CEMM Workshop. Dallas TX, November 2008.

ANALYTIC RESULTS
FOR TWO-FLUID TEARING MODES

E. Ahedo† and J. J. Ramos

M.I.T. Plasma Science and Fusion Center

† Permanent address: Universidad Politécnica de Madrid (Spain)

TWO-FLUID, HALL-MHD MODEL

$$\frac{\partial n}{\partial t} \;+\; \nabla \cdot (n\mathbf{u}) \;=\; 0$$

$$\mathbf{j} \;=\; \nabla \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} \;+\; \nabla \times \mathbf{E} \;=\; 0$$

$$\mathbf{E} \;=\; -\mathbf{u} \times \mathbf{B} \;+\; \frac{1}{en}(\mathbf{j} \times \mathbf{B} - \nabla p_e) \;+\; \eta \mathbf{j}$$

$$m_\iota n[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}] \;+\; \nabla(p_\iota + p_e) \;-\; \mathbf{j} \times \mathbf{B} \;=\; 0$$

$$p_s n^{-\Gamma_s} = \text{constant} \qquad (s = \iota, e)$$

ONE-DIMENSIONAL EQUILIBRIUM WITH CONSTANT DENSITY AND TEMPERATURES

$$n_0, \quad p_{\iota 0}, \quad p_{e0}, \quad B_0 \; = \; \text{constants}$$

$$\mathbf{B}_0(x) \; = \; \epsilon_B B_0 \tanh(x/L) \; \mathbf{e}_y \; + \; [B_0^2 - B_{0y}^2(x)]^{1/2} \; \mathbf{e}_z \quad \text{with} \quad \epsilon_B = O(1)$$

$$\mathbf{j}_0(x) = -B'_{0z}(x)\mathbf{e}_y + B'_{0y}(x)\mathbf{e}_z$$

$$\mathbf{u}_0 = 0$$

LINEAR NORMAL MODES INDEPENDENT OF z AND PERIODIC IN y

$$F(\mathbf{x}, t) = F_0(x) + F_1(x) \exp(iky + \gamma t)$$

DIMENSIONLESS PARAMETERS OF THE PROBLEM AND ADOPTED ORDERINGS

$$\epsilon_\eta = S^{-1} = \frac{\eta k}{c_A} = \frac{\eta k (m_\iota n_0)^{1/2}}{B_0} \qquad \epsilon_\eta \ll 1$$

$$\epsilon_\gamma = \frac{\gamma}{k c_A} \qquad \epsilon_\gamma \propto \epsilon_\eta^\nu \ll 1 \quad (0 < \nu < 1) \quad \textbf{as an output}$$

$$\alpha = k d_\iota = \frac{k m_\iota^{1/2}}{e n_0^{1/2}} \qquad \alpha \le O(\epsilon_\eta^{-1/5})$$

$$\beta = \frac{c_s^2}{c_A^2} = \frac{\Gamma_\iota p_{\iota 0} + \Gamma_e p_{e0}}{B_0^2} \qquad \beta \gg O(\epsilon_\gamma^2) \quad \textbf{i.e. } \gamma \ll k c_s$$

$$kL = O(1) \qquad \textbf{such that} \quad k^{-1}\Delta' = \frac{B'_{1x}(0+) - B'_{1x}(0-)}{k B_{1x}(0)} = 2[(kL)^{-2} - 1] \le O(1)$$

$$\epsilon_B = \frac{B_{0y}(\infty)}{B_0} \qquad \epsilon_B = O(1) \qquad \textbf{define} \quad kL_B = kL/\epsilon_B = O(1)$$

GENERAL SUBSONIC DISPERSION RELATION

In the considered $\gamma \ll kc_s$ and $\alpha = kd_\iota \lesssim S^{1/5}$ regime,

$$\frac{\epsilon_\gamma^{5/4}(kL_B)^{1/2}}{\epsilon_\eta^{3/4}k^{-1}\Delta'}\ D\left(\frac{\epsilon_\gamma^{1/2}\alpha}{\epsilon_\eta^{1/2}}, \frac{\epsilon_\gamma^{3/2}kL_B(1+\beta)}{\epsilon_\eta^{1/2}\beta}\right) = 1$$

where

$$D(\sigma,\tau) = \int_{-\infty}^{+\infty}[1\ -\ \bar{x}\ \bar{\xi}(\bar{x};\sigma,\tau)\ -\ \bar{x}\ \bar{Q}(\bar{x};\sigma,\tau)]\ d\bar{x}$$

and $\bar{\xi}(\bar{x};\sigma,\tau)$ and $\bar{Q}(\bar{x};\sigma,\tau)$ are the scaled x-component of the perturbed velocity and odd part of the z-component of the perturbed magnetic field, satisfying the non-ideal layer system:

$$\frac{d^2\bar{\xi}}{d\bar{x}^2} = \bar{x}^2(\bar{\xi}+\bar{Q}) - \bar{x}$$

$$\frac{d^2\bar{Q}}{d\bar{x}^2} = (\bar{x}^2+\tau)\bar{Q} + \sigma^2\frac{d^2\bar{\xi}}{d\bar{x}^2}$$

ASYMPTOTIC DOMAINS IN THE INPUT PARAMETER SPACE

Defining the two scaled input parameters,

$$\hat{\alpha} = \frac{\alpha}{\epsilon\eta^{1/5}}\left(\frac{\Delta'^2}{C^2k^3L_B}\right)^{1/5} \quad \text{and} \quad \hat{\beta} = \frac{\beta}{\epsilon\eta^{2/5}(1+\beta)}\left(\frac{C^8k^2}{\pi^5 L_B \Delta'^3}\right)^{2/5} \quad \text{with} \quad C = \frac{2\pi\Gamma(3/4)}{\Gamma(1/4)},$$

the following asymptotic regions in the $(\hat{\alpha}, \hat{\beta})$ plane are identified:

PR0 $\hat{\alpha} \sim \hat{\beta} \sim 1$

PR1 $\hat{\alpha} \ll 1$ or $\hat{\beta} \ll \hat{\alpha}^{-2}$

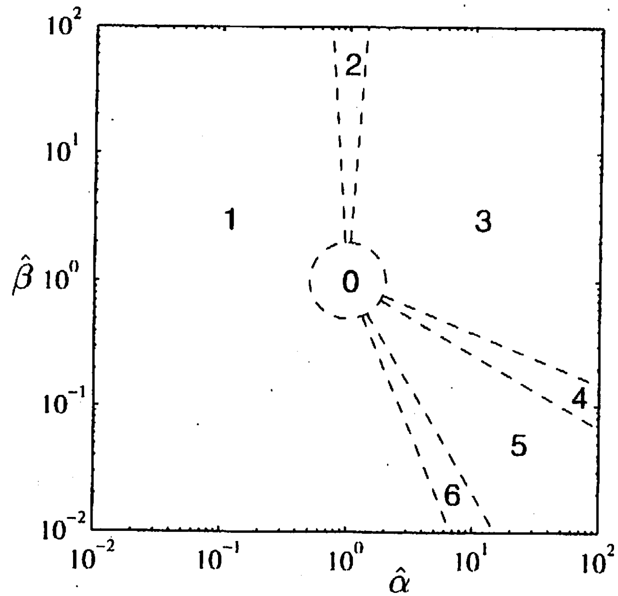
PR2 $\hat{\alpha} \sim 1$ and $\hat{\beta} \gg 1$

PR3 $\hat{\alpha} \gg 1$ and $\hat{\beta} \gg \hat{\alpha}^{-1/2}$

PR4 $\hat{\alpha} \gg 1$ and $\hat{\beta} \sim \hat{\alpha}^{-1/2}$

PR5 $\hat{\alpha}^{-2} \ll \hat{\beta} \ll \hat{\alpha}^{-1/2}$

PR6 $\hat{\alpha} \gg 1$ and $\hat{\beta} \sim \hat{\alpha}^{-2}$



CLASSIC TEARING MODE DISPERSION RELATIONS

In PR1:

$$D(\sigma,\tau) = C \qquad \Rightarrow \qquad \epsilon_\gamma = \epsilon_\eta^{3/5}\left(\frac{\Delta'^2}{C^2 k^3 L_B}\right)^{2/5} \quad (\textbf{single}-\textbf{fluid resistive})$$

In PR3:

$$D(\sigma,\tau) = C\sigma^{-1/2} \qquad \Rightarrow \qquad \epsilon_\gamma = \epsilon_\eta^{1/2}\left(\alpha\ \frac{\Delta'^2}{C^2 k^3 L_B}\right)^{1/2} \quad (\textbf{electron}-\textbf{MHD})$$

In PR5:

$$D(\sigma,\tau) = \pi\sigma^{-1}\tau^{1/2} \qquad \Rightarrow \qquad \epsilon_\gamma = \epsilon_\eta^{1/3}\left(\alpha\beta^{1/2}\ \frac{\Delta'}{\pi k^2 L_B}\right)^{2/3} (\textbf{semicollisional})$$

SOLUTIONS IN THE TRANSITION REGIONS OVERLAPPING THE CLASSIC DOMAINS

In PR1, PR2 and PR3:

$$D(\sigma,\tau) = C f_2(\sigma) \quad \textbf{with} \quad f_2(\sigma) = \frac{1}{2}\sum_{+,-}\left[1 \pm \left(1+4/\sigma^2\right)^{-1/2}\right]\left[1+\sigma^2/2 \pm \sigma\left(1+\sigma^2/4\right)^{1/2}\right]^{-1/4}$$

[new result]

In PR3, PR4 and PR5:

$$D(\sigma,\tau) = C\sigma^{-1/2} f_4(\tau/\sigma) \quad \textbf{with} \quad f_4(u) = \frac{2\pi\ \Gamma[(3+u)/4]}{C\ \Gamma[(1+u)/4]}$$

[derived by Mirnov et al.(2004) for $\epsilon_B \ll 1$; valid for $\epsilon_B \sim 1$]

In PR5, PR6 and PR1:

$$D(\sigma,\tau) = C f_6(\sigma^2/\tau) \quad \textbf{with a numerical } f_6(u) \textbf{ fitted by } f_6(u) \simeq \frac{1-u/4+\pi u^2/20}{1+Cu^{5/2}/20}$$

[new result]

NIMROD CODE BENCHMARK

Carried out by C. Sovinec and J. King [S. Jardin et al., 2008 IAEA Conference, TH/P9-29]

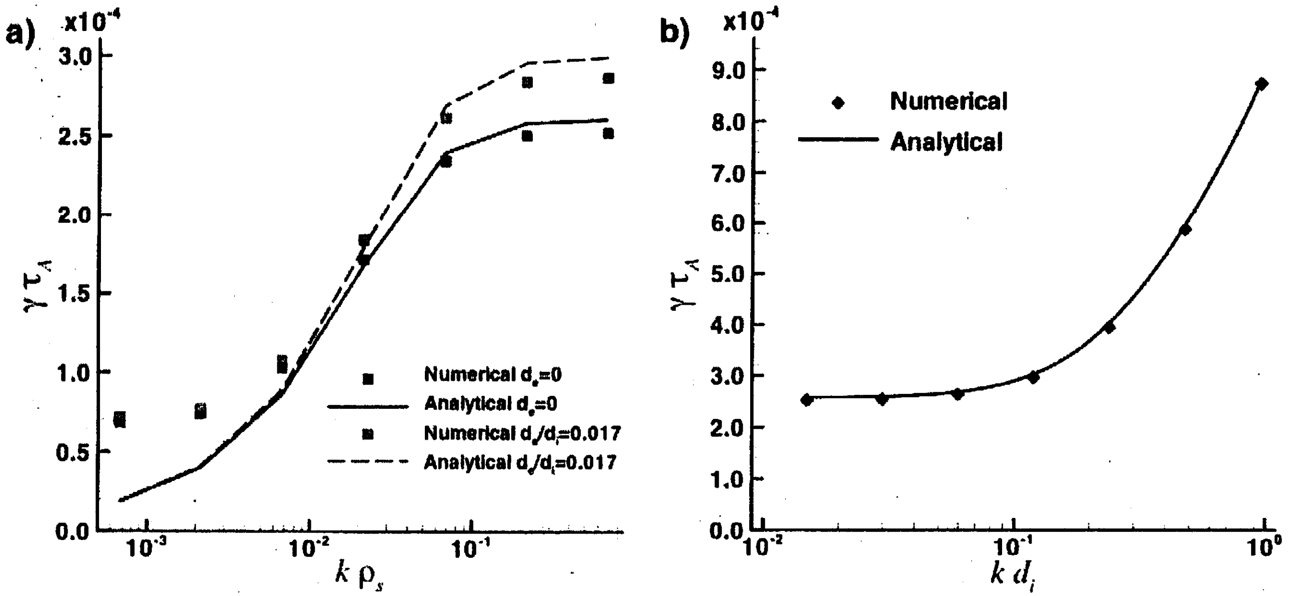


FIG. 7. Comparison of numerical tearing-mode growth rates at $S = 7.0\times10^5$ from the NIMROD code with a) asymptotic analysis in Ref. [18] for conditions with $\Delta'/k = 0.30$, $kL = 0.93$, and $kd_i = 2.3$ and b) the analysis in Ref. [20] for $\beta >> S^{-2/5}$ and $d_e = 0$ with $\Delta'/k = 1.45$, $kL = 0.76$, and $\beta = 0.083$. Here, $\tau_A \equiv \sqrt{\mu_0\rho}/kB_{y_\infty}$, where k is the wavenumber in the periodic direction, and B_{y_∞} is the magnitude of the reconnecting field; $S \equiv \mu_0/\eta k^2\tau_A$.