Two-fluid Tearing and Saturation in Pinch Profiles

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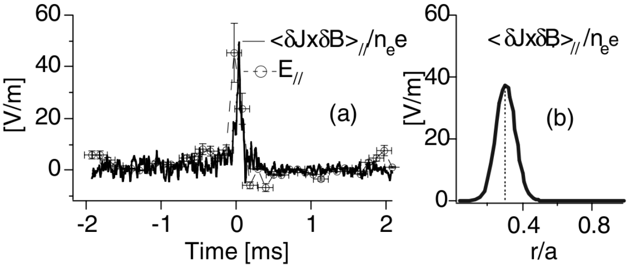
November 5, 2010

Measurements show the Hall effect is important in the Madison Symmetric Torus Reversed Field Pinch.



- Laser polarimetry is used to measurement magnetic field fluctuations during relaxation events in MST. (Ding et al. PRL 2004)
- The large Hall dynamo measurement indicates that ion-electron decoupling is significant.
- In MST, $\rho_s \sim 2 - 3cm$ and is at least comparable to a resistive skin depth.

Motivation

- We study linear and nonlinear single helicity tearing in pinch profiles to provide a basis for understanding these observations.

Outline

With sufficiently large FLR effects, the electron and ion fluids decouple in tearing modes.

- Drake and Lee (Phys. Fluids 1977) showed electrons slightly off the resonant surface are subject to an alternating $E_{\|}$ if ρ_i is large.
- Zakharov and Rogers (Phys. Fluids B 1992) and Rogers et al. (PRL 2001) recognized that decoupling occurs through KAW physics and ρ_s is the critical scale with a fluid model.
- $\rho_s = c_s/\omega_{ci} = \sqrt{\Gamma k\left(T_i + T_e\right) m_i}/eB = \sqrt{\Gamma\beta/2}d_i$
- This parameter is dependent on both the ion and electron temperatures and it is always greater the ion gyroradius.
- Mirnov et al. (PoP 2004) and Ahedo and Ramos (PPCF 2009) performed a detailed study of the β-d_i phase space for sheared slab equilibria.

Drift-tearing effects are also important for our pinch cases.

- The original work on drift-tearing (Coppi 1964) found significant stabilizing effects for large R/a tokamaks.
- Results described here show qualitatively similar effects with warms ions from $\nabla \mathbf{B}_0$ in pinch profiles even with $\nabla p_0 = 0$.
- Nonlinearly, cold ion island evolution is consistent with results from MHD as expected, (Biskamp 1979, Monticello and White 1980, Scott et al. 1985, Drake et al. 1983, Scott et al. 1987) but drift effects with warm ions do not vanish.

Our two fluid model includes the leading order effects from finite ion gyro-radii.

Generalized Ohms' Law

- $\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \mathbf{J} \times \mathbf{B}/ne - \nabla p_e/ne + \eta \mathbf{J} + \frac{m_e}{ne^2}\frac{\partial \mathbf{J}}{\partial t}$
- The Hall term captures the effect of fluid decoupling as $\mathbf{J}/ne = \mathbf{v}_i - \mathbf{v}_e$.
- We study cases in the experimentally relevant regime, where the resistive term dominates the contribution from electron inertia. (semi-collisional)

For warm ions computations, we also include a fluid ion gyroviscosity.

Momentum Equation

- $m_i n \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi}_{gyro} - \nabla \cdot \nu m_i n \underline{\underline{\mathbf{W}}}$
- Ion gyroviscosity is $\mathbf{\Pi}_{gyro} = \frac{m_i p_i}{4eB}\left[\hat{b} \times \mathbf{W} \cdot \left(\mathbf{I} + 3\hat{b}\hat{b}\right) - \left(\mathbf{I} + 3\hat{b}\hat{b}\right) \cdot \mathbf{W} \times \hat{b}\right]$.
  - It is important only with a warm ion population.
- The last term on the RHS is an isotropic viscosity, and we set $P_m = \nu/\eta = 0.1$.

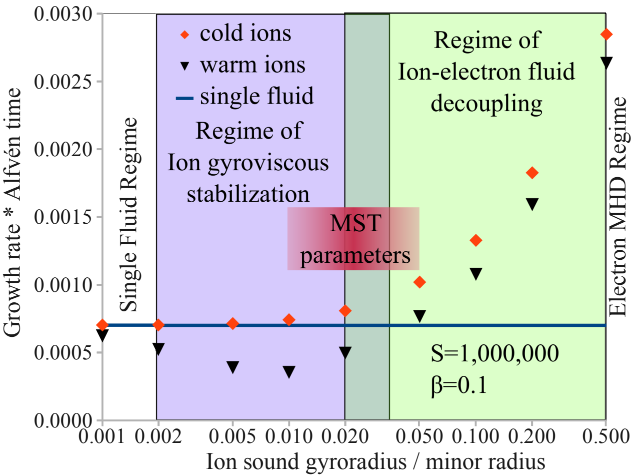
Our numerical studies examine linear tearing from the single fluid MHD limit to the electron MHD regime in cylindrical pinch profiles.

- We use a paramagnetic pinch equilibrium to study core resonant $m = 1$ tearing.
- There is no equilibrium pressure gradient - thus no diamagnetic flow.
- Cases have either $T_i = 0$ or $T_i = T_e$, but $\beta = 0.1$.
- We scan $\rho_s/a \left(= \sqrt{\Gamma\beta/2} d_i\right)$ by varying d_i and holding other dimensionless parameters fixed.

	MST	Simulation
$k_z a$	2	1.98
β	0.07	0.1
d_i/a	0.14	$0 - 1.7$
ρ_s/a	0.035	$0 - 0.5$

Our linear parameter scan shows three regimes.

The single fluid limit at small ρ_s, an intermediate regime of ion gyroviscous stabilization, and a regime with decoupled fluids at large ρ_s.



To understand the stabilizing effects with warm ions, we consider a reduced model with ion gyroviscosity.

- We use the $\left|r - r_s\right| / r_s \sim \epsilon$ as a small parameter and expand the ion gyroviscous tensor.
- We model $\tilde{v}$ with a stream function representation
  - $\tilde{v}_1 = \hat{\mathbf{b}}_0 \times \nabla \tilde{\phi}_1$
- And we note that radial derivatives are order $1/\epsilon$.
- Thus $\tilde{\phi}_1 \sim \mathcal{O}\left(1\right)$, $\tilde{\phi}_1' \sim \mathcal{O}\left(\epsilon^{-1}\right)$, $\tilde{\phi}_1'' \sim \mathcal{O}\left(\epsilon^{-2}\right)$, etc.

Terms associated with $\nabla \mathrm{B}_0$ contribute to gyroviscous effects in pinch profiles.

- Terms of $\mathcal{O}\left(\epsilon^{-3}\right) \sim \tilde{\phi}_1'''$ and lower order cancel.
- Direct contribution ($k_\perp = \hat{\mathbf{b}}_0 \times \hat{r} \cdot \mathbf{k}$, $b_\theta = B_\theta / B_0$)
  - $-\hat{\mathbf{b}}_0 \cdot \nabla \times \nabla \cdot \tilde{\mathbf{\Pi}}_{1gyro} \simeq -\frac{p_{i0}}{2\omega_{ci0}} \frac{b_\theta^2}{r} ik_\perp \tilde{\phi}_1'' - \frac{\partial}{\partial r}\left(\frac{p_{i0}}{\omega_{ci0}}\right) ik_\perp \tilde{\phi}_1'' + \mathcal{O}\left(\epsilon^{-1}\right)$
- Curvature contribution
  - $2\hat{\mathbf{b}}_0 \times \kappa_0 \cdot \nabla \cdot \tilde{\mathbf{\Pi}}_{1gyro} \backsimeq -\frac{p_{i0}}{\omega_{ci0}} \frac{b_\theta^2}{r} ik_\perp \tilde{\phi}_1'' + \mathcal{O}\left(\epsilon^{-1}\right)$

Linearized Vorticity Equation

- $i\omega m_i n_0 \tilde{U}_1 =$
  $-\mathbf{v}_0 \cdot \nabla \tilde{U}_1 + B_0 \hat{\mathbf{b}}_0 \cdot \nabla \left(\frac{\tilde{J}_{\parallel 1}}{B_0}\right) + 2\hat{\mathbf{b}}_0 \times \kappa_0 \cdot \left(\nabla \tilde{p}_1 + \nabla \cdot \tilde{\mathbf{\Pi}}_1\right) - \hat{\mathbf{b}}_0 \cdot \nabla \times \nabla \cdot \tilde{\mathbf{\Pi}}_1$

- Note $\tilde{U}_1 = \tilde{\phi}_1'' + \mathcal{O}\left(\epsilon^{-1}\right)$.
- The RFP has significant magnetic curvature and $\nabla \mathbf{B}_0$, unlike large aspect ratio tokamaks.

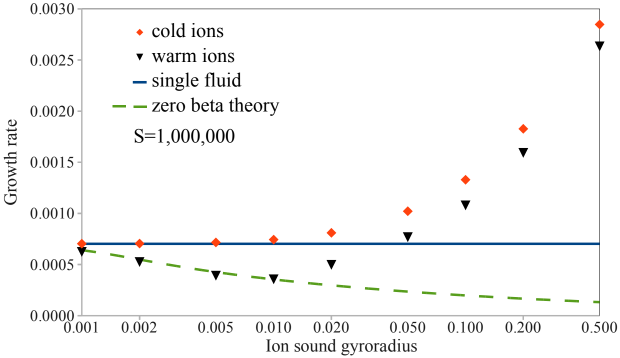
A dispersion relation with a resistive MHD Ohm's law illustrates the effects of ion gyroviscosity.

Heuristic Model Equations

- $(\gamma + i\omega_{*gyro})\,\tilde{U} = -v_A^2 \nabla_\| \nabla_\perp^2 \psi$
- $\frac{\gamma}{c}\psi + \nabla_\| \phi = \frac{\eta c}{4\pi}\nabla_\perp^2 \psi$

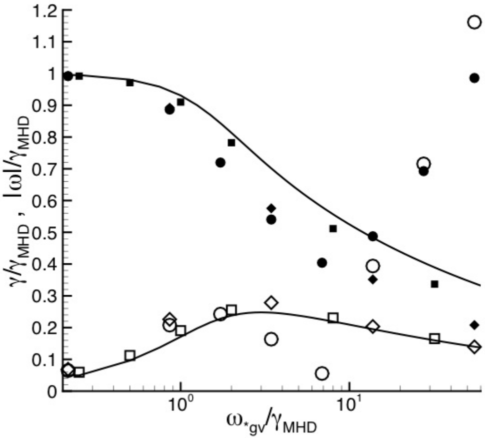
- It does not account for two-fluid electron dynamics or coupling to pressure.
- This yields $\omega^4\left(\omega - \omega_{*gyro}\right) = \omega_{MHD}^5$, where $\omega_{MHD} = i\gamma_{MHD}$ is the growth rate without the ion gyroviscous effects.
- $\omega_{*gyro} = \frac{k_\perp}{m_i n_0}\frac{p_{i0}}{\omega_{ci0}}\left[\frac{3}{2}\frac{b_\phi^2}{r} - \frac{B_0'}{B_0}\right] = k_\perp f_{Ti}\beta d_i v_A \left[\frac{3}{2}\frac{b_\phi^2}{r} - \frac{B_0'}{B_0}\right]$
- The contribution from p_i' cancels with the term $\mathbf{v}_{*i} \cdot \nabla \tilde{U}$.

The model dispersion relation exhibits the stabilizing behavior of our computations.



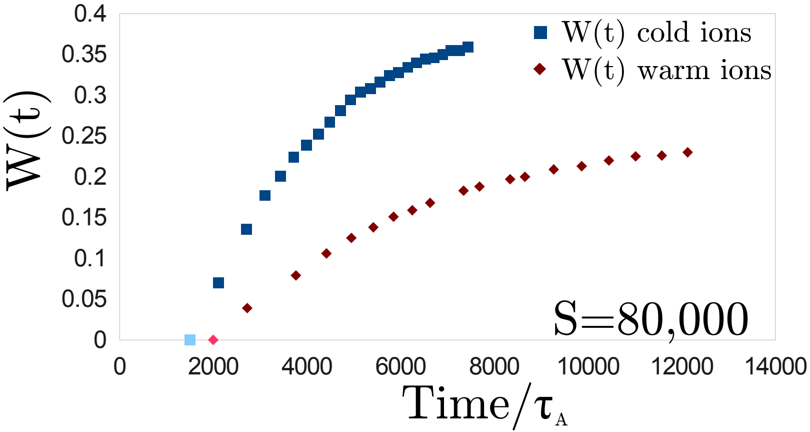
- Results with the simplified dispersion relation are consistent with our linear computations in the low and intermediate ρ_s regimes.

A more detailed comparison at zero β shows the ion gyroviscous effect is well captured by the model.



- Lines: model dispersion relation
- Squares: No perturbed pressure, ion G.V., resistive Ohm's law
- Diamonds: $\beta = 0.1$, ion G.V., resistive Ohm's law
- Circles: $\beta = 0.1$, ion G.V., two-fluid Ohm's law
- $Re\,(\omega)$ is the lower curve and unfilled symbols.
- $Im\,(\omega)$ is the upper curve and filled symbols.

Our nonlinear two-fluid tearing computations show a Rutherford phase prior to saturation.



For cold ion cases, we find the the saturation width is the same as single fluid MHD, regardless of ρ_s.

ρ_s	S	W (cold)
single fluid	5000	0.36
0.01	5000	0.36
0.05	5000	0.36
0.20	5000	0.36
0.05	8×10^4	0.36

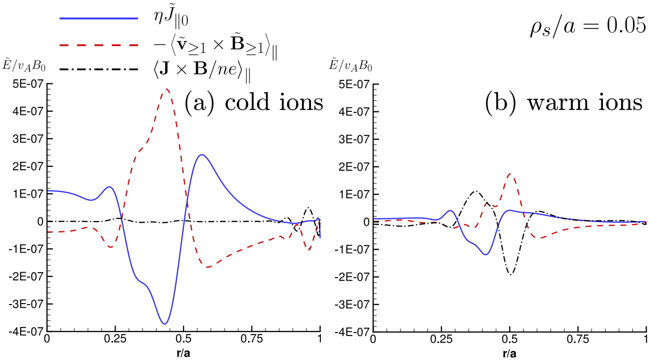
- This saturation width is consistent with previous drift tearing theory. (references earlier)
- We allow for the generation of higher m harmonics, however they are not observed to be large.

With realistic ρ_s/a, warm ion cases saturate at a smaller island width than the cold ion cases.

ρ_s	S	W (cold)	W (warm)
single fluid	5000	0.36	
0.01	5000	0.36	0.36
0.05	5000	0.36	0.24
0.05	8×10^4	0.36	0.24
0.20	5000	0.36	0.21

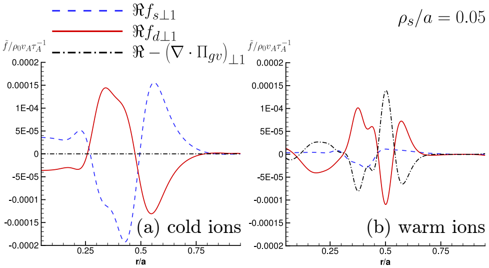
- The mode saturation width is reduced in warm ion cases at larger values of ρ_s.
- We find the saturation width is independent of Lundquist number for these configurations.

The dynamo electric fields modify the axisymmetric magnetic fields through the $m = 0$ induction equation.



- With cold ions, the saturation is equivalent to resistive MHD.
- With warm ions, there is a residual $\tilde{\mathbf{J}} \times \tilde{\mathbf{B}}$, hence a Hall dynamo.

The ion gyrovisous force does not vanish, and cases with warm islands saturate at a smaller island width.



$m = 1$ Momentum Equation

- $\imath\omega\rho\tilde{\mathbf{v}}_1 + (\rho\mathbf{v}\cdot\nabla\mathbf{v})_1 = \mathbf{f}_d - \nabla\cdot\mathbf{\Pi}_1 + \mathbf{f}_s$
- $\mathbf{f}_d = \mathbf{J}_{eq}\times\tilde{\mathbf{B}}_1 + \tilde{\mathbf{J}}_1\times\mathbf{B}_{eq} - \nabla\tilde{p}_1$ $\quad\quad$ $\mathbf{f}_s = \tilde{\mathbf{J}}_0\times\tilde{\mathbf{B}}_1 + \tilde{\mathbf{J}}_1\times\tilde{\mathbf{B}}_0$

- Ion gyroviscous forces add to the nonlinear secondary forces described by Rutherford.

Previous work finds that ion diamagnetic drift does not affect the island saturation width.

- These studies find that a finite island width eliminates the pressure gradient and thus the diamagnetic drift.
- See Biskamp 1979, Monticello and White 1980, Scott et al. 1985, Drake et al. 1983, Scott et al. 1987.
- In contrast, we have shown the stabilization from ion gyroviscosity is related to $\nabla \mathbf{B}_0$, which is still present in the final equilibrium state and significant in the RFP.
- $\omega_{*gyro} = \frac{k_\perp}{m_i n_0} \frac{p_{i0}}{\omega_{ci0}} \left[\frac{3}{2} \frac{b_\phi^2}{r} - \frac{B_0'}{B_0} \right] = k_\perp f_{Ti} \beta d_i v_A \left[\frac{3}{2} \frac{b_\phi^2}{r} - \frac{B_0'}{B_0} \right]$

Conclusions

- Our computations at parameters relevant to MST confirm the importance of the Hall dynamo and ion electron decoupling and find that effects from ion gyroviscosity are also likely significant.
- Ion gyroviscosity is linearly stabilizing and important in the RFP as a result of the relatively large magnetic curvature.
- The effect of ion gyroviscosity, unlike the effect of a diamagnetic drift, impacts the nonlinear island evolution and reduces the island saturation width.
- Multihelicity computations are needed to model the dynamic relaxation events in MST. (See our poster, Wednesday at 2PM, PP9.00070)

Additional Slides

Previous drift-tearing analysis applied large aspect ratio tokamak ordering.

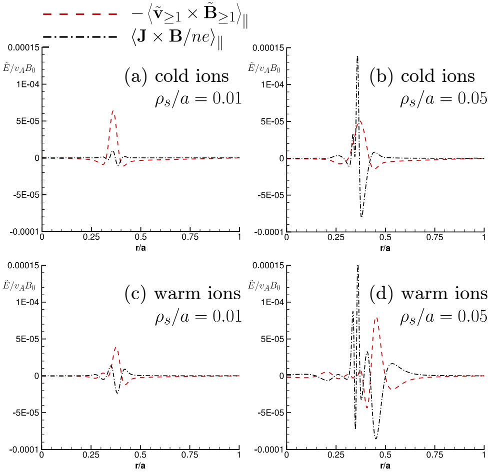
The reduced form of ion gyroviscosity cancels with the inertia term from the diamagnetic drift.

Linearized Vorticity Equation

- $i\omega m_i n_0 \tilde{U}_1 =$
  $-\mathbf{v}_0 \cdot \nabla \tilde{U}_1 + B_0 \hat{\mathbf{b}}_0 \cdot \nabla \left(\frac{\tilde{J}_{\|1}}{B_0} \right) + 2\hat{\mathbf{b}}_0 \times \kappa_0 \cdot \left(\nabla \tilde{p}_1 + \nabla \cdot \tilde{\mathbf{\Pi}}_1 \right) - \hat{\mathbf{b}}_0 \cdot \nabla \times \nabla \cdot \tilde{\mathbf{\Pi}}_1$

- The ion diamagnetic drift is $\mathbf{v}_0 = \mathbf{v}_{*i} = \hat{\mathbf{b}}_0 \times \nabla p_{i0} / \omega_{ci0} m_i n_0$.
  - Thus $\mathbf{v}_0 \cdot \nabla \tilde{U}_1 = i k_\perp p'_{i0} \tilde{U} / \omega_{ci0} m_i n_0$
- Coppi's drift-tearing analysis (1964) cancels leading-order ion gyroviscous effect with ion diamagnetic drift.
  - $\hat{\mathbf{b}}_0 \cdot \nabla \times \nabla \cdot \tilde{\mathbf{\Pi}}_1 \simeq -i k_\perp p'_{i0} \tilde{U} / \omega_{ci0} m_i n_0$
- The RFP has significant magnetic curvature and ∇B_0, unlike large aspect ratio tokamaks.

The Hall dynamo induced by the linear fluctuations is consistent with previous theories.



- The Hall dynamo calculated from the linear perturbation is nonzero and radially localized to rational surface as previously described.
- Mirnov et al. (Plasma Phys. Reports 2003) treated the Hall dynamo from the linear fluctuations in slab geometry.

A helical projection of the island magnetic fieldlines