Progress in Linear and Nonlinear Two-Fluid Resistive Plasma Response to Non-Axisymmetric Fields

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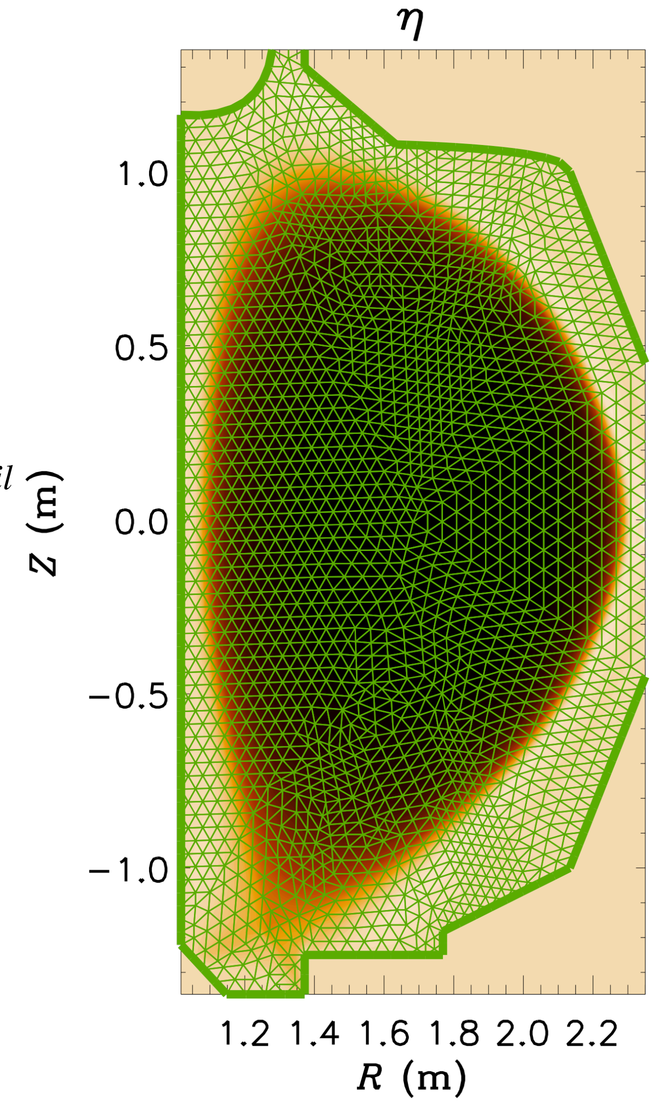
Outline

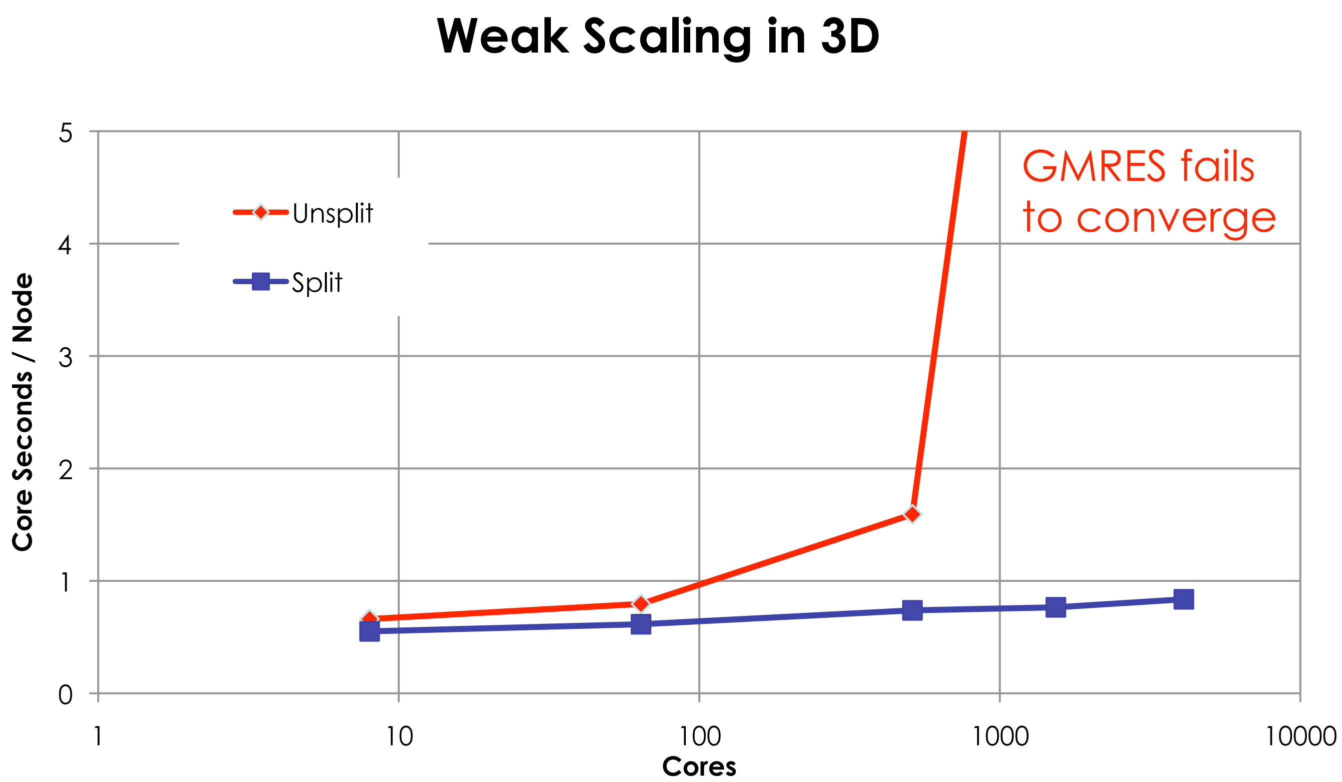
- **Miscellaneous Progress in M3D-C1**
- **Linear Results**
  - Influence of rotation
  - Ion rotation vs. Electron rotation
- **Linear vs. Nonlinear**

Miscellaneous Progress in M3D-C1

- **Ability to generate realistic mesh boundaries**
- **Ability to include non-axisymmetric coils inside of computational domain**
  - Fields are decomposed: $\mathbf{B} = \mathbf{B}_{eq}+\mathbf{B}_{plas}+\mathbf{B}_{coil}$
  - Terms involving $\mathbf{J}_{coil}$ are removed from eqns.
- **VisIt visualization (Sanderson)**
- **Transitioning focus from Linear → Nonlinear**



Split Method Scales Much Better Than Unsplit

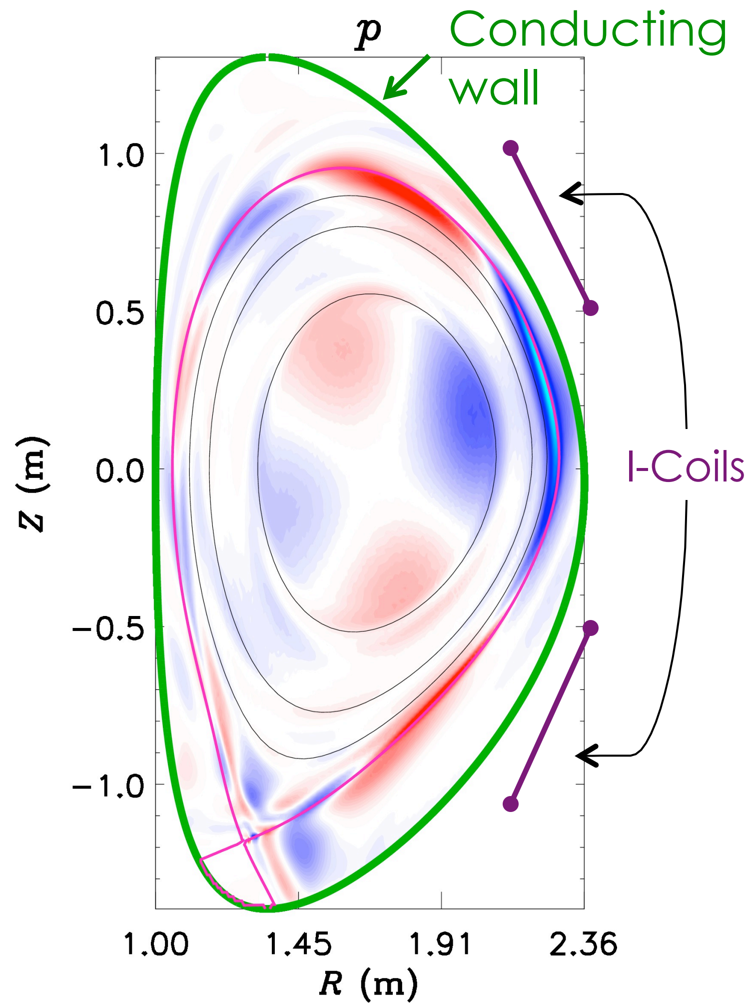


- **Unsplit method fails at large core counts**
  - (GMRES fails to converge)

3D Response

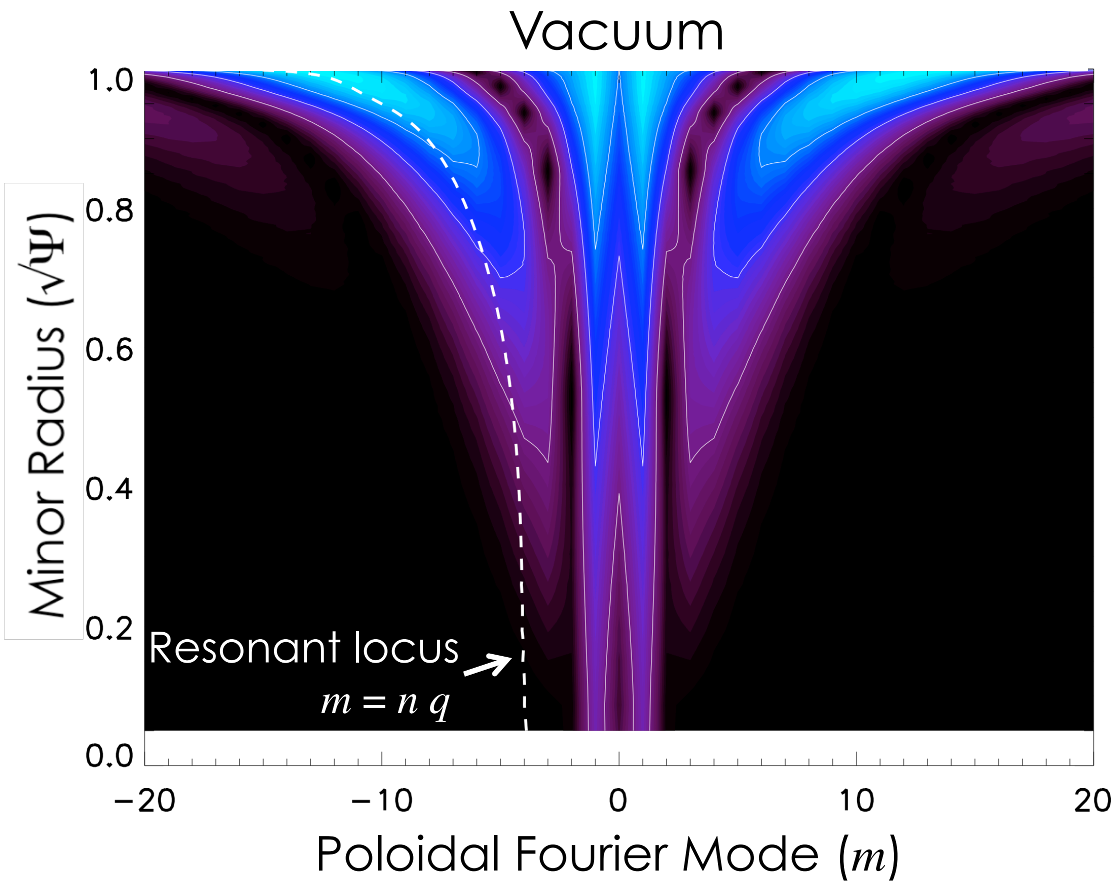
Analysis Considers Reconstructed DIII-D Equilibria

- **Vacuum fields generated by DIII-D I-coils**
- **Boundary conditions:**
  - Vacuum B_n is held constant at the boundary
  - No-slip ($\mathbf{v}=0$)
- **Realistic transport parameters**
  - Lundquist number ~ 10^9
- **Toroidal rotation**
  - Rotation is added self-consistently: $p \neq p(\psi)$

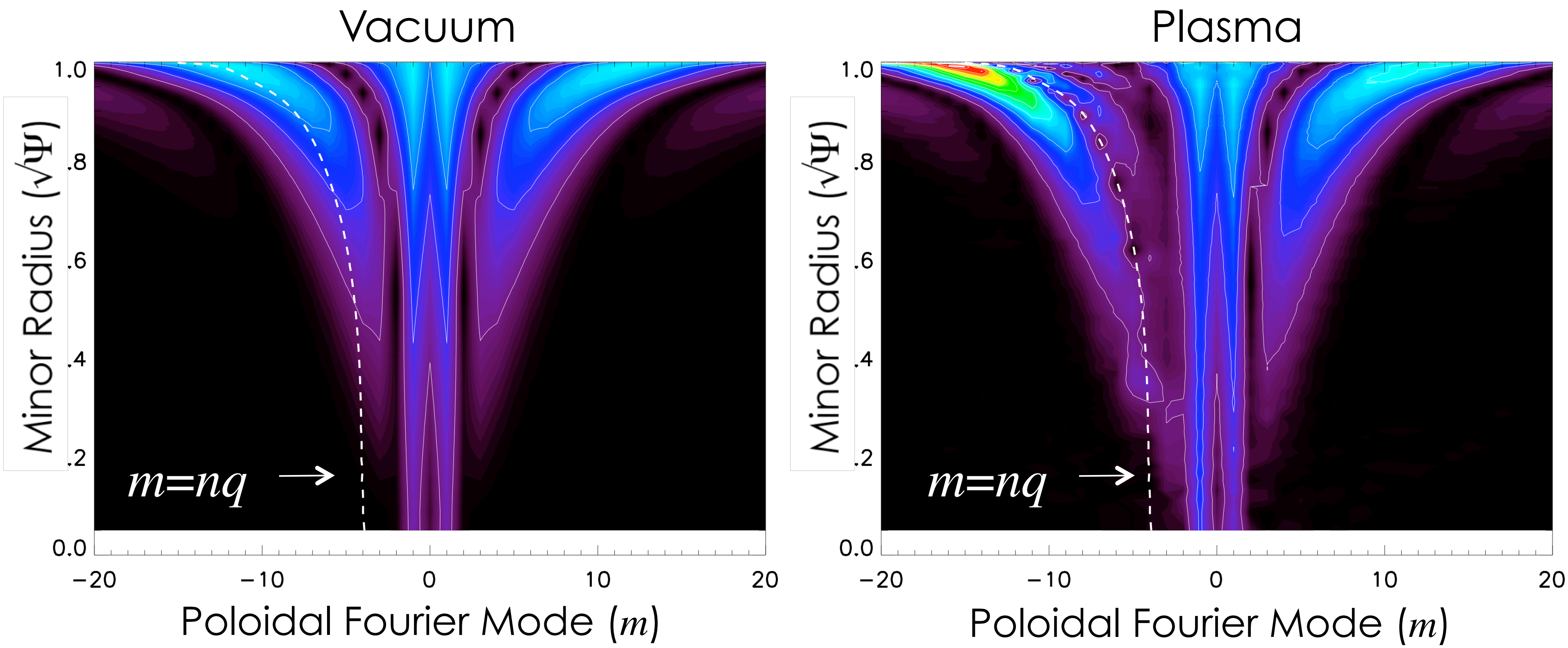


Non-Resonant Fields Bend Surfaces; Resonant Fields Tear Surfaces

- **Plot shows Fourier spectrum of B_n**
- B_n **= component of applied field normal to equilibrium magnetic surfaces**
- **Resonant components (along dashed line) cause islands**
- **Non-resonant components cause bending of surfaces**
- **Poloidal spectrum of B_n depends on Ψ**



Plasma Response Modifies Spectrum



- **Ideal response → no islands → reduction in resonant components**
- **Excited ideal modes → enhancement of non-resonant components**

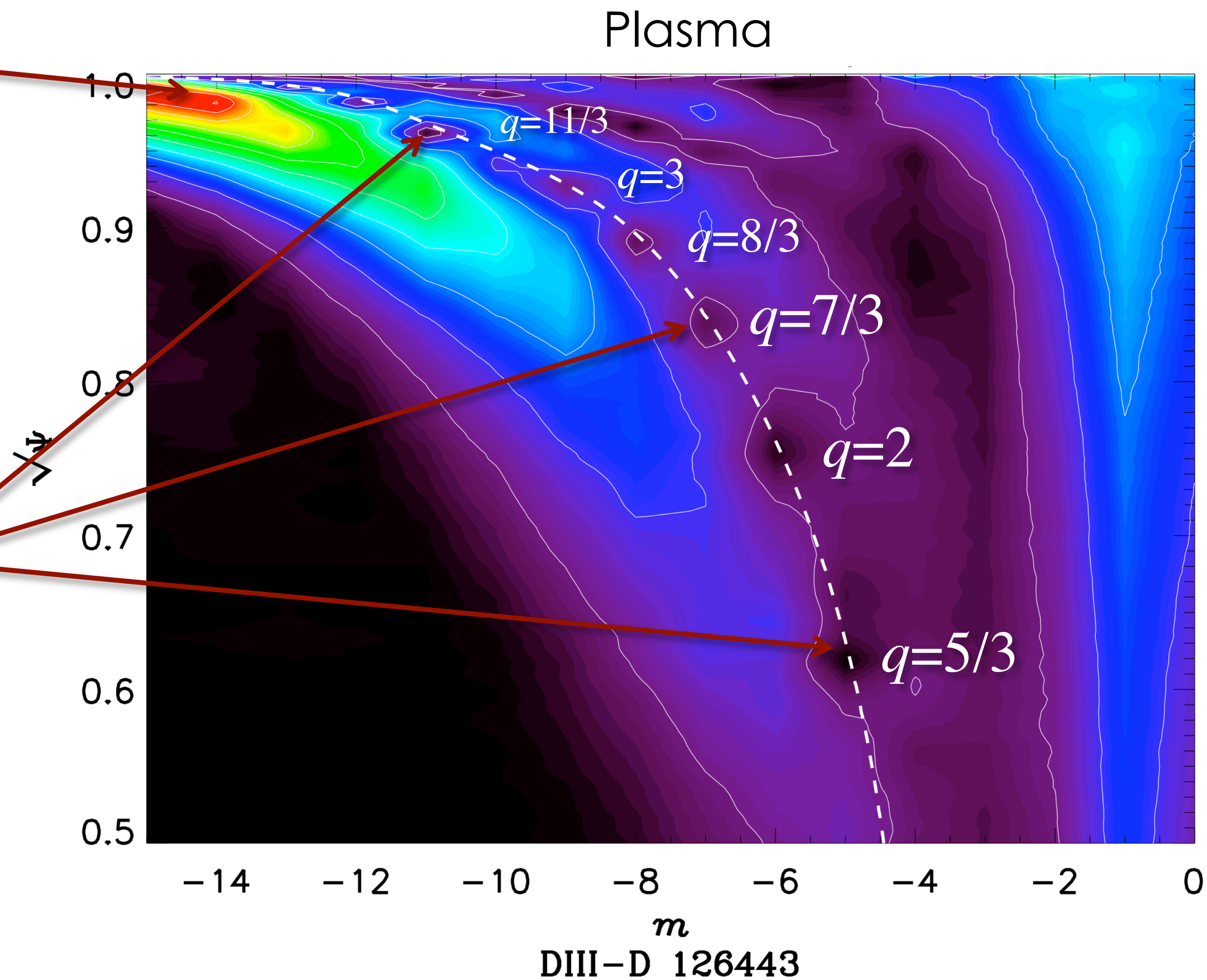
Plasma Can Kink and Screen

"Kinking"

- **Distorts surfaces**

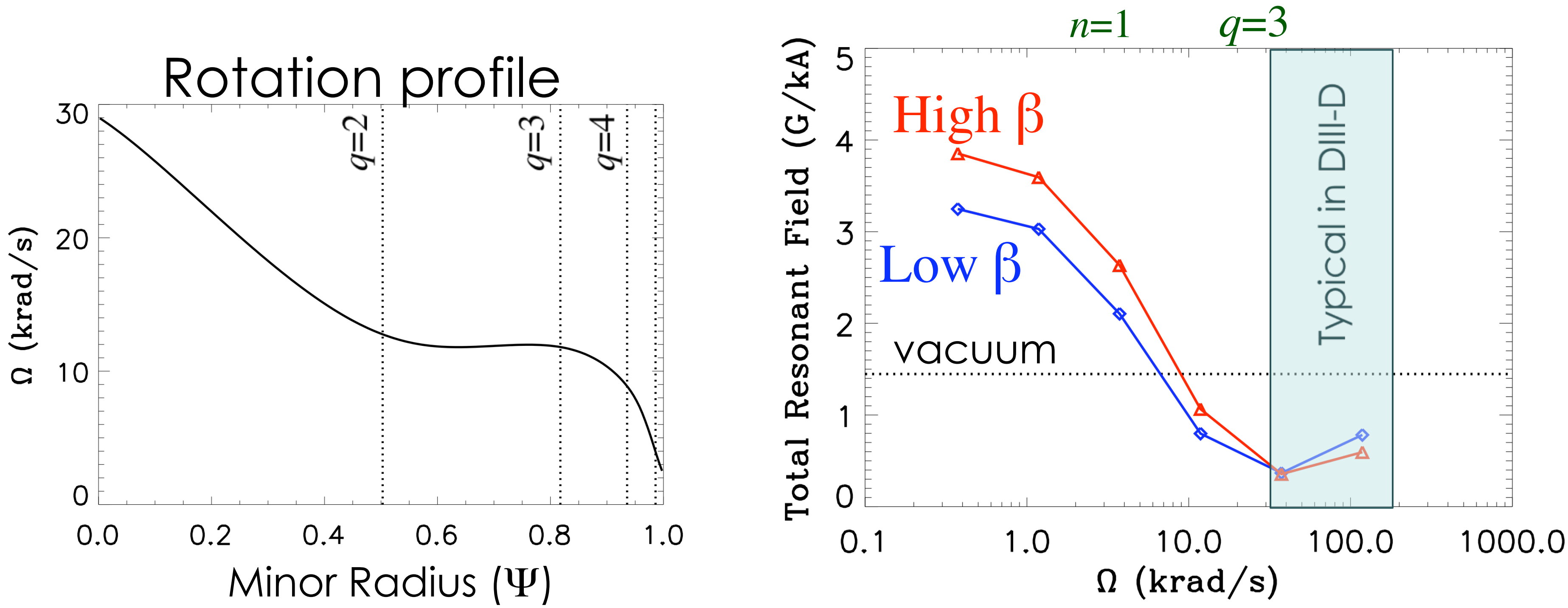
Screening

- **Eliminates islands**



Single-Fluid Results

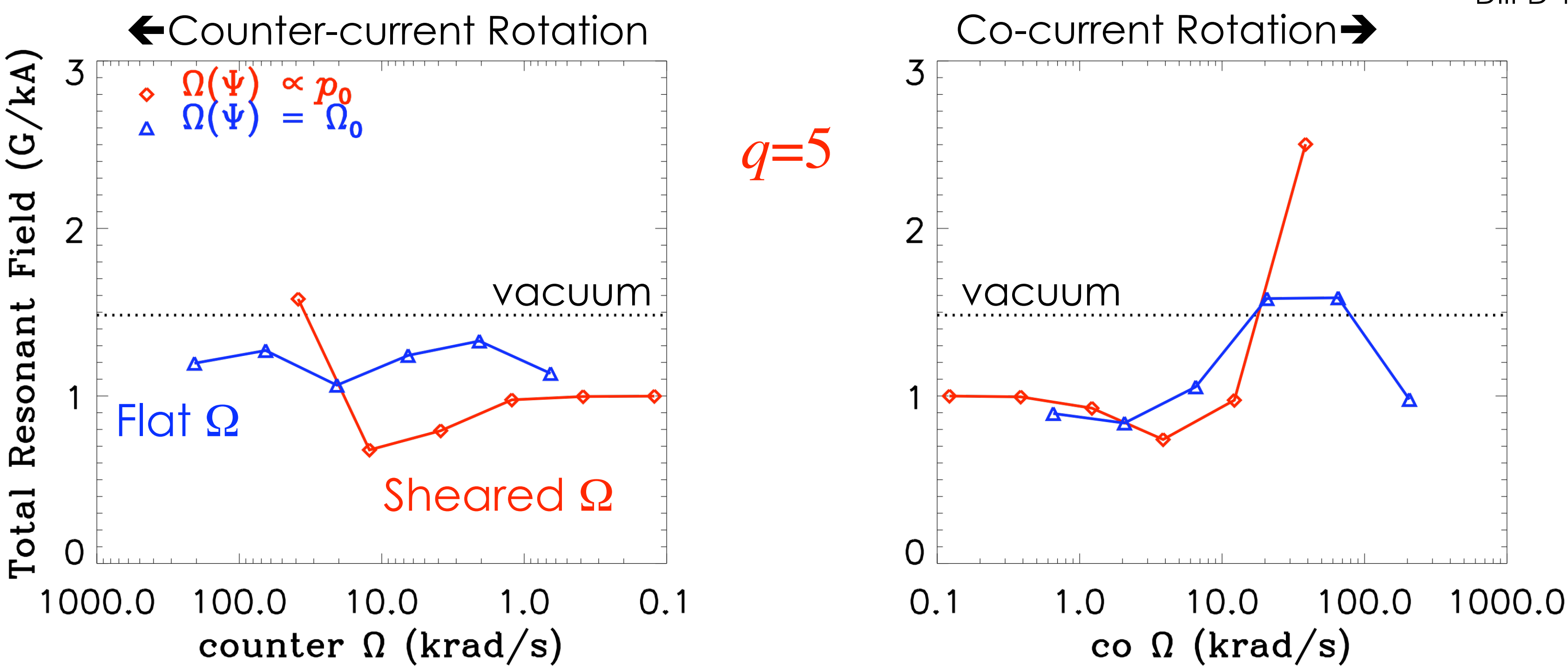
Single-Fluid Result:
Rotation (Usually) Improves Screening



- **Plasma may enhance resonant fields at low rotation**
- **Large rotation screens resonant fields**
- **Response depends on beta**

Single-Fluid Result: Rotation Shear Increases Edge Response

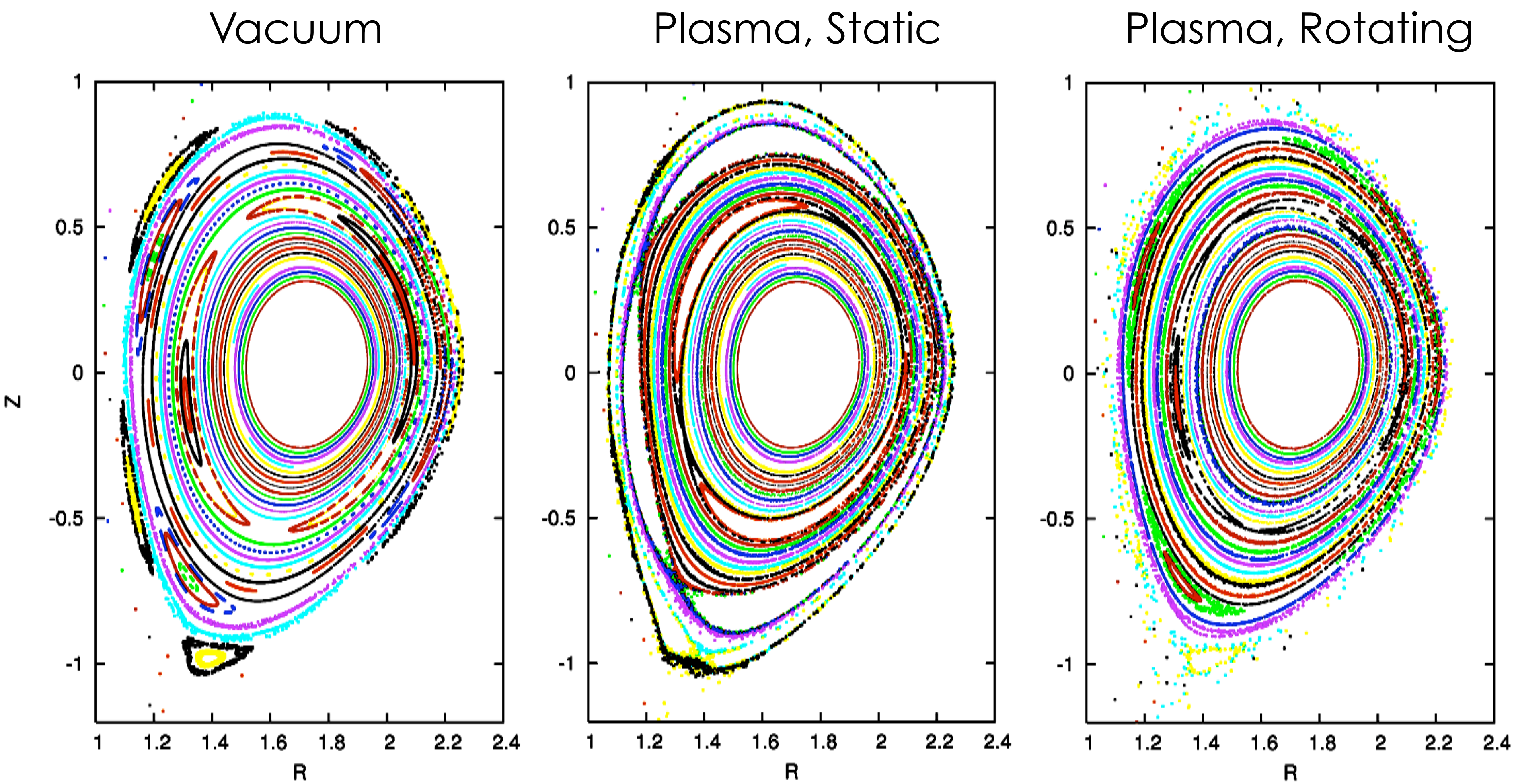
DIII-D 135762



- **Large rotation shear seems to increase edge response**
- **Why? Theory predicts Ω' is destabilizing to low-n peeling-ballooning modes***

* Snyder, *et al.* *Nucl. Fusion* **47** (2007)
Aiba, *et al.* *Nucl. Fusion* **50** (2010)
Ferraro, *et al.* *Phys. Plasmas* **17** (2010)

Rotation Improves Core Screening; But Sheared Rotation Stochasticizes Edge



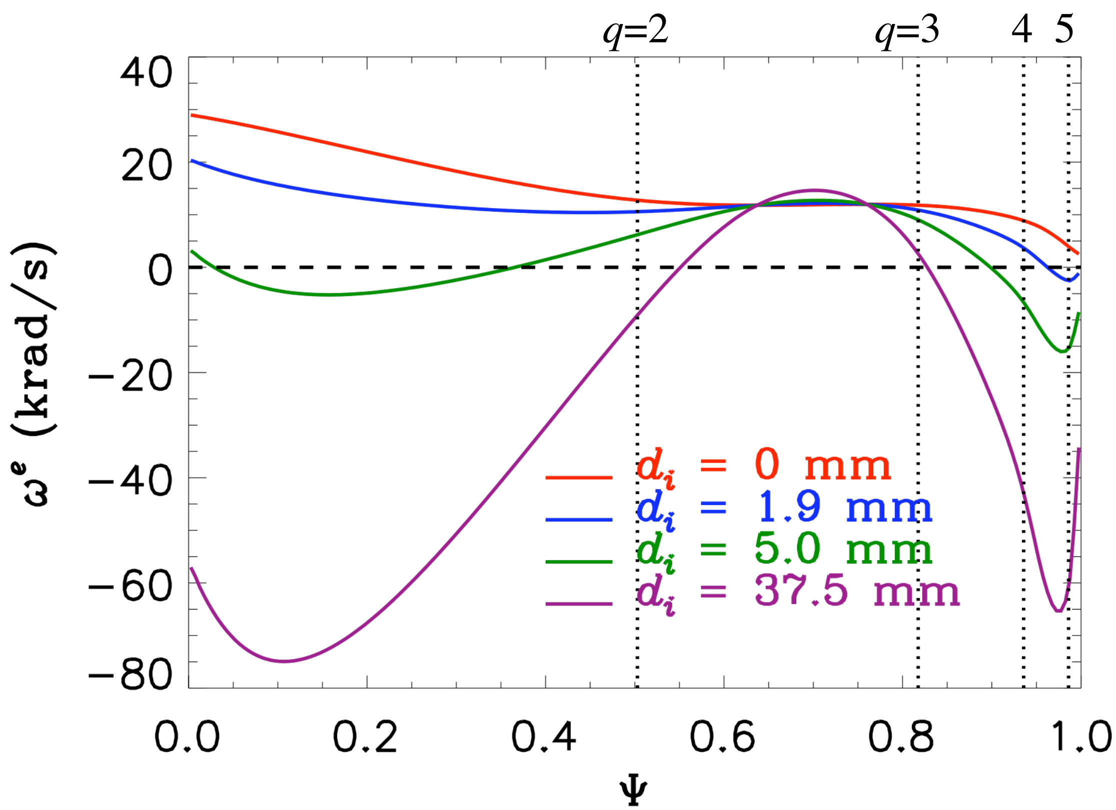
Ω_0=0

Ω_0=300 krad/s

Two-Fluid Results

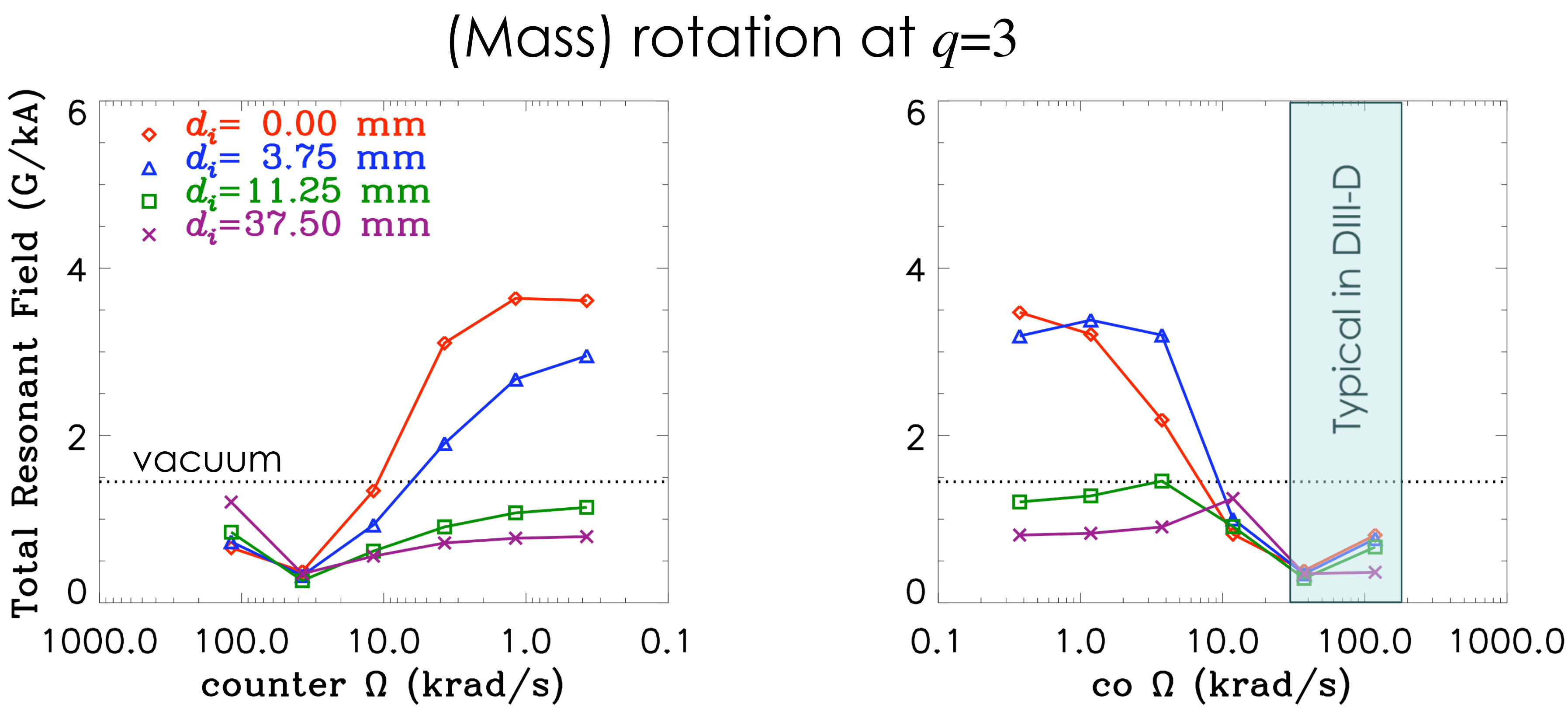
Two-Fluid Results: Ion and Electron Rotations are Distinct

- **Given Ω, we can change $\Omega^e = \Omega + \omega_*$ by adjusting** $\omega_* = d_i\, p'/n$



For this equilbrium, $d_i = 37.5$ mm is the physical value

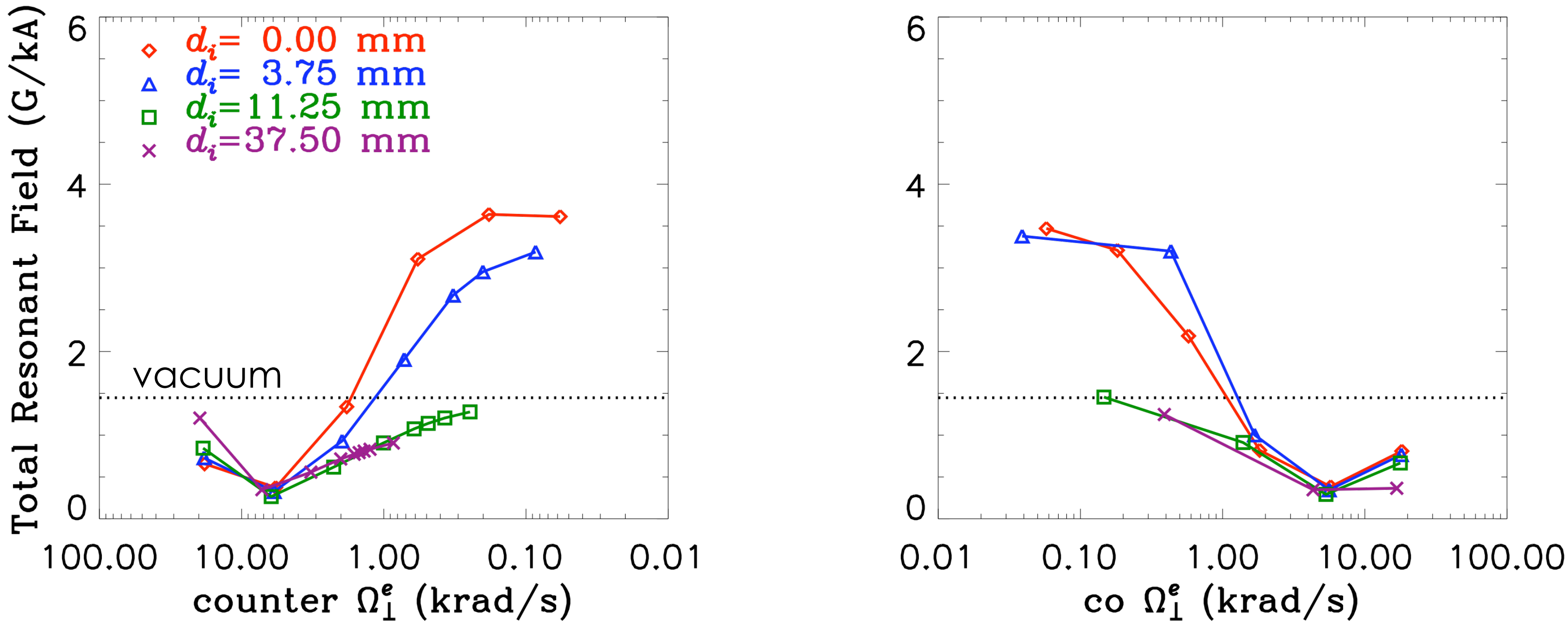
Two-Fluid Effects Shift Resonance



- **Strongest tearing no longer occurs at $\Omega = 0$**

Penetration In Core Depends on Electron Rotation

Perpendicular electron rotation at $q = 3$

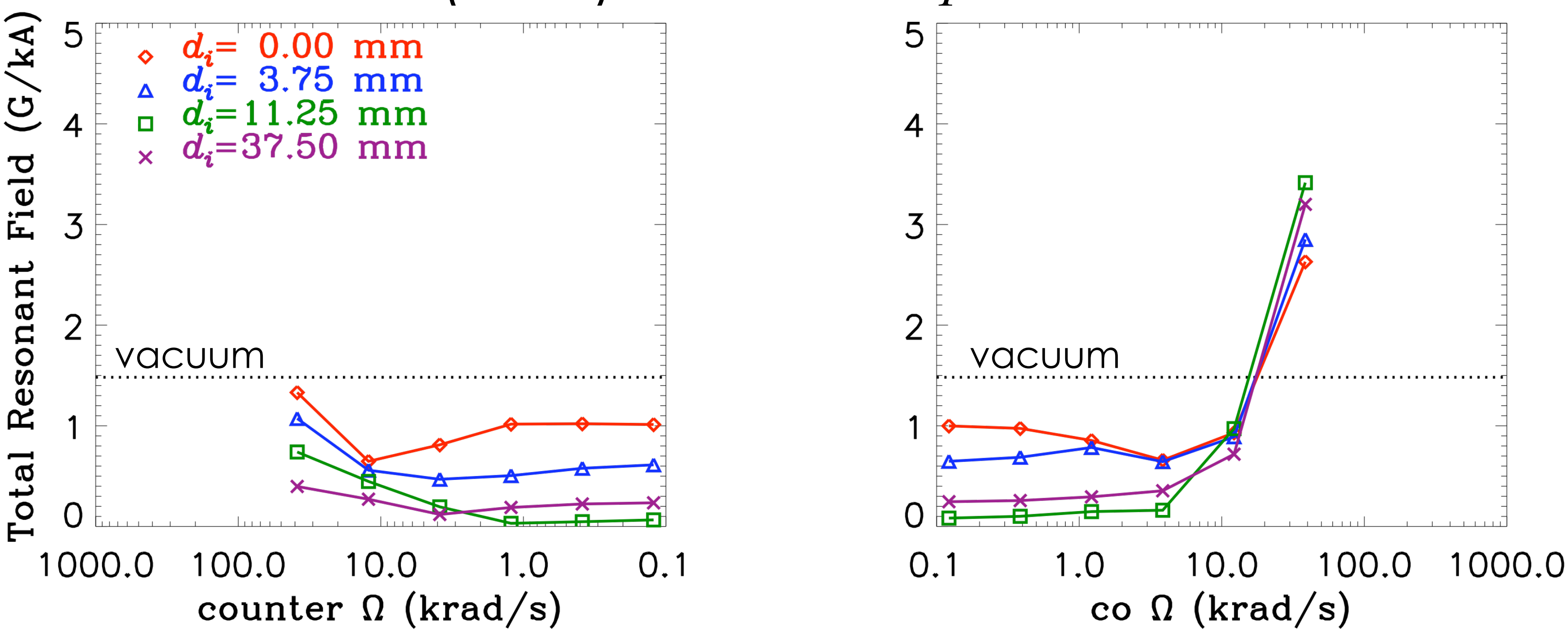


- **Screening of q=3 island clearly depends more on Ω^e than Ω**

Edge Response Depends on Mass Rotation Shear

- **Tearing of edge modes is dependent on ion, not electron, rotation shear**

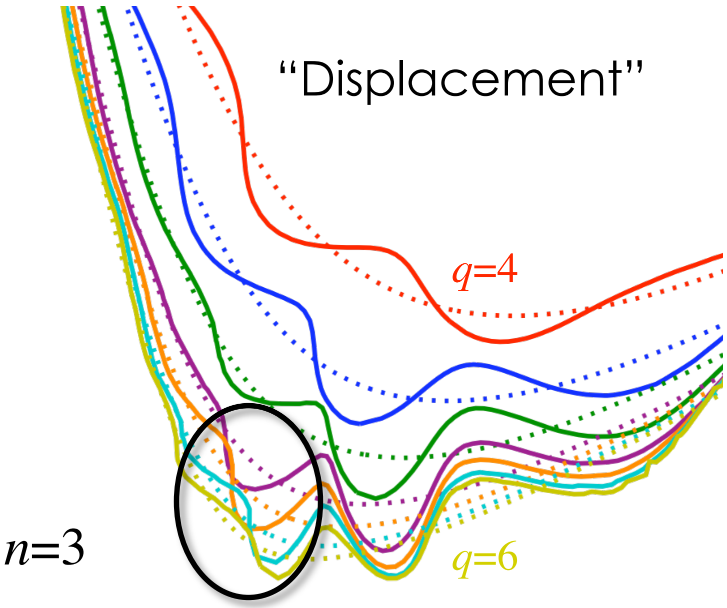
(Mass) rotation at q=5



Linear vs. Nonlinear

Is Linear Response Appropriate?

- **For typical experimental parameters, linear response may not be strictly valid in some regions**
  - Large current density near rational surfaces
  - Back-reaction on rotation is important



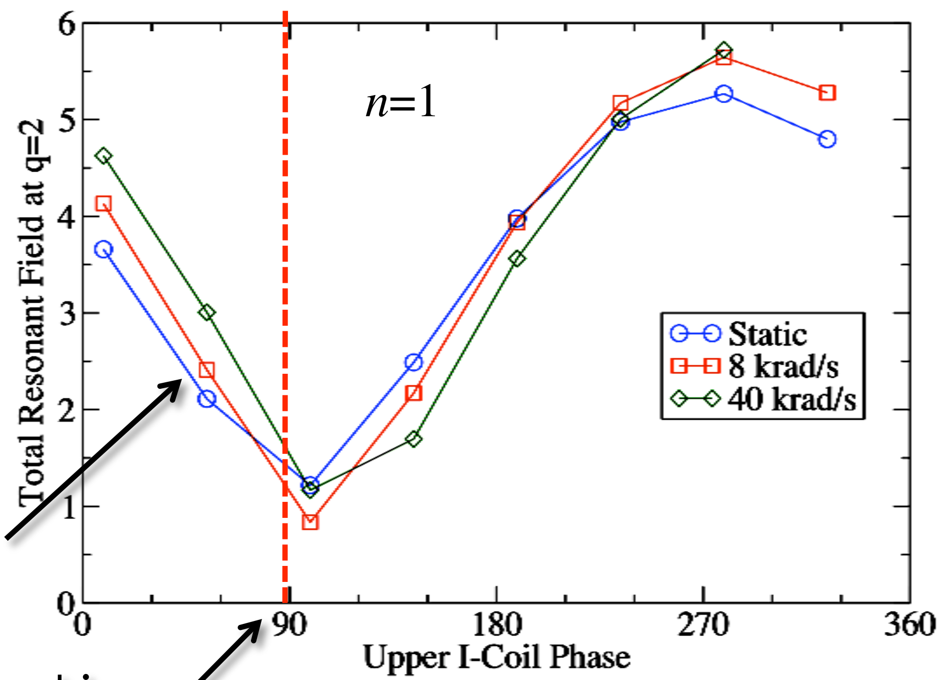
5 kAt even-parity I-Coil

  - "Displacement" shows overlapping surfaces near separatrix!

- **Quantitative predictions of island size, stochasticity from linear calculations are suspect**

Linear Response Gets Some Things Right

- **Which modes are most sensitive**
- **How parameters (rotation, viscosity, etc.) affect sensitivity**
- **How to optimize coil design**



Calculated resonant field (proxy for resonant torque)

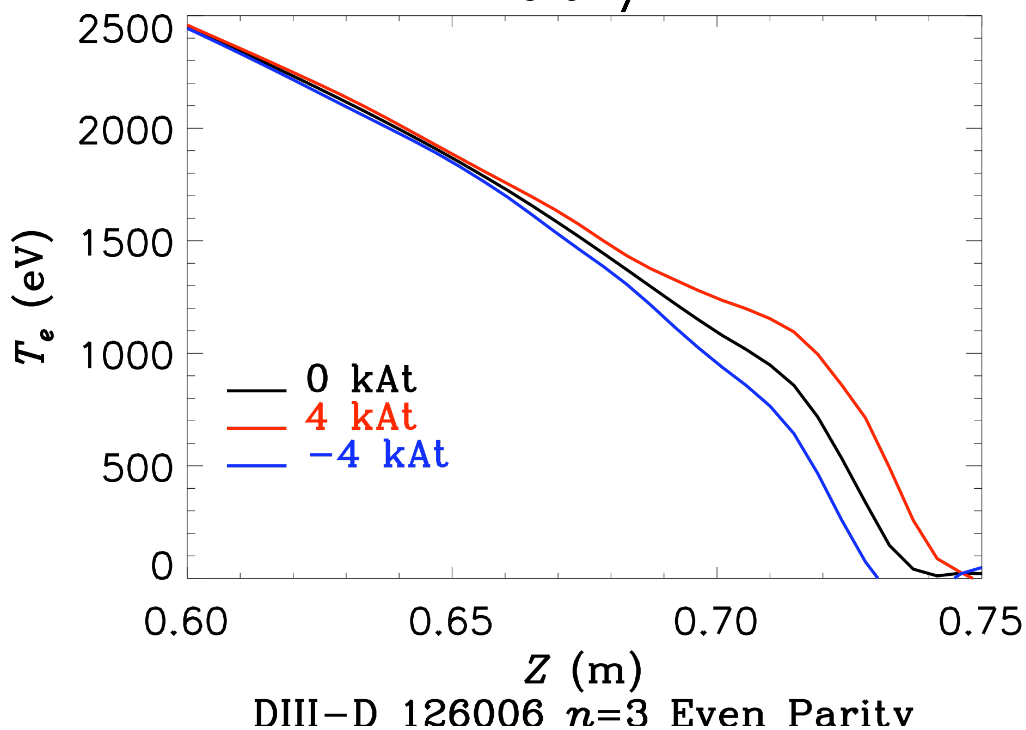
Empirical phase least prone to locking

DIII-D 144182

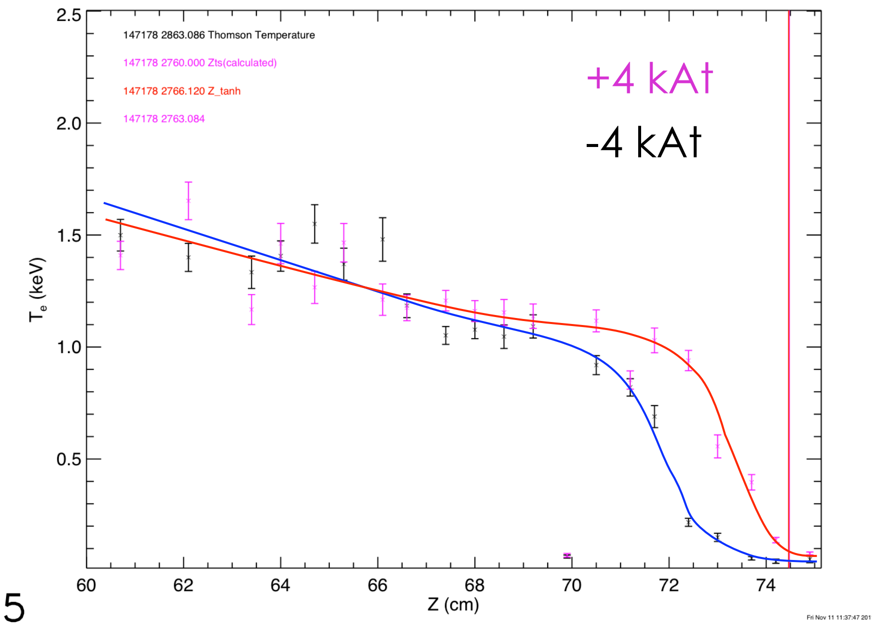
3D Fields Shift Edge T_e Profile

- **Linear calculations seem to capture T_e profile shift seen in experiment**

Theory



Experiment

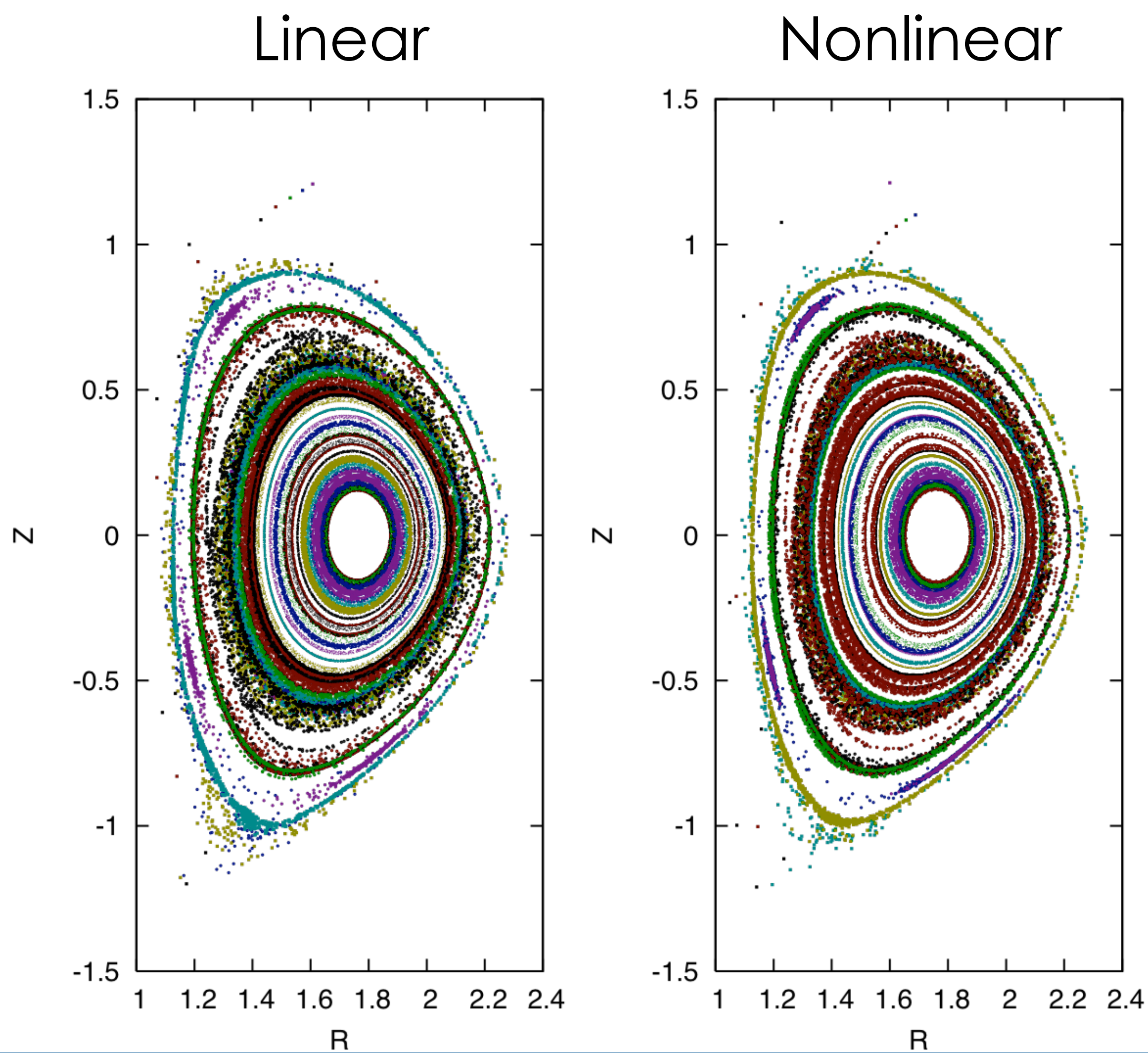


DIII-D 147178 n=3

Nonlinear Calculations are Underway

- **Nonlinear calculations are necessary for some things**
  - Rotation/locking
  - Transport
  - Large islands

- **Preliminary non-linear results agree with linear results for non-rotating plasma**



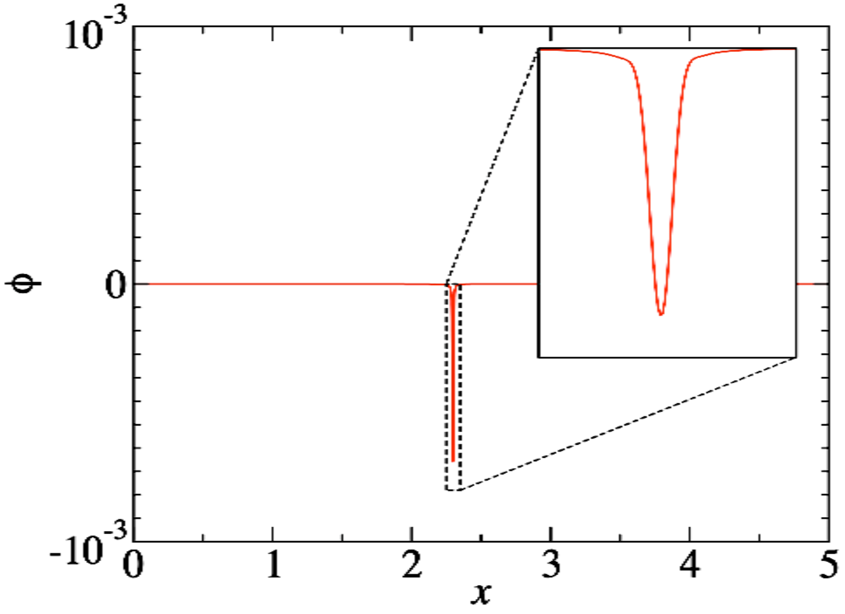
Opportunities For CEMM

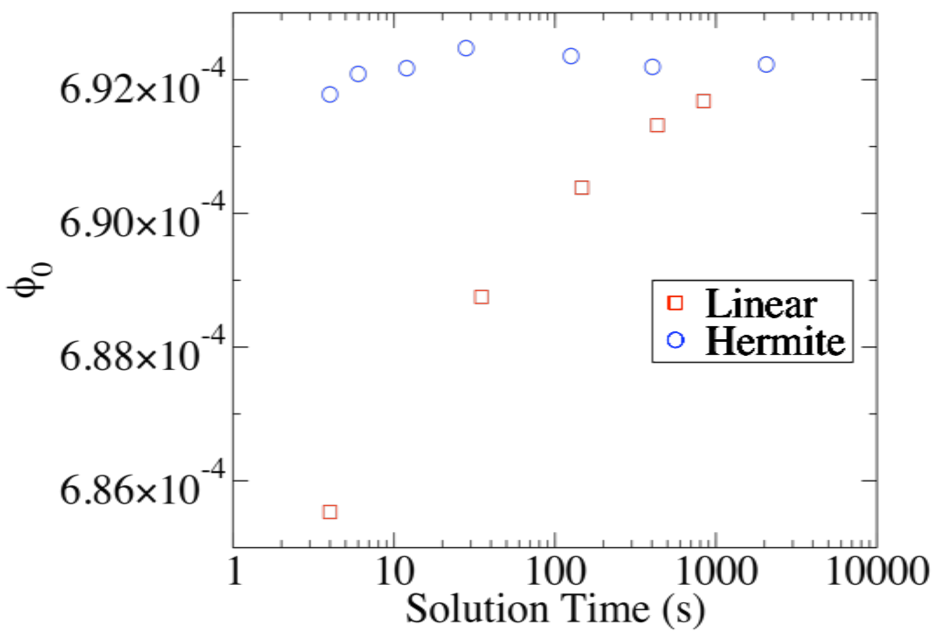
- **3D equilibrium properly requires nonlinear calculation**
  - $n = 0$ rotation and $n \neq 0$ response are strongly coupled
  - Island saturation is nonlinear
  - There is healthy debate how to do this efficiently!
- **Can we simulate a rotation bifurcation (locking)?**
- **M3D, M3D-C1, and NIMROD are well positioned to address these challenges**

Extra Slides

Hermite Elements Are More Efficient Than Linear Elements For Resolving A Boundary Layer in 1D

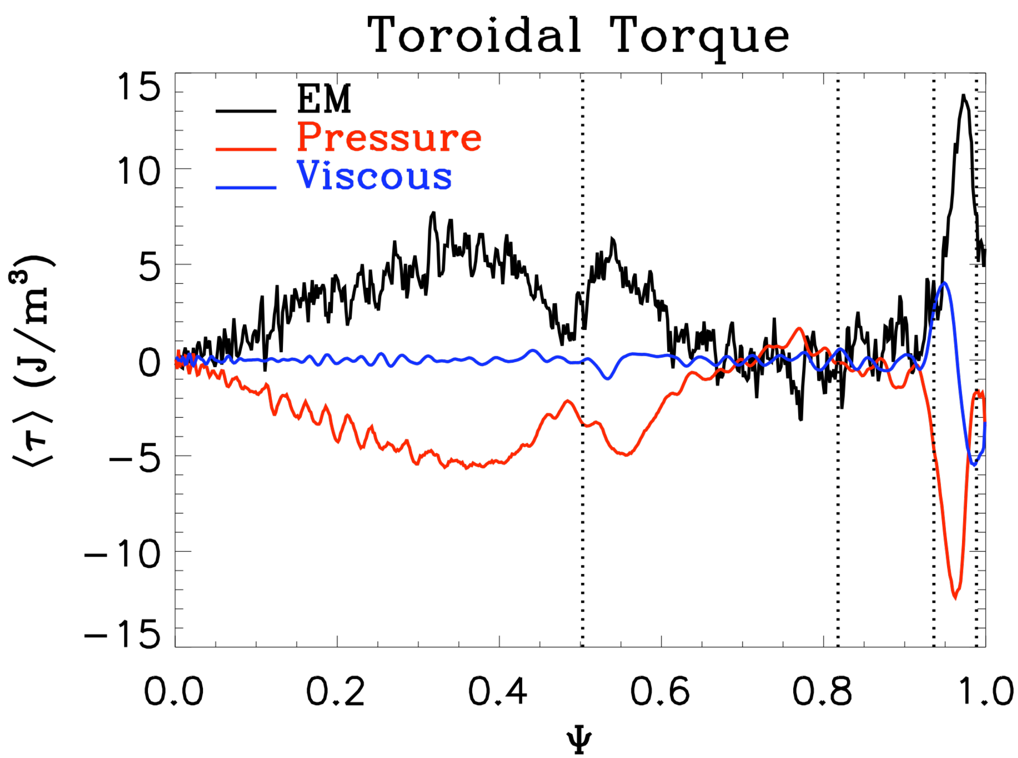
- **Simple equilibrium with one mode-rational surface**
- **Width of boundary layer determined by resistivity**
  - $\eta = 10^{-9}$, $\Delta\phi \sim \eta^{1/3}$





Dominant Balance is Between EM and Pressure Torques (At First Order)

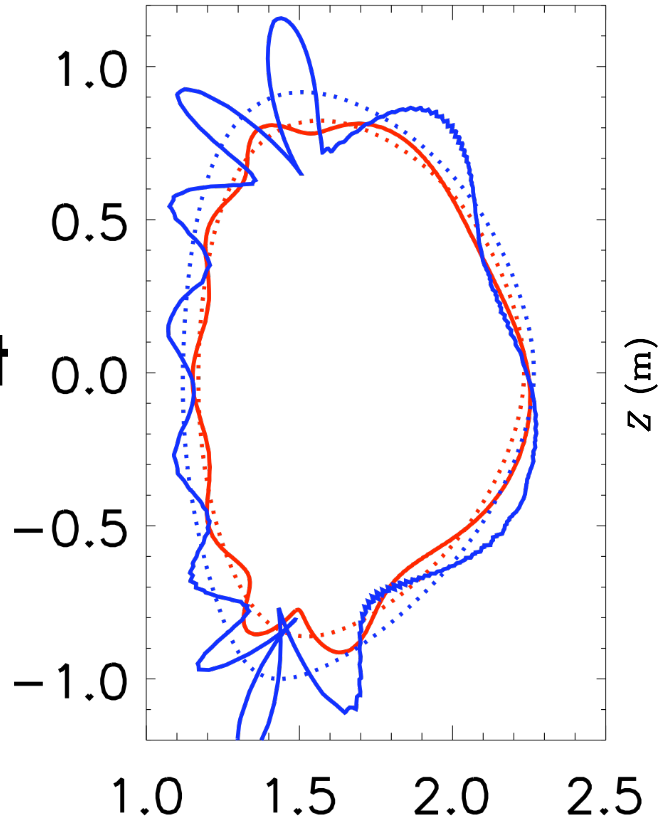
- **(Poloidal) surface average of first-order torques shows balance between EM and pressure torque densities**
  - This is for a low beta (β_N=1.14), high viscosity (3.6 m^2/s) case!
  - Full flux-surface averages would yield 0 at this order (due to $e^{in\phi}$ dependence)

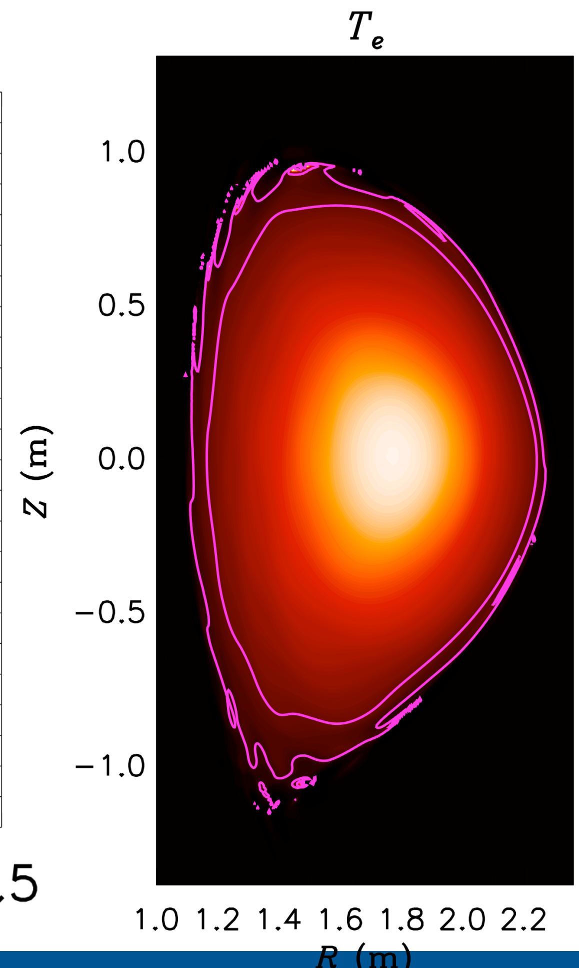


- **Second-order balance requires nonlinear calculation**

“Displacement” is Not Always an Accurate Measure of Surface Displacement

- **The ideal “displacement” can give a poor indication of perturbed surfaces**
- T_e **isosurfaces can deviate significantly from displacement**





What is "Perpendicular" Electron Velocity?

- The perpendicular angular velocity is defined as

$$\Omega_\perp^{e,i} = \frac{\mathbf{v}^{e,i}}{R} \cdot \frac{\mathbf{B} \times \nabla\psi}{\left|\mathbf{B} \times \nabla\psi\right|}$$

- To lowest order, $\mathbf{v}^{e,i} = R^2 \omega^{e,i}(\psi)\nabla\varphi + \lambda^{e,i}\mathbf{B}$. Thus:

$$\Omega_\perp^{e,i} = \frac{\left|\nabla\psi \times \nabla\varphi\right|}{\left|B\right|} \omega^{e,i}(\psi)$$

- From radial force balance: $\omega^{e,i}(\psi) = \phi'(\psi) + \dfrac{p_{e,i}{}'(\psi)}{n_{e,i} q_{e,i}}$

- $\Omega_\perp^e$ vanishes wherever ω^e vanishes, but also at $\mathbf{B}_{pol}$ nulls

Diamagnetic Drift

- The diamagnetic drift may be represented in the standard $\mathbf{v} = R^2\omega(\psi)\nabla\varphi + \lambda\mathbf{B}$ form if p is a flux function

$$\mathbf{v}_* = \frac{\mathbf{B}\times\nabla p}{neB^2}$$

$$= R^2\frac{p'}{ne}\nabla\varphi - \frac{p'}{ne}\frac{RB_\varphi}{B^2}\mathbf{B}$$

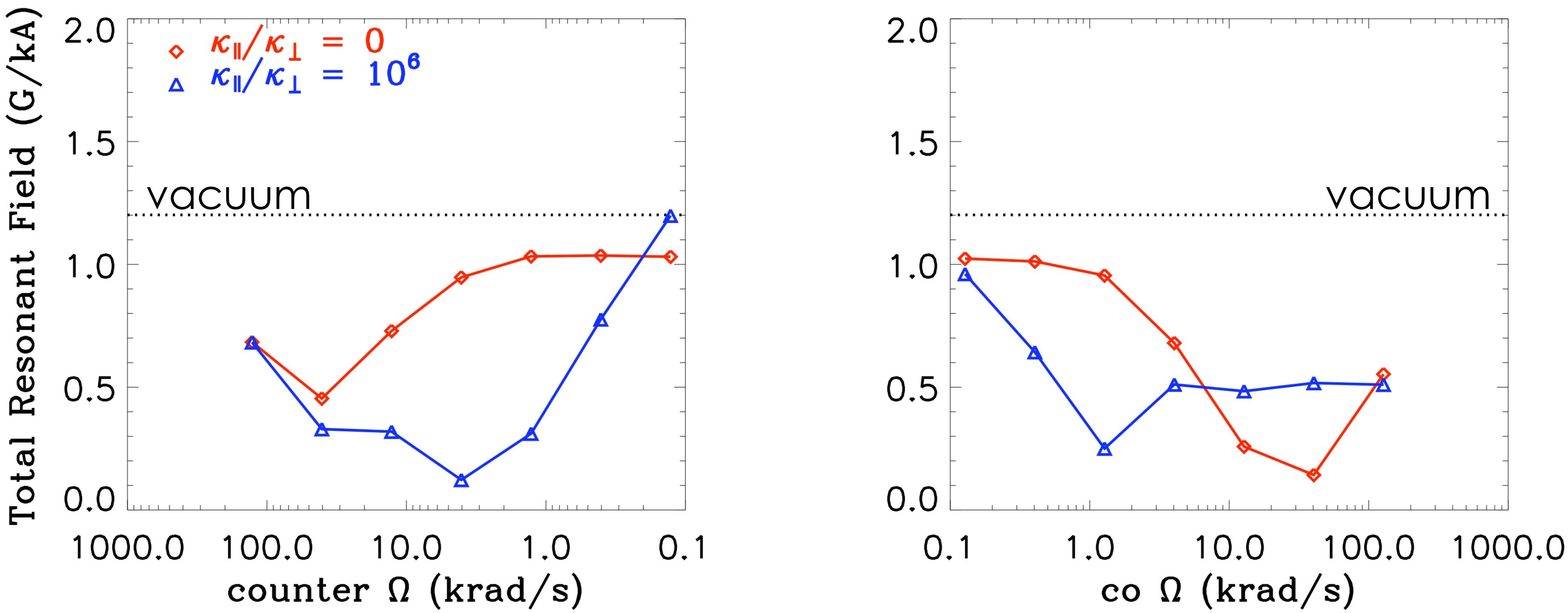
$$\Rightarrow$$

$$\omega_* = \frac{p'}{ne}$$

$$\lambda_* = \frac{p'}{ne}\frac{RB_\varphi}{B^2}$$

Parallel Thermal Conductivity Narrows Rotation Resonance

q=2



Single-fluid calculation

Two-Fluid Model

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$n\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\Gamma p \nabla \cdot \mathbf{u} - \boxed{\frac{d_i}{n}\mathbf{J} \cdot \left(\Gamma p_e \frac{\nabla n}{n} - \nabla p_e\right)} - (\Gamma - 1)\nabla \cdot \mathbf{q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} + \boxed{\frac{d_i}{n}\left(\mathbf{J} \times \mathbf{B} - \nabla p_e\right)}$$

$$\Pi = -\mu\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right]$$

$$\mathbf{q} = -\kappa \nabla\left(\frac{p}{n}\right) - \kappa_{\parallel} \mathbf{b}\mathbf{b} \cdot \nabla\left(\frac{p_e}{n}\right)$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\Gamma = 5/3$$

$$p_e = p/2$$

- Complete (not reduced) two-fluid model is implemented
- Time-independent equations may be solved directly for linear response