Background and Recent Progress on Simulation of Giant Sawteeth in Tokamaks with the NIMROD Code

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General Atomics

Long Term Goals

- Compute the *onset* and *nonlinear evolution* of a *Giant Sawtooth Crash* in a tokamak, including
  - Properties of relaxed state
  - Loss and destiny of stored energy
  - Coupling to/generation of MHD activity
  - Fate of energetic particles
  - *etc.*

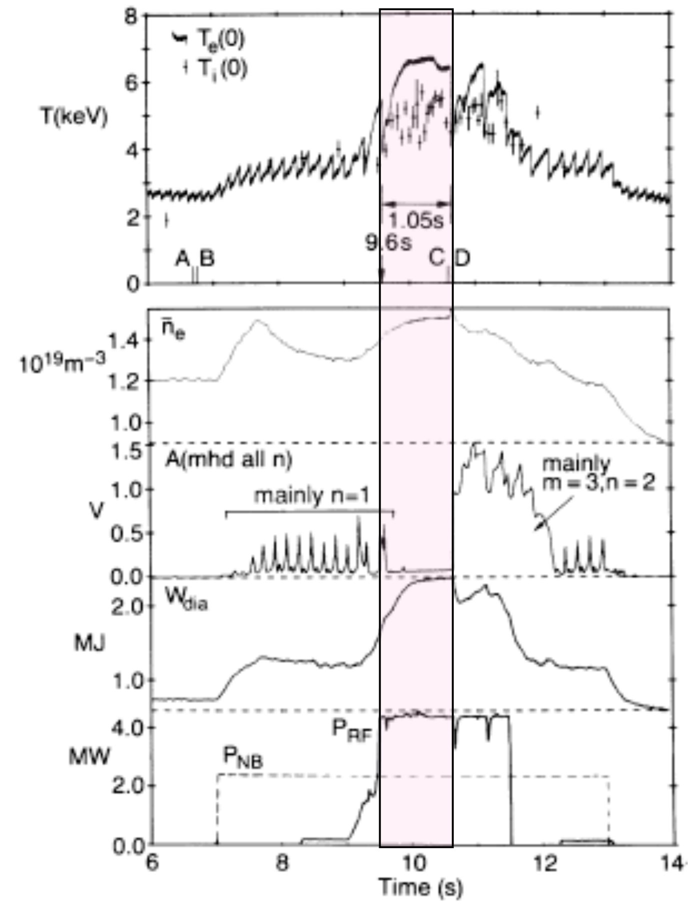
Short Term Goals

- *Demonstrate* and *validate* energetic particle capability in NIMROD by
  - Direct comparison with theory
  - Direct comparison with experiment (DIII-D 96043)
  - Direct comparison with previous numerical results (Choi, *et al*, PF **14**, 112517 (2007))

Sawtooth Stabilization

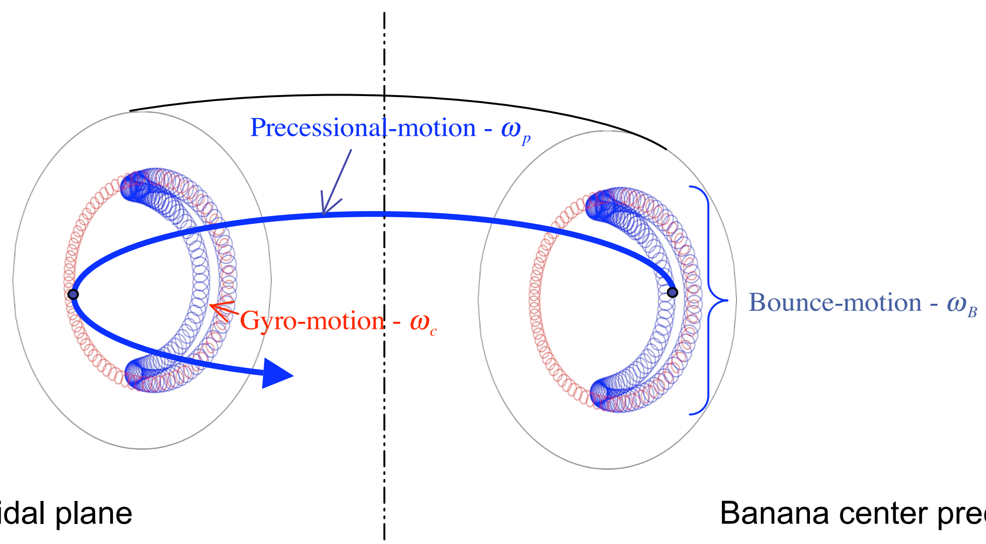
Campbell, et al., PRL **60**, 2148 (1988)

- JET - 1988
- "Sawtooth-free" period induced by NB and RF
  - Stored energy ~ doubles
  - Confinement improves by ~ 20%
  - Interaction between MHD and energetic particles?
- Terminated by "monster" or "giant" sawtooth crash
  - *m*=1, *n*=1 (++)
  - Loss of stored energy
  - Loss of energetic particles
  - MHD activity (triggering)



Physics: MHD/Energetic Particle Interaction

- How can high energy particles (E > 100 KeV) interact with low frequency MHD?
- Particle orbits in a tokamak



Projections of Orbits in Poloidal plane

Banana center precesses toroidally

$$\omega_c \gg \omega_B \gg \omega_p$$

Adiabatic Invariants

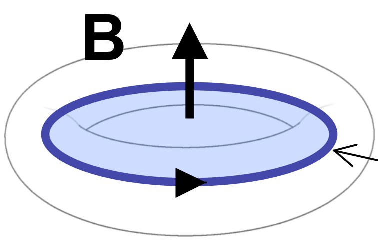
- "Almost" periodic motion with frequency ~ ω "almost" conserves "adiabatic invariants" on slower frequencies

$$\omega \ll \omega_c \sim \text{conserves } \mu = \frac{mv_\perp^2}{2B} \quad \text{magnetic moment}$$

$$\omega \ll \omega_B \sim \text{conserves } J = \oint v_{\|} ds \quad \text{"longitudinal invariant"}$$

$$\omega \ll \omega_D \sim \text{conserves } \Phi = \int \mathbf{B} \cdot d\mathbf{S} \quad \text{flux linked by precessing orbit}$$

"Third adiabatic invariant"



Precessing banana center

Particle Effect on Kink Mode

- MHD (kink) frequency less than precession frequency $\omega_A < \omega_p$ (sometimes <<)
  - MHD activity perturbs flux
- If $\omega_A << \omega_p$, kink perturbs flux on low frequency
  - Third adiabatic well conserved
  - Flux change resisted
  - *Stabilization of kink mode*
- Requires *enough* particles (threshold density, or hot particle β)
  - *Can you get enough energetic particles to stabilize kink without destabilizing fishbone?*

More Kink Stabilization (Slowing-down Distribution)

White, Romanelli and Bussac, PFB **2**, 745 (1990)

$$\underbrace{\delta W_{fluid}}_{\substack{\text{Outer sol'n} \\ \text{Ideal MHD}}} + \underbrace{\delta W_k}_{\substack{\text{Energetic} \\ \text{particles}}} = \underbrace{\frac{8i\Gamma\left[\left(\Lambda^{3/2}+5\right)/4\right]\left[\omega\left(\omega-\omega_{*i}\right)\right]^{1/2}}{\Lambda^{9/4}\Gamma\left[\left(\Lambda^{3/2}-1\right)/4\right]\omega_A}}_{\text{Inner layer solution; Resistive MHD}}$$

$\delta W_{fluid} \equiv -\gamma_I / \omega_A$ Ideal MHD growth rate

$$\Lambda = -i\left[\omega\left(\omega-\omega_{*i}\right)\left(\omega-\omega_{*e}\right)\right]^{1/3} / \gamma_R \quad , \quad \gamma_R \equiv S^{-1/3}\omega_A$$

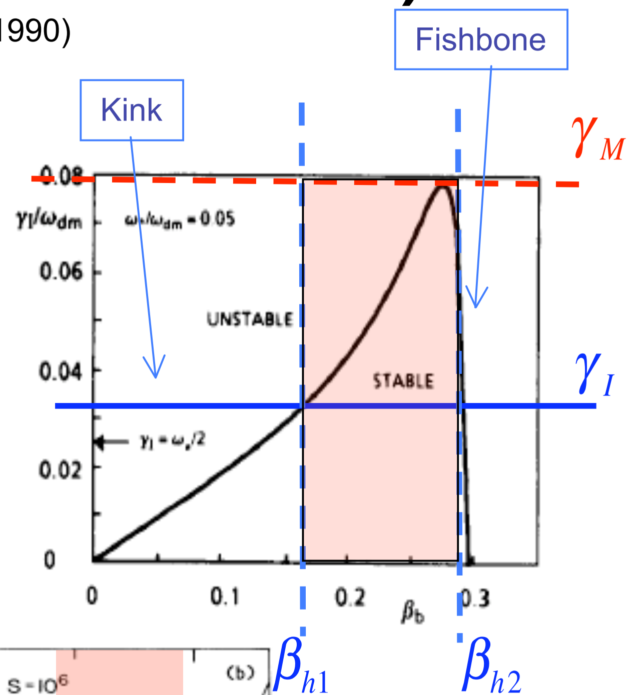
$\rightarrow$ Slowing-down distribution (E_m):

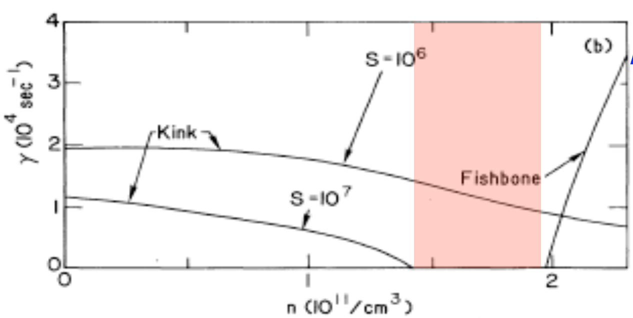
$$\delta W_k = \frac{\beta_h}{\varepsilon}\frac{\omega}{\omega_{dm}}\ln\left(1-\frac{\omega_{dm}}{\omega}\right) \quad , \quad \omega_{dm} = \frac{E_m q}{m_h \omega_{ch} r_1 R} \quad , \quad \omega < \omega_{dm}$$

Stabilization:

$$\frac{1}{\pi}\left(1-\frac{\omega_{*i}}{\omega_1}\right)^{1/2} < \frac{\beta_h}{\varepsilon}\frac{\omega_A}{\omega_{dm}} < \frac{1}{\pi} \quad ,$$

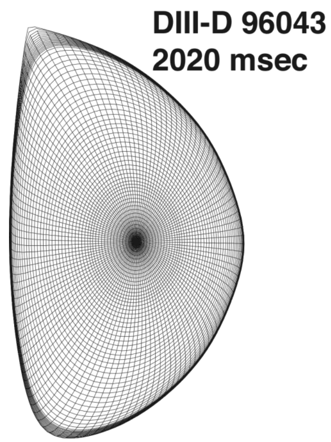
$$\gamma_I < \gamma_M \quad , \quad S > S_{crit} \approx \left(\frac{\omega_A}{\gamma_M}\right)^3$$

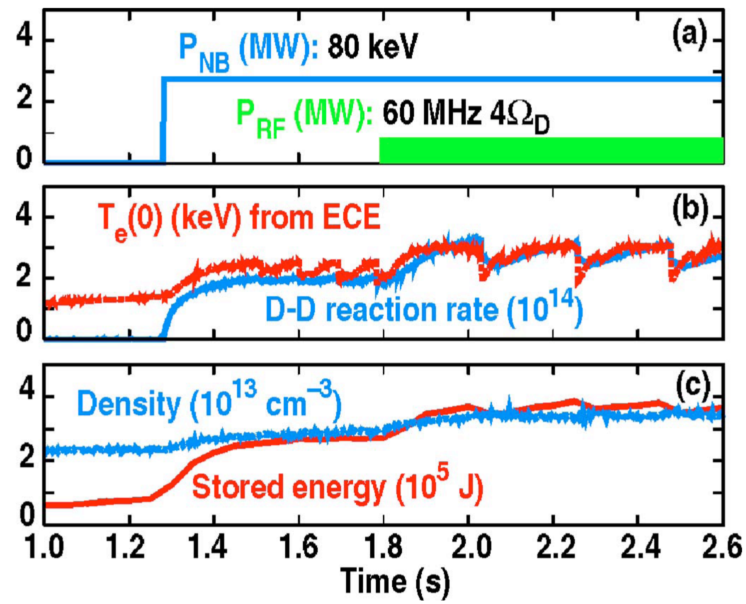


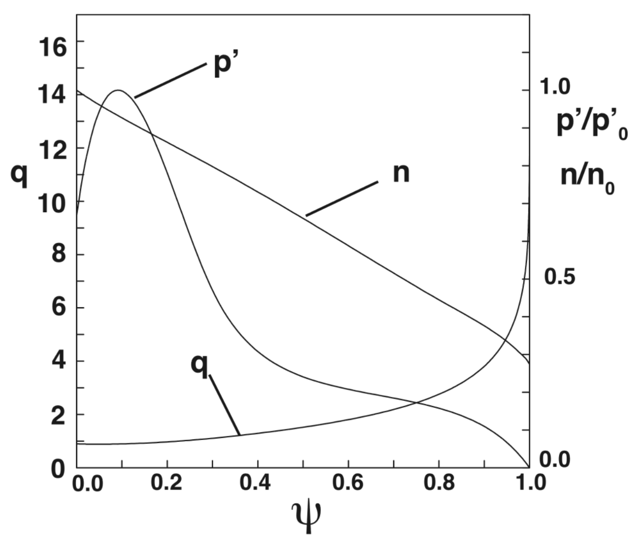


Are diagnetic effects required?

DIII-D Shot 96043

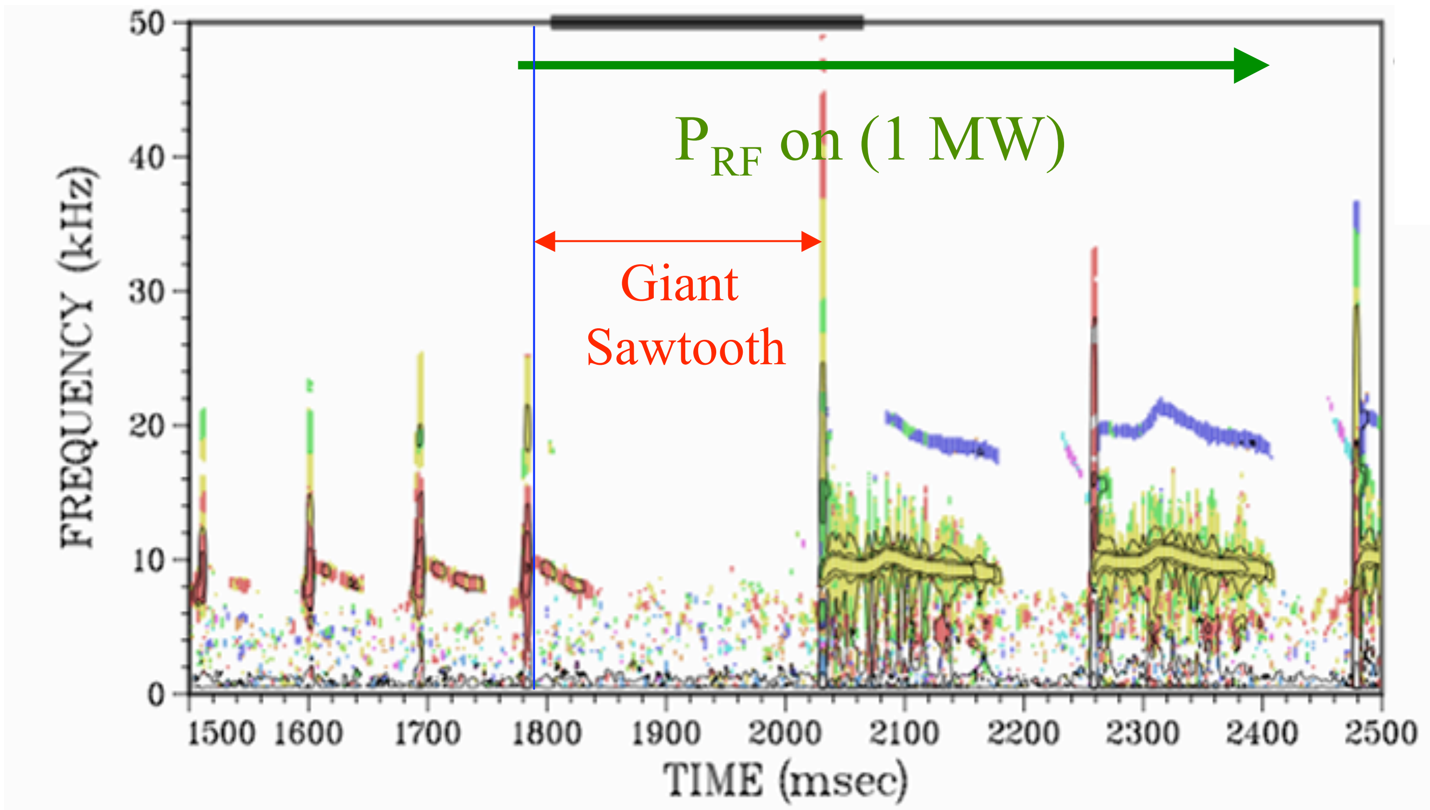






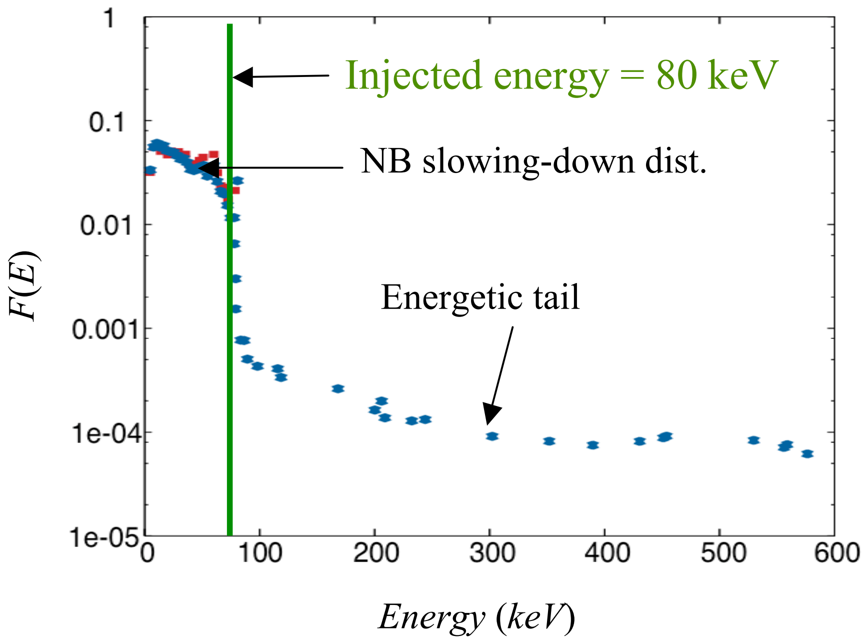
- Neutral beam heated
- RF produces energetic particles
- Sawtooth period increases with RF

“Giant Sawtooth” in DIII-D



Hot Particle Distribution Function

- Effect of RF computed with ORBIT-RF



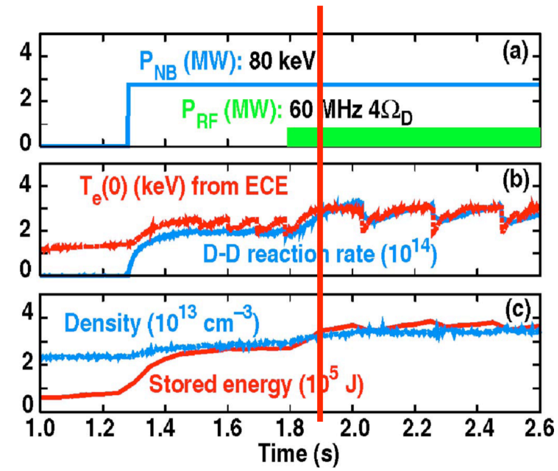
- Energetic particle distribution is sum of slowing-down + RF acceleration

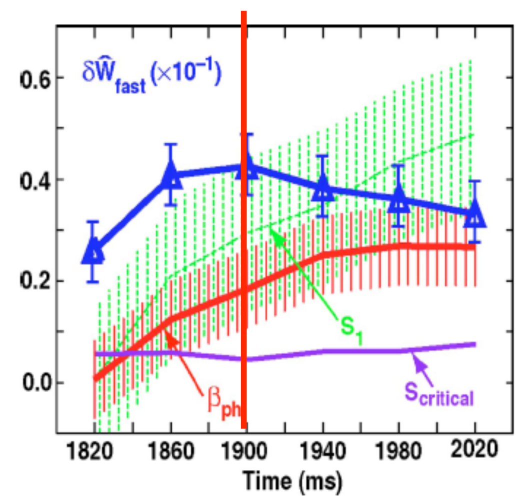
$$F_{hot}(E) \sim F_{S-D}(E) + \exp\left(-\frac{(\chi-\chi_0)^2}{\Delta\chi^2}\right) \quad , \quad \chi = \frac{v_{\parallel}}{v}$$

- Energetic tail can affect stabilization

NIMROD Calculations

- NIMROD extended MHD code has model for energetic particles
- Energetic particles do not affect MHD equilibrium
- *Present goal*: Examine linear stability of DIII-D shot 96043 at t = 1900 ms.
- Resistive MHD + Energetic particles (Slowing-down dist.)
- Look at linear stability as function of $\beta_{frac} = P_{hot} / P_{tot}$
- Nonlinear runs eventually





NIMROD Fluid Model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V}\right) = -\nabla P + \mathbf{J} \times \mathbf{B} + \nabla \cdot \left(\underbrace{\Pi_{visc}}_{\text{Braginskii viscoous stress}} + \underbrace{\Pi_{hot}}_{\text{Energetic particles}} + \underbrace{\Pi_{FLR}}_{\text{Gyro-viscosity}} \right)$$

$$\frac{\partial P}{\partial t} = -\mathbf{V} \cdot \nabla P - P \nabla \cdot \mathbf{V} + \eta J^2 - \nabla \cdot \mathbf{q} + \Pi : \nabla \mathbf{V}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad , \qquad \mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$\mathbf{E} = -\underbrace{\mathbf{V} \times \mathbf{B}}_{\text{Ideal MHD}} + \underbrace{\eta \mathbf{J}}_{\text{Resistive MHD}} + \underbrace{\frac{1}{ne}\left(-\nabla P_e + \mathbf{J} \times \mathbf{B}\right)}_{\text{2-fluid and diamagnetic effects}}$$

- Model is nonlinear; present study is linear
- Closures for viscous stress, FLR, energetic particle stress, and heat flux

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NIMROD Particle Model

Kim, et al., PP **15**, 072507 (2008)

$$\Pi_{hot} = \int d\mathbf{v} f_{hot}(\mathbf{v})(\mathbf{v}-\mathbf{V})(\mathbf{v}-\mathbf{V})$$

- f_{hot} is solution of kinetic equation for hot particle species
  - Drift kinetic approximation
  - δf PIC method (Parker and Lee, PF B **5**, 77 (1993))
  - Present application is linear (integrate δf along unperturbed orbits)

$$\dot{\mathbf{x}}_0 = v_{\|}\hat{\mathbf{b}}_0 + \frac{m}{eB_0^4}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)\mathbf{B}_0 \times \nabla \frac{B_0^2}{2} + \frac{\mu_0 m v_{\|}^2}{eB_0^2}\mathbf{J}_{\perp 0}$$

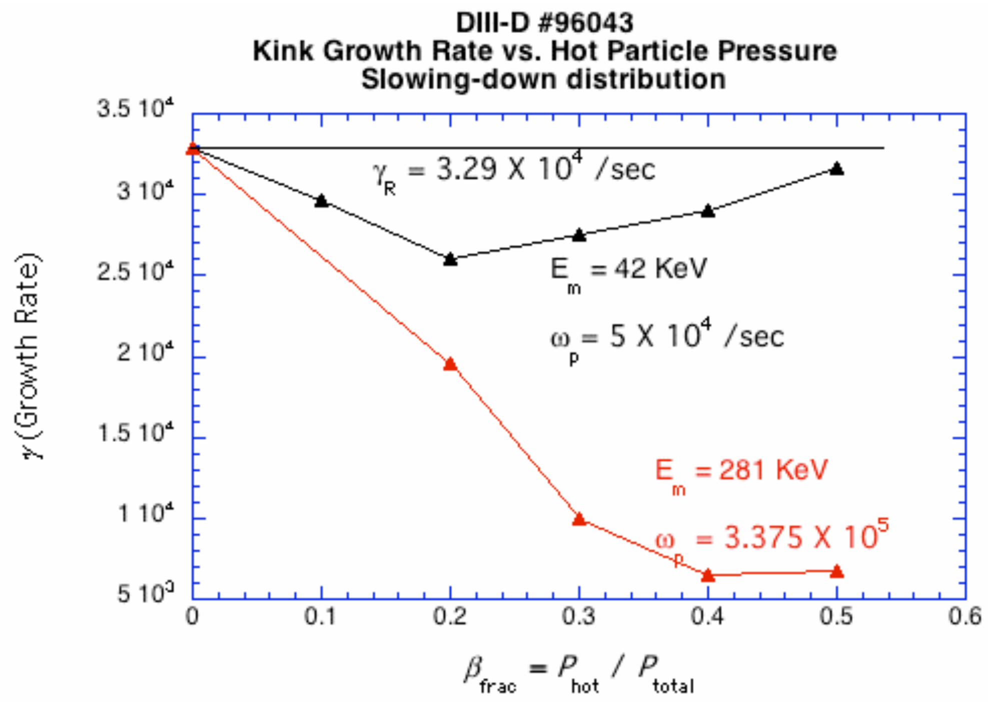
$$m\dot{v}_{\|0} = -\widehat{\mathbf{b}}_0 \cdot \mu \nabla B_0 \quad , \quad \mu = \frac{1}{2}\frac{mv_{\perp}^2}{B_0}$$

$$\delta \dot{f} = -\delta\mathbf{v}\cdot\nabla F_0 - e\mathbf{v}_0 \cdot \delta\mathbf{E}\frac{\partial F_0}{\partial E} \qquad F_0 = \begin{cases} \dfrac{C}{E^{3/2}+E_c^{3/2}} & \text{for } E < E_m \\ 0 & \text{for } E > E_m \end{cases}$$

Slowing-down distribution

- Particles subcycled each fluid timestep
- NIMROD has demonstrated agreement with M3D and kink mode stabilization

NIMROD Results

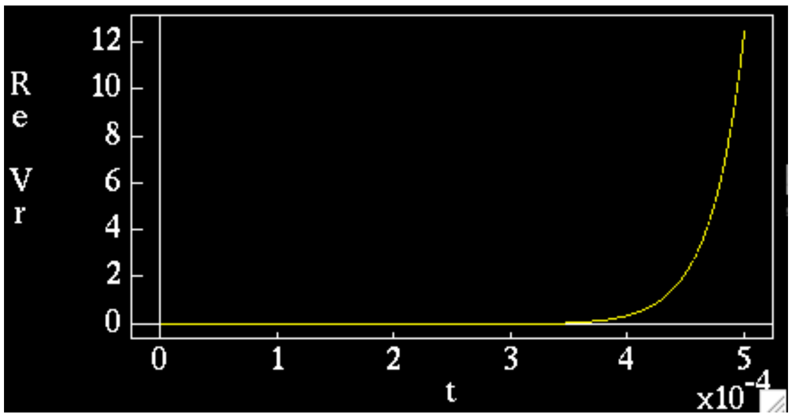


- t = 1900 ms.
- S = 1.7 X 10^7
- At 281 KeV, $\gamma_R/\omega_p \sim 0.1$
  - Sufficient separation?
- $S > S_{crit}$?
- Need RF tail?
- Need 2-fluid?

- Resistive MHD + energetic particles
  - No diamagnetic or FLR effects
- Slowing-down distribution
  - No RF tail
- Transition from kink to fishbone
- 2 X 10^6 - 4 X 10^7 particles

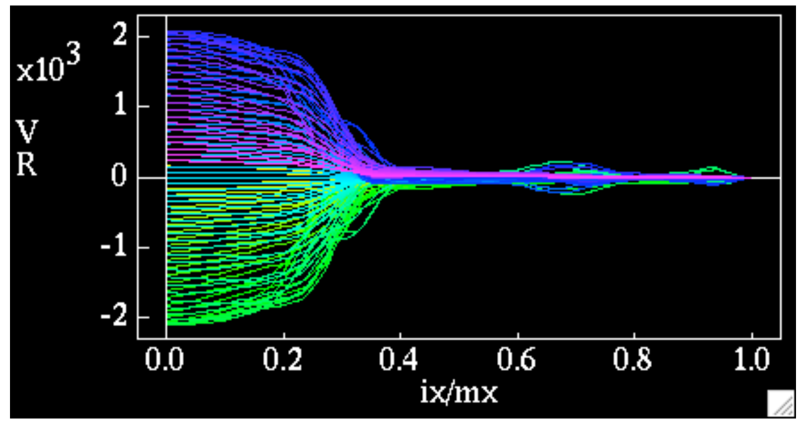
At constant energy, β_{frac} measures hot particle density

Kink Mode ($\beta_{frac} = 0$)



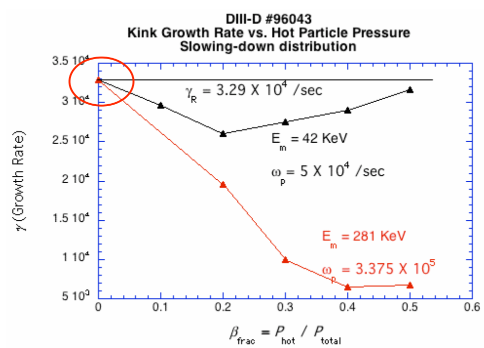
V_r vs. time

Pure exponential growth



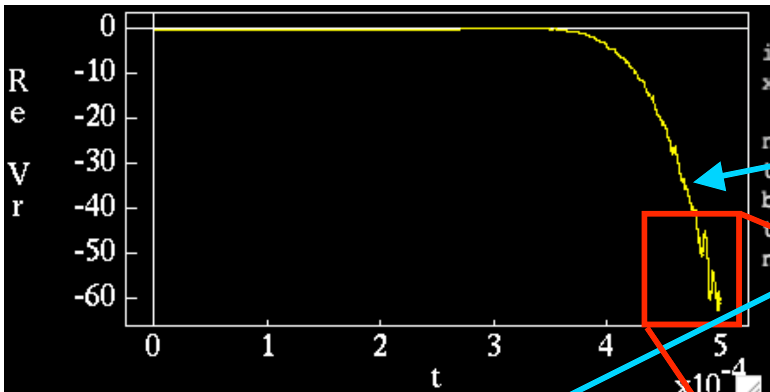
V_r eigenfunction

"Top hat" structure

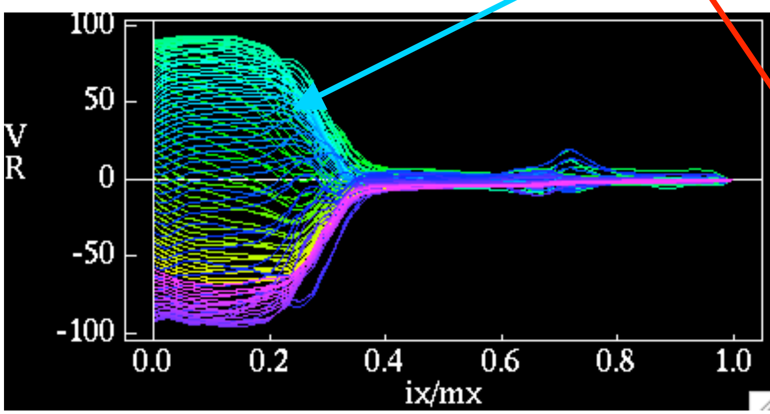


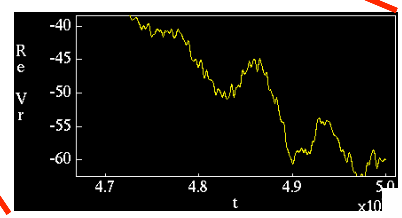
E_m = 41.75 KeV, β_{frac}=0.2

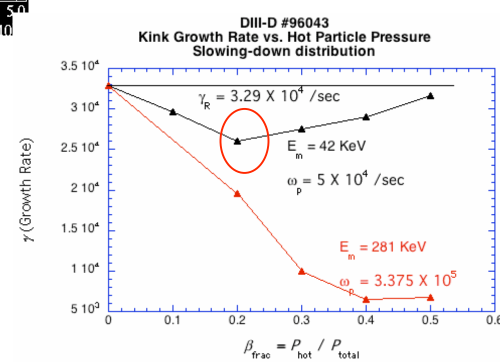
Energetic particle effects on kink



- Kink with particle effects
- Real frequency
- Distortion due to rotation

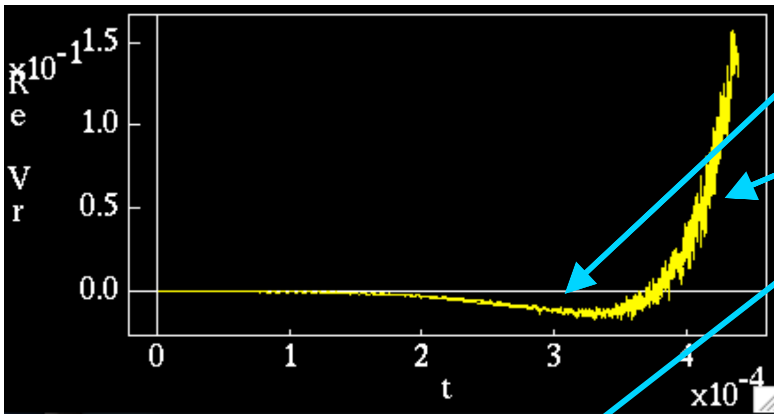




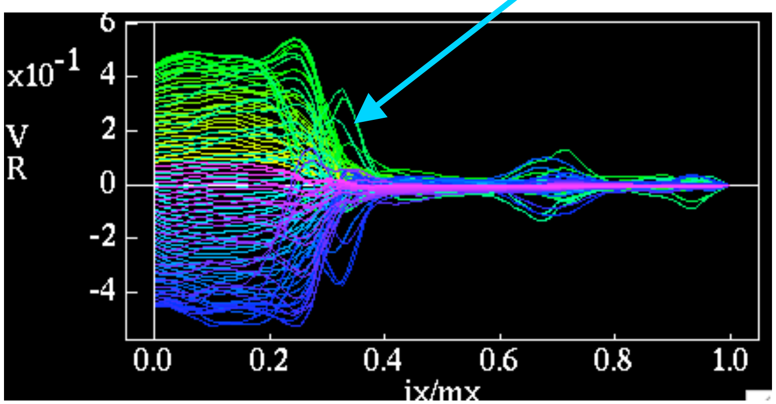


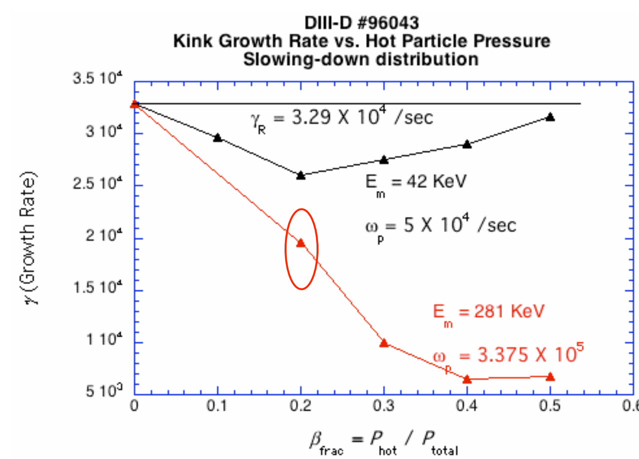
E_m = 281 KeV, β_{frac} = 0.2

Energetic particle effects on kink



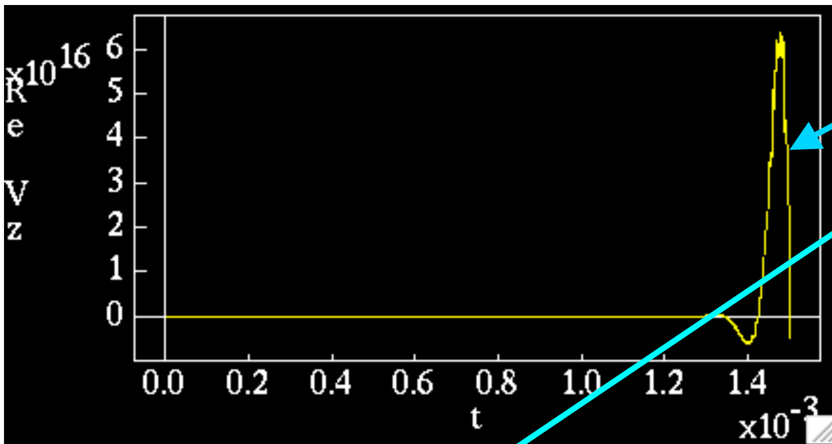
- Slow rotation
- Noise due to particles
- Distortion near rational surface
- “Looks like kink mode”

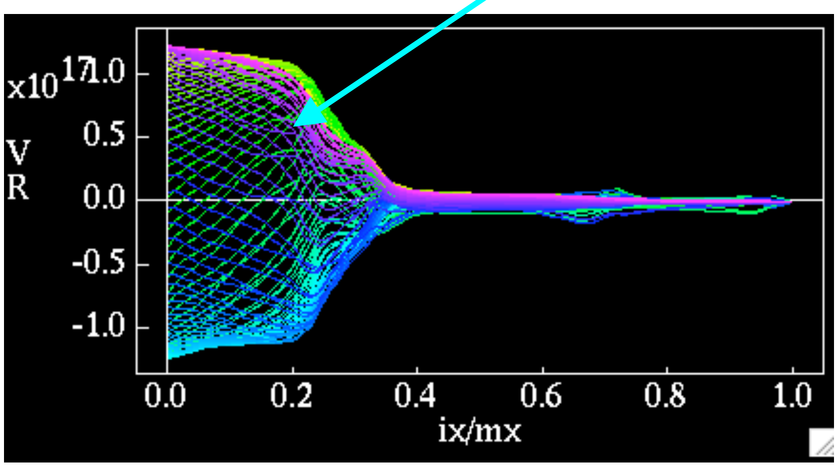




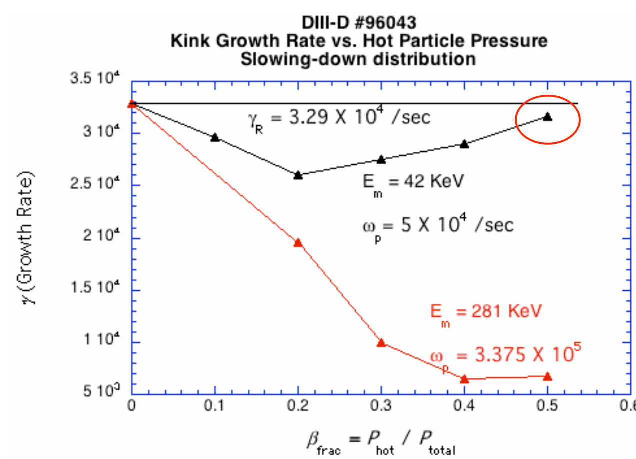
E_m = 41.75 KeV, β_{frac}=0.5

Energetic particle effects on kink



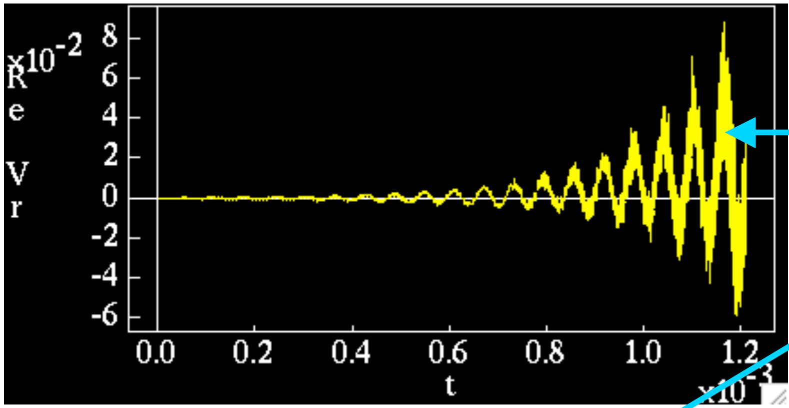


- Real frequency
- Distortion due to rotation
- Transition to fishbone

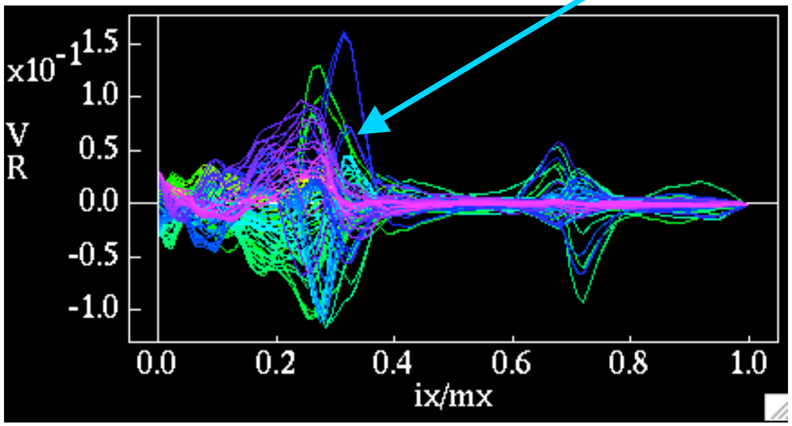


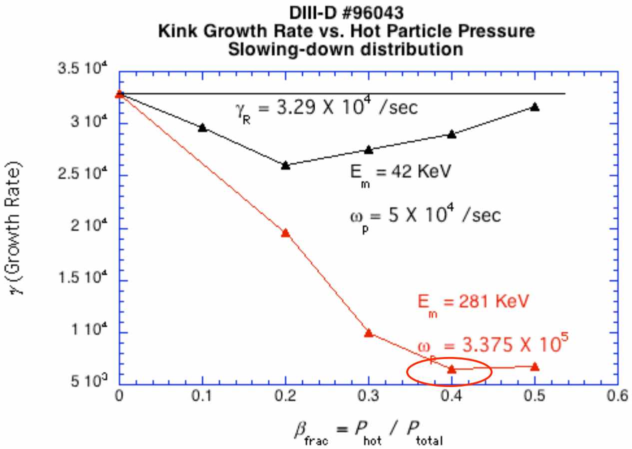
E_m = 281 KeV, β_{frac} = 0.4

Energetic particle effects on kink



- Real frequency
- Large modification of radial eigenfunction
- Transition to fishbone?



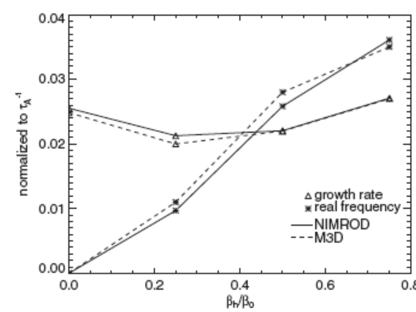


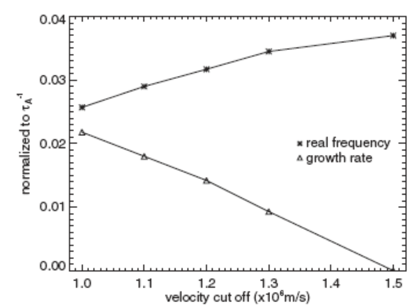
Discussion

- Need high energy particles for conservation of third adiabatic invariant
  - Precession frequency must be >> MHD frequency
  - How high?
  - NIMROD includes both passing and trapped particles Π_{hot}. Comparison with theory?
- Role of RF tail
  - Need tail for stabilization?
  - Stabilize with SD distribution?
- Role of diamagnetic effects
  - Need 2-fluid for stabilization?
  - Can we simulate “ion-kinetic” regime? Just need “some” reconnection mechanism?
- Is *S* large enough?
- What about thermal trapped particles (Kruskal-Oberman)?
  - Need closure?
  - Second “Maxwellian” particle species?
- Effect of energetic particles on equilibrium?
  - Anisotropic pressure?
- Can NIMROD exhibit same stability properties as experiment?
  - The ultimate validation?
- What happens non-linearly?
  - A *real* FSP problem!

NIMROD Integrated Modeling Status

- Resistive MHD
  - Extensive V&V
    - Comparison and agreement with known solutions, other codes, and experiment
  - Astrophysical problems
    - Extragalactic jets, MRI, simulation of dynamo experiments
- Two-fluid/FLR
  - Scaling to 10,000 processors
  - Verification
    - *g*-mode in slab
  - Non-linear calculations
- Energetic (kinetic) ion species
  - Comparison with M3D on kink-fishbone transition
  - Sawtooth stabilization
  - V&V (DIII-D) underway





Required Development for Kinetic Ions

- More efficient parallel implementation
  - Use more than nlayers = 1
  - ???
- Anisotropic equilibrium pressure
  - Energetic particles don't contribute to equilibrium force balance
- Extended Ohm's law in particle advance
- Modification to equilibrium distribution function
  - RF tail
  - ??

The Porcelli Model

Porcelli, Boucher and Rosenbluth, Plasma Phys. Cont. Fusion **38**, 2163 (1996)

- A "predictive" model for the sawtooth "trigger"
- Based on "zero-dimensional" formulas
- Can be applied to evolving profiles in a transport code
- Based on normalized energy $\delta W \Leftrightarrow -4\delta W / (s_1 \xi_0^2 \varepsilon_1^2 R B^2)$, $s_1 = (rq')_{r=r_1}$

$$\delta W = \underbrace{\delta W_{tor}}_{\text{Fluid}} + \underbrace{\delta W_{el}}_{\text{Shaping}} + \underbrace{\delta W_{KO}}_{\text{Thermal trapped particles}} + \underbrace{\delta W_{k}}_{\text{Energetic trapped particles}}$$

- Everything evaluated inside the $q = 1$ surface
- Sawtooth crash is triggered whenever any of the following is satisfied:

$$-\delta W_{tor} - \delta W_{el} > c_h \omega_{pm} \tau_A$$ Few precessional orbits in MHD growth time

$$-\delta W > 0.5 \omega_{*i} \tau_A$$ Loss of two-fluid stabilization: $\omega_* < 2\gamma_I$

$$-c_\rho \rho_i < -\delta W < 0.5 \omega_{*i} \tau_A \text{ and } \omega_{*i} < c_* \gamma_\rho$$ Unstable in “ion-kinetic” regime

“Even though feasible, it is impractical to interface” a linear stability code “with a transport code.”

- "Incomplete relaxation" model for post-crash profiles

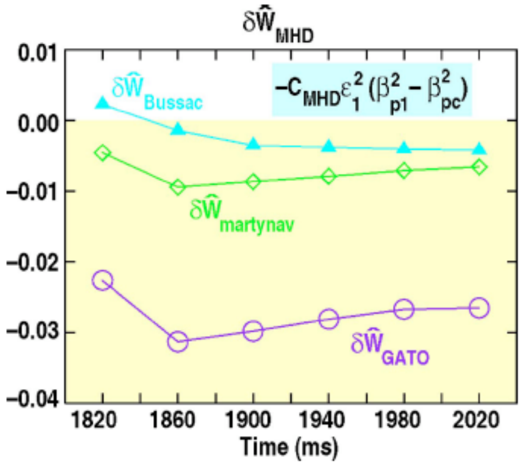
“Kadomtsev’s model is not always consistent with experimental data, even though observations with different tokamak experiments are somewhat conflicting.”

Testing the Porcelli Model

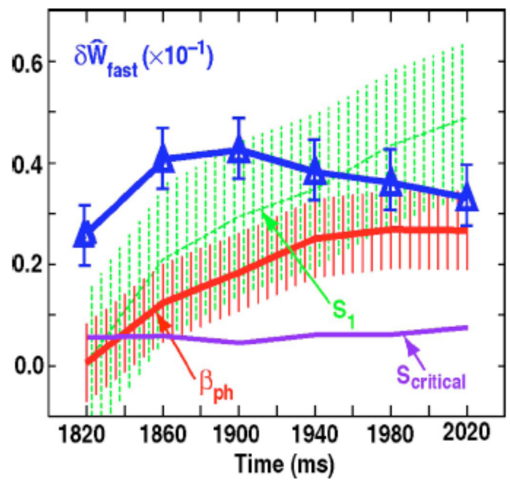
Choi, Turnbull, Chan, et al., PP **14**, 112517 (2007)

- Experiments on DIII-D (shot 96043)
- Induce sawtooth-free period
  - NB
  - RF
- Reconstruct profiles at time intervals (EFIT)
- Compute terms in Porcelli model
  - δW_{MHD} computed with GATO
- Compare predicted “trigger” with onset of crash

δW Evolution before Crash

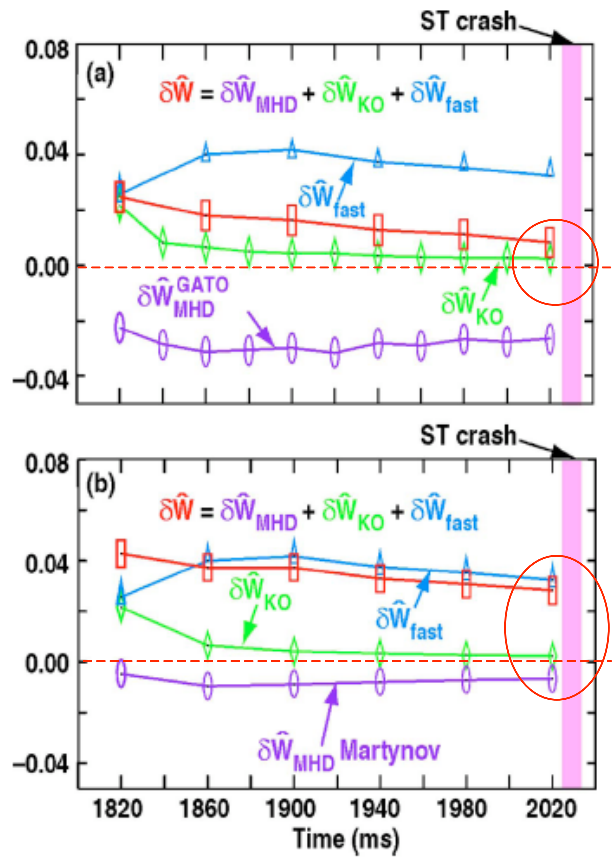


Porcelli always underestimates MHD instability



δW_k increases, then decreases

Shear increases, lowering normalized energy



Total potential energy using GATO and models