NUMERICAL CALCULATION OF THE NEOCLASSICAL ELECTRON DISTRIBUTION FUNCTION IN AN AXISYMMETRIC TORUS

B.C. LYONS (PPPL), S.C. JARDIN (PPPL), J.J. RAMOS (MIT PSFC)

ANNUAL CEMM MEETING
SALT LAKE CITY, UT
SUNDAY, NOVEMBER 13, 2011

Outline

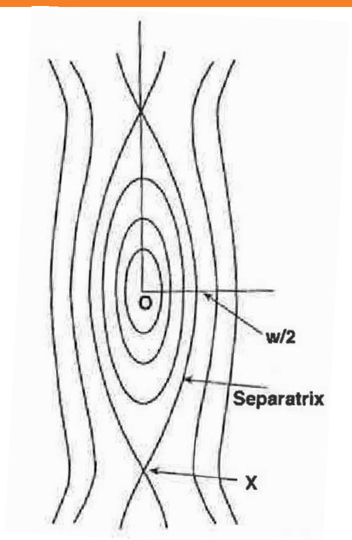
- Introduction and Motivation
- Analytic Model
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

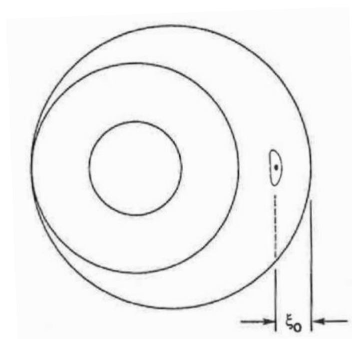
Outline

- Introduction and Motivation
- Analytic Model
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

Neoclassical tearing mode (NTMs)

- Density, temperature, pressure, etc. tend to equilibrate across an island width
- Difference in current at O-point and X-point can drive island growth
  - Without these gradients, there can be no bootstrap current within the island
  - Bootstrap current at the X-point can drive island growth
- Large islands allow hot, dense plasma near core to be transported outward, reducing confinement
- Modifications to magnetic topology can result in macroscopic instability and disruption





Images taken from The Theory of Toroidally Confined Plasmas by R. White, 2006

NTM stability modeling

- NTM stability place a severe limit on maximum β
- NTMs incorporate a lot of physics
  - Cause: Neoclassical kinetic theory
  - Effect: MHD destabilization
  - Requires a hybrid model
- High-fidelity simulations required for prediction, control, avoidance, and understanding of NTMs
  - Especially important for ITER operation, where very few disruptions can be tolerated

Framework for hybrid solver

What's needed

Solve the drift kinetic equation in a general, 3D, toroidal geometry for the ion and electron perturbed distribution functions in parameter regimes relevant to ITER and reactors and couple to an MHD solver

Current work

Solve the drift kinetic equation in a 2D, large aspect ratio tokamak for the neoclassical, electron perturbed distribution function

Outline

- Introduction and Motivation
- Analytic Model[1]
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

[1] Ramos, J.J. 2010. Phys. Plasmas. 17, 082502.

Electron DKE

$$\frac{\partial \bar{f}_e}{\partial t} + \dot{\mathbf{x}} \cdot \frac{\partial \bar{f}_e}{\partial \mathbf{x}} + \dot{v}_{\parallel} \frac{\partial \bar{f}_e}{\partial v_{\parallel}} + \dot{v}_{\perp} \frac{\partial \bar{f}_e}{\partial v_{\perp}} = \langle C_{ee}[f_e, f_e] + C_{ei}[f_e, f_i] \rangle$$

- Derived from average of Fokker-Planck equation over gyromotion
  - Determines form of $\dot{\mathbf{x}}, \dot{v}_{\parallel}$, and $\dot{v}_{\perp}$
  - Collision operators taken in their linearized Landau form
- Two expansion parameters for high-temperature fusion plasmas

$$\boxed{\delta \sim \rho_i / L \ll 1} \qquad \boxed{\nu_* \sim L/\lambda_{\mathrm{mfp}} \sim \delta}$$

- Four common subsidiary parameters

$$\boxed{m_e/m_i \sim \delta^2} \quad \boxed{\delta_e \sim \rho_e/L \sim \delta^2} \quad \boxed{u_e/v_{the} \sim \delta^2} \quad \boxed{f_{NMe}/f_{Me} \sim \delta^2}$$

- Equations maintained to third order in δ
  - Scale at which collisional dynamics first become important
  - Terms of order $\delta_e \nu_*$ will be kept, but δ_e^2 dropped
  - Equivalent to neoclassical electron banana regime

Stationary, Axisymmetric Equilibrium

- Nested flux surfaces labeled by $\psi = \psi_{PF}/2\pi$

- Fields: $\mathbf{B} = \nabla\psi \times \nabla\zeta + RB_\zeta \nabla\zeta$

$$\mathbf{E} = -\nabla\phi - \frac{V_{loop}}{2\pi}\nabla\zeta$$

- Lowest-order fluid equations result in

$$n = n(\psi) \qquad T_s = T_s(\psi) \qquad \phi = \phi(\psi)$$

$$RB_\zeta = I(\psi) \qquad \mathbf{u}_s = U_s(\psi)\mathbf{B} + R^2\left[\frac{d\phi}{d\psi} + \frac{1}{e_s n}\frac{dP_s}{d\psi}\right]\nabla\zeta$$

Resulting DKE

- Given these assumptions, it is convenient to write

$$\bar{f}_e = f_{Me} + (g_{e,0} + g_{e,1}\cos\chi) f_{Me} + h_e$$

where $g_{e,0}$ and $g_{e,1}$ have analytic forms

- Then, the DKE for h_e can be reduced to

$$\boxed{v_{||}(\mathbf{b}\cdot\nabla\theta)\frac{\partial h_e}{\partial\theta} - C_e[h_e] = S_e v_{||}}$$

where $\quad v_{||}(\psi,\theta,v,\lambda) = \pm v\left[1 - \lambda B(\psi,\theta)/B_{max}(\psi)\right]^{1/2}$

$$\lambda(\psi,\theta,\chi) = \sin^2\chi B_{max}(\psi)/B(\psi,\theta)$$

Source Term

Source contains Ohmic drive, interaction with ion flow, and pressure and temperature gradient bootstrap drive

$$S_e = \left\{ \frac{eV_{loop}I}{2\pi T_e B R^2} + \nu_e \left(U_i B + \frac{I}{enB}\frac{dP}{d\psi} \right) \frac{v_{the}}{v_{thi}^2 v} \xi \left(\frac{v}{v_{thi}} \right) \right.$$
$$+ \frac{\nu_e m_e I}{eBT_e}\frac{dT_e}{d\psi}\frac{v_{the}}{v} \left[2\varphi \left(\frac{v}{v_{the}} \right) - 10\xi \left(\frac{v}{v_{the}} \right) \right.$$
$$\left.\left. + \frac{1}{2}\varphi \left(\frac{v}{v_{thi}} \right) - \frac{5v_{the}^2}{2v_{thi}^2} \xi \left(\frac{v}{v_{thi}} \right) \right] \right\} f_{Me}$$

Solubility Condition

- Standard solution method for neoclassical theory

$$h_e = \underbrace{\varsigma(v_{\|})H(1-\lambda)K_e(\psi, v, \lambda)}_{h^{odd}} + h_e^{(3)}$$

- DKE becomes $v_{\|}(\mathbf{b}\cdot\nabla\theta)\dfrac{\partial h_e^{(3)}}{\partial\theta} - C_e[h^{odd}] = S_e v_{\|}$

- Solubility condition: $\boxed{\oint_{\psi,v,\lambda}\frac{dl}{v_{\|}}C_e[h^{odd}] = -\oint_{\psi,v,\lambda} dl S_e}$

- Contour integrals taken along one poloidal turn of magnetic field line

Integral over Collision Operator

$$\oint_{\psi,v,\lambda} \frac{dl}{v_{\|}} C_e[h^{odd}] = \frac{2\nu_D}{v}\frac{\partial}{\partial\lambda}\left(\eta_1\lambda\frac{\partial}{\partial\lambda}K_e\right)$$

$$+\nu_e\eta_2 v_{the}^3\left[\frac{1}{v^3}\frac{d}{dv}\left\{\begin{array}{c}\xi\left(\frac{v}{v_{the}}\right)\left[v\frac{d}{dv}+\frac{v^2}{v_{the}^2}\right]\\ +\xi\left(\frac{v}{v_{thi}}\right)\left[v\frac{d}{dv}+\frac{m_e v^2}{m_i v_{thi}^2}\right]\end{array}\right\}+\frac{4\pi f_{Me}}{nv}\right]K_e$$

$$-\frac{\nu_e v_{the}}{nv} f_{Me}\int_0^{2\pi} JB\left[1-\lambda\frac{B}{B_{max}}\right]^{-\frac{1}{2}}\Phi d\theta$$

$$+\frac{\nu_e v}{n v_{the}} f_{Me}\frac{d^2}{dv^2}\int_0^{2\pi} JB\left[1-\lambda\frac{B}{B_{max}}\right]^{-\frac{1}{2}}\Psi d\theta$$

where

$$\eta_1(\psi,\lambda) = B_{max}\int_0^{2\pi} J\left[1-\lambda\frac{B}{B_{max}}\right]^{\frac{1}{2}} d\theta$$

$$\eta_2(\psi,\lambda) = \int_0^{2\pi} JB\left[1-\lambda\frac{B}{B_{max}}\right]^{-\frac{1}{2}} d\theta$$

Outline

- Introduction and Motivation
- Analytic Model
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

Expansions

- Expand Rosenbluth Potentials in Legendre and Fourier series

$$\begin{pmatrix} \Phi(\psi, \cos\chi, \theta, v) \\ \Psi(\psi, \cos\chi, \theta, v) \end{pmatrix} = \sum_{m=0}^{M} \sum_{l=1,odd}^{2L-1} \begin{pmatrix} \Phi_{l,m}(\psi, v) \\ \Psi_{l,m}(\psi, v) \end{pmatrix} P_l(\cos\chi) \cos m\theta$$

- Then expand K_e , $\Phi_{l,m}$, and $\Psi_{l,m}$ in finite elements in v and λ, as necessary

$$K_e(\psi, v, \lambda) = \sum_{i=0}^{N} \sum_{j=0}^{J} K_{i,j}(\psi) \varphi_i(v) \varphi_j(\lambda)$$

$$\begin{pmatrix} \Phi_{l,m}(v) \\ \Psi_{l,m}(v) \end{pmatrix} = \sum_{i=0}^{N} \begin{pmatrix} \Phi_{i,l,m} \\ \Psi_{i,l,m} \end{pmatrix} \varphi_i(v)$$

Expanded Form

$$\sum_{i=0}^{N}\sum_{j=0}^{J}\Bigg\{2v^2\nu_D\frac{d}{d\lambda}\left(\eta_1\lambda\frac{d\varphi_j(\lambda)}{d\lambda}\right)K_{i,j}\varphi_i(v)$$

$$+\nu_e\eta_2 v_{the}^3\left[\frac{d}{dv}\left\{\begin{array}{c}\xi\left(\frac{v}{v_{the}}\right)\left[v\frac{d\varphi_i(v)}{dv}+\frac{v^2}{v_{the}^2}\varphi_i(v)\right]\\ +\xi\left(\frac{v}{v_{thi}}\right)\left[v\frac{d\varphi_i(v)}{dv}+\frac{m_e v^2}{m_i v_{thi}^2}\varphi_i(v)\right]\end{array}\right\}+\frac{4\pi v^2}{n}f_{Me}\varphi_i(v)\right]K_{i,j}\varphi_j(\lambda)$$

$$-\frac{\nu_e v_{the} v^2}{n}f_{Me}\sum_{m=0}^{M}\sum_{l=1,odd}^{2L-1}a_{l,m}\Phi_{i,l,m}\phi_i(v)$$

$$-\frac{\nu_e v^4}{n v_{the}}f_{Me}\sum_{m=0}^{M}\sum_{l=1,odd}^{2L-1}a_{l,m}\Psi_{i,l,m}\frac{d^2\phi_i(v)}{dv^2}\Bigg\} \quad = \quad -v^3\oint_{\psi,v,\lambda}dlS_e$$

$$\left[\frac{d}{dv}\left(v^2\frac{d\varphi_i(v)}{dv}\right)-l(l+1)\varphi_i(v)\right]\begin{pmatrix}\Phi_{i,l,m}\\ \Psi_{i,l,m}\end{pmatrix}=\begin{pmatrix}v^2\varphi_i(v)\sum_{j=0}^{J}D_{j,l,m}K_{i,j}\\ v^2\Phi_{i,l,m}\varphi_i(v)\end{pmatrix}$$

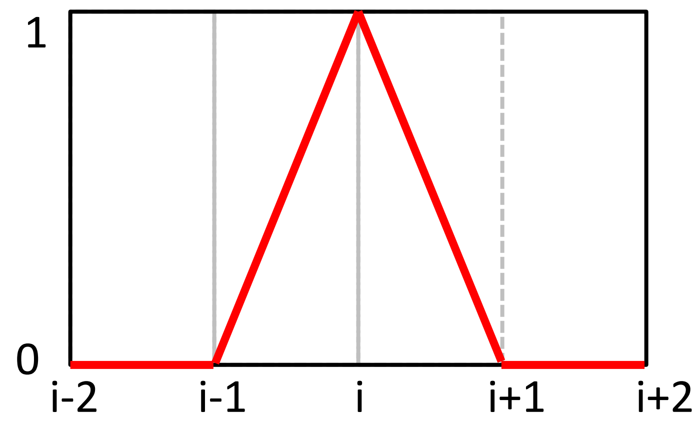
$$a_{l,m}=\int_0^{2\pi}d\theta JB\left[1-\lambda\frac{B}{B_{max}}\right]^{-1/2}P_l\left(\left[1-\lambda\frac{B}{B_{max}}\right]^{1/2}\right)\cos m\theta$$

$$D_{j,l,m}=-\frac{2(2l+1)}{1+\delta_{m,0}}\int_0^{2\pi}d\theta\cos m\theta\int_0^1 d\lambda P_l\left(\left[1-\lambda\frac{B}{B_{max}}\right]^{1/2}\right)\varphi_j(\lambda)\frac{B}{B_{max}}\left[1-\lambda\frac{B}{B_{max}}\right]^{-1/2}$$

Galerkin Method

- Take the inner product of the previous equations with each finite element
- Use linear tent functions:
- Only overlap with their two nearest neighbors and themselves
- DKE becomes tridiagonal in both v and λ
- Rosenbluth Potential eqs. are tridiagonal in v and dense in λ

$$\varphi_p(x) = \begin{cases} \frac{x - x_{i-1}}{x_i - x_{i-1}} & : x_{i-1} \le x \le x_i \\ \frac{x_{i+1} - x}{x_{i+1} - x_i} & : x_i \le x \le x_{i+1} \end{cases}$$



Block Tridiagonal Algorithm

- Since all equations are tridiagonal in v , we rewrite the coupled set as a block tridiagonal matrix eq.

$$\mathbf{A}_i \cdot \mathbf{U}_{i+1} - \mathbf{B}_i \cdot \mathbf{U}_i + \mathbf{C}_i \cdot \mathbf{U}_{i-1} = \mathbf{D}_i$$

- Size of each block matrix is $(J + 2L(M+1))^2$
- Given appropriate boundary conditions, there exists a straightforward algorithm to solve for $\mathbf{U}$
- Computation time required is O($N\,(J + 2L(M+1))^3$)

Boundary Conditions

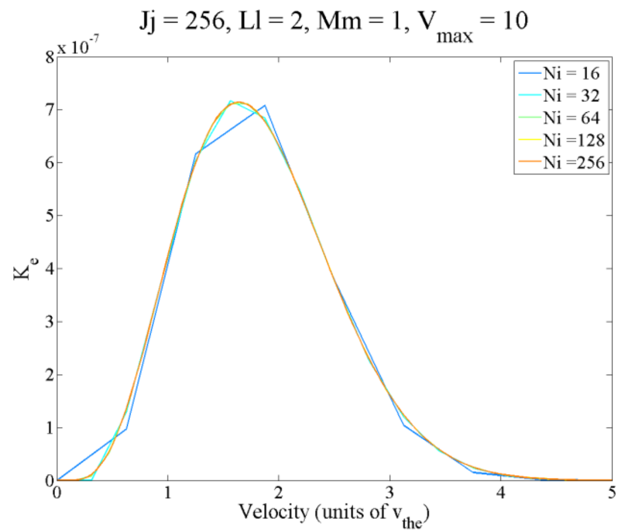
- For v
  - $K_e(0,\lambda) = 0$ $\qquad$ $K_e(\text{``}\infty\text{''} = v_{max}, \lambda) = 0$
  - $\Phi_{l,m}(0) = 0$ $\qquad$ $\dfrac{d\Phi_{l,m}}{dv}(v_{max}) = -(l+1)\dfrac{\Phi_{l,m}(v_{max})}{v_{max}}$
  - $\Psi_{l,m}(0) = 0$ $\qquad$ $\dfrac{d\Psi_{l,m}}{dv}(v_{max}) = -(l-1)\dfrac{\Psi_{l,m}(v_{max})}{v_{max}}$
- For λ
  - $K_e(v,0)$ must be regular
  - $K_e(v,1) = 0$
  - $\dfrac{\partial K_e}{\partial \lambda}(v,1) = 0$ requires boundary layer (future work)

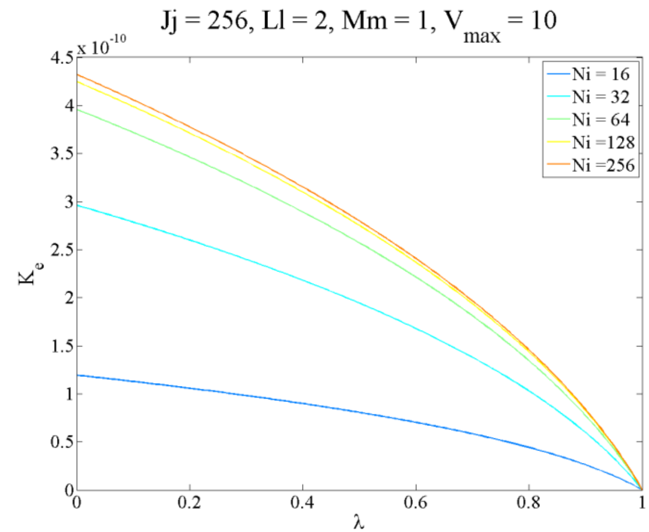
Outline

- Introduction and Motivation
- Analytic Model
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

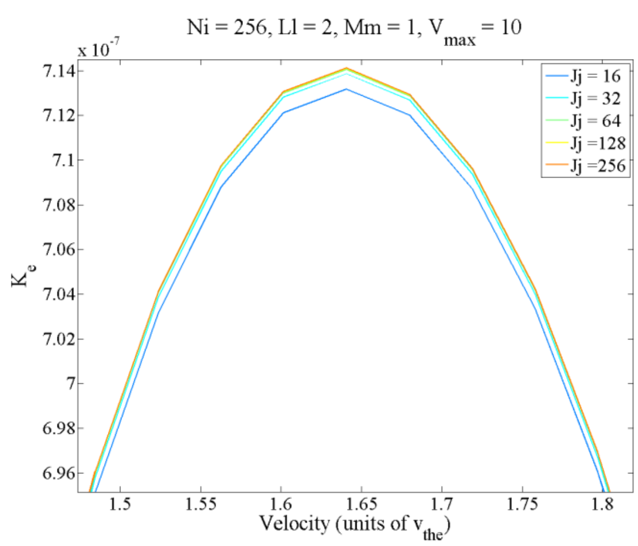
Convergence (1)

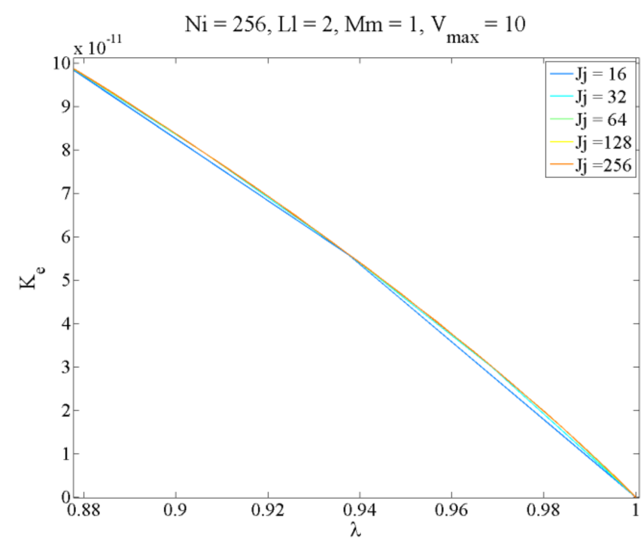
Velocity Finite Elements





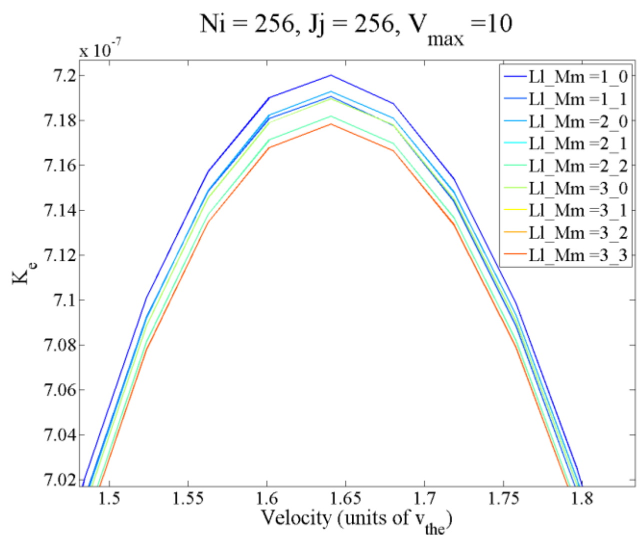
Lambda Finite Elements

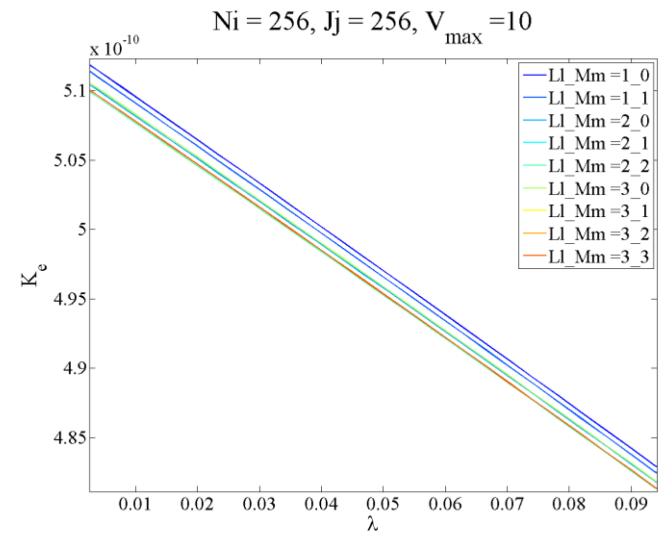




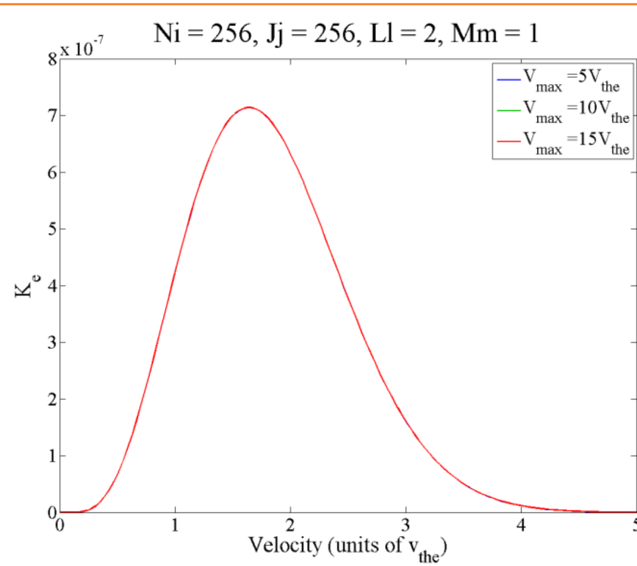
Convergence (2)

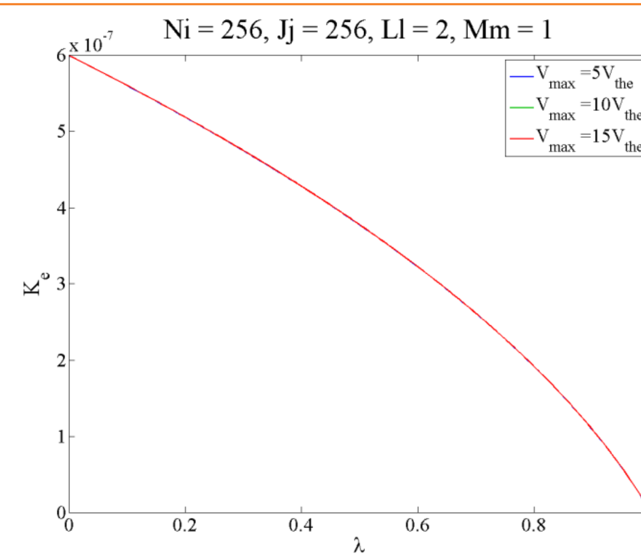
Legendre & Fourier Terms





Maximum Velocity



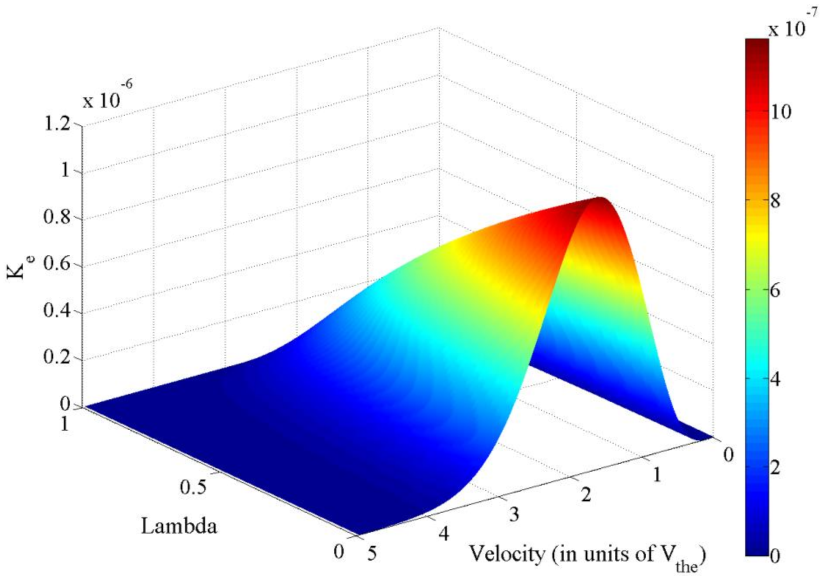


Outline

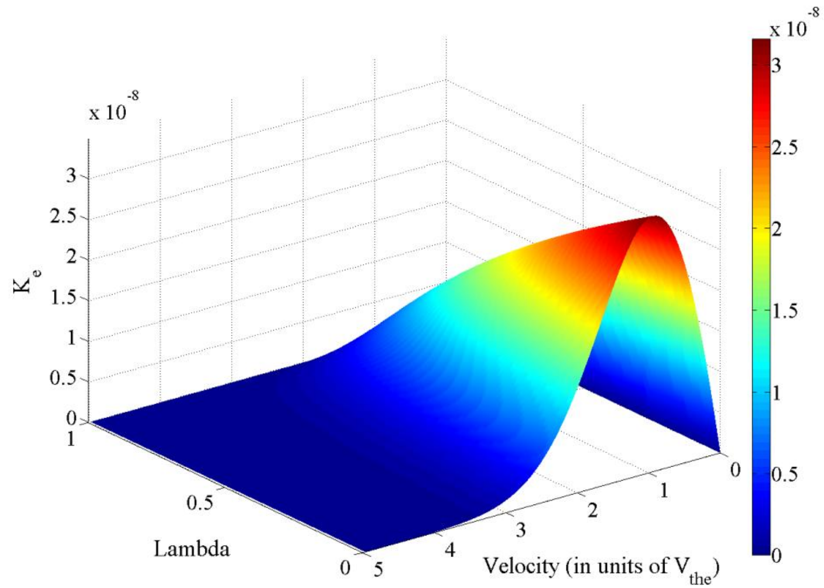
- Introduction and Motivation
- Analytic Model
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

Example Distribution Functions

Ohmic Drive



Density-Gradient Drive



Calculating Current

- Need part of distribution function that is odd wrt $v_{||}$

$$\bar{f}^{odd}_{NMe} = \varsigma(v_{||})H(1-\lambda)K_e - v_{||}\left[\frac{m_e U_e B}{T_e} + \frac{m_e I}{2eBT_e}\left(\frac{m_e v^2}{T_e} - 5\right)\frac{dT_e}{d\psi}\right] f_{Me}$$

- Requiring $\int d^3\mathbf{v} v_{||} \bar{f}^{odd}_{NMe} = 0$, we find that

$$U_e(\psi) = \frac{2\pi}{nB_{max}} \int_0^{\infty} dv v^3 \int_0^1 d\lambda K_e(\psi, v, \lambda)$$

- Then we use $j_{||} = (U_i - U_e)B + \dfrac{I}{B}\dfrac{dP}{d\psi}$

Ohmic Drive Conductivity

- On-axis: $\sigma_{||} = \sigma_{Spitzer} \approx 1.96\sigma_{\perp}$

$$\sigma_{\perp} = \frac{e^2 n}{m_e \nu_{Brag}}$$

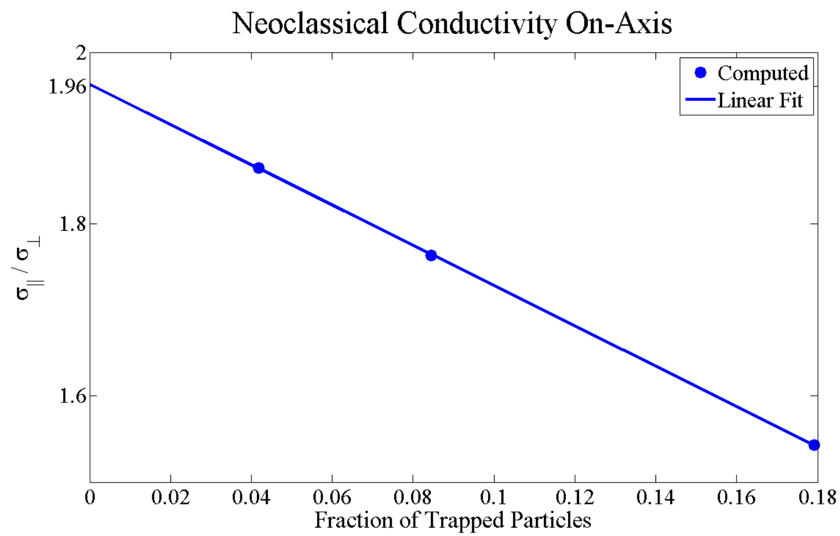
- Trapped particles carry no current
  - To lowest order, expect conductivity to decrease linearly with the trapped particle fraction

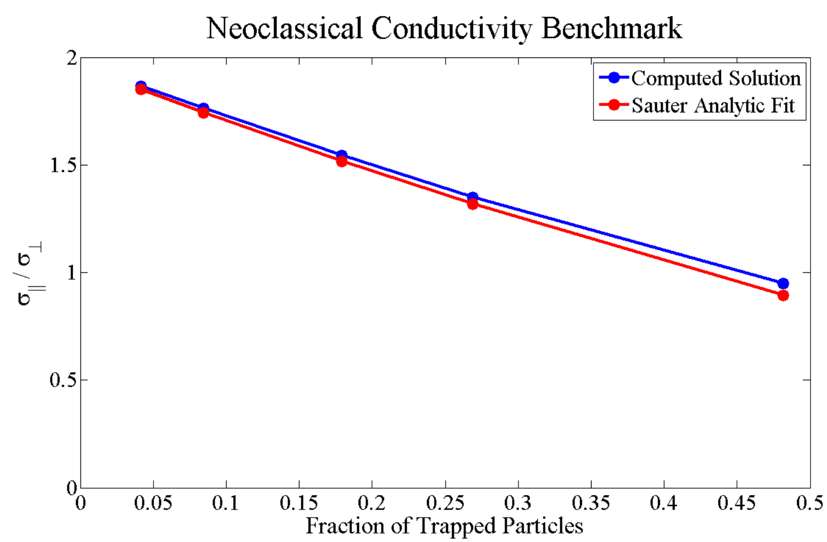
$$f_t = 1 - \frac{3}{4}\left\langle \frac{B^2}{B_{max}^2} \right\rangle \int_0^1 \frac{\lambda d\lambda}{\left\langle \sqrt{1 - \lambda B / B_{max}} \right\rangle}$$

- Sauter analytic fit[1]

$$\frac{\sigma_{neo}}{\sigma_{\perp}} \approx 1.96\left(1 - 1.36 f_t + 0.59 f_t^2 - 0.23 f_t^3\right)$$

[1] Sauter, O et al. (1999) Phys. Plasmas: 6,7.



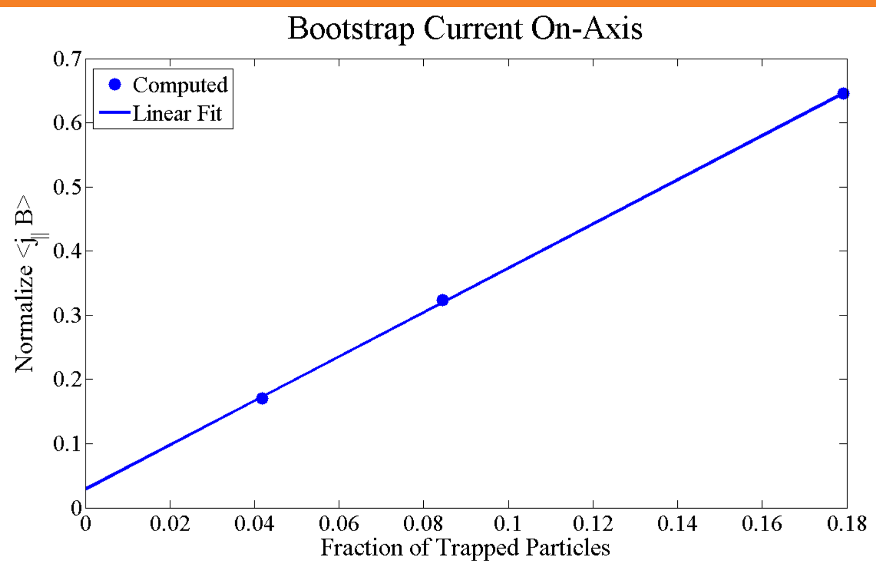


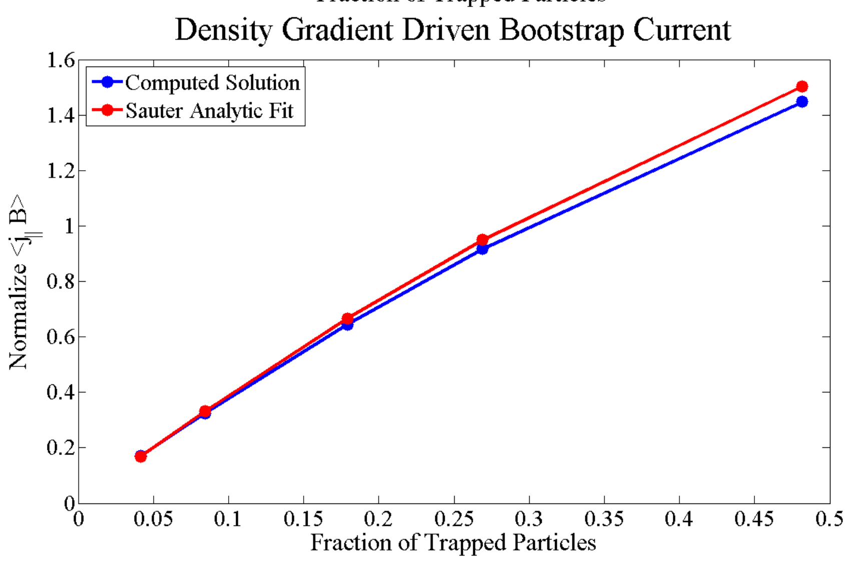
Density-Gradient Bootstrap Current

- Expect zero current for $f_t = 0$
  - To lowest order, current should decrease linearly with trapped particle fraction
- Sauter Analytic fit

$$\langle j_{\|} B \rangle = -I \mathcal{L}_{31} \frac{dP}{d\psi}$$

$$\mathcal{L}_{31} = 1.7 f_t - 0.95 f_t^2 + 0.15 f_t^3 + 0.1 f_t^4$$





Outline

- Introduction and Motivation
- Analytic Model
- Computational Methods
  - Expansions and Algorithm
  - Convergence Studies
- Initial Results
- Conclusion

Summary

- A code has been written to solve for the component of the non-Maxwellian electron distribution function necessary to compute the current in an axisymmetric toroidal plasma
- Code demonstrates good convergence
- Ohmic and density-gradient sources have been benchmarked against the Sauter analytic fits for a large-aspect ratio expansion equilbrium

Future Work

- Short Term
  - Complete implementation of temperature-gradient driven source
  - Generalize geometry to use solution from Grad-Shafronov solver
  - Benchmark against NEO and NCLASS
  - Implement simple (i.e., lowest-order) ion code
  - Couple with MHD code (e.g., M3D-C1)
- Long Term
  - Generalize to 3D Geometry
  - Develop ion theory to appropriate ordering
  - Implement fully 3D, coupled ion-electron code
  - Couple with MHD code (e.g., M3D-C1)
  - Perform NTM and sawtooth studies

Acknowledgements

- This research was supported in part by an award from the Department of Energy (DOE) Office of Science Graduate Fellowship Program (DOE SCGF). The DOE SCGF Program was made possible in part by the American Recovery and Reinvestment Act of 2009. The DOE SCGF program is administered by the Oak Ridge Institute for Science and Education for the DOE. ORISE is managed by Oak Ridge Associated Universities (ORAU) under DOE contract number DE-AC05-06OR23100. All opinions expressed in this paper are the author's and do not necessarily reflect the policies and views of DOE, ORAU, or ORISE.
- This presentation has been authored by Princeton University and collaborators under Contract Number DE-AC02-09CH11466 with the U.S. Department of Energy.

Additional Slides

Coefficient functions

- Reference frame of the macroscopic flow
- $$\boxed{\dot{\mathbf{x}} = \mathbf{u}_e - \mathbf{u}_{De} + v_\| \mathbf{b} + \frac{v_\perp^2}{2} \nabla \times \left(\frac{\mathbf{b}}{\Omega_{ce}} \right) + \left(v_\|^2 - \frac{v_\perp^2}{2} \right) \frac{\mathbf{b} \times \kappa}{\Omega_{ce}}}$$
- $$\boxed{\begin{aligned} \dot{v}_\| =& \frac{\mathbf{b} \cdot (\nabla \cdot P_e^{CGL} - \mathbf{F}_e^{coll})}{m_e n} - v_\| \mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)(\mathbf{u}_e - \mathbf{u}_{De})] \\ & - \frac{v_\perp^2}{2} \mathbf{b} \cdot \nabla \ln B + \frac{v_\| v_\perp^2}{2} \nabla \cdot \left(\frac{\mathbf{b} \times \kappa}{\Omega_{ce}} \right) \end{aligned}}$$
- $$\boxed{\begin{aligned} \dot{v}_\perp =& \frac{v_\perp}{2} \{ \mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)(\mathbf{u}_e - \mathbf{u}_{De})] - \nabla \cdot (\mathbf{u}_e - \mathbf{u}_{De}) \\ & + v_\| \mathbf{b} \cdot \nabla \ln B - v_\|^2 \nabla \cdot \left(\frac{\mathbf{b} \times \kappa}{\Omega_{ce}} \right) \} \end{aligned}}$$

where $\mathbf{u}_{De} = \frac{1}{m_e n \Omega_{ce}} [\mathbf{b} \cdot \nabla p_{e\perp} + (p_{e\|} - p_{e\perp})(\mathbf{b} \times \kappa)]$

$$\mathbf{b} \cdot (\nabla \cdot P_e^{CGL}) = \mathbf{b} \cdot \nabla p_{e\|} - (p_{e\|} - p_{e\perp}) \mathbf{b} \cdot \nabla \ln B$$

$$\kappa = (\mathbf{b} \cdot \nabla) \mathbf{b}$$

Electron –Electron Collision Operator

$$C_{ee}[f_e, f_e] = C_{ee}[f_{Me}, f_{NMe}] + C_{ee}[f_{NMe}, f_{Me}]$$

$$C_{ee}[f_{Me}, f_{NMe}] = \frac{\nu_e v_{the}}{n} f_{Me}(v) \left\{ 4\pi v_{the}^2 f_{NMe}(\mathbf{v}) - \Phi[f_{NMe}](\mathbf{v}) + \frac{\mathbf{vv}}{v_{the}^2} : \frac{\partial^2 \Psi[f_{NMe}](\mathbf{v})}{\partial \mathbf{v} \partial \mathbf{v}} \right\}$$

$$C_{ee}[f_{NMe}, f_{Me}] = \frac{\nu_e v_{the}^3}{v^3} \left\{ \left[\varphi\left(\frac{v}{v_{the}}\right) - \xi\left(\frac{v}{v_{the}}\right) \right] \mathcal{L}[f_{NMe}](\mathbf{v}) + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{v}} \left[\xi\left(\frac{v}{v_{the}}\right) \mathbf{v} \cdot \frac{\partial f_{NMe}(\mathbf{v})}{\partial \mathbf{v}} + \frac{v^2}{v_{the}^2} \xi\left(\frac{v}{v_{the}}\right) f_{NMe}(\mathbf{v}) \right] \right\}$$

where $\mathcal{L}[f](\mathbf{v}) = \frac{v^3}{2} \frac{\partial}{\partial \mathbf{v}} \cdot \left\{ \frac{1}{v} \frac{\partial f(\mathbf{v})}{\partial \mathbf{v}} - \frac{1}{v^3} \left[\mathbf{v} \cdot \frac{\partial f(\mathbf{v})}{\partial \mathbf{v}} \right] \mathbf{v} \right\}$

$$\frac{\partial}{\partial \mathbf{v}} \cdot \frac{\partial \Phi[f](\mathbf{v})}{\partial \mathbf{v}} = -4\pi f(\mathbf{v})$$

$$\varphi\left(\frac{v}{v_{the}}\right) = \frac{v}{n} \Phi[f_{Me}](v) = \sqrt{\frac{2}{\pi}} \int_0^{v/v_{the}} dt \exp(-t^2/2)$$

$$\frac{\partial}{\partial \mathbf{v}} \cdot \frac{\partial \Psi[f](\mathbf{v})}{\partial \mathbf{v}} = \Phi[f](\mathbf{v})$$

$$\xi\left(\frac{v}{v_{the}}\right) = \frac{1}{nv} \mathbf{vv} : \frac{\partial^2 \Psi[f_{Me}](v)}{\partial \mathbf{v} \partial \mathbf{v}} = \frac{v_{the}^2}{v^2} \left[\varphi\left(\frac{v}{v_{the}}\right) - \sqrt{\frac{2}{\pi}} \frac{v}{v_{the}} \exp\left(-\frac{v^2}{2v_{the}^2}\right) \right]$$

$$\nu_e = 3\sqrt{\frac{\pi}{2}} \nu_{Braginskii}$$

Electron-Ion Collision Operator

$$C_{ei}[f_e, f_i] = C_{ei}[f_{Mi}, f_{NMe}] + C_{ei}[f_i, f_{Me}]$$

$$C_{ei}[f_{NMe}, f_{Mi}] = \frac{\nu_e v_{the}^3}{v^3}\left\{\left[\varphi\left(\frac{v}{v_{thi}}\right) - \xi\left(\frac{v}{v_{thi}}\right)\right]\mathcal{L}[f_{NMe}](\mathbf{v}) + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{v}}\left[\xi\left(\frac{v}{v_{thi}}\right)\mathbf{v}\cdot\frac{\partial f_{NMe}(\mathbf{v})}{\partial \mathbf{v}} + \frac{m_e v^2}{m_i v_{thi}^2}\xi\left(\frac{v}{v_{thi}}\right) f_{NMe}(\mathbf{v})\right]\right.$$

$$C_{ei}[f_{Me}, f_i] = \nu_e v_{the} f_{Me}(v)\left[\left(\frac{T_e}{T_i} - 1\right)\frac{4\pi v_{thi}^2}{n} f_{Mi}(v) + \frac{\mathbf{v}\cdot(\mathbf{u}_i - \mathbf{u}_e)}{v_{thi}^2 v}\xi\left(\frac{v}{v_{thi}}\right) + \frac{m_e}{m_i}\left(\frac{T_e}{T_i} - 1\right)\frac{v}{v_{thi}^2}\xi\left(\frac{v}{v_{thi}}\right)\right]$$

Gyro-average of the collision operators

- Maxwellian-test part of C_{ei} has analytic solution

$$\langle C_{ei}[f_{Me}, f_i]\rangle (v,\chi) = \mathcal{D}_{e,0}(v) + \mathcal{D}_{e,1}(v)\cos\chi$$

where $$\mathcal{D}_{e,0}(v) = \nu_e v_{the} f_{Me}\left(\frac{T_e}{T_i} - 1\right)\left[\frac{4\pi v_{thi}^2}{n} f_{Mi} - \frac{v}{v_{the}^2}\xi\left(\frac{v}{v_{thi}}\right)\right]$$

$$\mathcal{D}_{e,1}(v) = \nu_e f_{Me}\frac{v_{the} j_{\|}}{v_{thi}^2 en}\xi\left(\frac{v}{v_{thi}}\right)$$

- And the remainder is simple

$$\langle C_{ee}[f_{Me}, f_{NMe}] + C_{ee}[f_{NMe}, f_{Me}] + C_{ei}[f_{NMe}, f_{Mi}]\rangle = C_e[\bar{f}_{NMe}](v,\chi)$$

Or $$\langle C_e[f]\rangle = C_e[\bar{f}]$$