Novel Experiments on the H-Regime and Theory of the Quasi-Coherent Mode Consistent with Them

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Foreword

There are several well identified plasma confinement regimes. Three of them involve excellent confinement properties and can be characterized by the modes that are excited at the edge of the plasma column.

The considered regimes are

— the EDA-H-Regime

— the I-Regime

— the ELMy-H-Regime

An interpretation is provided for the corresponding three types of the modes that have been observed combining both the theory and the relevant experimental information.

Plasma Confinement Regimes and Collective Modes Characterizing Them

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See also posters	*TP*9.00041	for the Quasi-Coherent Mode,
	*TP*9.00040	for the I-mode,
	9:30 AM, Thursday	.

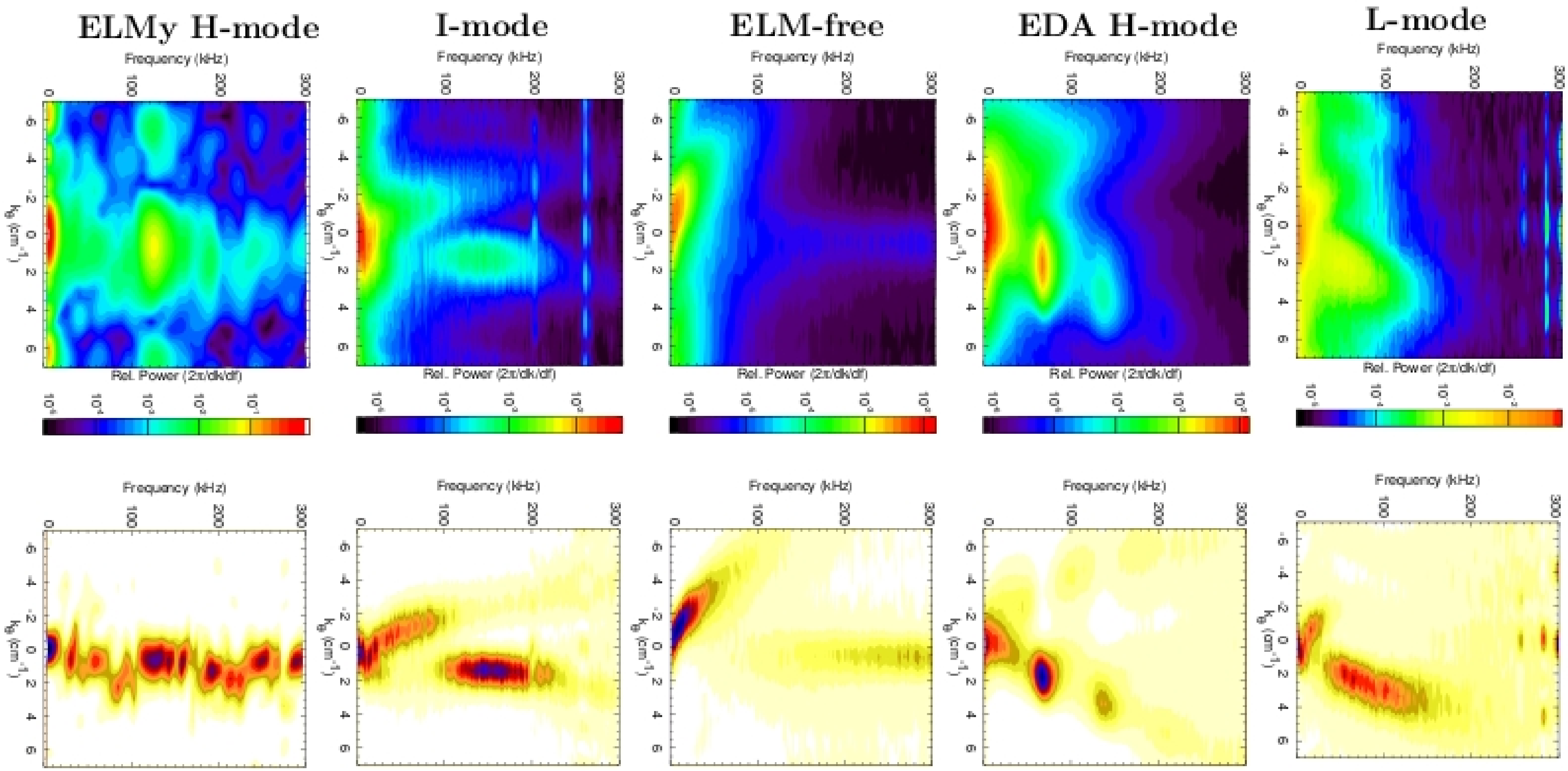

Figure: Confinment regimes and fluctuations associated [1].

[1] I. Cziegler, MIT Ph. D. thesis, June 2011.

Experimental observations on the QCM

- The so-called Quasi-Coherent Mode (QCM) is observed in the EDA-Regime with frequency $\omega_L = 80 - 150$ kHz in the lab frame.
- The radial electric field $\boldsymbol{E}_r$ has a deep negative well close to the last closed flux surface.

Around the E_r well close to the plasma edge,

$$E_r \simeq -\frac{\mathrm{v}_{\theta_\mathrm{i}}}{c} B_\phi,$$

$$\frac{1}{eZ_i n_i}\frac{dp_i}{dr} \simeq \frac{\mathrm{v}_{\phi \mathrm{i}}}{c} B_\theta .$$

So the ions $\left(\text{e.g., } B^{5+}\right)$ are not electrostatically confined.

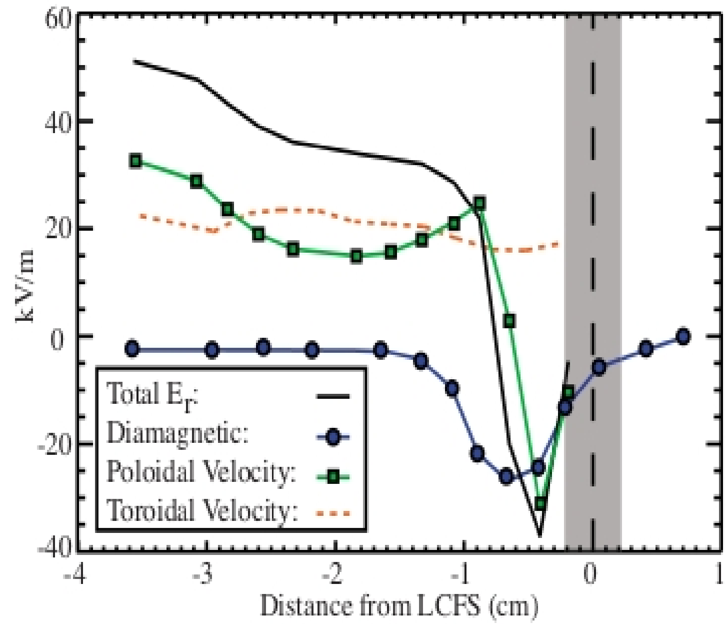

Figure: E_r well. [a]

[a] R. McDermott, B. Lipschultz, J. Hughes, et al., Phys. Plasmas, **16** (2009) 056103.

Experimental observations on the QCM (cont.)

- The $E \times B$ drift velocity $\mathrm{v}_E = -cE_r/B_\phi$ is in the direction of the electron diamagnetic drift $\mathrm{v}_{de}^{\theta} \equiv -\left[c/\left(en_e B_\phi\right)\right] dp_e/dr$.
- The phase velocity in the Lab frame ω_L/k_θ is in the direction of v_{de}^{θ} while the phase velocity in the plasma frame $\omega'/k_\theta = \omega_L/k_\theta - \mathrm{v}_E$ is in that of the ion diamagnetic velocity $\mathrm{v}_{di}^{\theta} \equiv \left[c/\left(en_i B_\phi\right)\right] dp_i/dr$.

Table: Comparison of the QCM and WCM mode propagation in the lab frame and the plasma frame [1]

	$\mathrm{v}_{\mathrm{lab}}^{\mathrm{mode}}$	E_r	$\mathrm{v}_{\mathrm{plasma}}^{\mathrm{mode}}$
QCM	2.5km/s	-44.5 ± 15kV/m	-4.5 to -13.5km/s
WCM	10.1km/s	-20 to 18kV/m	4.5 to 14.5km/s

[1] I. Cziegler, MIT Ph. D. thesis, June 2011.

Experimental observations on the QCM (cont.)

- Estimated $k_\theta \mathrm{v}_E$ and ω_{di}.

$$k_\theta \mathrm{v}_E \simeq 4.0 \times 10^6 \mathrm{sec}^{-1} \left[\frac{E_r}{40\,\mathrm{kV/m}}\right] \left[\frac{5\mathrm{T}}{B_\phi}\right] \left[\frac{m^0}{100}\right] \left[\frac{0.2\mathrm{m}}{r_0}\right].$$

$$|\omega_{di}| \equiv k_\theta \left|\mathrm{v}_{\mathrm{di}}^\theta\right| \simeq 3.1 \times 10^6 \mathrm{sec}^{-1} \left[\frac{T_i}{300\,\mathrm{eV}}\right] \left[\frac{5\mathrm{T}}{B}\right] \left[\frac{m^0}{100}\right] \left[\frac{0.2\,\mathrm{m}}{r_0}\right] \left[\frac{1\,\mathrm{cm}}{r_{pi}}\right].$$

- The $E \times B$ drift velocity shifts the mode frequency close to ω_{di}.

Experimental observations on the QCM (cont.)

- As the "Spontaneous Rotation" of the core plasmas is in the direction of the ion diamagnetic velocity, angular momentum is considered to be scattered out of the plasma column in that of electron diamagnetic velocity in the lab frame, according to the Accretion Theory [2].

[2] B. Coppi, Nucl. Fusion, **42** (2002) 1.

Simplified Plane Geometry Model

1. Equilibrium fields:

$$\begin{aligned} \text{Uniform magnetic fields:} \quad \boldsymbol{B} &\simeq B_y \boldsymbol{e}_y + B_z \boldsymbol{e}_z, \qquad \left|B_y\right| \ll \left|B_z\right|; \\ \text{electric fields:} \quad \boldsymbol{E} &= E\left(x\right) \boldsymbol{e}_x, \qquad E\left(x\right) < 0; \\ \text{uniform gravity:} \quad \boldsymbol{g} &= g \boldsymbol{e}_x \quad \text{to simulate the effects of} \\ & \qquad\quad \text{magnetic field line} \\ & \qquad\quad \text{curvature and gradient.} \end{aligned}$$

2. We find it simple to analyze an equilibrium configuration in which the ions are not electrostatically confined and have a diamagnetic velocity $\boldsymbol{u}_{di} \equiv -\left[c/\left(en_i B_y\right)\right] dp_i/dx \, \boldsymbol{e}_z$ besides the $E \times B$ drift velocity $\boldsymbol{u}_E \equiv -\left(cE/B_z\right) \, \boldsymbol{e}_y$.
3. Plasma inhomogeneities are in x-direction.
4. Perturbations are standing in the direction of $\boldsymbol{B}$ and propagating in the perpendicular direction.

"Viscous" ballooning modes

- We can take propagating solution

$$\hat{\mathrm{v}}_x = \tilde{\mathrm{v}}_{0x} \exp\left(-i\omega t + ik_y y + ik_z z\right)$$

as the mode dispersion relation is independent of the sign of $k_{||}$.

- The perturbed momentum equation

$$-i\omega' \rho \hat{\mathrm{v}}_i \quad = \quad -\nabla\left(\hat{p} + \frac{1}{4\pi}\widehat{\boldsymbol{B}}\cdot\boldsymbol{B}\right) + \nabla\cdot\widehat{\boldsymbol{\Pi}}^i_\mu + \boldsymbol{g}\hat{\rho} + \frac{1}{4\pi}\boldsymbol{B}\cdot\nabla\widehat{\boldsymbol{B}}$$

leads to

$$-i\rho\left(\omega' + i\gamma_\mu\right)\hat{\mathrm{v}}_{ix} = i\frac{k_{||}}{4\pi}B\widehat{B}_x + g\hat{\rho}, \tag{1}$$

"Viscous" ballooning modes (cont.)

$$\omega' \equiv \omega - k_y u_E \text{ — mode frequency in the plasma frame,}$$
$$\widehat{\mathbf{\Pi}}^i_\mu \quad \text{— ion collisional viscous stress tensor,}$$
$$\gamma_\mu \equiv \left(k_\perp^2 \mu_\perp^i / \rho\right), \quad \mu_\perp^i = \frac{3}{10} \frac{n_i T_i \nu_{ii}}{\Omega_{ci}^2}.$$

Note: the contribution from $\boldsymbol{u}_{di}$ on the LFHS of the momentum equation cancels that from the gyro-viscous tensor on the RHS [3].

- $c\widehat{\boldsymbol{E}} + \widehat{\mathsf{v}_\mathbf{i} \times \boldsymbol{B}} \simeq 0$ and $c\nabla \times \widehat{\boldsymbol{E}} = i\omega \widehat{\boldsymbol{B}}$ give

$$\widehat{B}_x = -\frac{k_{||} B}{\omega' - \omega_{di}} \hat{\mathrm{v}}_{ix}, \qquad (2)$$
$$\omega_{di} \equiv k_y \mathsf{v}_{di}^y, \quad \mathsf{v}_{di}^y \equiv \frac{c}{e n_i B_z} \frac{dp_i}{dx}.$$

[3] B. Coppi, Phys. Rev. Lett. **12** (1964) 417.

"Viscous" ballooning modes (cont.)

- mass conservation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_i \cdot \nabla\right)\hat{\rho} + \hat{\mathbf{v}} \cdot \nabla \rho + \hat{\rho} \nabla \cdot \boldsymbol{u}_i + \rho \nabla \cdot \hat{\mathbf{v}} = 0$$

  yields

$$\hat{\rho} \simeq \frac{1}{i\left(\omega' - \omega_{di}\right)} \frac{d\rho}{dx} \hat{\mathrm{v}}_{ix} \tag{3}$$

  for $\nabla_\perp \cdot \widehat{\mathbf{v}}_\perp \simeq 0$ (valid for low β plasmas) and $\left|\nabla_{\shortparallel} \hat{\mathrm{v}}_{i\shortparallel}\right| \ll \left|\hat{\mathrm{v}}_x d\rho/dx\right|/\rho$.

- Eqs. (1), (2) and (3) give rise to the dispersion relation

$$\boxed{\left(\omega' + i\gamma_\mu\right)\left(\omega' - \omega_{di}\right) = k_{\shortparallel}^2 \mathrm{v}_A^2 - \gamma_{RT}^2}\,, \tag{4}$$

$$\mathrm{v}_A^2 \equiv \frac{B^2}{4\pi\rho} \quad \text{and} \quad \gamma_{RT}^2 \equiv -\frac{g}{\rho}\frac{d\rho}{dx}.$$

Discussion of the dispersion relation

- For the dispersion relation

$$\left(\omega' + i\gamma_\mu\right)\left(\omega' - \omega_{di}\right) = k_{\shortparallel}^2 \mathrm{v}_A^2 - \gamma_{RT}^2$$

we consider cases in which $\gamma_\mu / \left|\omega_{di}\right| < 1$,

1. $0 < \gamma_{RT}^2 - k_{\shortparallel}^2 v_A^2 < \omega_{di}^2/4$, we have

$$\omega' \simeq \omega_{di} + \delta\omega', \quad \delta\omega' \simeq \frac{k_{\shortparallel}^2 v_A^2 - \gamma_{RT}^2}{\omega_{di}^2}\left(\omega_{di} - i\gamma_\mu\right),$$

$$\mathrm{Im}\delta\omega' \simeq \frac{\gamma_{RT}^2 - k_{\shortparallel}^2 v_A^2}{\omega_{di}^2}\gamma_\mu > 0.$$

2. $\gamma_{RT}^2 - k_{\shortparallel}^2 v_A^2 = \omega_{di}^2/4$, i.e., marginally stable without γ_μ,

$$\omega' \simeq \frac{1}{2}\left[\omega_{di} + i\left(\left|\omega_{di}\right|\gamma_\mu\right)^{1/2}\right].$$

- In both cases, the mode phase velocities are in the direction of $\mathrm{v}_{\mathrm{di}}^{\mathrm{y}}$, the ion diamagnetic velocity.

Radially localized modes

- We take

$$\hat{\mathrm{v}}_x = \tilde{\mathrm{v}}_x(x)\exp\left(-i\omega t + ik_y y + ik_z z\right)$$

- Derivation of the radial differential equation.
  Combine the x and y-component of the momentum equation

$$\begin{aligned} -i\omega'\rho\hat{\mathrm{v}}_x &= -\frac{\partial \hat{p}_t}{\partial x} + g\hat{\rho} + \frac{i}{4\pi}k_{\shortparallel}B\widehat{B}_x, \quad \hat{p}_t \equiv \hat{p} + \frac{1}{4\pi}\boldsymbol{B}\cdot\widehat{B}, \\ -i\omega'\rho\hat{\mathrm{v}}_y &= -ik_y\hat{p}_t + \frac{i}{4\pi}k_{\shortparallel}B\widehat{B}_y, \end{aligned}$$

with $\nabla_\perp \cdot \hat{\mathrm{v}}_\perp \simeq 0$ and $\nabla_\perp \cdot \widehat{\boldsymbol{B}}_\perp \simeq 0$ (valid for low β plasmas),

$$\widehat{B}_x = -\frac{k_{\shortparallel}B}{\omega' - \omega_{di}}\hat{\mathrm{v}}_x$$

Radially localized modes (cont.)

and

$$\hat{\rho} \simeq \frac{1}{i\left(\omega' - \omega_{di}\right)} \frac{d\rho}{dx} \hat{\mathrm{v}}_x$$

we obtain

$$\left[\rho\omega'\left(\omega' - \omega_{di}\right) - \rho k_{\shortparallel}^2 \mathrm{v}_A^2\right] \left(1 - \frac{1}{k_y^2} \frac{d^2}{dx^2}\right) \tilde{\mathrm{v}}_x + \rho\gamma_{RT}^2 \tilde{\mathrm{v}}_x = 0,$$

where $\gamma_{RT}^2 \equiv -\left[1/\left(\rho R_c\right)\right] dp/dx$.

Radially localized modes (cont.)

- Expand $E(x)$, $\rho\,(x)$ and $p\,(x)$ around the minimum $x = x_0$ of $E(x)$,

$$
\begin{aligned}
E(x) &\simeq E(x_0)\left[1-\frac{x^2}{(r_E)^2}\right], &\qquad \frac{1}{(r_E)^2} &\equiv -\frac{1}{2E\,(x_0)}\frac{d^2E(x_0)}{dx^2};\\
\rho\,(x) &\simeq \rho\,(x_0)\left(1-\frac{1}{r_n}\right), &\qquad \frac{1}{r_n} &\equiv -\left[\frac{1}{\rho}\frac{d\rho}{dx}\right]_{x_0};\\
\frac{dp}{dx} &\simeq \left.\frac{dp}{dx}\right|_{x_0}\left(1-\frac{x}{r_p}\right), &\qquad \frac{1}{r_p} &\equiv \left[\frac{d^2p/dx^2}{dp/dx}\right]_{x_0}.
\end{aligned}
$$

Then the radial differential equation becomes

$$
\frac{2}{\left(\gamma_{RT}^2\right)_0}\left\{\omega_D\delta\omega+\frac{\omega_0'^4}{16\omega_D\left|\omega_E^0\right|}\left(\frac{r_E}{r_*}\right)^2\left[1+\frac{r_*}{r_p}\frac{\left(k_{\shortparallel}\mathrm{v}_A\right)_0^2}{\omega_0'^2}\right]^2\right\}\tilde{\mathrm{v}}_x
-\frac{2}{\left(k_y r_E\right)^2}\frac{\omega_D\left|\omega_E^0\right|}{\left(\gamma_{RT}^2\right)_0}\bar{x}^2\,\tilde{\mathrm{v}}_x+\frac{d^2}{d\bar{x}^2}\tilde{\mathrm{v}}_x=0,
$$

Radially localized modes (cont.)

where

$$\bar{x} \equiv k_y x + \frac{(k_y r_E)^2}{4 k_y r_*} \frac{\omega_0'^2}{\omega_D \left|\omega_E^0\right|} \left[1 + \frac{r_*}{r_p} \frac{(k_{\shortparallel} \mathrm{v_A})_0^2}{\omega_0'^2} \right],$$

$$\omega_D^2 \equiv \left(\frac{\omega_{di}^0}{2} \right)^2 + (k_{\shortparallel} \mathrm{v_A})_0^2 - \left(\gamma_{RT}^2\right)_0,$$

quantities with 0-script are evaluated at $x = x_0$ and $r_*/r_p \equiv r_n/\left(r_p - r_n\right)$.

- The radial differential equation has localized solution

$$\tilde{\mathrm{v}}_x = \tilde{\mathrm{v}}_{x0} \exp\left(-\frac{\alpha}{2} \bar{x}^2 \right)$$

with

$$\alpha = \frac{1}{\left|k_y\right| r_E} \frac{\sqrt{2 \omega_D \left|\omega_E^0\right|}}{\left(\gamma_{RT}\right)_0}$$

Radially localized modes (cont.)

and

$$\delta\omega = \frac{1}{|k_y|\, r_E}\sqrt{\frac{|\omega_E^0|}{2\omega_D}}\,(\gamma_{RT})_0 - \frac{1}{16}\left(\frac{r_E}{r_*}\right)^2 \frac{\omega_0'^4}{\omega_D^2\,|\omega_E^0|}\left[1 + \frac{r_*}{r_p}\frac{(k_{||}\mathrm{v_A})_0^2}{\omega_0'^2}\right]^2 .$$

when $\omega_0' = \omega_{di}^0/2 + \omega_D$ is real and

$$0 < \omega_D^2 < \left(\frac{\omega_{di}^0}{2}\right)^2 .$$

So it is the E-well that localizes the mode.

Toroidal Modes

- Simplified toroidal geometry

$$\boldsymbol{B} = \frac{1}{1 + r\cos\theta/R_0}\left[B_\zeta(r)\boldsymbol{e}_\zeta + B_\theta(r)\boldsymbol{e}_\theta\right], \quad |B_\zeta| \gg |B_\theta|\,.$$

- "Disconnected" mode approximation [4]

$$\hat{\mathrm{v}} \simeq \tilde{\mathrm{v}}(r_0,\theta)\exp\left\{-i\omega t + in^0\left[\zeta - q(r)\theta\right] + in^0\left[q(r) - q_0\right]F(\theta)\right\};$$

$$F(\theta) \text{ is odd in } \theta \; F(\theta) = \begin{cases} \pi, & \theta = \pi \\ 0, & -\pi < \theta < \pi \\ -\pi, & \theta = -\pi \end{cases}.$$

- "Quasi-flute" modes [5] are not represented by the "disconnected" approximation.

[4] B. Coppi and G. Rewoldt, *Advances in Plasma Physics*, **6**, 433 [Publ. John Wiley & Sons, Inc., (1976)]

[5] B. Coppi, Phys. Rev. Lett. **39** (1977) 939.

Toroidal Modes (cont.)

- Modes are even in θ and subject to the boundary conditions

$$\tilde{\mathrm{v}}_r\Big|_{\theta=\pm\pi} = 0 \qquad \text{and} \qquad \frac{d\tilde{\mathrm{v}}_r}{d\theta}\Big|_{\theta=\pm\pi} = 0.$$

- The equilibrium flow velocity is similar to that in the plane geometry,

$$\begin{aligned} \boldsymbol{u}_i &= \mathrm{v}_E\, \boldsymbol{e}_\theta + \mathrm{v}_{di}\, \boldsymbol{e}_\zeta, \\ \mathrm{v}_E &\equiv -\frac{cE_r(r)}{B_\zeta}, \\ \mathrm{v}_{di} &\equiv -\frac{c}{eB_\theta n}\frac{dp_i}{dr}. \end{aligned}$$

Ballooning mode equation

- Radial component of the momentum equation

$$
\begin{aligned}
- i\omega' \rho \hat{\mathrm{v}}_r &= -\boldsymbol{e}_r \cdot \nabla \hat{p}_t + \frac{1}{4\pi} \boldsymbol{e}_r \cdot \left(\boldsymbol{B} \cdot \nabla \widehat{\boldsymbol{B}} + \widehat{\boldsymbol{B}} \cdot \nabla \boldsymbol{B} \right), \\
\omega' &\equiv \omega + \frac{q^0 n^0}{r_0} \mathrm{v}_{\mathrm{E}}.
\end{aligned}
$$

- Terms involving the magnetic perturbations

$$
\begin{aligned}
\boldsymbol{e}_r \cdot \left(\boldsymbol{B} \cdot \nabla \widehat{\boldsymbol{B}} + \widehat{\boldsymbol{B}} \cdot \nabla \boldsymbol{B} \right) &\simeq 2\widehat{B}_{\shortparallel} B \kappa_r + \boldsymbol{B} \cdot \nabla \widehat{B}_r, \\
\kappa_r &\quad \text{— normal curvature,}
\end{aligned}
$$

for negligible magnetic shear and parallel equilibrium current around $r = r_0$.

Ballooning mode equation (cont.)

- Similar to the plane geometry, we have

$$\widehat{B}_r \simeq \frac{i}{\omega' - \omega_{di}} \boldsymbol{B} \cdot \nabla \hat{\mathrm{v}}_r \quad \text{and} \quad \hat{\rho} \simeq \frac{1}{i\left(\omega' - \omega_{di}\right)} \frac{d\rho}{dr} \hat{\mathrm{v}}_r .$$

- The adiabatic equation then gives

$$\hat{p} \simeq \frac{1}{i\left(\omega' - \omega_{di}\right)} \frac{dp}{dr} \hat{\mathrm{v}}_r .$$

- To the leading order, $\hat{p}_t = \hat{p} + B\widehat{BB}_{\shortparallel} / \left(4\pi\right) \simeq 0$. Thus,

$$B\widehat{B}_{\shortparallel} \simeq -\frac{4\pi}{i\left(\omega' - \omega_{di}\right)} \frac{dp}{dr} \hat{\mathrm{v}}_r .$$

- The radial component of the momentum equation leads to the ballooning mode equation

$$\boxed{4\pi\rho\omega'\left(\omega' - \omega_{di}\right) \hat{\mathrm{v}}_r = -\left[8\pi\kappa_r \frac{dp}{dr} \hat{\mathrm{v}}_r + \left(\boldsymbol{B} \cdot \nabla\right)^2 \hat{\mathrm{v}}_r\right]} .$$

Solution of the ballooning equation

- When the corrections from both B_θ and from r_0/R_0 are neglected, the ballooning equation can be rewritten as [6,7,8]

$$\boxed{\frac{d^2}{d\theta^2}\tilde{\mathrm{v}}_r + \left(G\cos\theta - \Gamma^2\right)\tilde{\mathrm{v}}_r = 0}\,, \tag{5}$$

where

$$G \equiv -\frac{8\pi\left(q^0\right)^2 R_0}{B_0^2}\frac{dp}{dr} > 0 \text{ and } \Gamma^2 \equiv -\frac{\left(q^0 R_0\right)^2}{\mathrm{v}_\mathrm{A}^2}\omega'\left(\omega' - \omega_{di}\right).$$

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[6] B. Coppi, M. Rosenbluth, *Plasma Physics and Controlled Nuclear Fusion Research*, **1** (1966) 617.

[7] B. Coppi, M. Rosenbluth, S. Yoshikawa, Phys. Rev. Lett. **20** (1968) 190.

[8] B. Coppi, Phys. Rev. Lett. **39** (1977) 939.

Solution of the ballooning equation (cont.)

- It is the standard Mathieu equation[9]

$$\boxed{\frac{d^2}{d\Theta^2}\tilde{\mathrm{v}}_r + \left[\sigma - 2\alpha\cos\left(2\Theta\right)\right]\tilde{\mathrm{v}}_r = 0},$$

$$\Theta \equiv \left(\pi - \theta\right)/2, \quad \sigma \equiv -4\Gamma^2 \quad \text{and} \quad \alpha \equiv 2G.$$

  But the standard even periodic Mathieu functions are not the solution of the ballooning modes for general G since they are subject to different boundary conditions.

- Although mathematically a solution of Eq. (5) for small G, "Quasi-flute" modes are not suitable ballooning solution since they are strongly affected by the magnetic shear and by the average magnetic curvature.

[9] *Handbook of Mathematical Functions*, Eds. M. Abramowitz, I. Stegun, Publ. Dover Publ. Inc..

Solution of the ballooning equation (cont.)

- Solution close to the marginal stability value of $G = G_c$,

$$G_c = \frac{\int_{-\pi}^{\pi} d\theta \, (d\tilde{\mathrm{v}}_r/d\theta)^2}{\int_{-\pi}^{\pi} d\theta \cos\theta \, (\tilde{\mathrm{v}}_r)^2}.$$

For trial function

$$\tilde{\mathrm{v}}_r = \tilde{\mathrm{v}}_{r0} \left(1 + \cos\theta\right),$$

$$G_c = \frac{1}{2} \quad \text{and} \quad \Gamma^2 \simeq \frac{2}{3}\left(G - \frac{1}{2}\right)$$

for $0 < G - G_c \ll 1$.

Solution of the ballooning equation (cont.)

- Solution in the strong ballooning case where $G \gg 1$.
  The asymptotic ballooning equation is

$$\frac{d^2}{d\theta^2}\tilde{\mathrm{v}}_r + \left[G\left(1 - \frac{\theta^2}{2}\right) - \left(\Gamma_0^2 + \delta\Gamma^2\right)\right]\tilde{\mathrm{v}}_r = 0$$

where $\Gamma_0^2 = G$, and the solution

$$\tilde{\mathrm{v}}_r = \tilde{\mathrm{v}}_{r0}\exp\left(-\sigma_\theta\frac{\theta^2}{2}\right), \quad \sigma_\theta^2 = G/2,$$

$$\delta\Gamma^2 = -\sigma_\theta, \quad \text{and} \quad \Gamma^2 = \sqrt{G}\left(\sqrt{G} - \frac{1}{\sqrt{2}}\right).$$

This strongly ballooning solution is the asymptotic form of the Mathieu cosine function $\mathrm{ce}_0\left[\left(\pi - \theta\right)/2, 2G\right]$ for large values of G.

Solution of the ballooning equation (cont.)

- Close to G_c, the mode is FLR stabilized

$$\omega' \left(\omega' - \omega_{di}\right) \simeq -\gamma_G^2, \quad \gamma_G^2 \equiv (2/3)\left(G - 1/2\right)\omega_A^2$$

where $4\ \gamma_G^2 < \omega_{di}^2$ because of the sharp pressure gradient at the plasma edge; but weakly unstable when the effects of the transverse viscosity is included,

$$\omega' \simeq \omega_{di} + i\frac{\gamma_G^2}{\omega_{di}^2}\gamma_\mu\ .$$

Resistive ballooning modes: considered as candidates explaining the observed ELMS in the ELMy H-Regimes.

- Consider the limit of large electron thermal conductivity, i.e., $\omega \ll k_{\shortparallel}\lambda_e k_{\shortparallel} \mathrm{v}_{\mathrm{the}}$, where λ_e is the electron mean free path, and $|\delta\omega \equiv \omega - \omega_{*e}| < D_m k_y^2$, where $D_m \equiv c^2\eta/(4\pi)$ and η is the plasma resistivity. The relevant mode is nearly electrostatic and is represented by the dispersion relation

$$\omega_{*e}\left(\omega_{*e} - \omega_{di}\right) + \gamma_{RT}^2 \simeq i\frac{\delta\omega\left(k_{\shortparallel}\mathrm{v}_{\mathrm{A}}\right)^2}{D_m k_y^2}.$$

Clearly, this mode has intrinsically different characteristics from those of the modes considered for the EDA-H Regime.

Resistive ballooning modes (cont.)

- A mode that is of of special interest is the strongly resistive mode that is found for $|\omega - \omega_{*e}| > D_m k_y^2$ is represented by the dispersion relation

$$\omega^2 \simeq -\gamma_{RT}^2 + k_{\shortparallel}^2 \mathrm{v}_A^2 - i\frac{k_y^2 D_m}{\omega} k_{\shortparallel}^2 \mathrm{v}_A^2$$

and corresponds to the ideal MHD marginal stability $\gamma_{RT}^2 = (k_{\shortparallel}\mathrm{v}_\mathrm{A})^2$. In this case

$$\omega = i\left(k_y^2 D_m k_{\shortparallel}^2 \mathrm{v}_\mathrm{A}^2\right)^{1/3}$$

as anticipated in the first analysis of resistive plasma instability [10], and can have phase velocities in either directions.

[10] B. Coppi, 1962.

Final Remarks

1. We have constructed a consistent theoretical picture of the edge modes that are the signature of three important confinement regimes.

2. While further experimental information (e.g., on the mode characteristics, the relevant spontaneous rotation and associated transport) will be needed to solidify this picture, a complementary theoretical effort will be devoted to analyze the modes that are excited in the main body of the plasma column.

3. A computational effort employing two-fluid codes will play an important role in providing a description of the non-linear evolution of the considered modes.

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[13] B. Coppi, J. Filreis, F. Pegoraro, Annals of Physics **121** (1979) 1.

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