Some Thoughts on New Directions

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ELM Simulations bring up new questions

- Solutions found in the nonlinear simulation
  - Smooth regular solutions?
  - Chaotic or partially chaotic: what structure?
  - Meaning of “small” perturbations
- Many developments in mathematics and other fields in past few decades, many open problems
  - Largely ignored by fusion community
  - Many specific solutions can only be addressed by numerical simulation – fits CEMM approach and SciDAC mission

Are solutions smooth?

- ELM magnetic field is exactly Hamiltonian at each instant in time
  - 2D (axisymmetric or 1 degree of freedom for toroidal field): Exact solutions exist, no chaos
  - 3D: generally chaotic for any perturbation from 2D
- Plasma interaction -> changed field at next time step is not a Hamiltonian or near-Hamiltonian perturbation from equilibrium field
  - Does an average field exist, from which it is the perturbation?
  - For problem with fast and slow motions, yes in 2D, or 3D when the perturbation scales decouple from the slow variation (BBGKY). No in 3D when the fast and slow motions are coupled.

Are solutions smooth?

- Navier Stokes equation
  - Smooth, regular, unique solutions in 2D (J. Leray 1933)
  - In 3D, yes for small enough initial conditions (proof used fractal nature of NS flows, T. Kato 1984)
  - General existence/non-existence of smooth, regular, unique soln in 3D is one of the Clay Institute Millenium problems (worth $1M).
  - Related to behavior of vorticity $w = \nabla \times v$. (Vorticity convected by fluid in 2D.)
- Plasmas also have the current density $J = \nabla \times B$, similar question.
  - Important for fluid models of fusion plasmas at realistic resistivity
- Non-smooth solutions: volume, or even curved surface area are not conserved if divided into (many, but finite number of) unmeasurable pieces

Small perturbations

- Effect of nominally 'small' perturbation is not always small !
  - L. Trefethen and pseudospectra (book: Spectra and Pseudospectra, Princeton Univ Press, 2003)
- Non-normal matrices or operators
  - Extensions to ideal MHD: non-self-adjoint (flows, neoclassical stress tensor, two-fluid, GK particles)
  - W. Kerner, D. Borba et al., resistive MHD spectrum and eigenvalues
- Many applications, originally from numerical methods
  - Advection-diffusion, upwind advection
  - Krylov subspace convergence, GMRES non-Hermitian matrix solutions
  - Why spectral methods may have problems
- Failing: analysis mostly 1D and 2D. 3D introduces chaos -> not done.

Summary

- Rethink the physics of desired solutions to simulation problems
  - Smooth regular solutions?
  - Chaotic or partially chaotic
  - Small perturbations <-> small changes?
  - Is there a meaningful average as basis for analysis?
- Need better connections to mathematics and developments outside fusion at higher level than existing applied math/computer science ties
- Other needs: Visualization for next-generation large scale simulations
  - Two levels: interactive/debug and analysis/discovery
  - Visualization is essential (and difficult) for ELM simulations