The effects of applied 3-D magnetic fields and resistive wall boundary conditions on nonlinear MHD simulations

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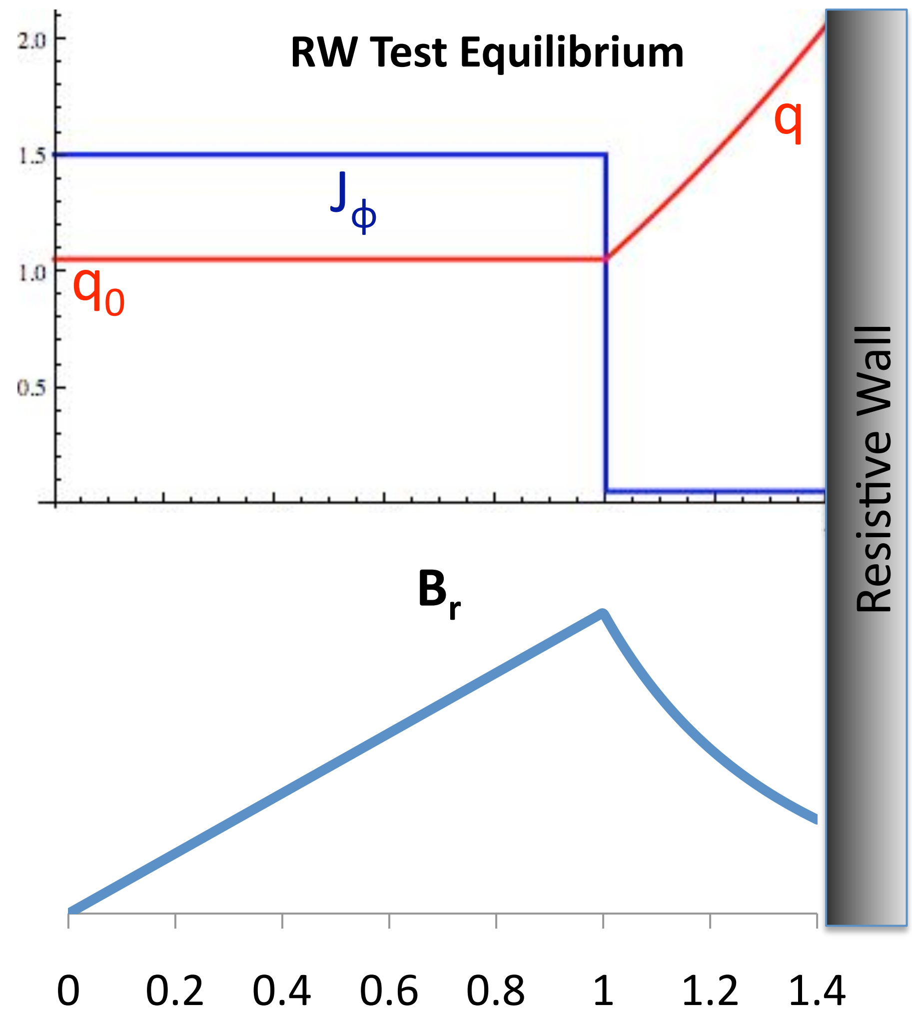
Motivation

- Toroidal rotation is important for stabilizing tokamak plasmas (in the high-beta regime)
- One mechanism for significant slowing of toroidal rotation is locking of a mode (i.e. island) to a stationary external "error field" (nonaxisymmetric field due to coil misalignment) – seen at DIII-D
- The ability to model this in NIMROD would be advantageous – starting with adding resonant error fields to the resistive wall boundary condition for a cylinder
- After proof-of-principle in cylinder, the error field boundary condition can be implemented in a toroidal geometry

Resistive wall boundary condition allows non-zero B_r at the wall

- this boundary condition allows RWMs to grow and other modes to couple to the wall
- the wall-time can be changed to reflect experimental parameters
- resistive wall boundary condition in a periodic cylinder benchmarked for a simple equilibrium
- additional terms can be included in the boundary condition to allow for external 3D magnetic fields which leak into the plasma over the wall-time.



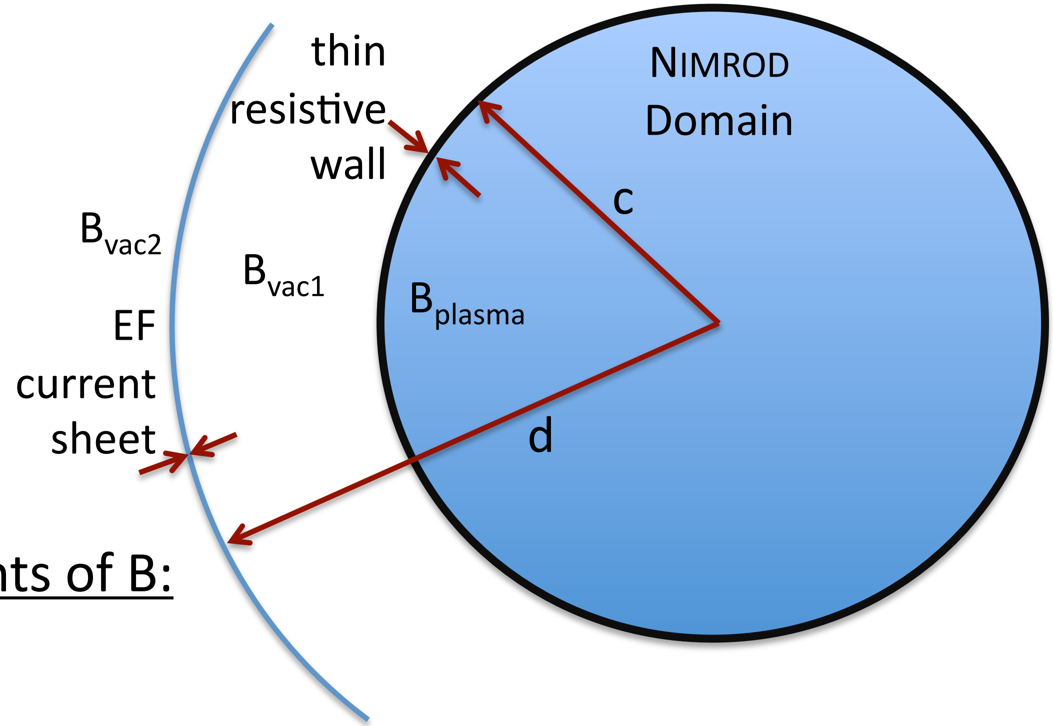
External resonant field is modeled as a helical current sheet that affects the B-field at the wall

$$\mathbf{B}_{vac} = -\nabla\chi \qquad k = \frac{-n}{R}$$

$$\nabla^2\chi = 0$$

$$\chi_1 = p_1 K_m(|k|r)$$

$$\chi_2 = \left[p_2 K_m(|k|r) + p_3 I_m(|k|r)\right]$$



Matching Conditions on components of B:

$$\hat{\mathbf{r}} \cdot \left[\mathbf{B}_{out} - \mathbf{B}_{in}\right]\Big|_{c,d} = 0$$

$$\hat{\mathbf{r}} \times \left[\mathbf{B}_{v1} - \mathbf{B}_{plasma}\right]\Big|_{c} = \mu_0 \mathbf{K} = \frac{\mu_0 \delta_{wall}}{\eta_{wall}} \mathbf{E}_T$$

$$\hat{\mathbf{r}} \times \left[\mathbf{B}_{v2} - \mathbf{B}_{v1}\right]\Big|_{d} = \mu_0 \mathbf{K}^{EF} = \mu_0 \left(\frac{nd}{mR} + 1\right) K_\phi^{EF}$$

Tangential electric field dependent on perturbed plasma fields and input parameters

Electric field boundary condition:

$$\mathbf{E} = v_w \left\{ \left(\underbrace{\tilde{B}_{pz} - ik\tilde{B}_{pn}C}_{\text{RW}} + \frac{dk}{m}\mu_0 K_{EF}^{\phi} G_{cd} \right)\hat{\theta} - \left(\underbrace{\tilde{B}_{p\theta} - i\frac{m}{c}\tilde{B}_{pn}C}_{\text{RW}} + \frac{d}{c}\mu_0 K_{EF}^{\phi} G_{cd} \right)\hat{\phi} \right\}$$

Geometric factors:

$$\begin{cases} G_{cd} = \left[K_m\left(|k|c\right)\dfrac{I'_m\left(|k|c\right)}{K'_m\left(|k|c\right)} - I_m\left(|k|c\right) \right] \Big/ K_m\left(|k|d\right)\left[1 - \dfrac{I'_m\left(|k|d\right)}{K'_m\left(|k|d\right)}\right] \\ C = \dfrac{K_m\left(|k|c\right)}{K'_m\left(|k|c\right)} \end{cases}$$

$K_{EF}^{\phi} \equiv$ magnitude of external 3-D field surface current density

$$v_w \equiv \frac{\eta_{wall}}{\mu_0 \delta_{wall}} \qquad \tau_w \equiv \frac{r_{wall}}{v_{wall}}$$

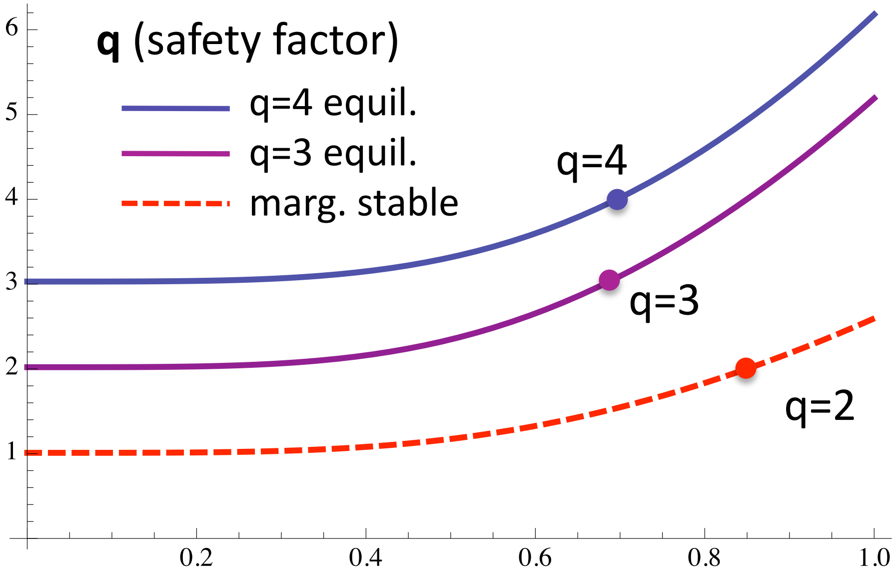
Normal magnetic field boundary condition:

$$B_r^{t+1} = B_r^t + \Delta t\left(\frac{-im}{c}E_z + ikE_\theta\right)$$

Velocity boundary condition:

$$v_r = v_\theta = v_\phi = 0$$

Stable q=4 resonant equilibrium with zero-beta is used for error-field studies



$$q = q_0 \left[1 + \left(\frac{r}{r_0} \right)^{2\lambda} \right]^{1/\lambda}$$

	q_0	r_0	λ	Δ'
q=4 equilibrium	3.03	0.75	2	-2.24

r_{wall}	1 m
R_0	5 m
$S(r_s)$	2.6×10^6
$P_m(r_s)$	1
δ_L	0.005 m
τ_A	1.9×10^{-5} s
τ_L	$2.2 \times 10^3\ \tau_A = 4.2 \times 10^{-2}$ s
τ_w	$105\ \tau_A = 2 \times 10^{-3}$ s
β	0.

Rotation sustains eddy currents at resonant surface which shield the error field

- Toroidal flow (along the axis of the cylinder in cylindrical geometry) is present in tokamaks
- Considering only linear terms, toroidal rotation maintains eddy currents at the resonant surface which shield the interior plasma ($r<r_s$) from the external error field.
- Rotation above a critical value will shield the plasma interior to the rational surface from the error-field.

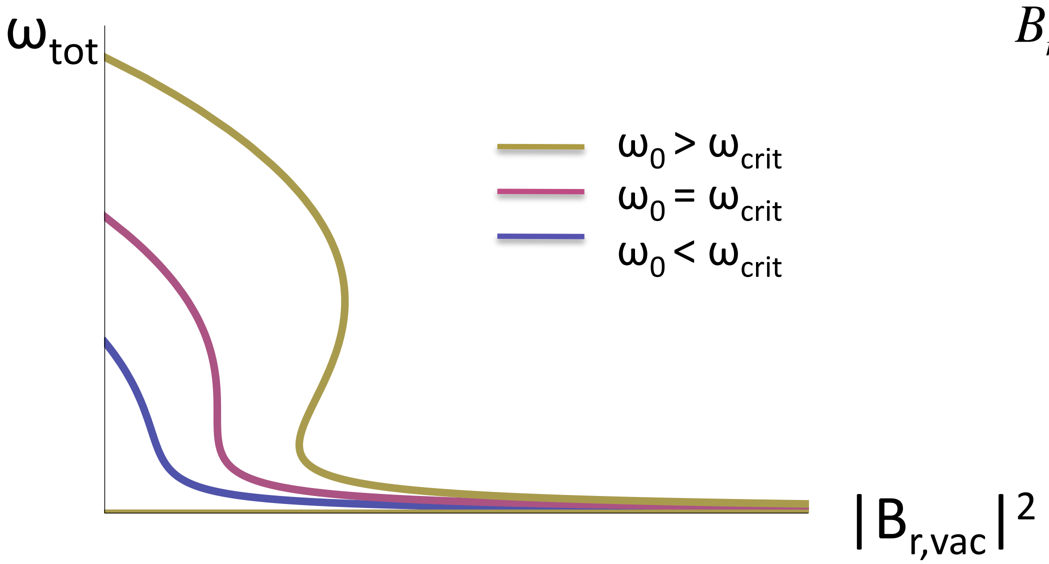
$$\omega_{crit} = \frac{V_{crit}}{R_0} = 3\sqrt{3}\,\frac{-\Delta'}{\tau_L} \sim \eta^{5/6}$$

- A **uniform** equilibrium axial flow is implemented in error-field test cases.

EM torque and opposing viscous torques at rational surface balance

• When non-linear terms are included, the balance between electromagnetic and viscous torques at the rational surface determine the net rotation of the plasma.

$$T_{\phi EM} + T_{\phi VS} = 0 \longrightarrow \left(\frac{m^2}{r_s^2} + \frac{n^2}{R_0^2}\right) \frac{2\tau_L \left(\int_{r_s}^{a} dr/r\rho\nu\right) \left|\tilde{B}_{r,vac}\right|^2}{\mu_0 (-\Delta')^2} \frac{\omega}{1 + \left(\omega\tau_L/(-\Delta')\right)^2} = \omega_0 - \omega$$

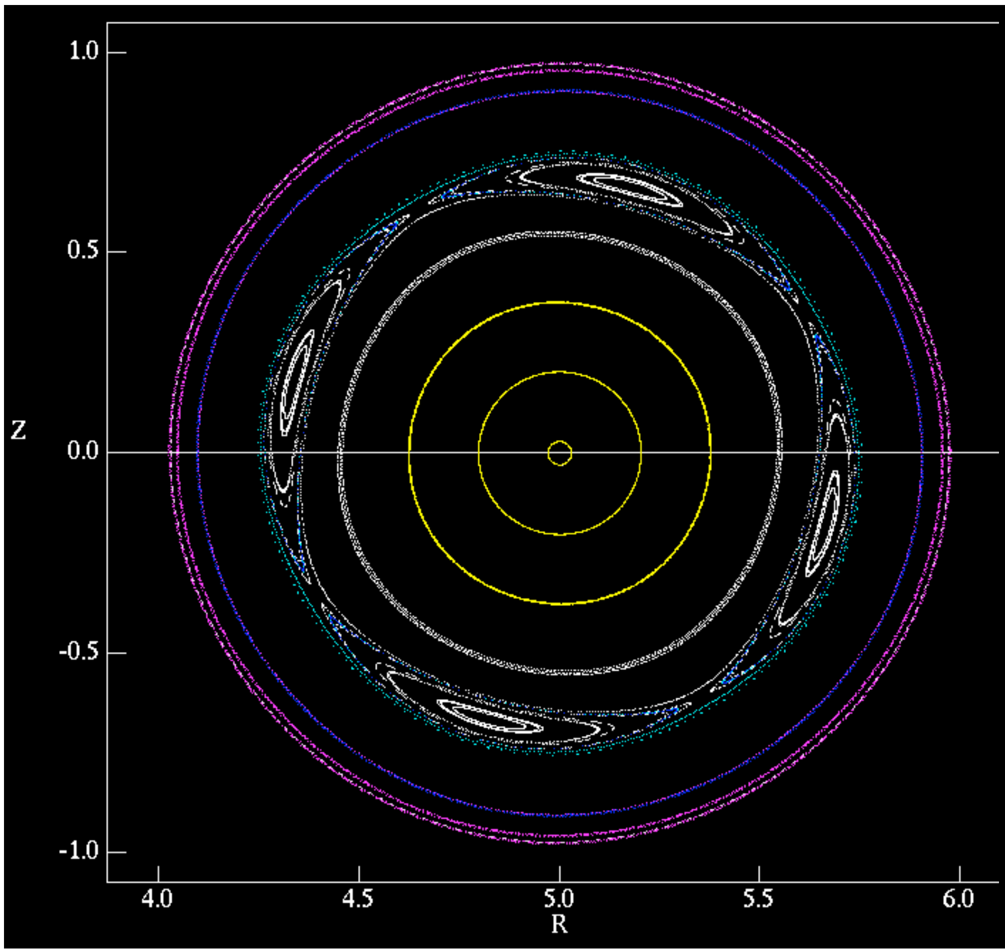


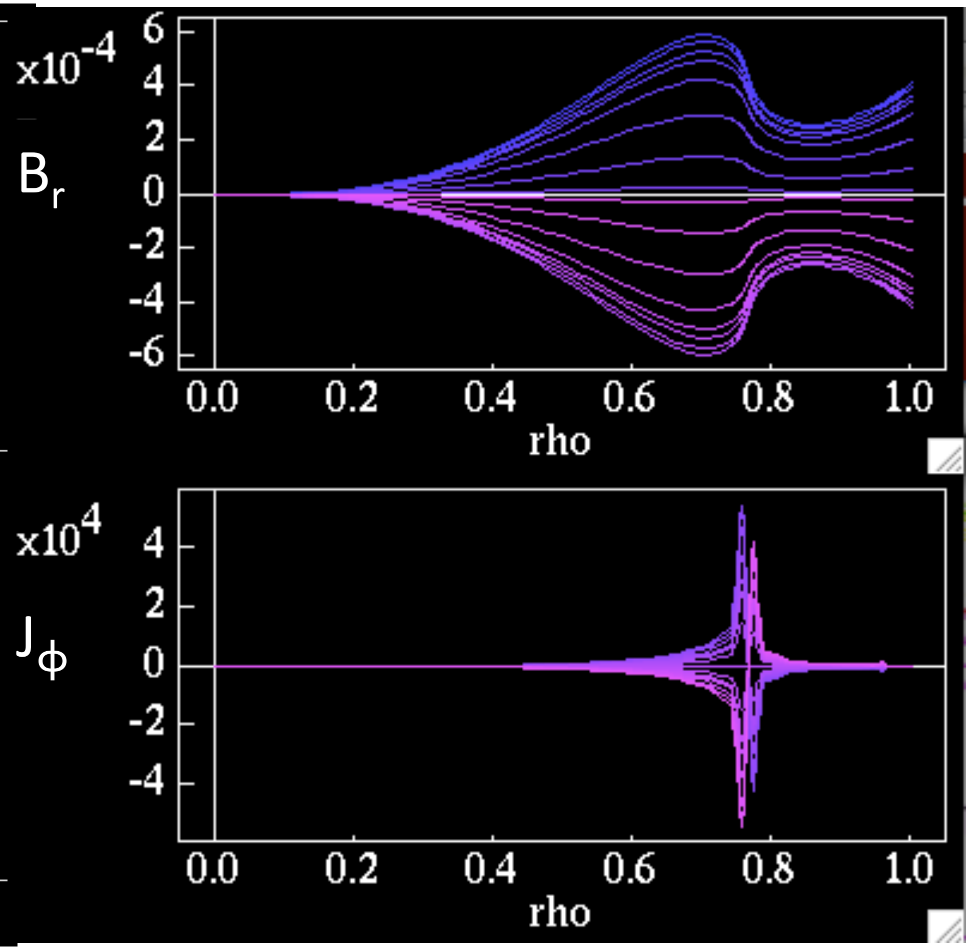
$$\tilde{B}_{rlock} = \frac{\omega_0}{2r_s} \sqrt{\left(\frac{m^2}{r_s^2} + \frac{n^2}{R_0^2}\right)^{-1} \frac{\mu_0 \tau_L}{2\left(\int_{r_s}^{a} dr/r\rho\nu\right)}}$$

• Plasma rotation must be above the critical value to see an abrupt lo cking or hysteresis as error-field is lowered.

•Theory assumes linear island evolution – not necessarily the case

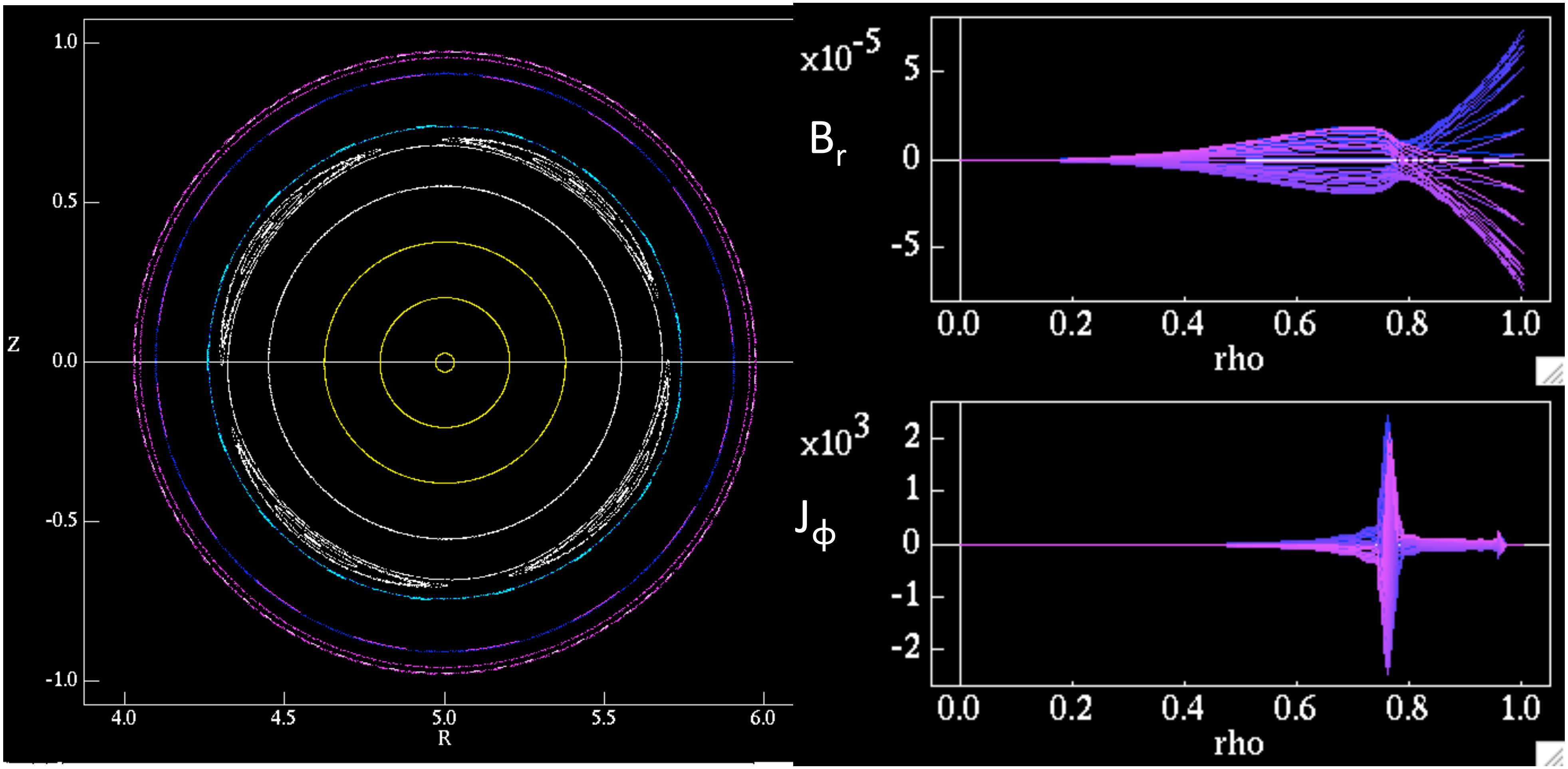
Linear result: error-field penetrates plasma to q=4 surface and forms island





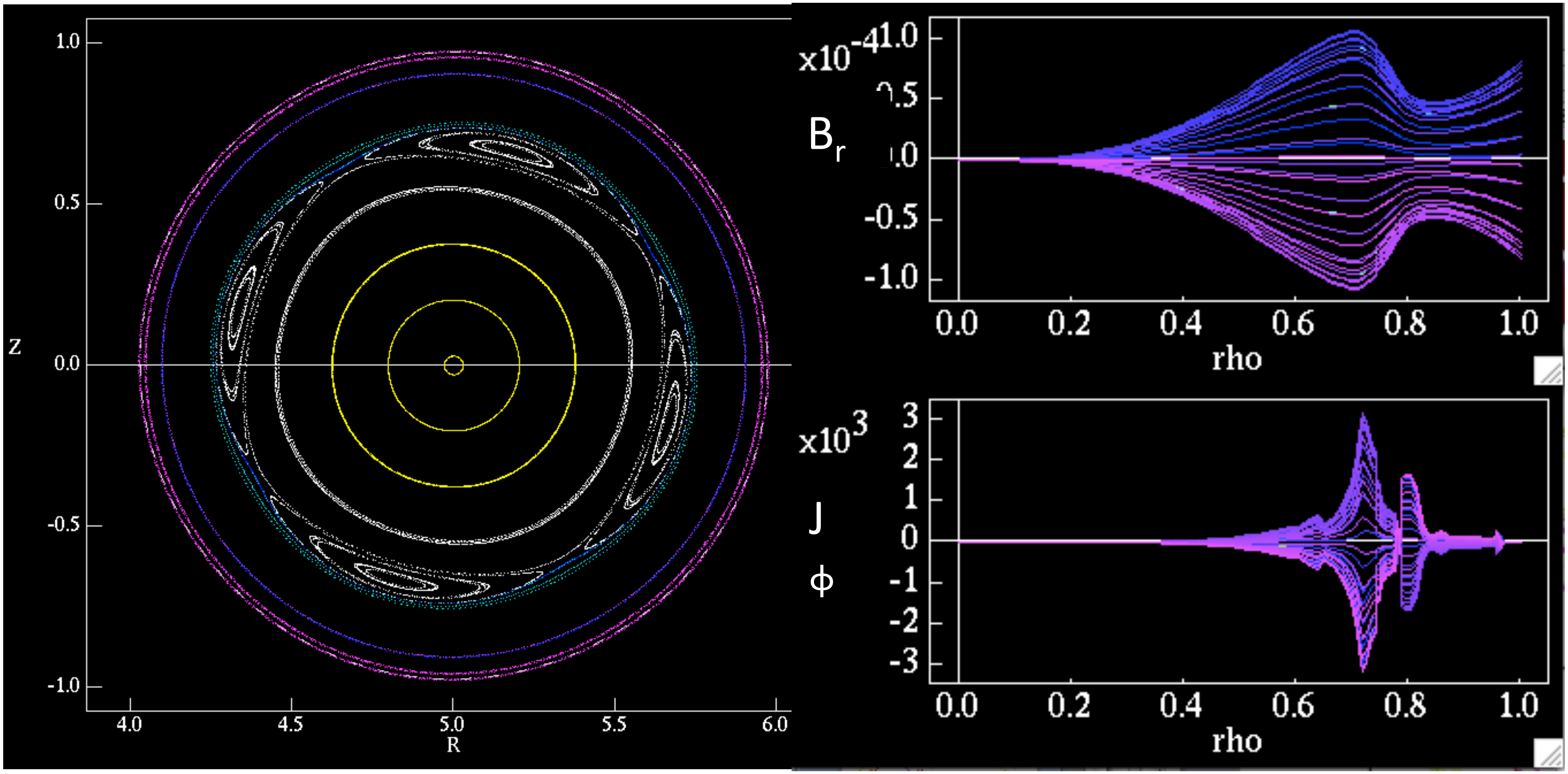
ve0=0 m/s, B_{rvac}(rs)=2.8 x 10^{-5} T

Linear result: equilibrium flow shields plasma by maintaining an eddy current



ve0=1500 m/s > v_crit=1390 m/s, $B_{rvac}(rs)=2.8 \times 10^{-5}$ T

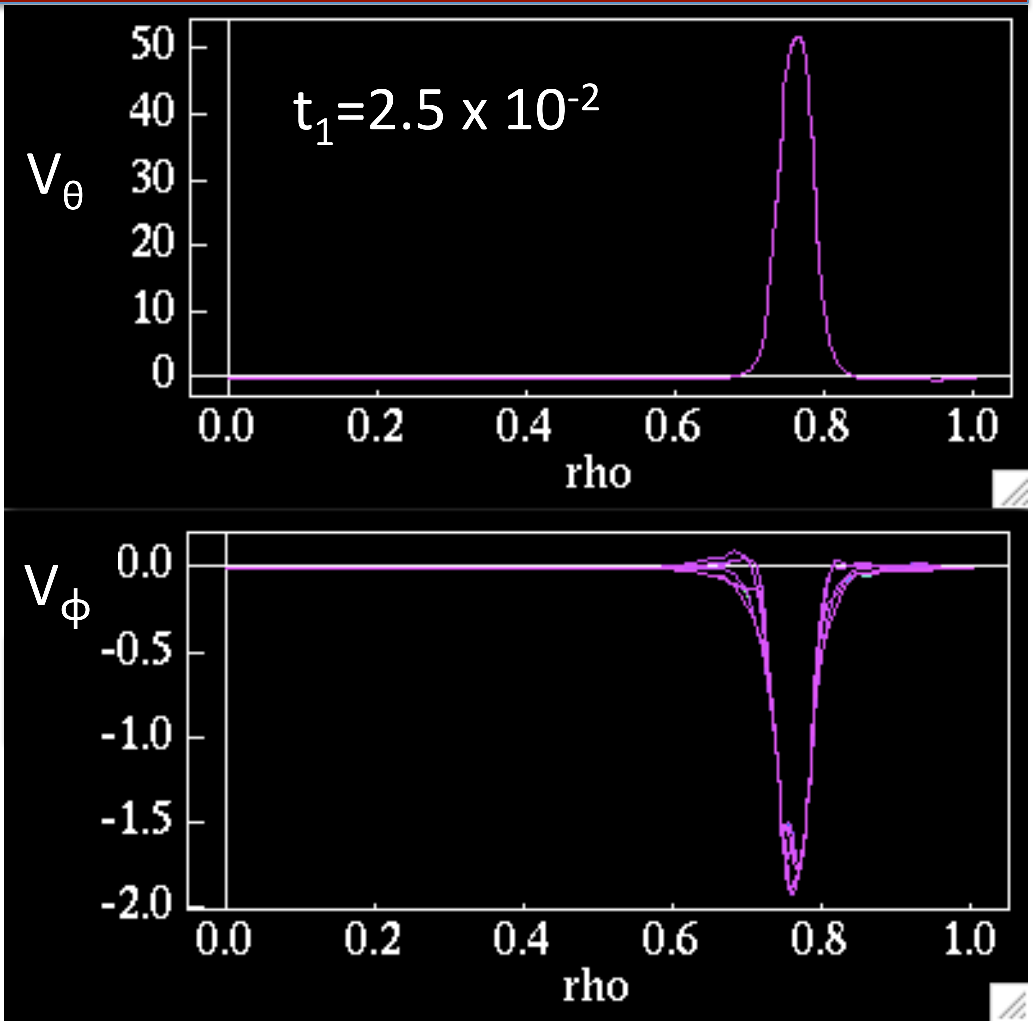
Non-linear result: large error-field can penetrate flowing plasma



ve0=1500 m/s > v_crit=1390 m/s, $B_{rvac}(rs)=2.8 \times 10^{-5}$ T

Non-linear result: large error-field can change plasma flow

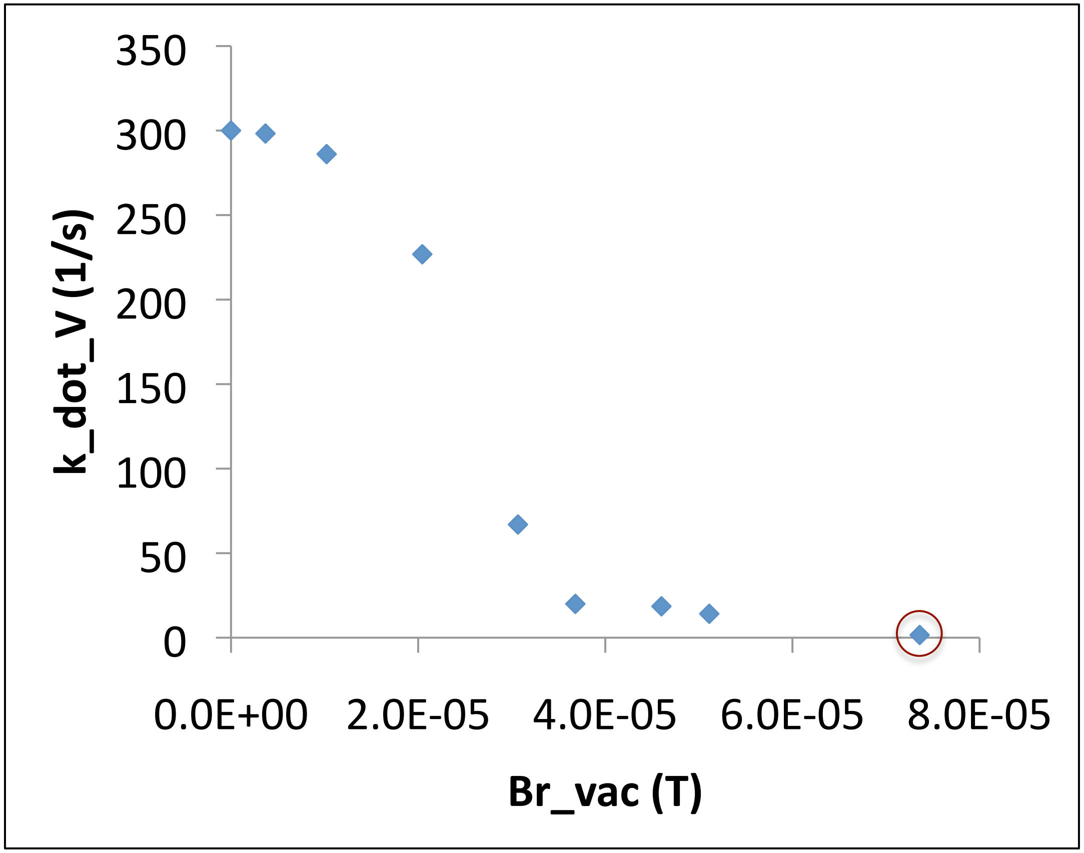
- Increase in poloidal flow decreases the net rotation k•V, and is the dominant term (besides the background equilibrium flow)



ve0=1500 m/s > v_crit=1390 m/s, B_{rvac}(rs)=2.8 x 10^{-5} T

Non-linear result: large error-field can produce islands that lock to the wall

- Equilibrium flow above the critical value for linear island theory, but not necessarily for non-linear island theory.
- Do not see an abrupt mode locking, but a gradual change in total rotation.
- At large enough error-field magnitude, island is locked to the wall.



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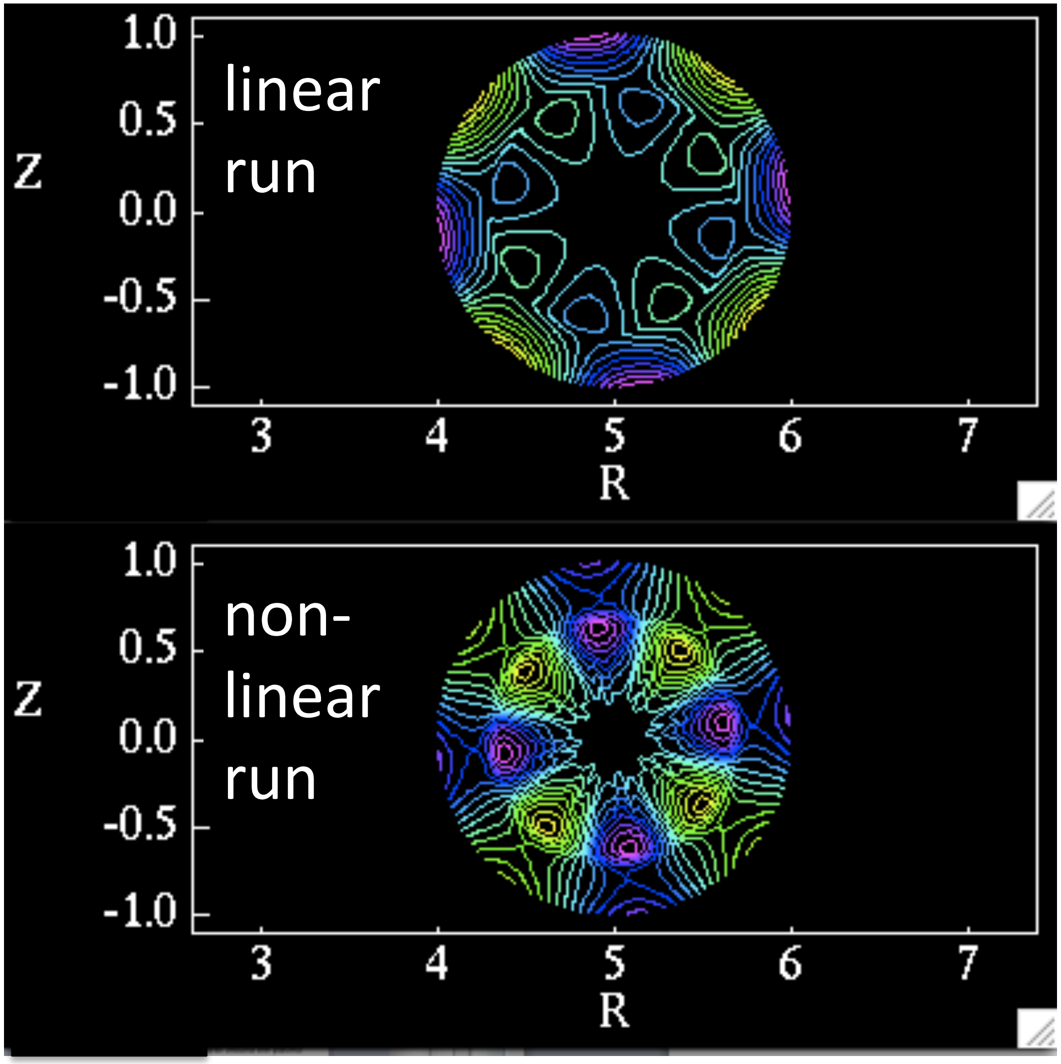
Contours of Re[B_r]



Conclusions

- Resonant error-field boundary conditions in NIMROD allow the formation of driven islands in a stable equilibrium
- Linear shielding of the plasma by adding equilibrium flow is demonstrated
- Nonlinear error-field penetration is demonstrated, including the 'slowing down' of the plasma in response to EM torque and locking of islands to the wall