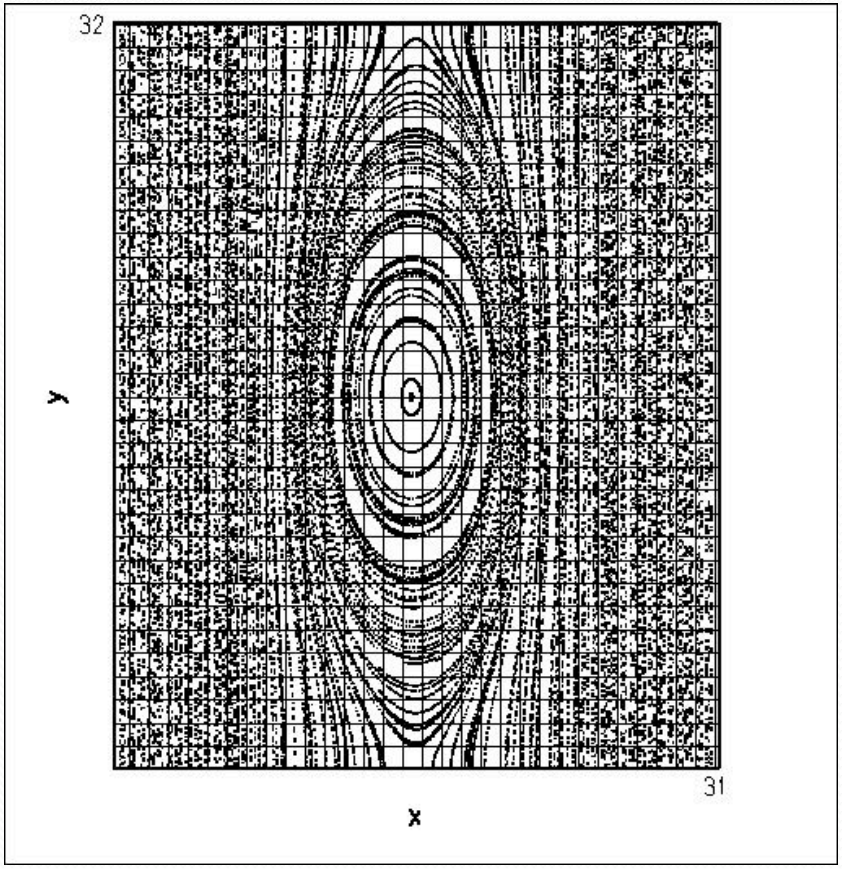
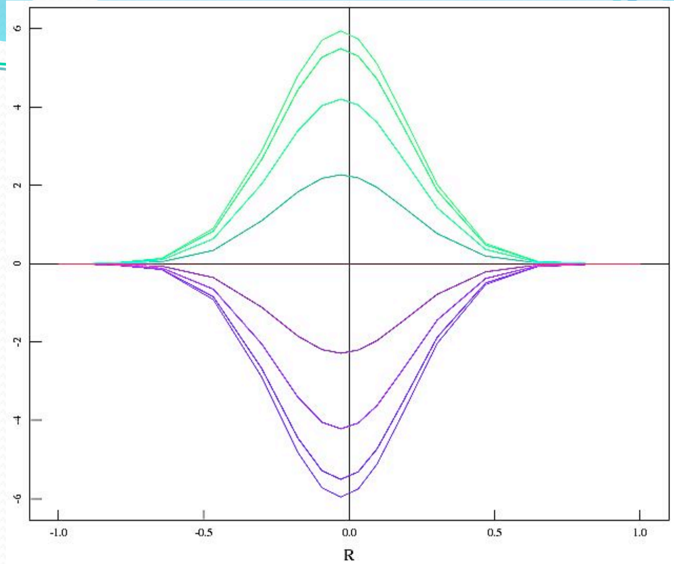


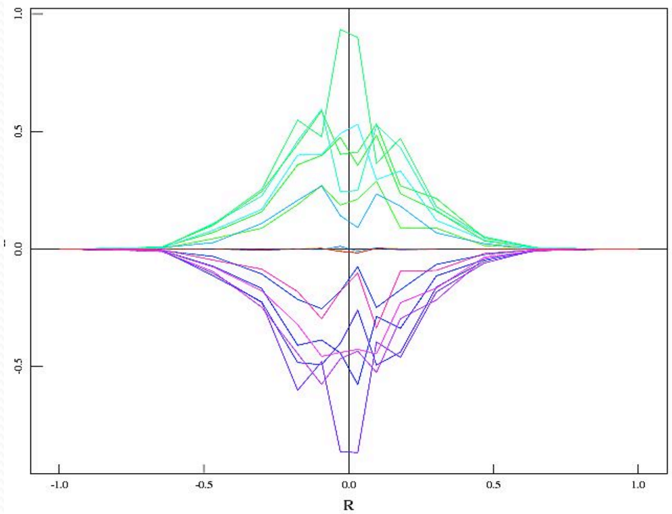
200 sub-randomly distributed starting points for integration along magnetic field, Braginskii heat flow sampled along field lines according to steps of ODE integrator.

~ 30,000 points folded into crude Monte Carlo integration of basis functions times Jacobian times Braginskii heat flow, should approximate real heat flow shown earlier



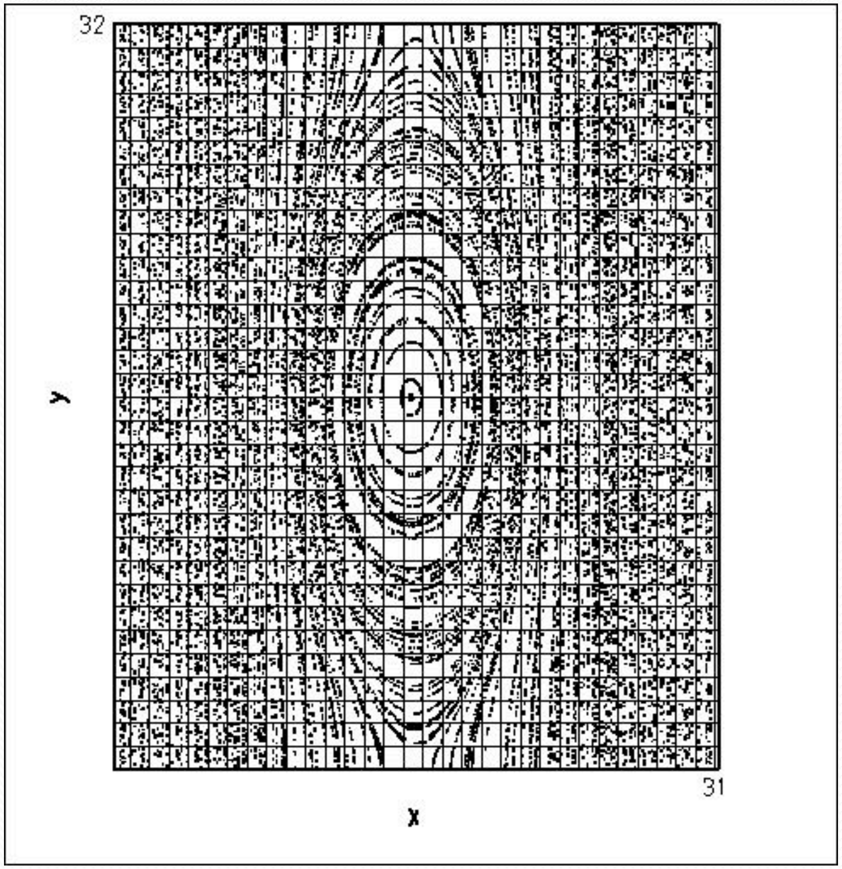


actual heat flow

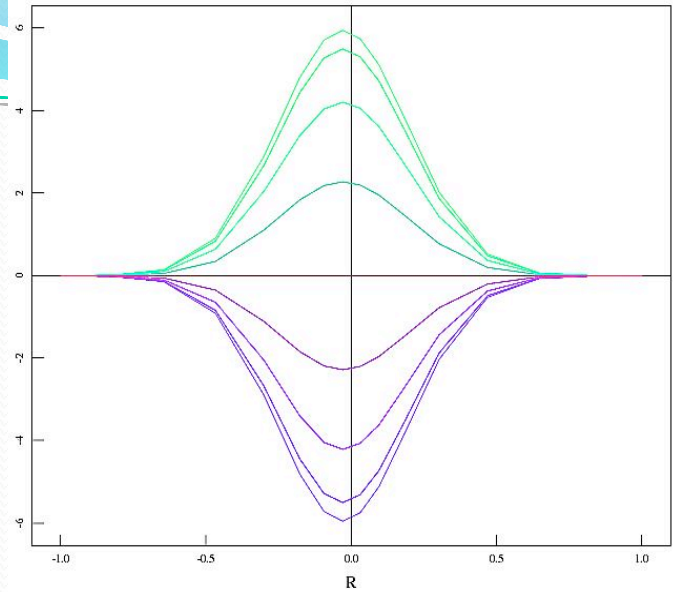


resultant heat flow from crude Monte Carlo integration and projection onto finite-element basis functions

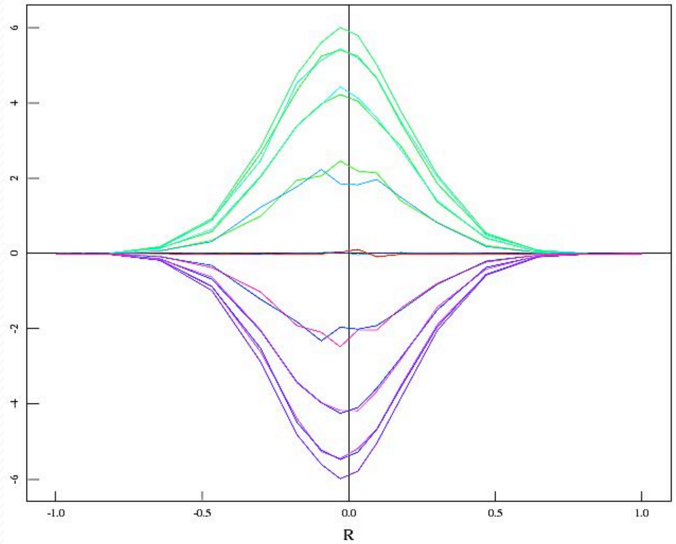
~ 20,000 points folded into importance-sampled Monte Carlo integration of basis functions times Jacobian times Braginskii heat flow, importance sampling based on basis function only; could also fold in Jacobian and analytic estimate of heat flow data



Approximately a third of the points are thrown out for basis function α(x,y) = x y.

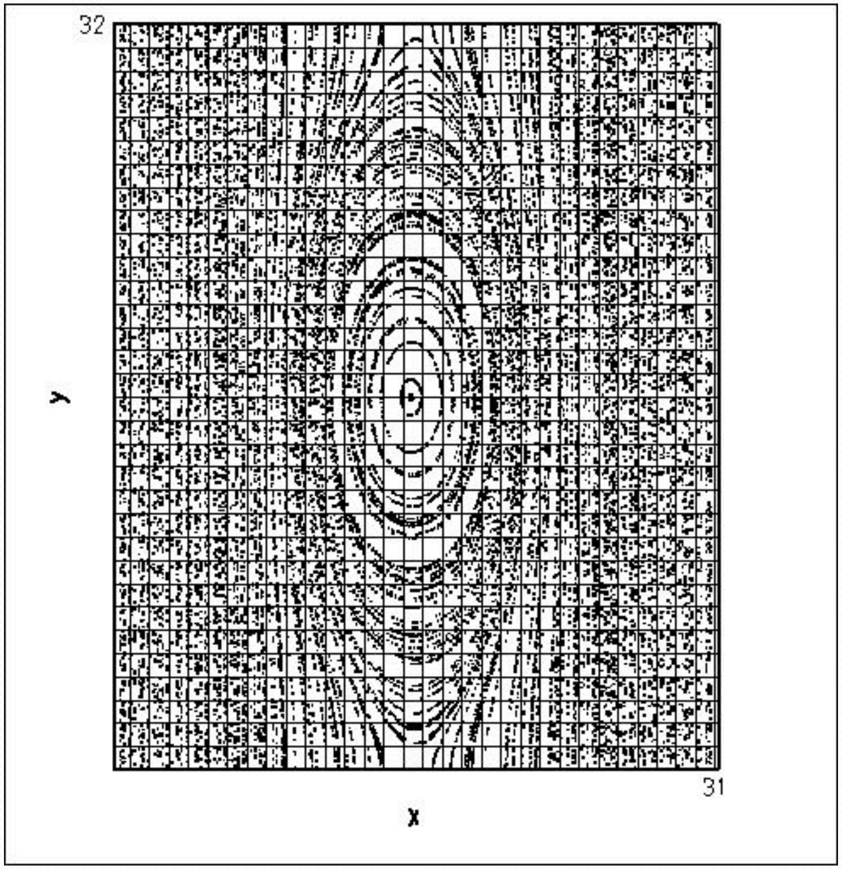


actual heat flow

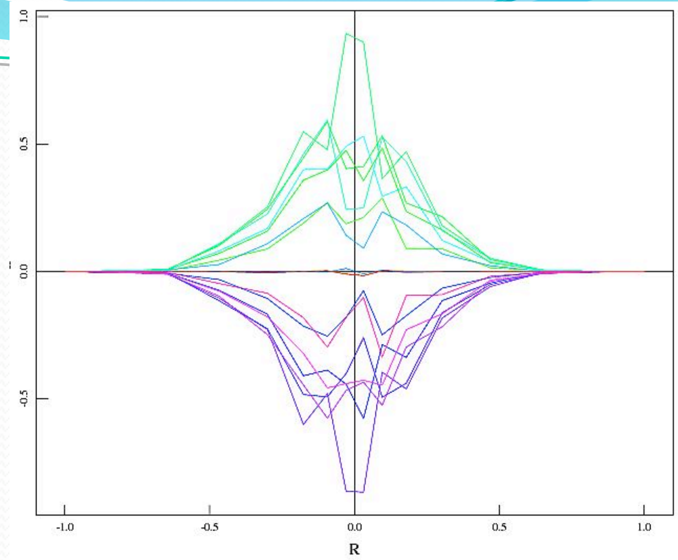


Resultant heat flow from importance-sampled Monte Carlo integration

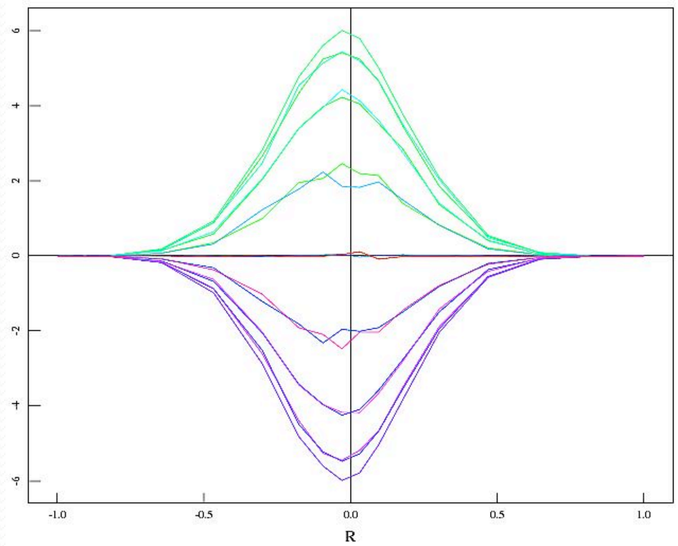
~ 20,000 points folded into importance-sampled Monte Carlo integration of basis functions times Jacobian times Braginskii heat flow, importance sampling based on basis function only; could also fold in Jacobian and analytic estimate of heat flow data



approximately a third of the points are thrown out for basis function, $\alpha(x,y) = xy$.



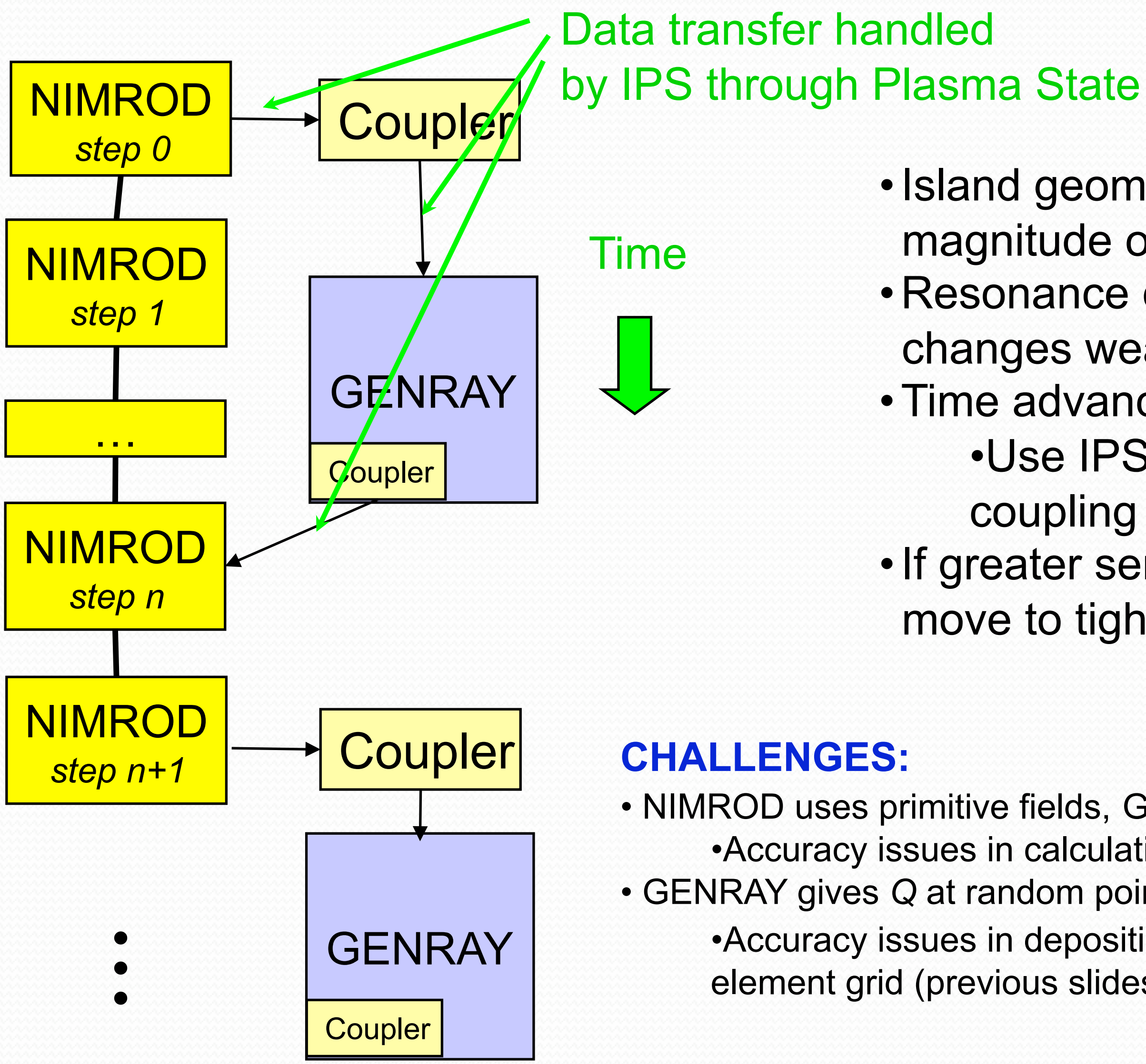
crude Monte Carlo



importance-sampled Monte Carlo

GENRAY + NIMROD coupling – initial approach

Initial Slow MHD Scenario

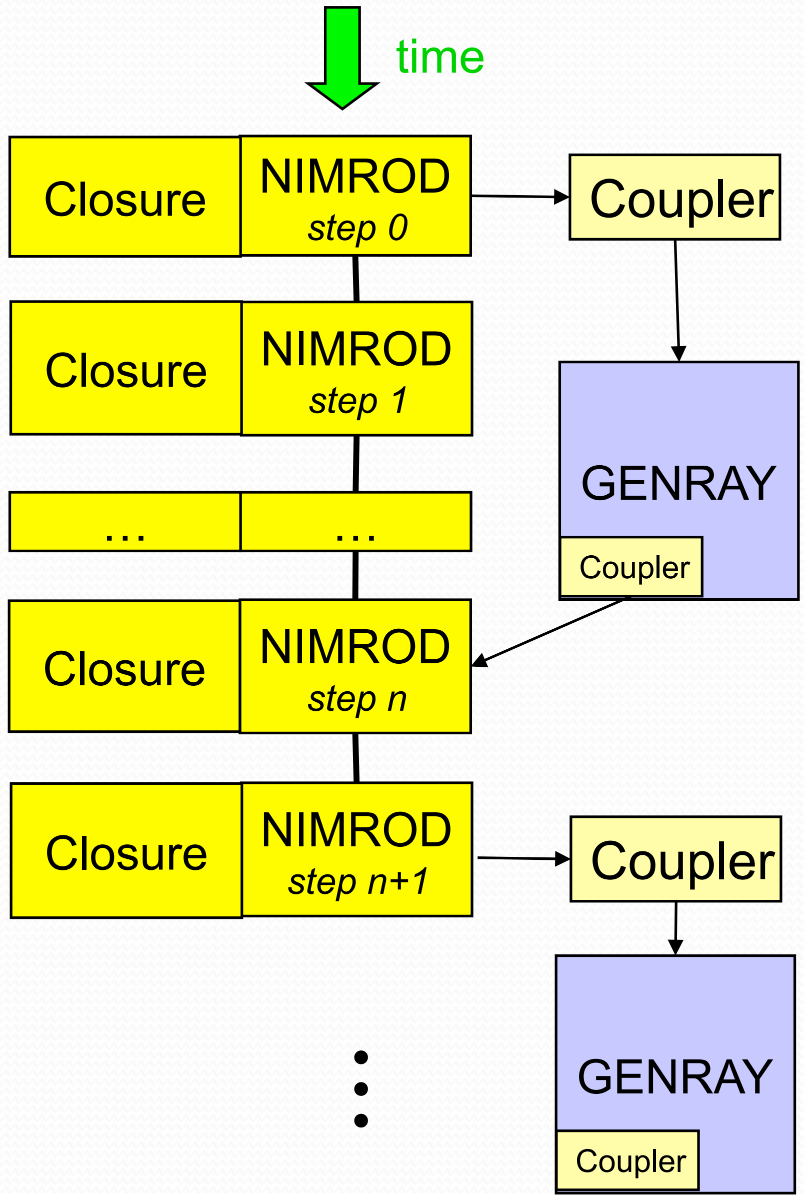


- Island geometry only weakly modifies magnitude of magnetic field
- Resonance condition of GENRAY changes weakly
- Time advance done via files
    - Use IPS framework to manage coupling
- If greater sensitivity discovered, can move to tighter coupling

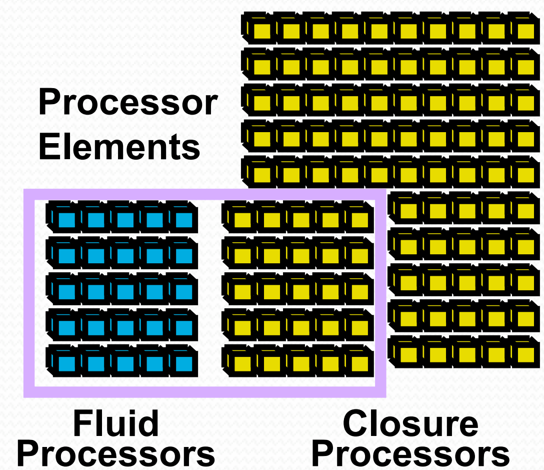
CHALLENGES:

- NIMROD uses primitive fields, GENRAY wants flux function
    - Accuracy issues in calculation and interpolation
- GENRAY gives Q at random points on NIMROD grid
    - Accuracy issues in depositing quantities onto high-order finite element grid (previous slides)

GENRAY + NIMROD coupling through IPS using neoclassical closures



- NIMROD runs on M processors and writes out dump files periodically
- IPS detects dump file, and executes NIMROD-GENRAY coupler
- IPS launches parallel GENRAY job
- IPS detects GENRAY completed, moves files into NIMROD directory
- NIMROD detects files and reads in sources, and writes out dump file.
- Cycle repeats.



Plans for coming year – physics in Slow MHD campaign

✔ Done

- **Phase 0:**
  Use axisymmetric, phenomenological model for the RF interaction; I.e., $F_{RF}= F_{RF}(R,Z)$ is specified as an analytic function in NIMROD.

✔ Published

- **Phase 1:**
  Use non-symmetric phenomenological model for RF interaction ($F_{RF}= F_{RF}(R,Z,\phi)$) and include equilibrium toroidal flow.

✔ Done

- **Phase 2:**
  Pass the NIMROD equilibrium data to RF ray tracing codes, and fit the ray data generated by these codes to the parameters of the phenomenological model (e.g. Gaussian half-width, amplitude, spatial location, etc.)

✔ Done

- **Phase 3:**
  Use quasilinear diffusion tensor from GENRAY and calculate F_{RF}.
  Update GENRAY sources in time while code is running using IPS.

✔ In progress

- **Phase 4:**
  Fully couple the RF and MHD codes such that F_{RF} is calculated at every time step using MPMD approach.

✔ In progress

- **Phase 5:**
  Incorporate more advanced closures and neoclassical effects.

Good progress achieved on project milestones thus far

- First simulation results with ad hoc J_{RF} term being written up for publication, demonstrating proof-of-principle concepts and agreeing well with existing literature

- Full coupling of GENRAY and NIMROD is nearly completed; bugs are rapidly being worked out.

- Studies in progress – effect of misalignment of RF deposition with rational surface, physics of RF equilibration across flux surface, discrete-to-continuum data mapping (with importance sampling).

- Near-term studies will focus on effects of time modulation when the RF is tightly toroidally localized.