

Dynamic Behavior of Peeling-Ballooning Modes in a Shifted-Circle Tokamak Equilibrium

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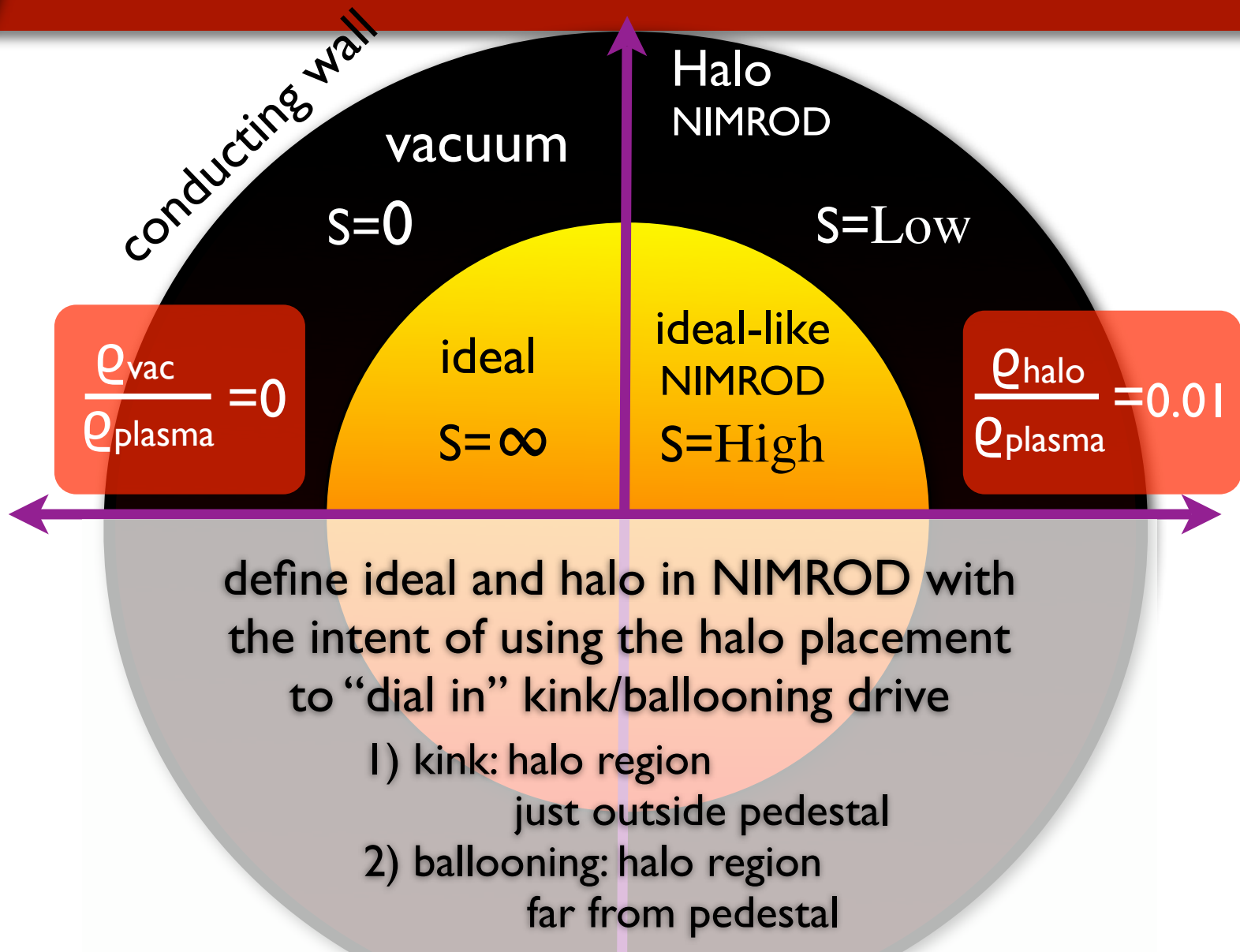


ELM onset and growth linked to the coupling between ballooning and kink modes

- “Ideal-like”/ “Halo” defined in NIMROD
- ELITE benchmarks with NIMROD
- Single Linear case examined as precursor to NL studies
- Technique developed to isolate ballooning and kink drives
- Preliminary nonlinear results guide future analysis
- Summary



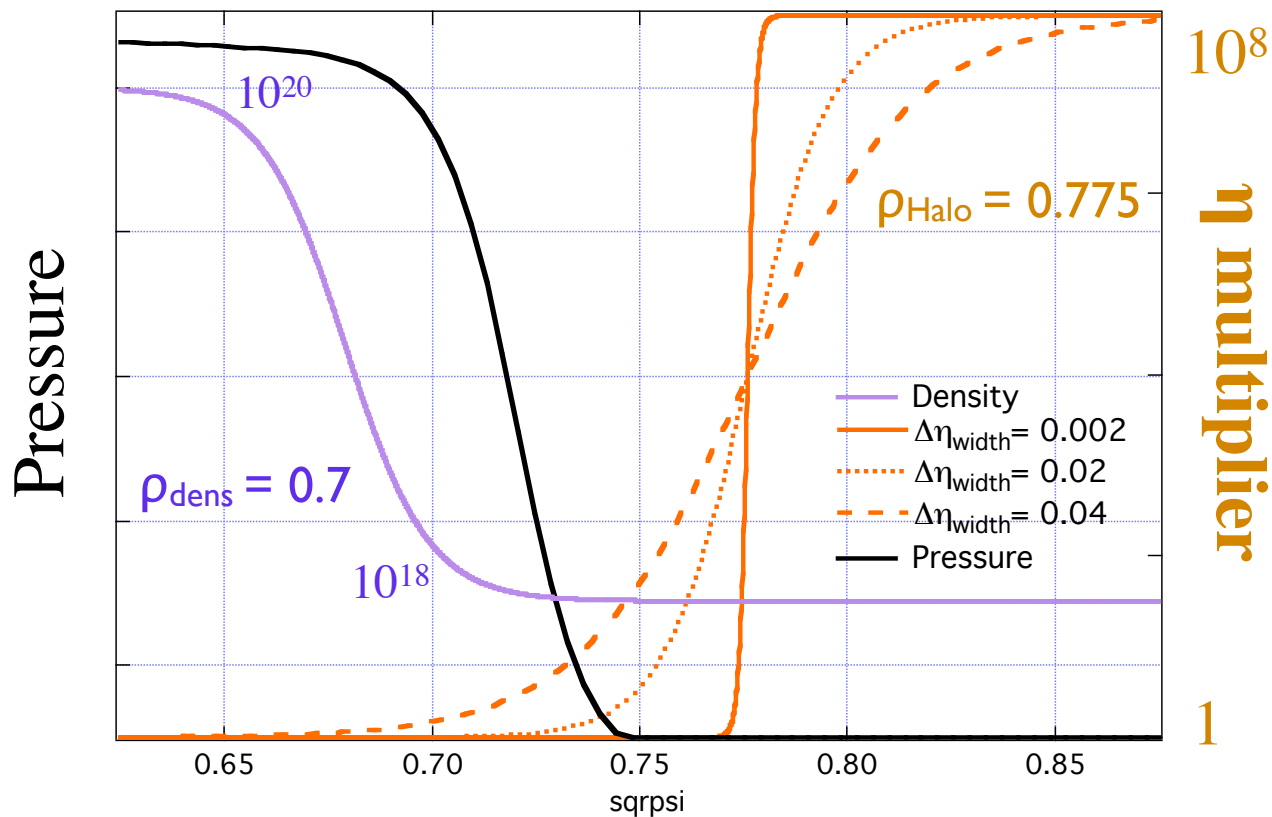
Detailed “ideal” study in NIMROD





Halo region defined with an imposed resistivity and density transition

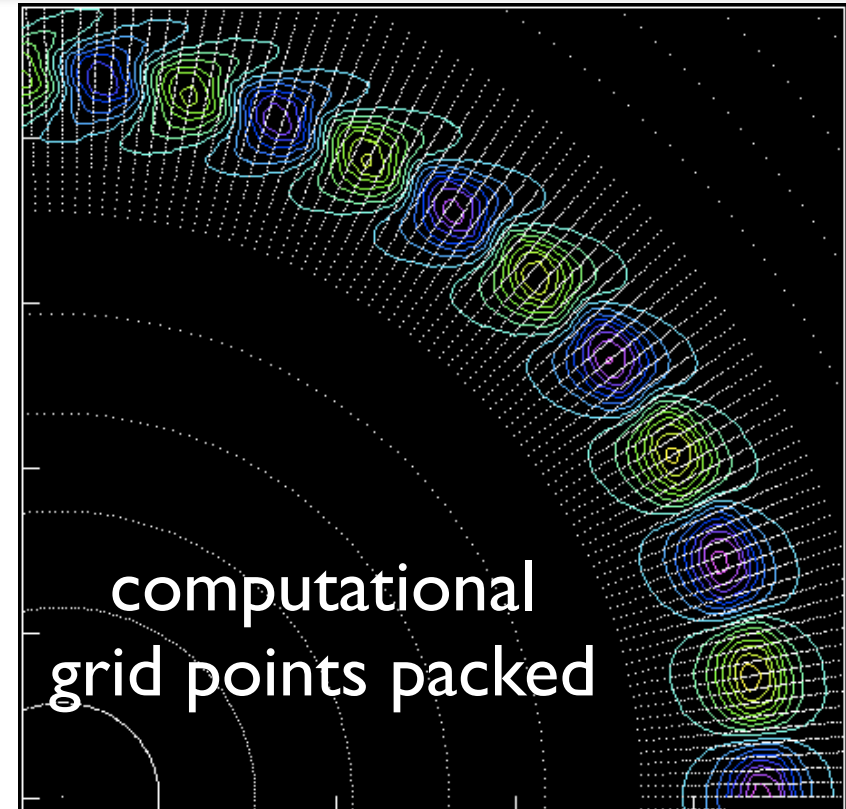
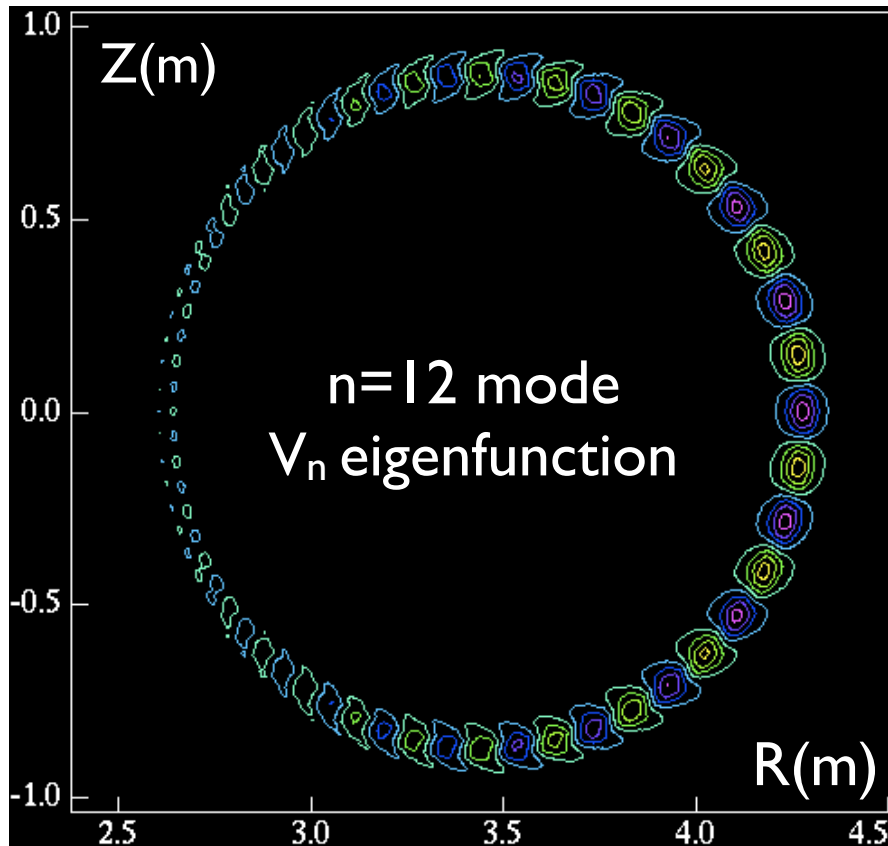
- η transitions from low, “ideal” to a large value at a specified ρ_{Halo}
 - ✦ Tanh function used as an η multiplier
- Density decreased by a factor of 100, transitions at a specified ρ_{dens}
 - ✦ Tanh function also used, sharp transition not possible





Quantifying “Ideal” in NIMROD requires high spatial resolution

- Start with purely ideal case
 - ✦ $S = \infty$ everywhere
 - *no halo region*

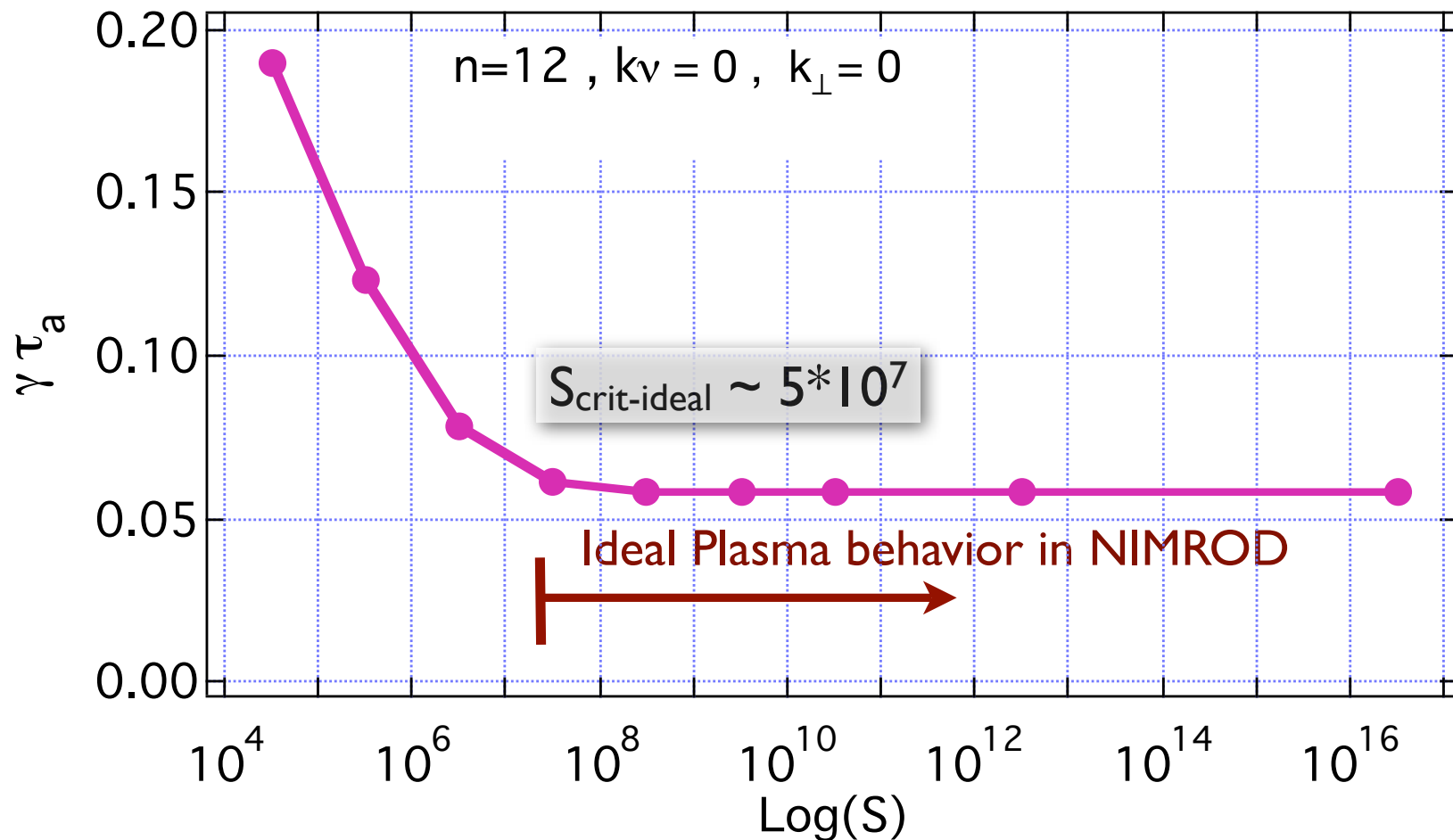


- ✦ linear ideal MHD, $n = 12$
- ✦ no dissipation in system
- ✦ $k_{\text{visc}}, k_{\text{perp}} = 0$



Lundquist scans define critical, “ideal-like” value in NIMROD

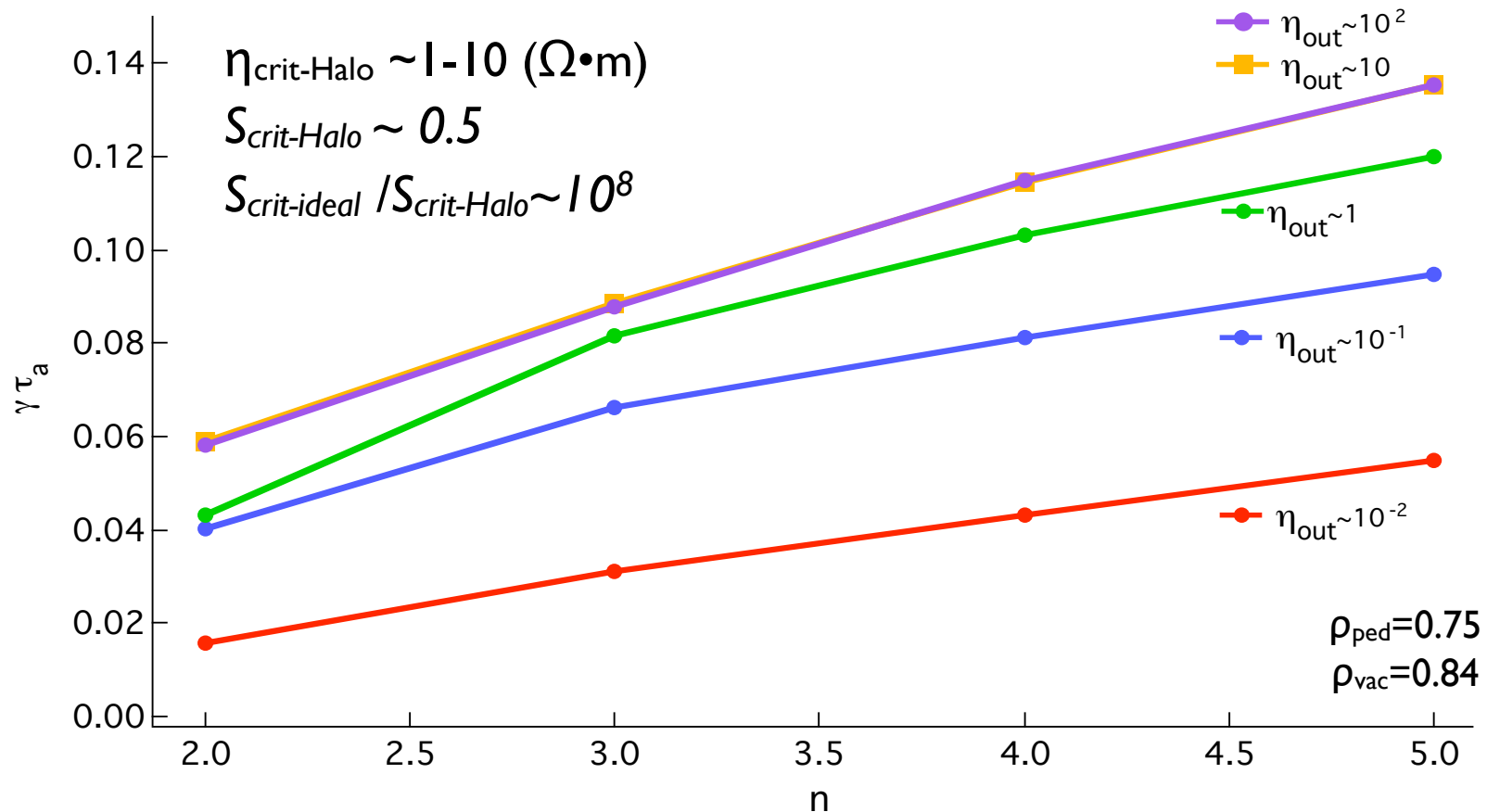
- systematically decreased S from ∞
- critical value defined, below which plasma behaves ideally





Beyond a critical halo-resistivity the modes are not affected

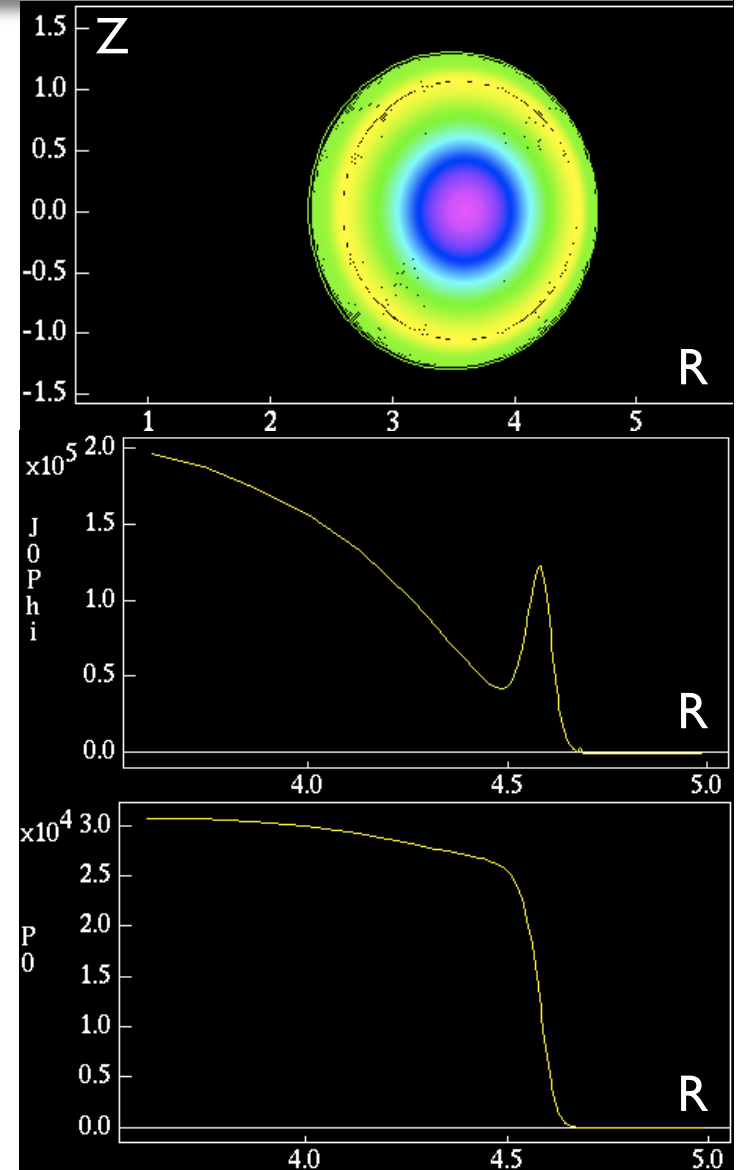
- Define $S_{\text{crit-Halo}}$: increase vacuum resistivity until no effect is produced
 - ✦ Lundquist ratio not a good characterization parameter
 - ✦ Introduction of halo region doesn't affect $S_{\text{crit-ideal}}$





Ballooning unstable equilibrium generated for ELITE benchmarking

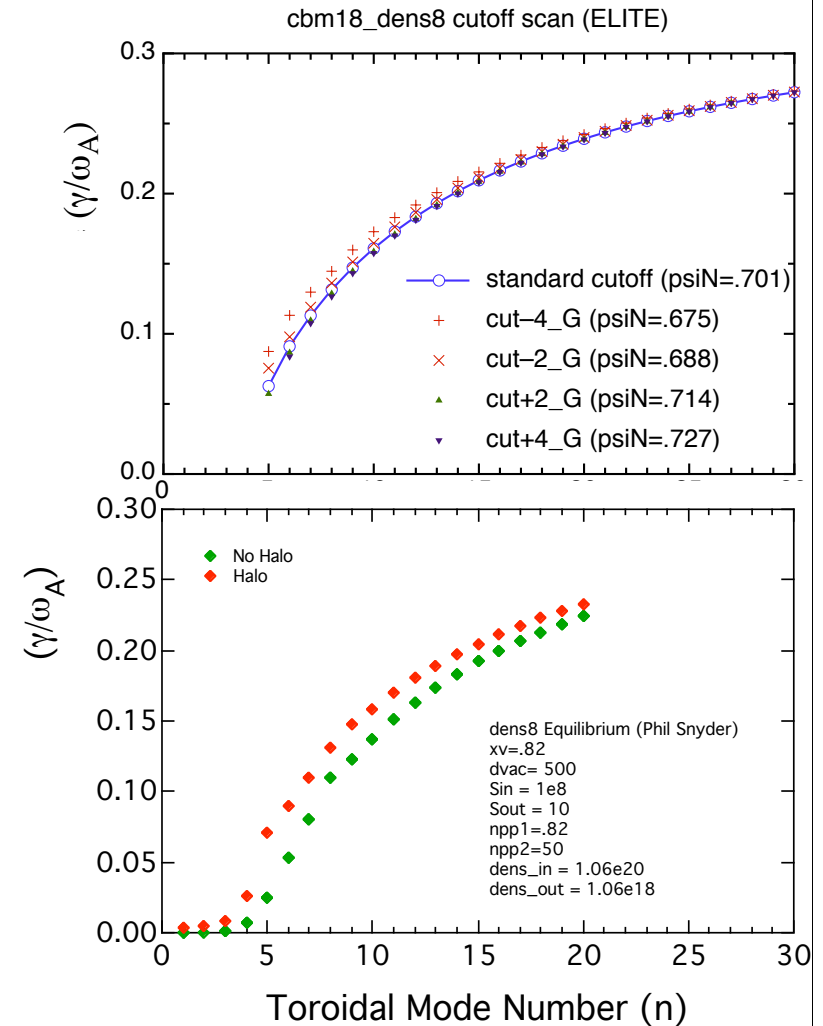
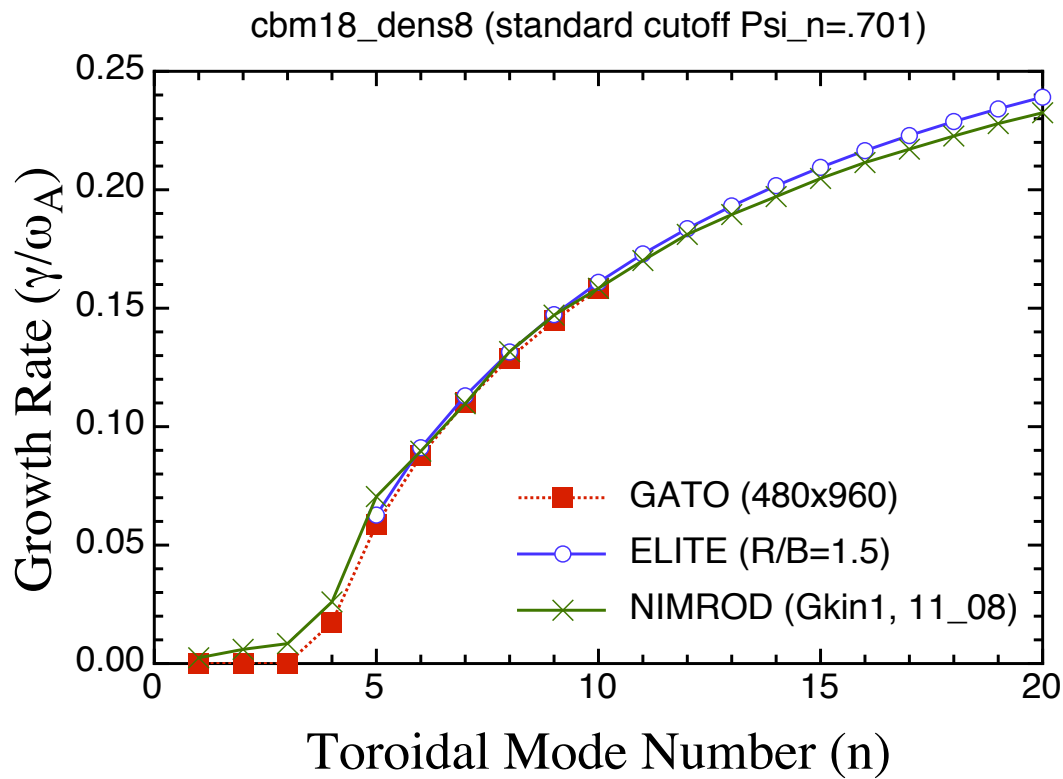
- TOQ-generated series of equilibria scanning across stability boundary
- shape = simple circle
- pedestal is wide
- interface at $\Psi_{iN} \sim 0.7$ (P' and $J_{||} = 0.0$)
- plasma vacuum interface has 0 pressure and current
- ELITE growth rates weakly sensitive to vacuum location





Results show excellent spectral agreement with ELITE

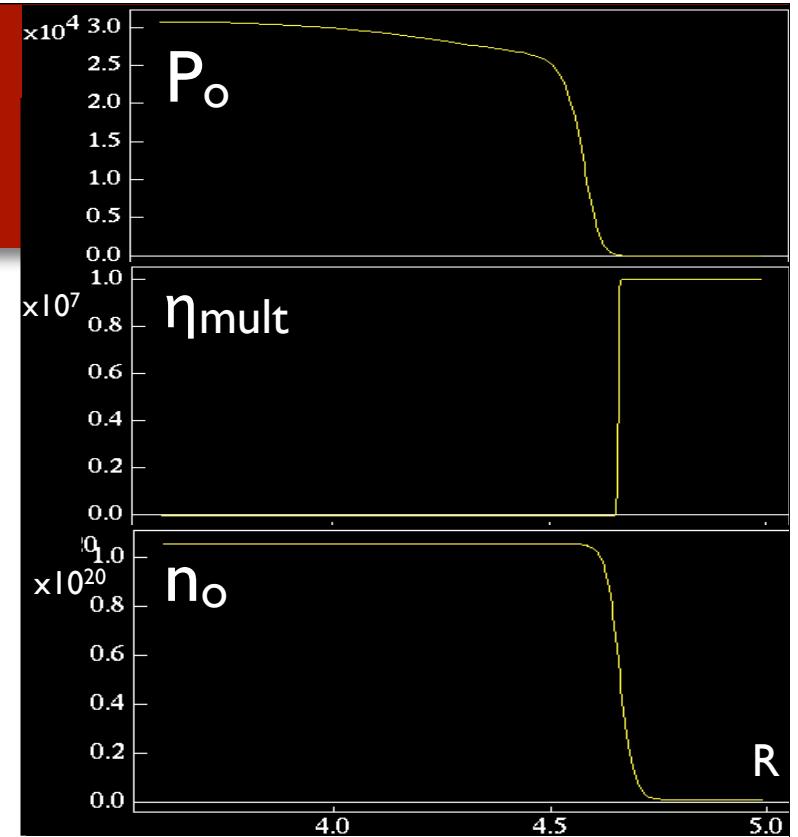
- Equilibrium generated to have little variation with vacuum placement
- results without halo region show little variation at high- n



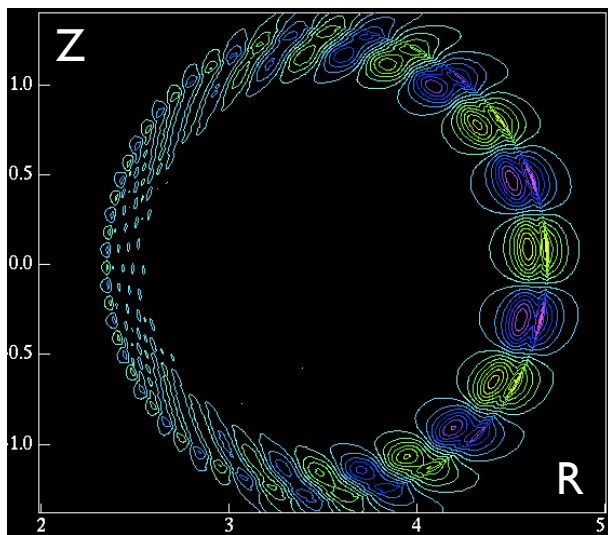


$n = 8$ mode structure in NIMROD and ELITE

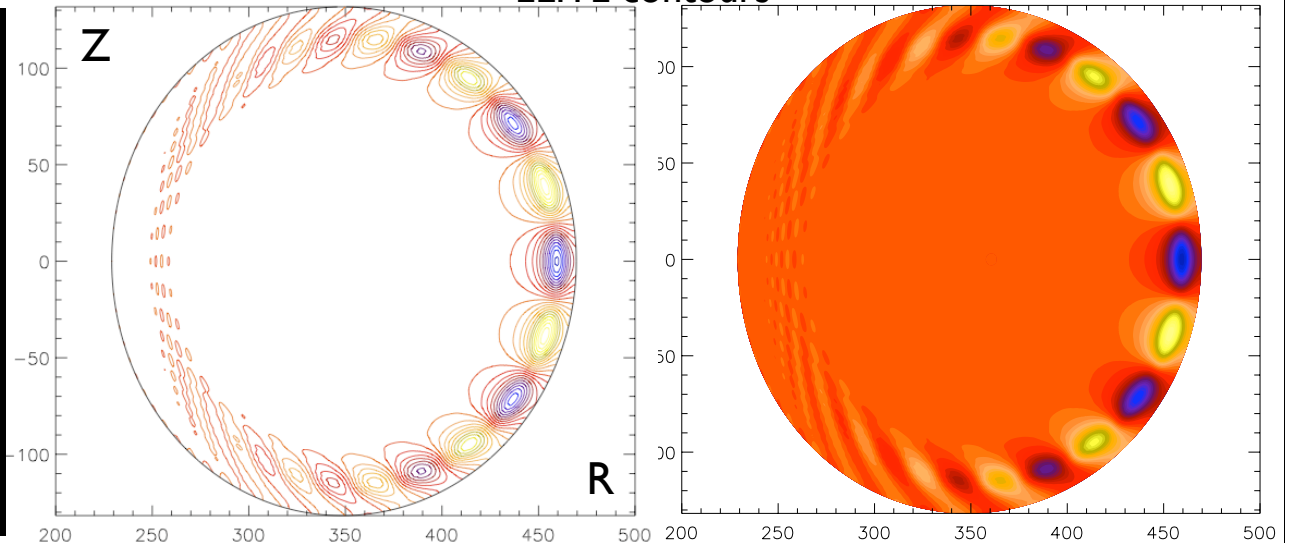
- $\Upsilon T_{aELITE} \sim 0.132$
- $\Upsilon T_{aNIMROD} = 0.132$
- $S_{in} = 1e8, S_{out} = 10$
- $\rho_{halo} = 0.82, dvac = 500$
- $\rho_{dens} = 0.82, npp(1) = 50$
- $k_{perp} = k_{visc} = 0$



Vn contours NIMROD



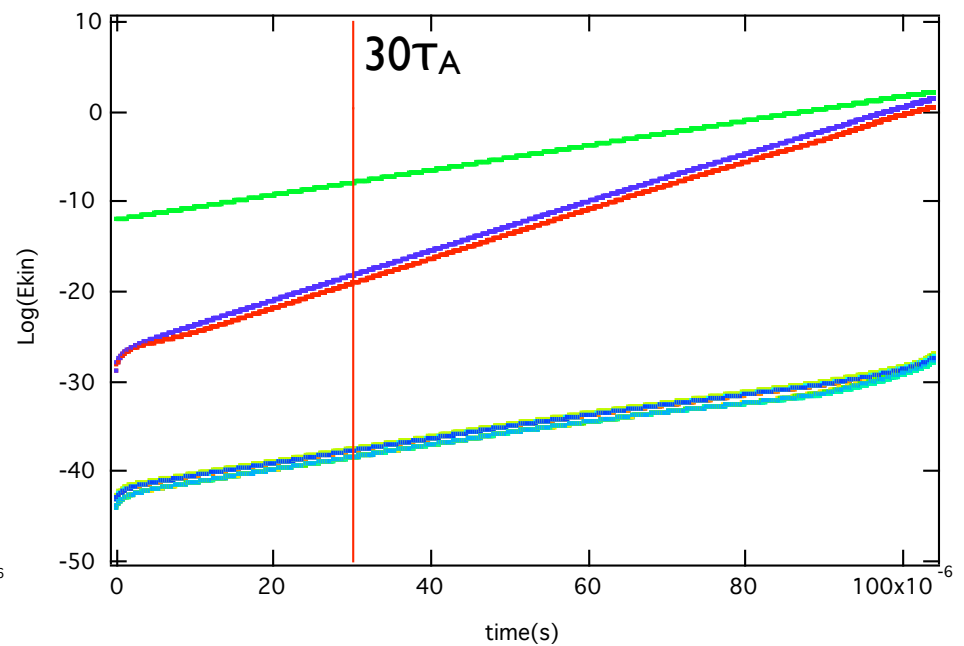
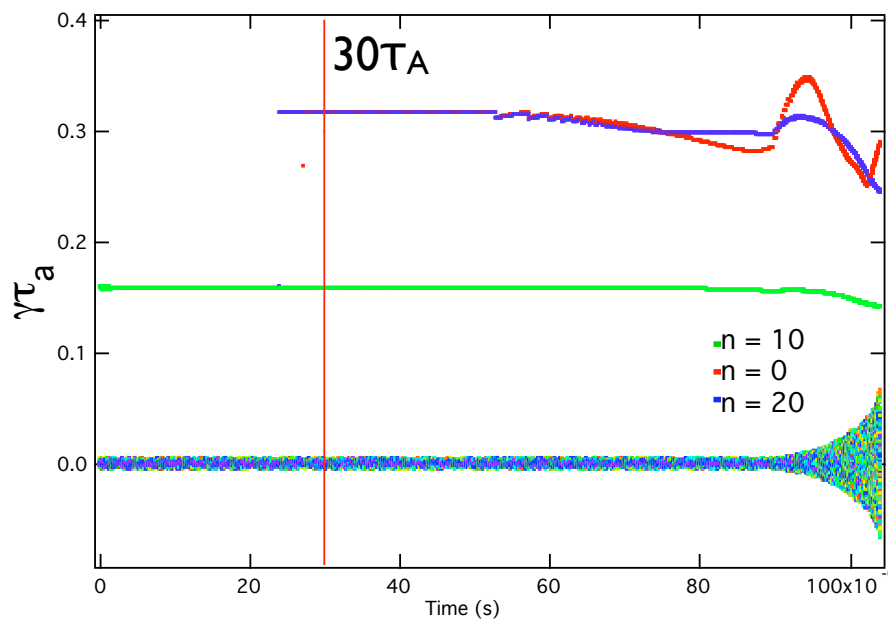
ELITE contours





Nonlinear calculations of $n = 10$ duration = $100\tau_A$

- 22 modes included: $n=0-21$, initialized with linear $n=10$ mode
- nonlinear $n=0$ & $n=20$ mode growth at twice linear $n=10$ rate expected
- The transition to nonlinear dynamics is expected when $\frac{\xi}{\Delta x} \sim O(1)$
 - ✦ For an initial velocity perturbation $V_0 \sim 1 \times 10^{-4}$ this occurs after $\sim 30\tau_A$

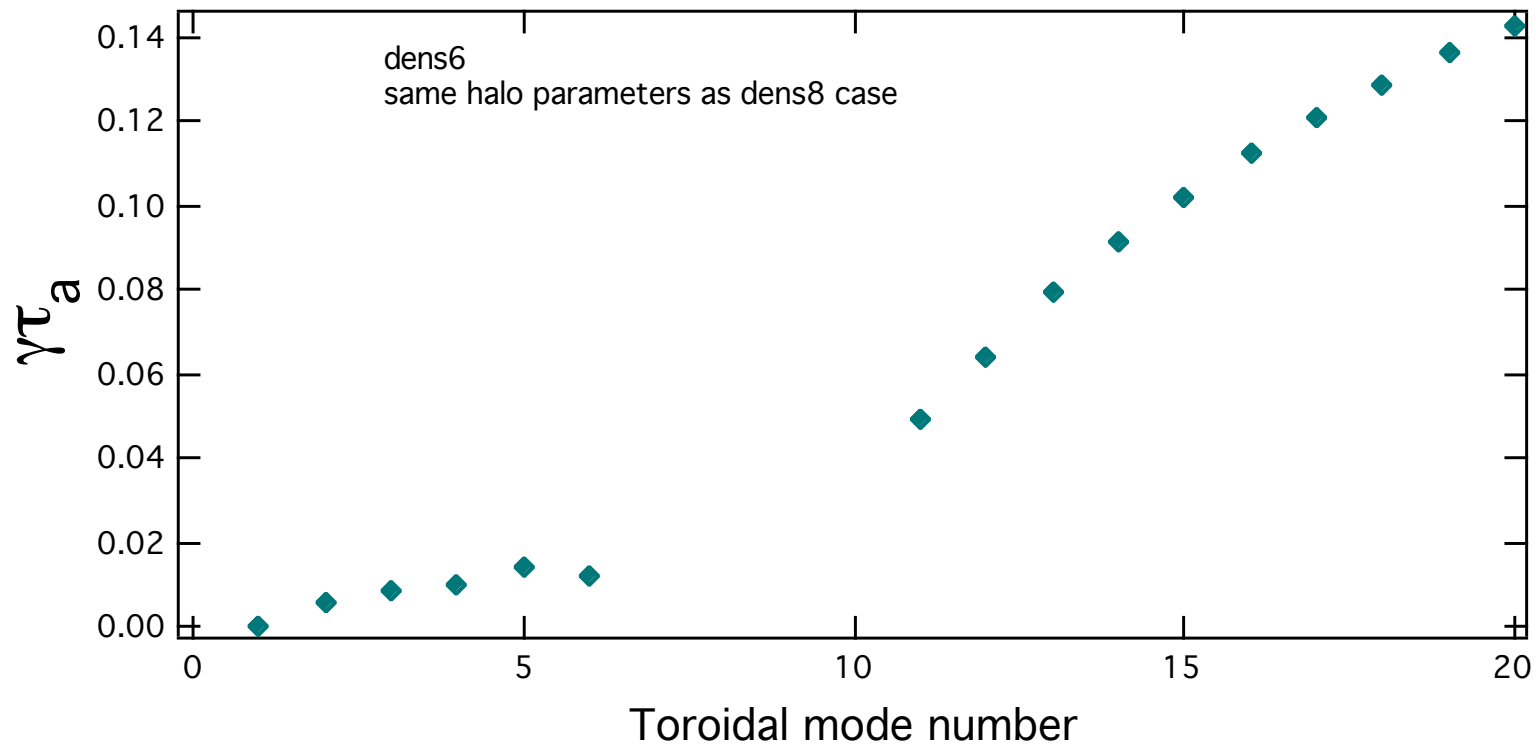


- Results show linear growth rates well into NL regime,
 - ✦ (as expected, Ping Zhu -- see APS poster)



Currently working on benchmarking the stability threshold with ELITE

- Similar equilibrium
- Lower pedestal pressure and edge current
- $n > 10$ converged
- n (1-10) appears to be slowly growing oscillating modes... in progress





New equilibrium allows the study of peeling & ballooning mode drives

TOQ-generated shifted-circle tokamak equilibrium

~S. Kruger & P. Snyder

$$R_o = 3\text{m}, a = 1\text{m}$$

$$\mathbf{B}_o = 2\text{T}$$

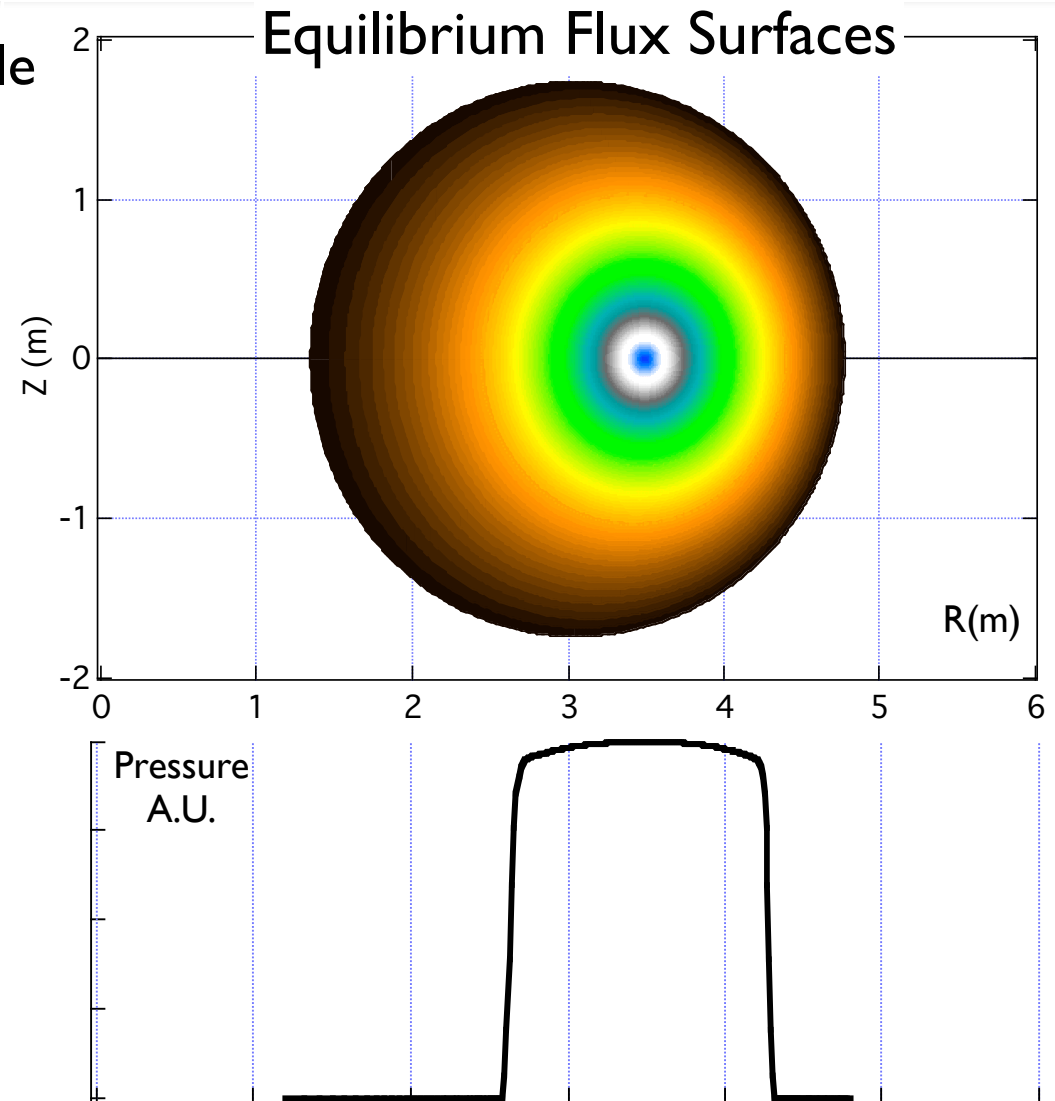
$$\beta_{to} = .005$$

$$n = 1.06 \times 10^{20} (\text{m}^{-3})$$

- no density transition

- Modified TOQ

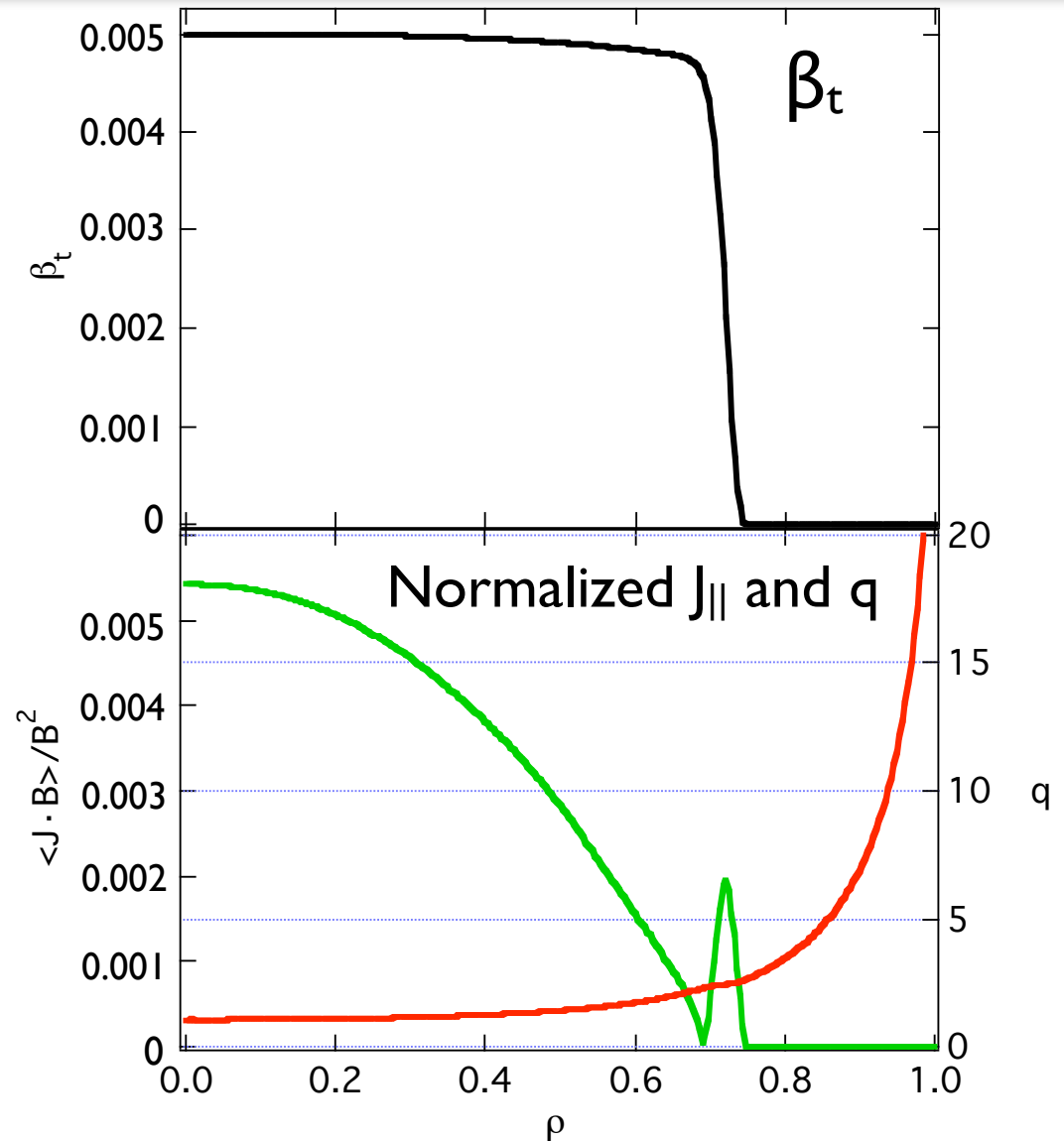
- ✦ currents in edge set to 0
- ✦ minimizes numerical errors (no separatrix)
- ✦ pedestal region
- ✦ ~67-75cm on midplane





Equilibrium profiles show peeling-ballooning instability drive source

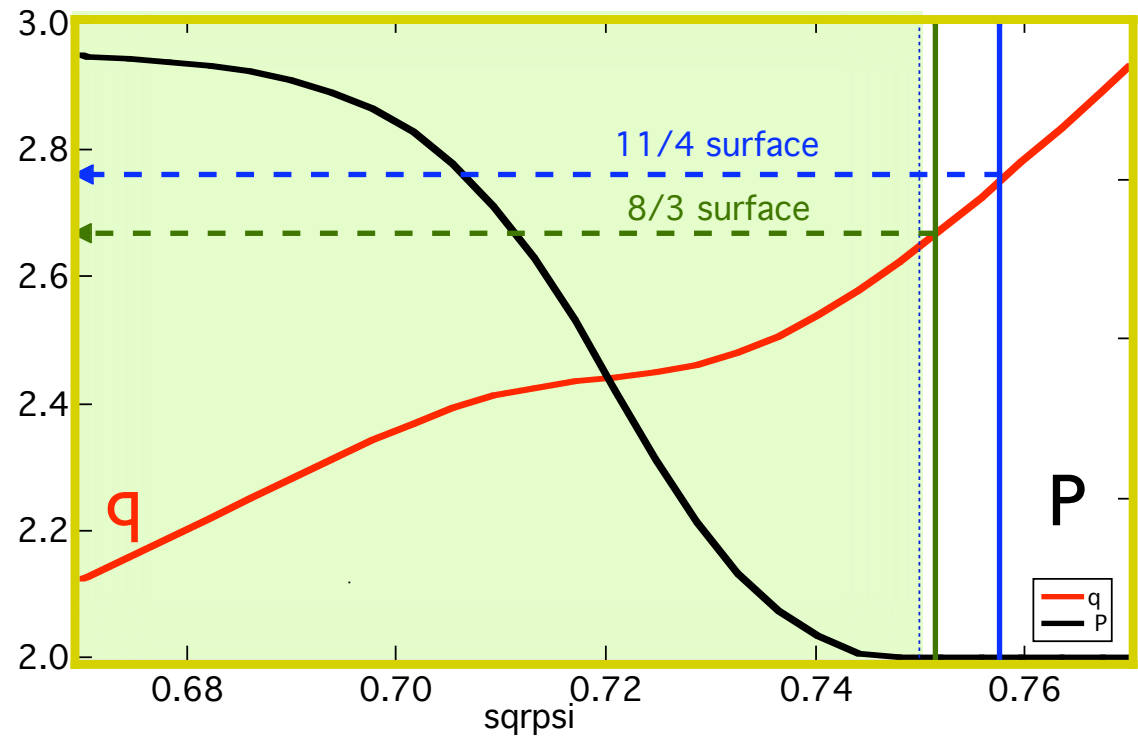
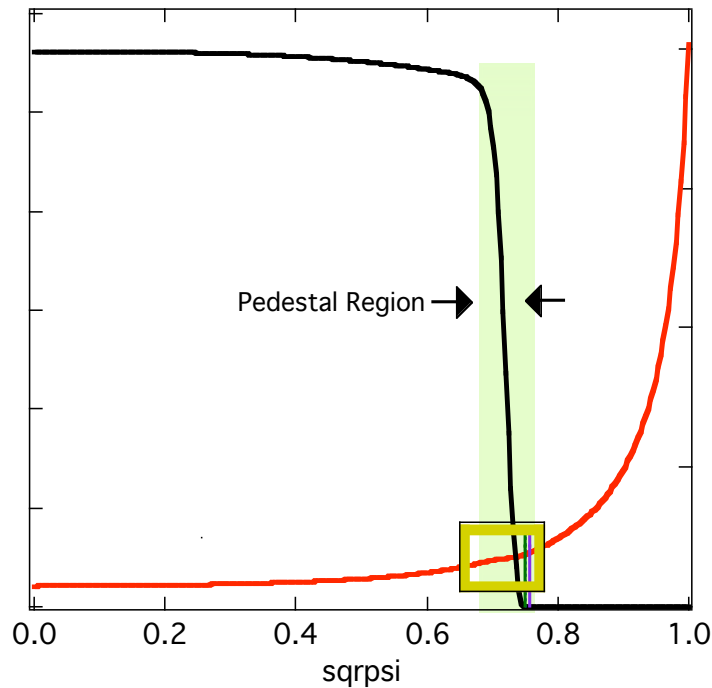
- Steep pressure gradients drive ballooning modes (DCON)
 - ✦ Pedestal width twice experimental value, simplify vacuum transition region
- Self-consistent edge currents & $2 < q_{\text{edge}} < 5$ to provide increased kink drive
 - ✦ comparable to ballooning drive





Halo location relative to the q rational surfaces affects instability drives

- using q profile identify mode rational surfaces

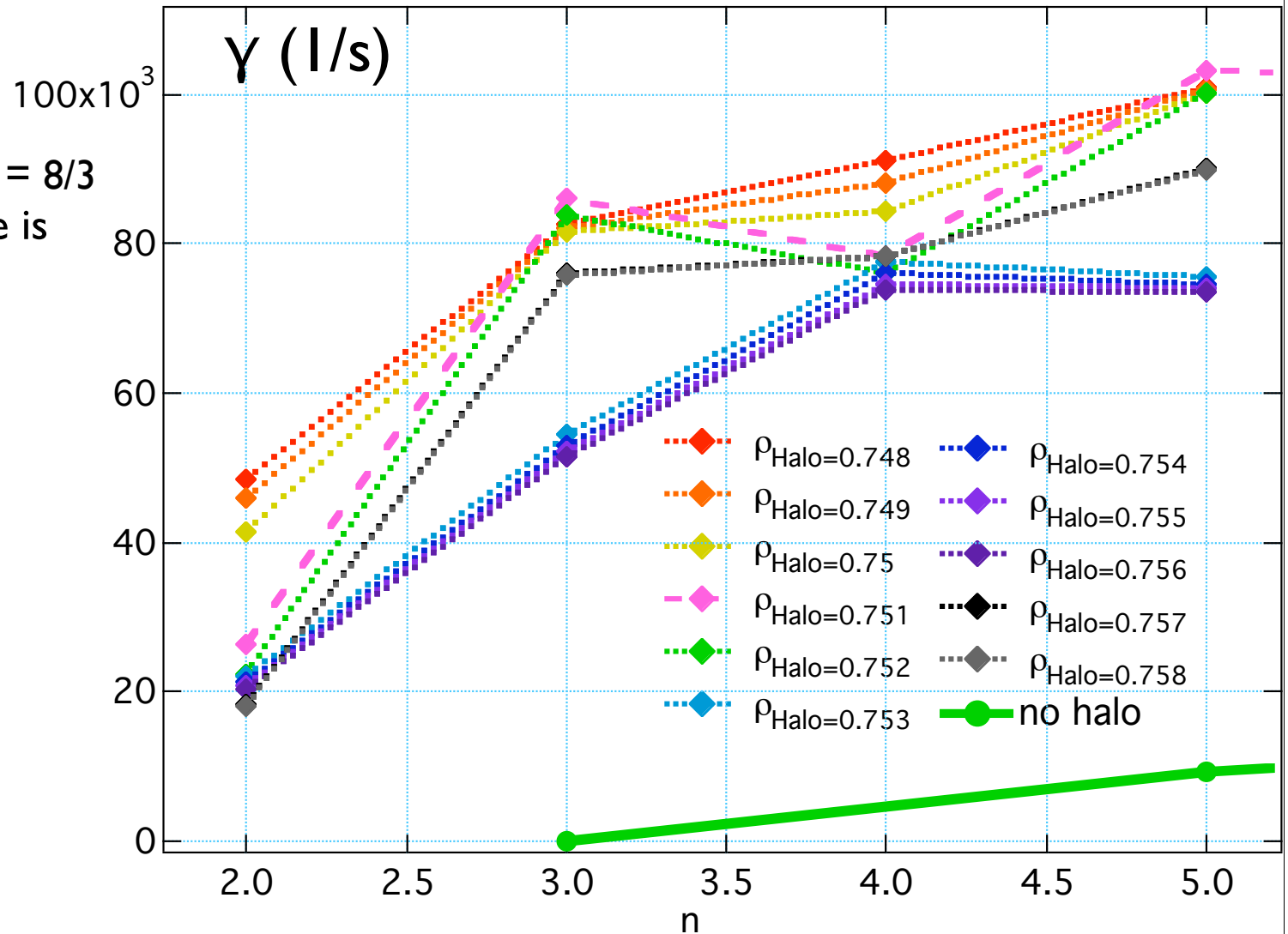


- adjusting the halo location “dials in” kink, ballooning, & peeling-ballooning behavior



Low-n modes are sensitive to location of halo transition

- When $\rho_{\text{ped}} < \rho_{\text{Halo}} < \rho_{\text{rat}} = 8/3$
 $n = 3$ kink mode is driven
- compared to ideal spectrum:
more clearly ballooning dominant
- convergence challenging

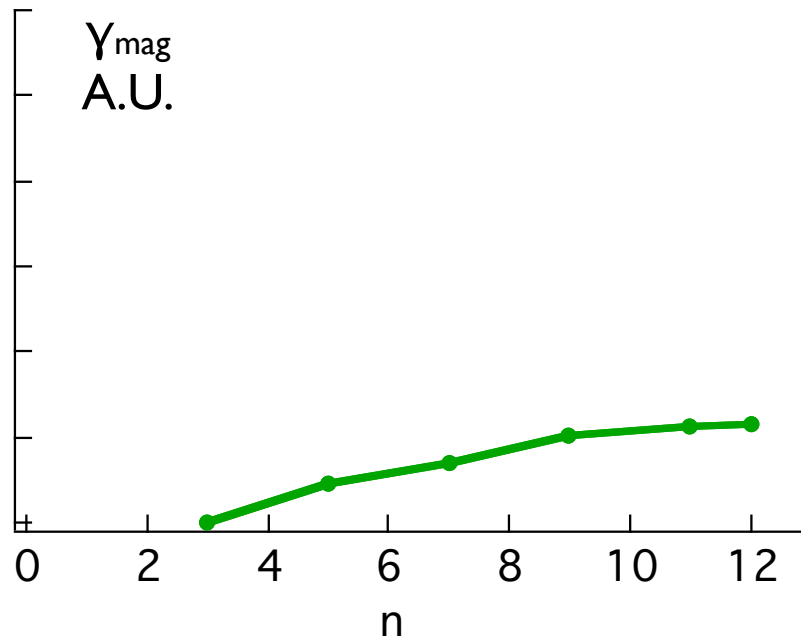




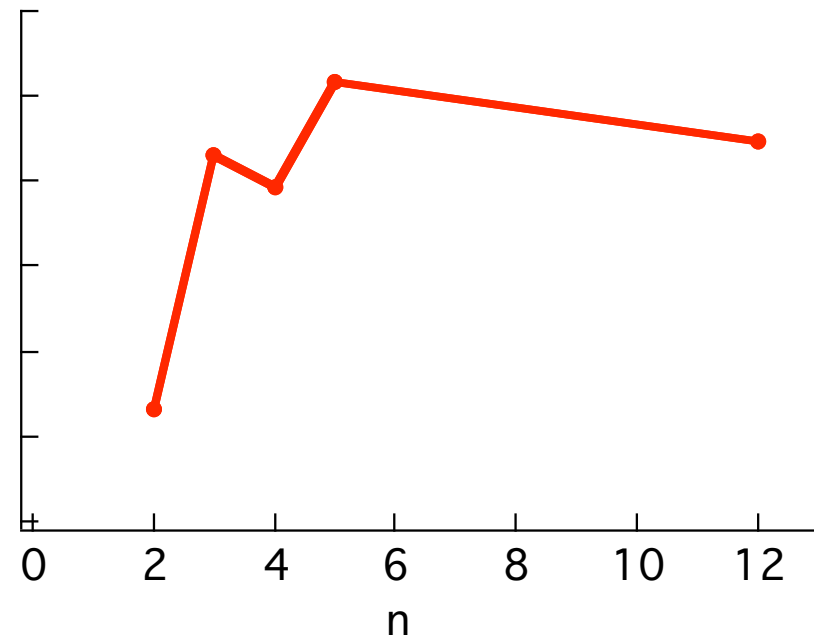
Kink & ballooning drives are adjusted within a single equilibrium

- Developed a technique where relative rates of ballooning / kink drive are changed by adjusting the location of the halo region relative to the plasma pedestal region
 - *actual NIMROD calculations*

No Halo: Ballooning Dominant Spectra



Halo: Increases Low-n Kink Drive





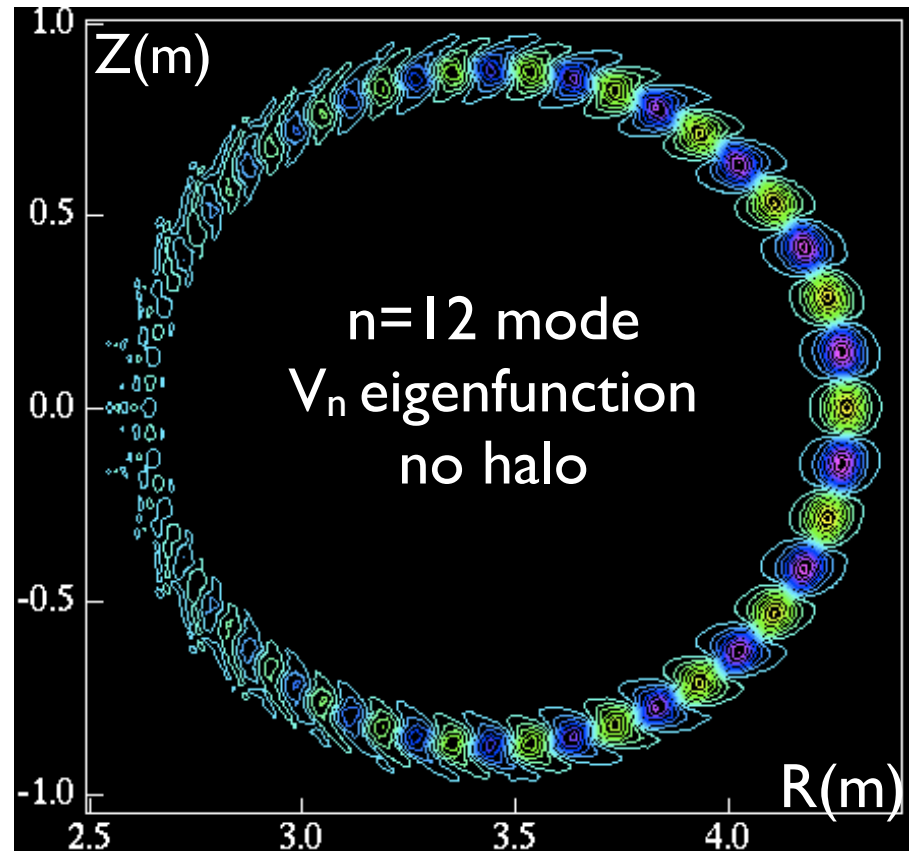
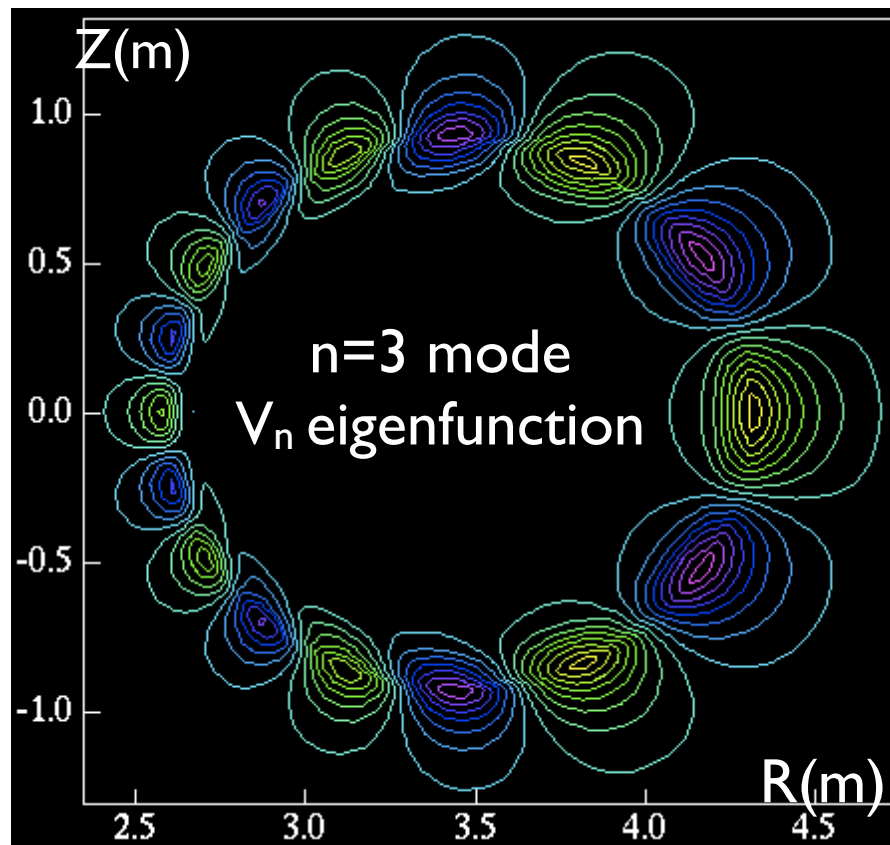
Summary

- Currently developing/documenting detailed linear peeling-ballooning analysis in NIMROD
 - ✦ Defined critical Lundquist values for defining an “ideal-like” plasma and halo region in NIMROD
 - ($S_{\text{crit-ideal}} \sim 5 * 10^7$; $S_{\text{crit-halo}} \sim 0.5$)
 - Ratio of these values are greater than in experiment
 - ✦ Demonstrated a technique that varies the linear spectral properties of a single equilibrium
 - scans show extreme spectral sensitivity to halo location
 - * convergence in this region is quite challenging
 - *(especially when $\rho_{\text{Halo}} \sim \rho_{\text{qmn}}$)*
 - edge ballooning & kink effects can be “dialed in” by using a sharp resistivity transition region located at relevant flux positions
- Preliminary NL results show qualitatively needed resolution and expected energy growth rates for a single NL filament growth



Eigenfunctions have peeling-ballooning structure

- $n=12$ Halo-free mode structure, ballooning
- $n=3$ $\rho_{vac}=0.751$ mode structure, peeling-ballooning





Preliminary NL runs in NIMROD

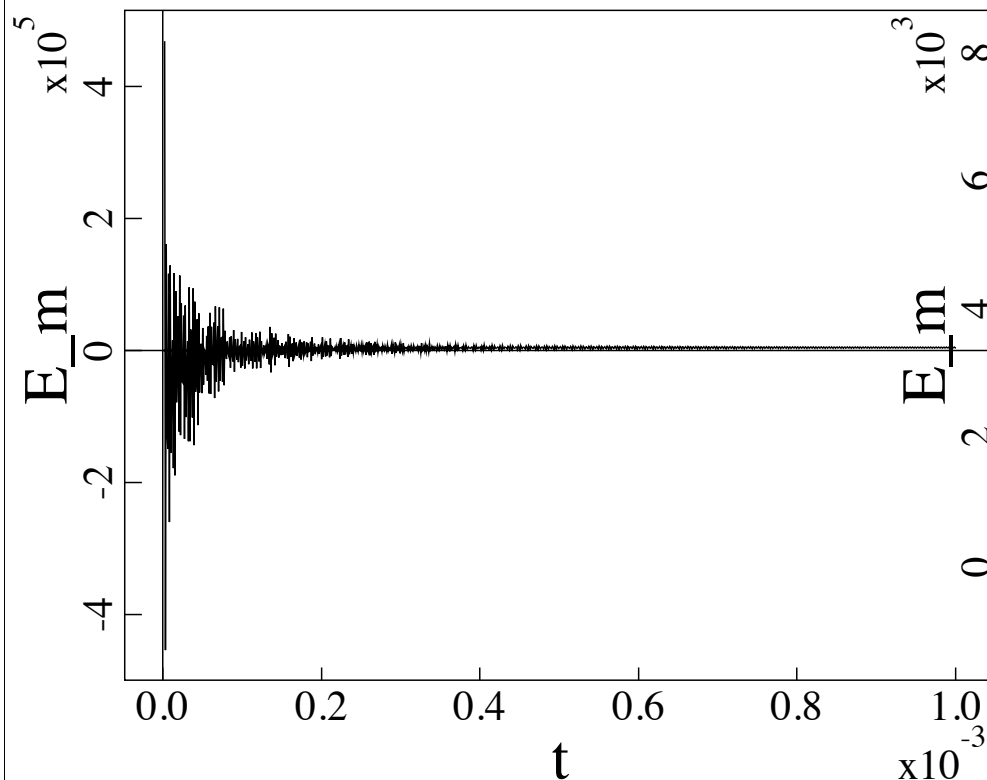
- In addition to the linear, began preliminary NL calculations in NIMROD
- Purely a demonstration of technique
 - ✦ not ideal S_{crit} : $S_{\text{in}} \sim 5 \cdot 10^5$
 - ✦ not resistivity independent halo: $\eta_{\text{out}} \sim 10^{-2} (\Omega \cdot \text{m})$
 - $S_{\text{out}} \sim 5 \cdot 10^2$
 - ✦ $\rho_{\text{vac}} = 0.84$
 - ✦ calculation grid points not packed
- Used to:
 - ✦ guide future studies
 - ✦ use results to design analysis tools
 - develop method to estimate transition between NL stages
 - determine growth regime to compare with analytic studies



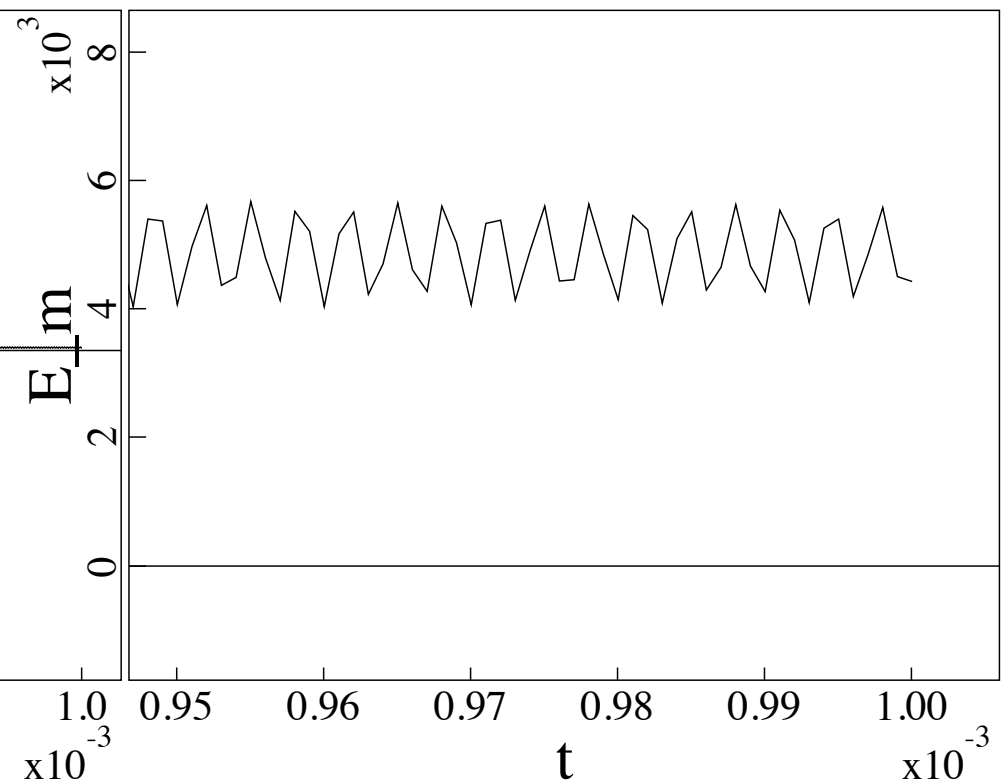
Increasing S (constant) increases problem

- For $S = 10^6$ the sawtoothing is seen up to $n = 3$ mode
- Not entirely sure if it is the exact same behavior
- Ping and Chris believe this is converged growth, I am not sure

Magnetic Energy vs. t



Magnetic Energy vs. t

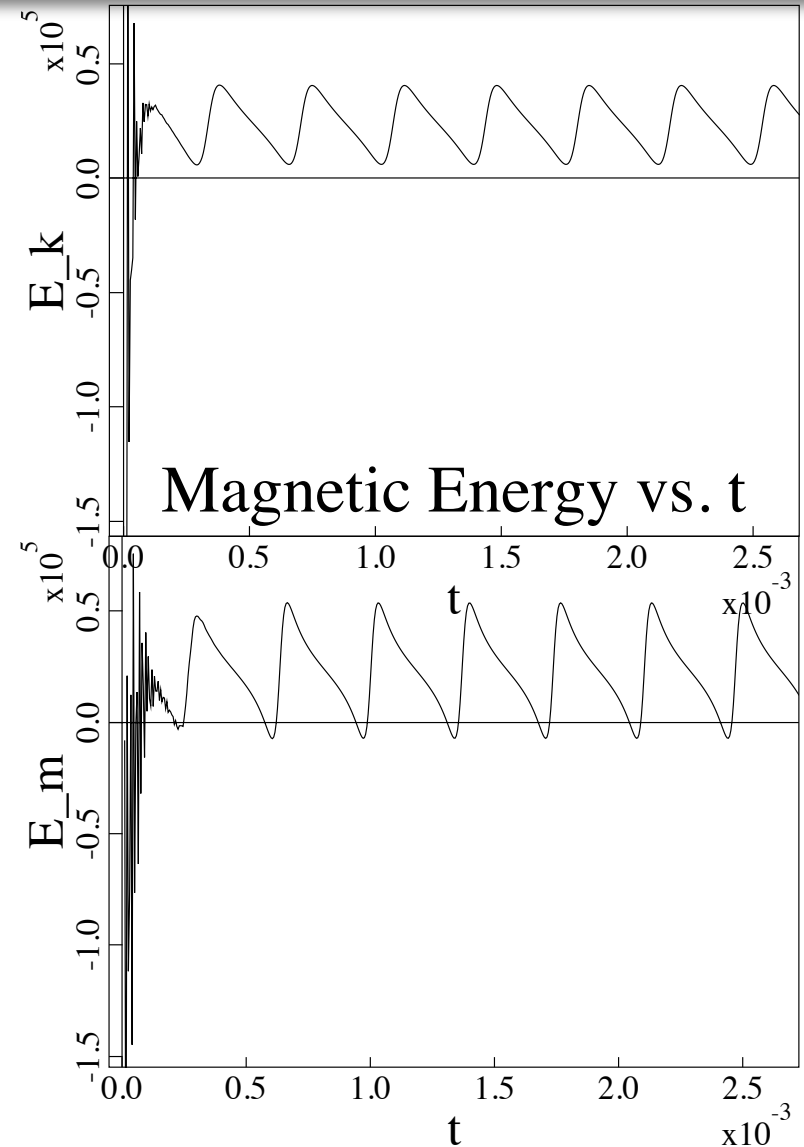




Low-n modes have oscillatory behavior

Kinetic Energy vs. t

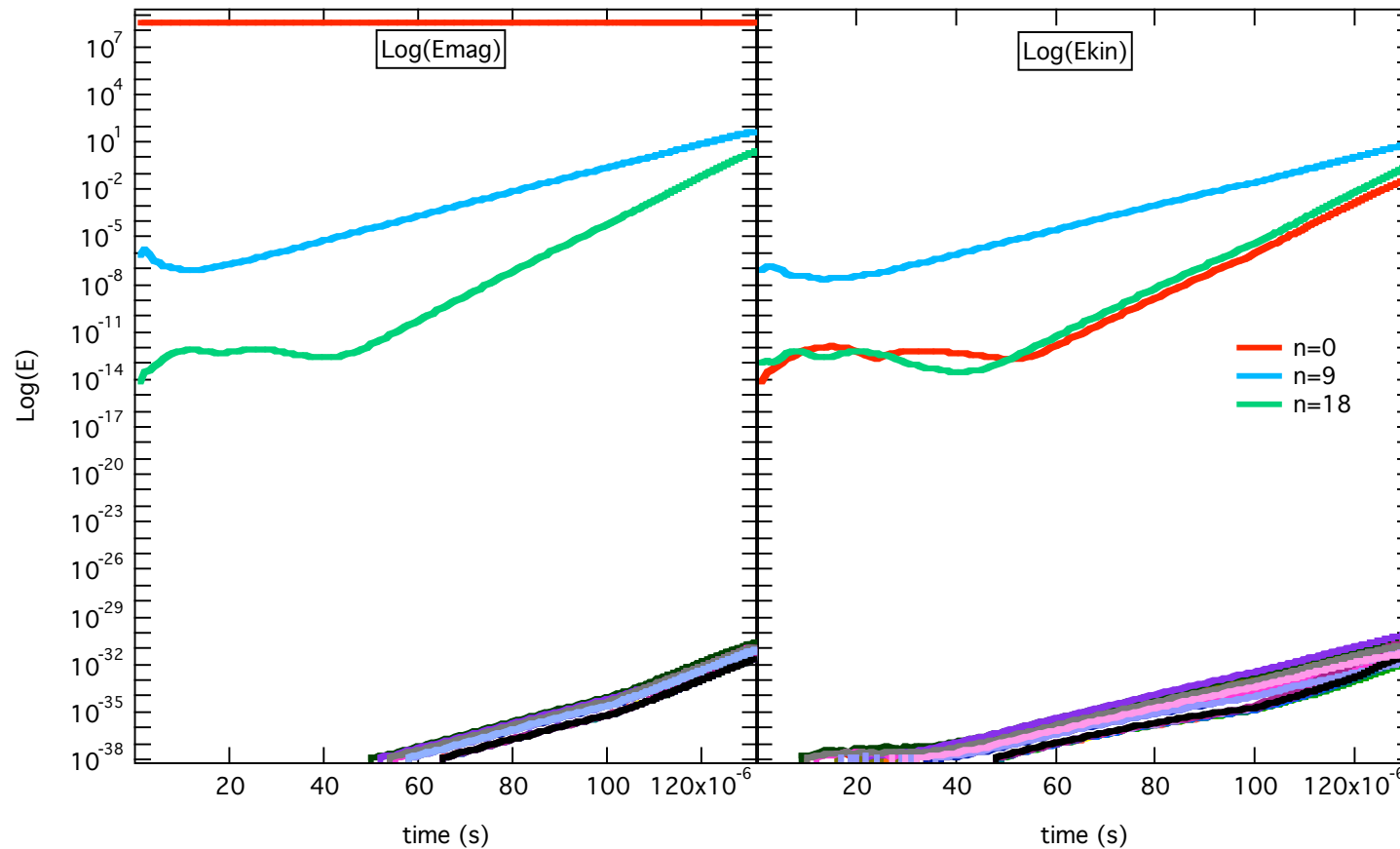
- Strange “saw-tooth-like” growth
 - ✦ occurs in low-n (stable?) modes
 - ✦ may be real physics
 - two modes (resistive & ideal) may simultaneously exist
 - Scott also saw this sawtoothing
 - perhaps nimrod bounces between two solutions
 - ✦ Moving the vacuum region out seems to eliminate the issue...
 - without a vacuum modes don't grow
 - ✦ Modes appear to be rotating/oscillating?





Linear $n=9$ eigenmode used to excite NL growth

- 22 modes included: $n=0-21$
- initialized with linear $n=9$
- nonlinear beating expected to produce $n=0$ & $n=18$ mode growth at twice linear $n=9$ rate





Lundquist/Resistivity ratio is not a good characterization parameter

