MHD and compressibility - Sawteeth and Snakes

L. Sugiyama

Laboratory for Nuclear Science
M.I.T.
Cambridge MA 02139-4307

CEMM meeting
APS-DPP
Providence RI
October 28, 2012

Topics

- MHD model: compressibility and toroidal aspect ratio
- 1/1 sawtooth fast crash, fast onset at low resistivity
  - Not RMHD, not Sweet-Parker reconnection!
- 1/1 snakes - update

Sawtooth and MHD

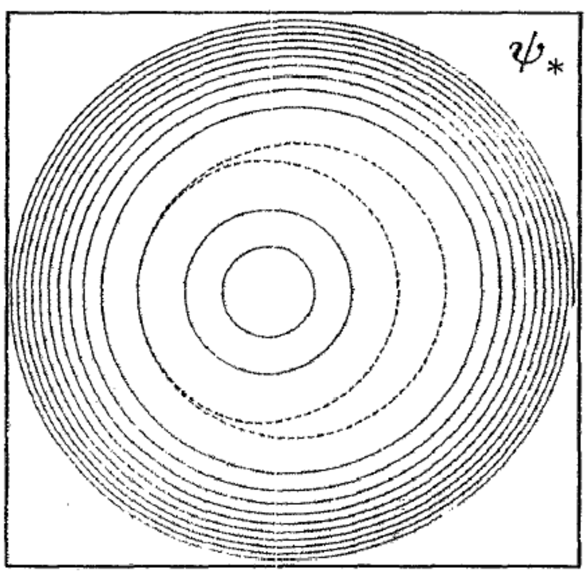
- Sawtooth crash still raises major theoretical questions:
  - Fast crash at low resistivity
  - Fast crash onset
  - Temperature loss through X-point (recent observations)
  - Kadomtsev sawtooth model?
- 1/1 mode is clearest choice to investigate MHD model issues - large scale structure and strong toroidal mode coupling
  - Theory is incomplete (both linear and nonlinear)
  - Large aspect ratio approximation, but toroidal effects are crucial (linear mode: Bussac 1975, Wahlberg 1999)
  - MHD compressibility couples to finite aspect ratio effects: sawtooth is best model to investigate

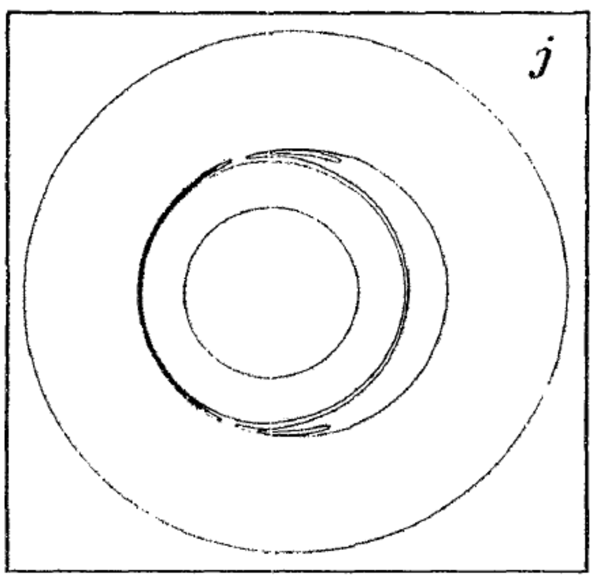
Existing nonlinear sawtooth models are pressure-only and strongly RMHD

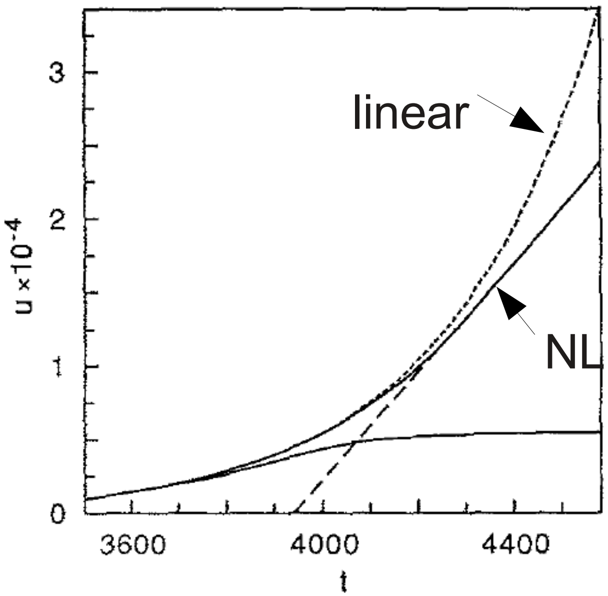
- Crash is determined by helical flux conservation; plasma tied to helical flux tubes (Kadomtsev 1975). Not RMHD.
  - Reconnection time $\tau_K = (\tau_R \tau_A / q')^{1/2}$
- How to get fast crash? Early nonlinear stage of sawtooth:
  - 1/1 internal kink instability + small 1/1 magnetic island (Hazeltine et al 1986, Waelbrock 1988, Biskamp 1991)
  - Modified Rutherford equation dW/dt at q=1 surface: current density in reconnection layer determines island growth (Wesson 1986, Smolyakov 1993)
- Hazeltine: basic 1/1 linear eigenfunction form for ψ; ignore current layer at q=1 $\rightarrow$ ψ grows exponentially at slightly less than linear γ_L.
- Waelbrock: Current sheet. Island dynamics determined by helical flux conservation (Kadomtsev)$\rightarrow$narrow, poloidally elongated reconnection layer, $W \sim \eta t^2$

Pressure-only and RMHD -2-

- Biskamp: current sheet + island + linear m=1 eigenfunctions→quasi-static, modified Sweet-Parker reconnection in narrow, poloidally elongated layer, momentum balance in r,θ directions: $W \sim K\eta t^2$, $V_{inflow} \sim K\eta t$
  - Good approximation to RMHD numerical solution
  - But, $V \sim \eta t$ was slower than linear growth γ_L. Too slow to explain later sawteeth at lower η but similar fast crash times!







Existing nonlinear sawtooth models are pressure-only and strongly RMHD -3-

- Fast crash “requires” non-MHD or non-kink nonlinear effects
  - Two fluid parallel electron compressibility (Aydemir 1991, Wang-Bhatacharjee 1992)
  - Electron inertia (Porcelli 1991, Biskamp 1994 (partial))
  - Continuing search: MHD/non-MHD stochasticity, ...
- Experimental observations of 2D T, etc, don't really resemble Kadomtsev model – why?
- 1/1 density snakes are common at or near q=1.They usually coexist with periodic sawtooth oscillations – how?

Fast crash occurs naturally in full MHD

- Slow crash is an artifact of RMHD assumptions
  - Infinite aspect ratio limit, incompressible, pressure-only
- Large aspect ratio (finite) introduces compressional velocity (χ), density evolution (no longer dn/dt=0)

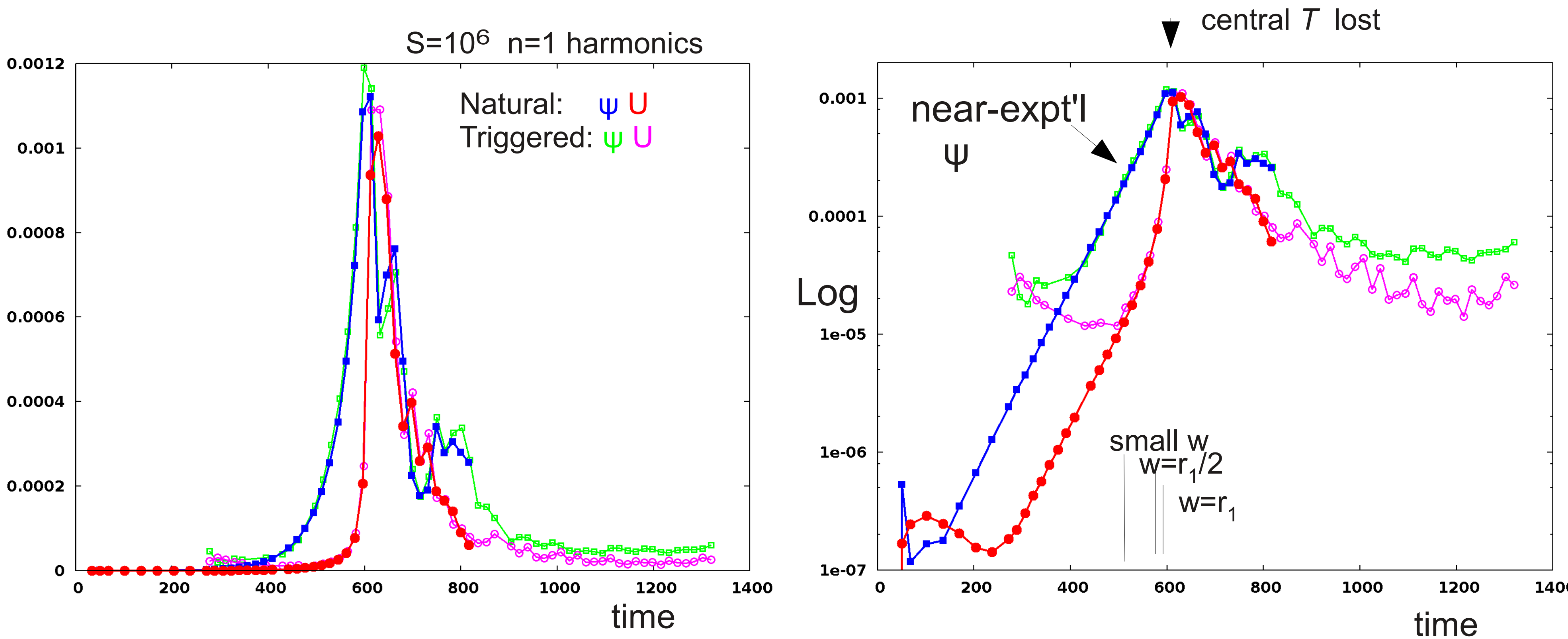
$$\mathbf{v} = \epsilon R \nabla_\perp U \times \hat{\boldsymbol{\phi}} + \nabla_\perp \chi + v_\phi \hat{\boldsymbol{\phi}}$$

  - Density evolution: density is not tied to magnetic flux tubes
    - Not Kadomtsev
    - Temperature and magnetic field (large parallel thermal conductivity) can evolve faster/slower than *n* and v
- Chain of cause and effect breaks down
  - “Fast” reconnection ↔ helical magnetic flux conservation ↔ thin, poloidally elongated reconnection layer ↔ Sweet-Parker quasi-static balance

Fast crash occurs naturally in full MHD

- Slow crash is an artifact of RMHD assumptions
  - Infinite aspect ratio limit, incompressible, pressure-only
- Large aspect ratio (finite) introduces compressional velocity, density evolution (no longer dn/dt=0)
  - Density evolution: density is not tied to magnetic flux tubes
    - Not Kadomtsev
    - Temperature and magnetic field (large parallel thermal conductivity) can evolve faster/slower than *n* and v
- Chain of cause and effect breaks down
  - "Fast" reconnection ↔ helical magnetic flux conservation ↔ thin, poloidally elongated reconnection layer ↔ Sweet-Parker quasi-static balance
- Compressible effects on linear internal kink (m=2) change growth rate scaling (J. Hastie personal comm.2000, Wahlberg 1999; many numerical solns)

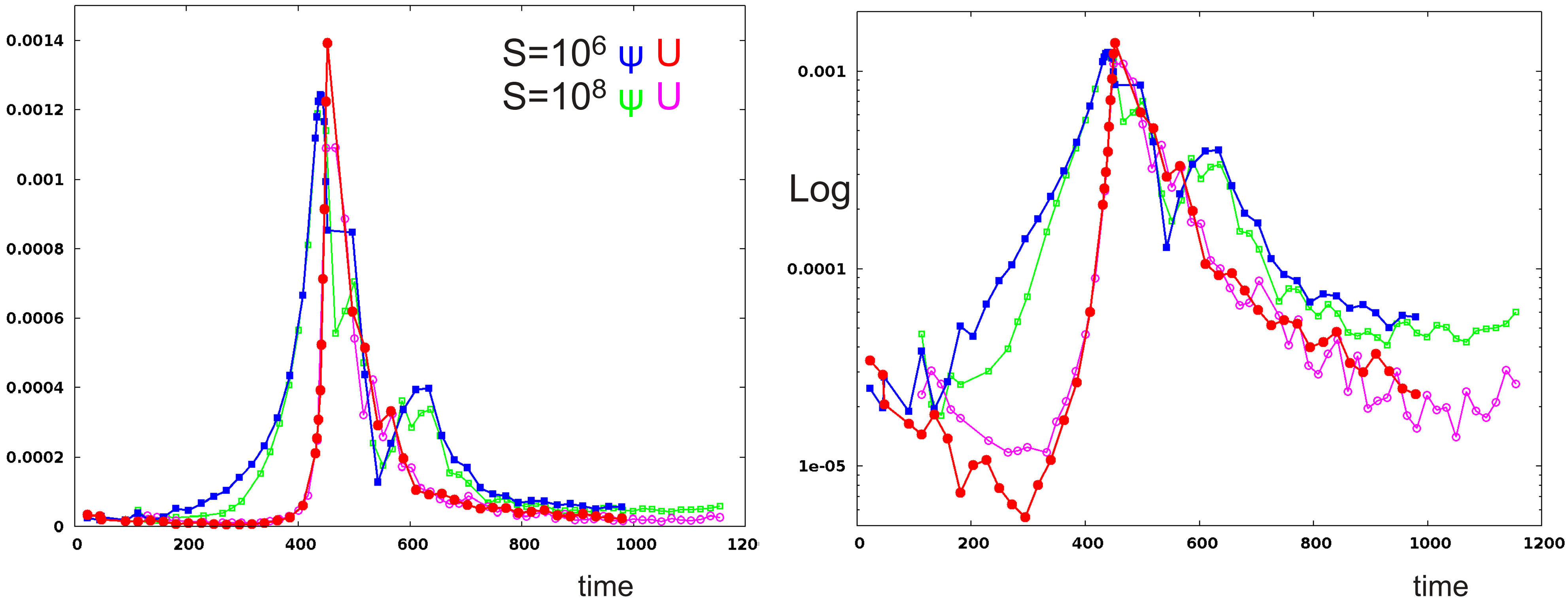
MHD simulation (M3D)



- Sawtooth crash from small amplitude ($S=10^6$) shows fast crash with rapid acceleration roughly indept of η for $S=10^6$-10^8
  - ψ grows slightly faster than linear exponential rate, U much faster. Ratio χ/U approx constant ~1/10 (L2 norm, vol-average)
  - Comparison to triggered crashes (large initial 1/1 δn/n) suggests there is a critical amplitude to trigger onset of fast crash

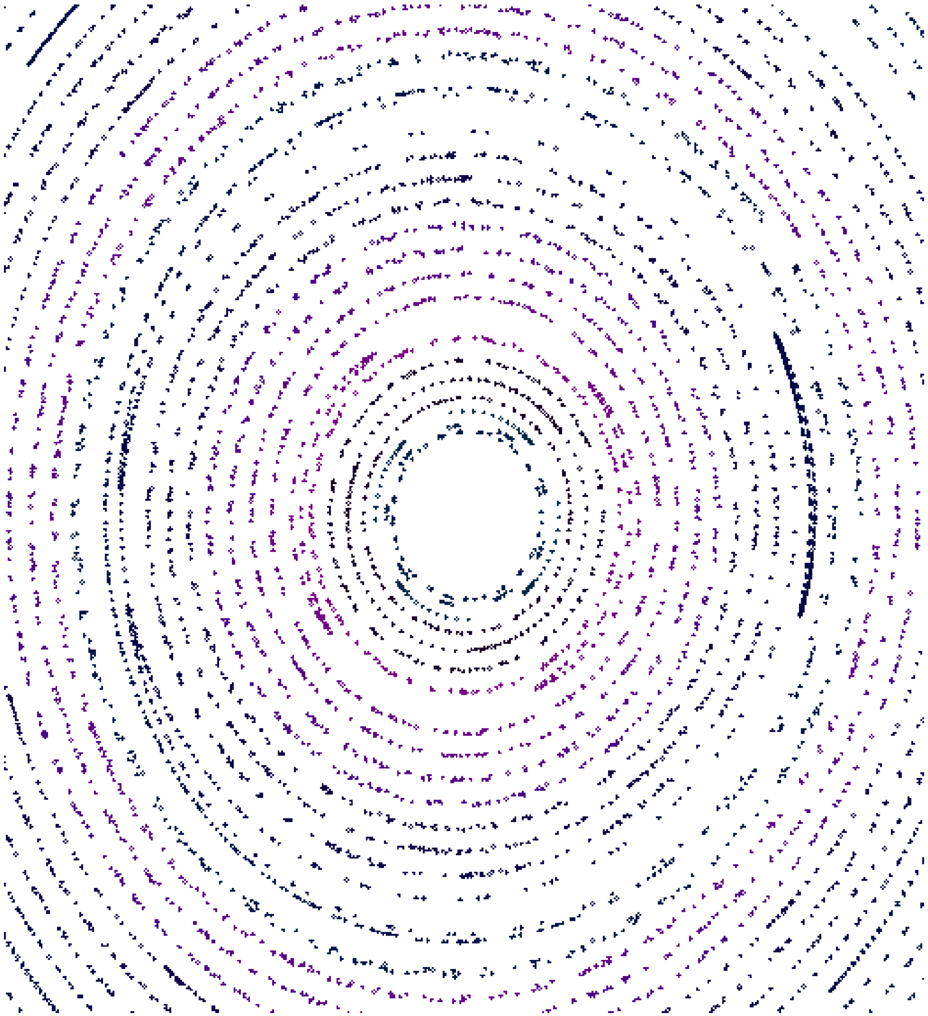
C-Mod large q<1 sawtooth (JRT2011 edge equilibrium

MHD simulation (M3D)

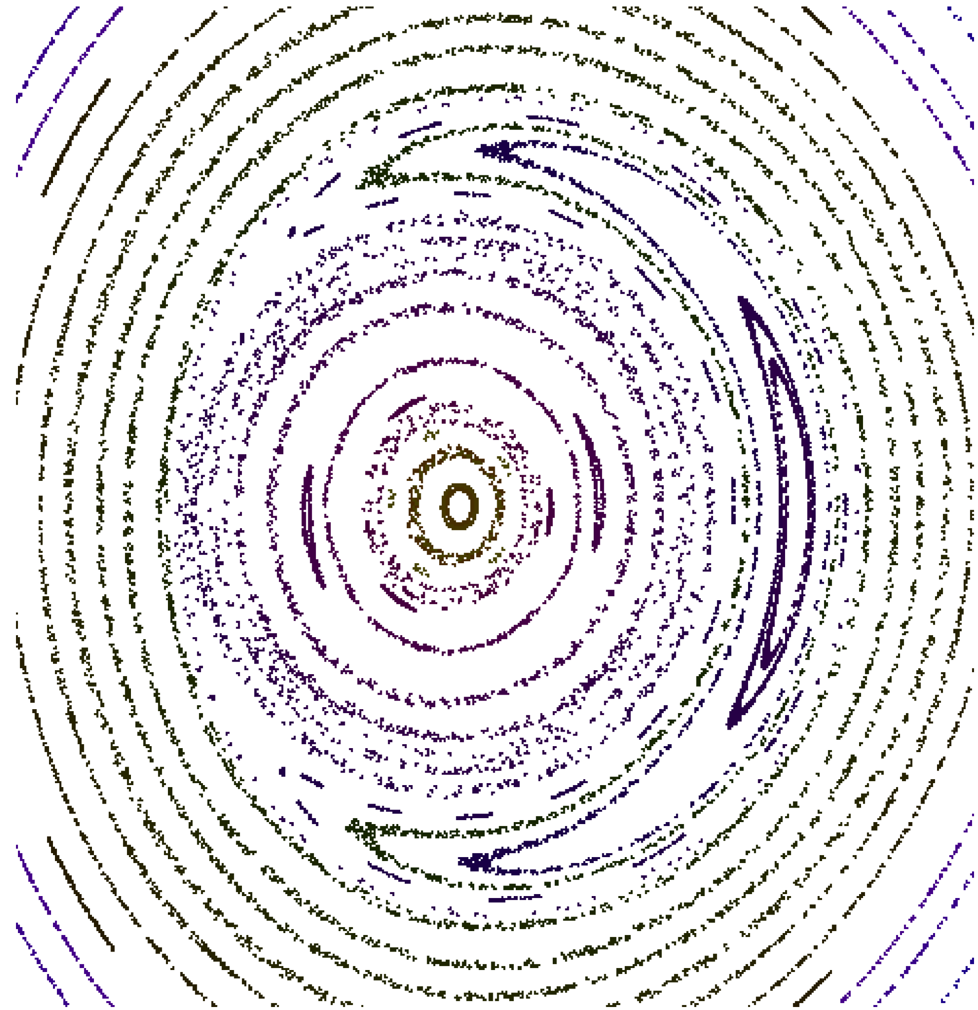


- Sawtooth crashes (triggered) show fast crash with rapid acceleration roughly independent of η for $S=10^6$-10^8
- Natural (non-triggered) sawtooth crash onset time varies inversely with η, as expected
- Time-traces shifted rigidly in time to overlay peak of fast crash.

MHD simulation (M3D)



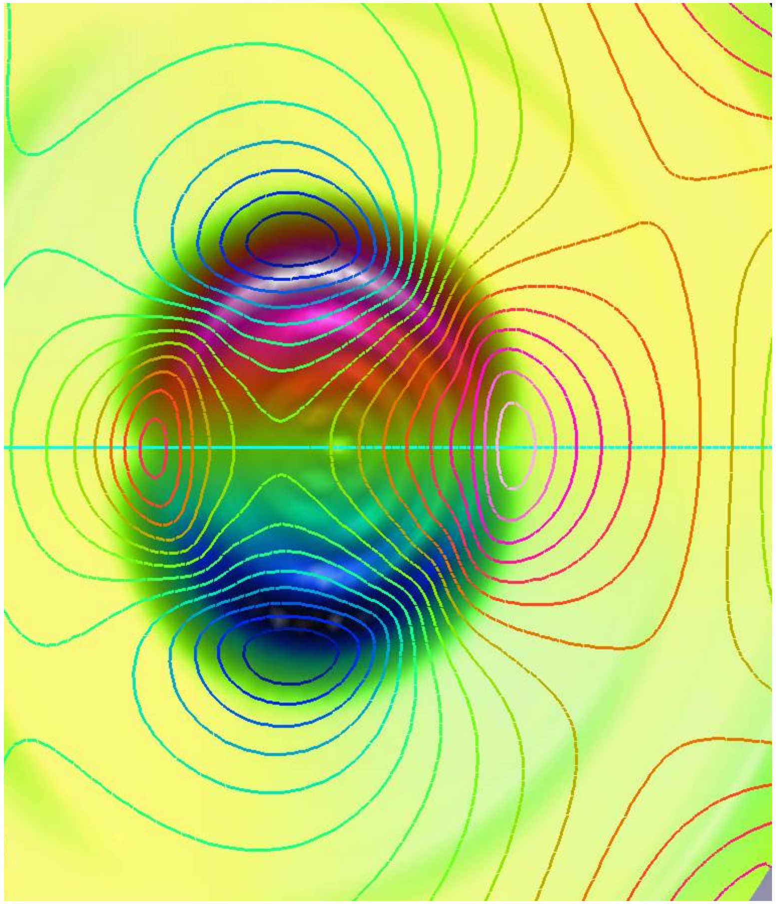
$S=10^8$ early



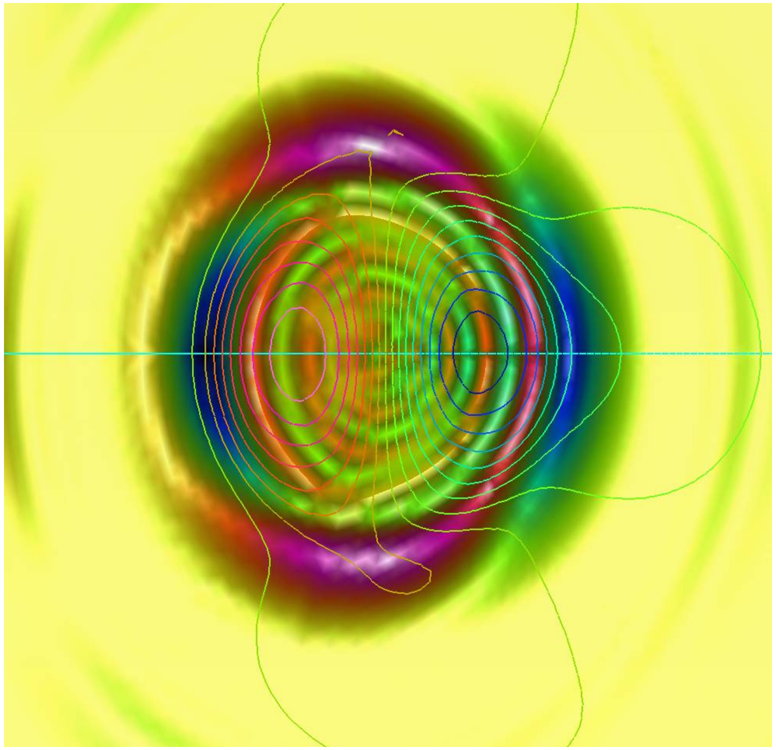
$S=10^6$ later

- Sawtooth crashes (natural and triggered) do not have thin elongated reconnection layer of RMHD, from beginning

MHD compressibility



χ contour lines, U red/blue shaded



$\nabla_\perp^2\chi$ (red/blue) on ψ lines

- n=1 compressibility terms in nearly-linear mode are strongly m=2 and concentrated around q=1 (case $S=10^6$)

(kink to right)

Compressible J_φ in $\perp$ momentum balance

- MHD $\quad \nabla_\perp(p + v_\perp^2/2) + \mathbf{J}\times\mathbf{B}|_\perp \simeq \partial\mathbf{v}_\perp/\partial t$
- RMHD momentum balance in q=1 layer (Biskamp '91)

$$\nabla_\perp(p + v_\perp^2/2 + B^2/2) \simeq \partial\mathbf{v}_\perp/\partial t$$

  - Sweet Parker reconnection balance in layer in r, θ directions

$$\nabla_\perp(p + v_\perp^2/2 + \cancel{I^2/2}) + \nabla_\perp(|\nabla_\perp\psi|^2/R^2) \simeq 0$$

  - $v_\perp$ and $B \approx B_{hel}$ are approximately 1/1 eigenfunctions, so p is $m=n=2$

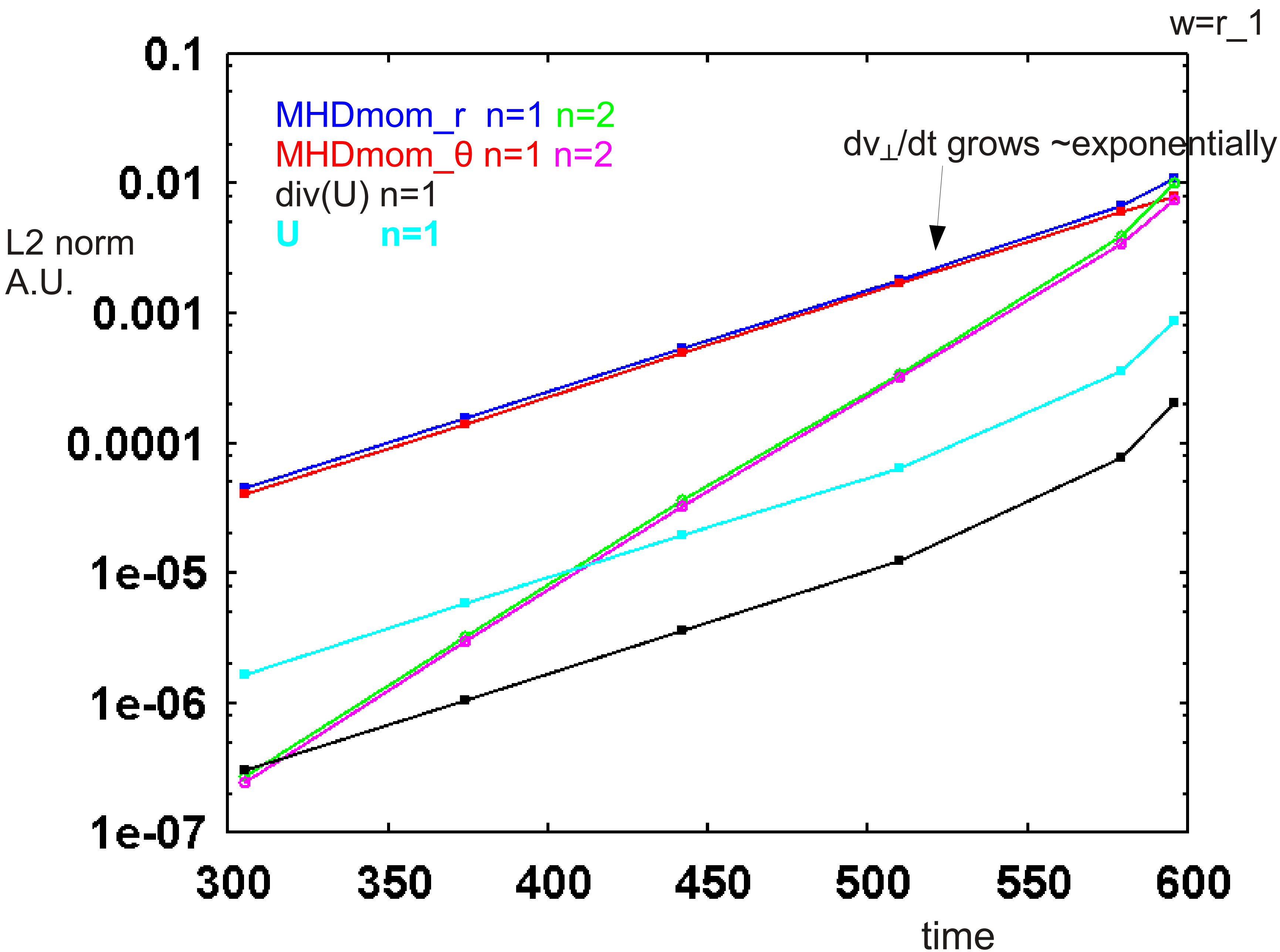
- MHD $\nabla_\perp[p + v_\perp^2/2 + (R_o I/R)^2/2] + (J_\phi/R)\nabla_\perp\psi \simeq \partial\mathbf{v}_\perp/\partial t$

- Difference MHD-RMHD is same order as RMHD term. MHD term grows with mode n=1 ψ, so $\partial v_\perp / \partial t$ grows exponentially!

- MHD effect begins from linear kink instability (also seen in other numerical simulations; analytical U-only J.Hastie 2000, private communication)

Note: $\perp$ is perpendicular to φ

Compressible ⊥ momentum balance



Compressibility II

- Advection terms are important source of nonlinearity in large aspect ratio
  - In RMHD, *only* NL terms are advection and $\mathbf{B} \cdot \nabla J_\phi$ in d**w**/dt
- χ introduces radial layer width into advection

$$v_r = \partial\chi/\partial r + imU/r_1 \simeq \chi/\delta_1 + iU/r_1$$

- Separate density and temperature evolution equations, rather than just pressure become important
  - Crucial to explain late stages of crash
    - Fast temperature and magnetic loss through X-point, followed by slower density rearrangement
    - Fast *T*, *J* re-symmetrization around axis, while density still evolving, asymmetric

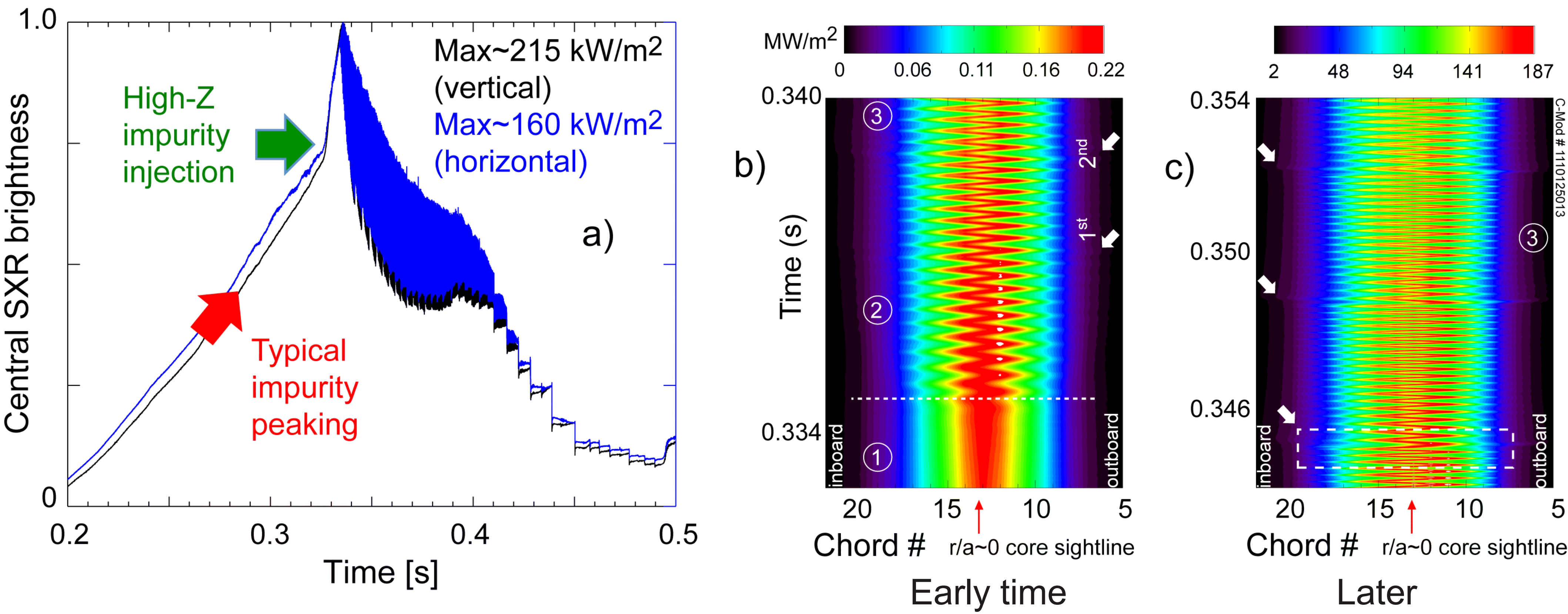
Other MHD effects

- Density evolution is crucial to model late sawtooth crash
  - Temperature evolution matches high resolution expt'l observations of fast localized flow through X-point to created a transient higher temperature layer just outside q=1 (H. Park, etc; LS CEMM spring'12)
- Magnetic stochasticity is enhanced in a torus compared to cylinder or RMHD, due to stronger mode coupling
  - Larger effective (nonlinear) reconnection region, more magnetic mixing can speed up reconnection; effect stronger at low resistivity (seen in simulations)
- Recent results for slab reconnection also show fast reconnection at low η. Large plasmoids form (Sherwood'12).
- Discussion here covers only some of arguments for the effects of compressibility
- Simulation at higher spatial resolution underway

Snakes - update

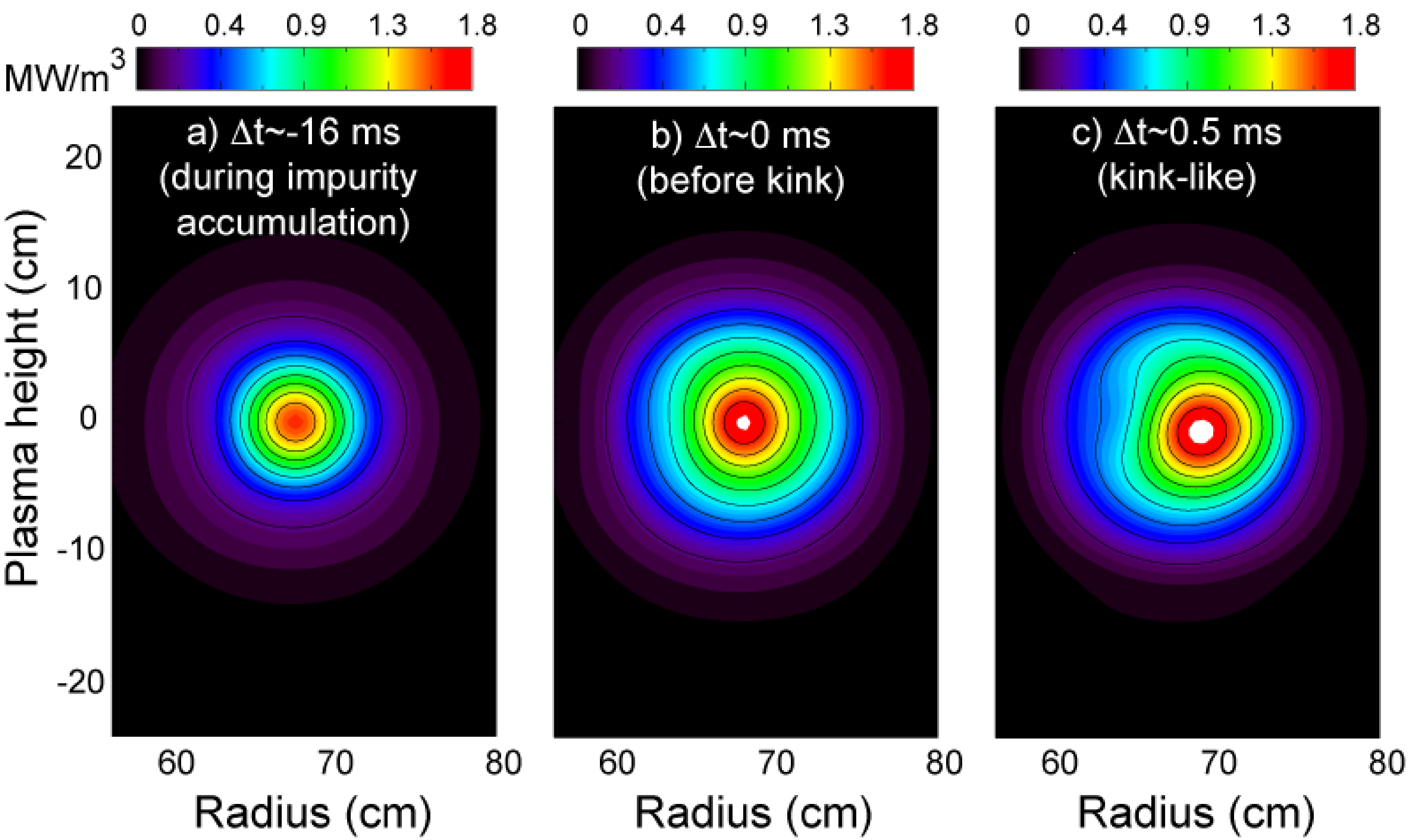
- Alcator C-Mod 1/1 snakes with heavy impurities
  - Original SXR measurements of Mo impurities
  - New experimental results – Ar snakes
  - Two stages: early very broad, near-circular density kink
    later crescent-shaped 1/1 magnetic “island”
    - New data: early stage can last 500 ms
    - Indications that transition between the two depends on early sawteeth and possibly density source due to continued impurity pinching
- M3D simulations of early impurity snake with dn/dt, dT/dt
  - 1/1 helical density concentration around q=1: quasi-steady state
  - Interaction with slowly growing 1/1 internal-kink-like mode inside q<1
  - (Slow) toroidal rotation of background plasma appears important
  - First dynamic interaction of snake+kink => orientation shows kink mode sawtooth crash may preserve early snake
  - Not the Wesson 1986 snake model based on modified Rutherford equation
  - Phys Plasmas paper submitted in June; exptl+sim L. Delgado-Aparicio, Rev Sci Instr 2012, submitted PRL 2012, IAEA FEC/NF 2012.

Long-lived (1,1) snakes in Alcator C-Mod are routinely observed on a number of diagnostics



- Heavy impurity snakes during ohmic current ramp-up phase or early current flattop.
  - High-T_{edge} increases Mo erosion from limiters; on-axis impurity peaking.
- Distinct phases: Impurity peaking → Broad 1/1 kink → Crescent “island”+sawteeth
- Strongly resembles old Doublet-III type-O discharges (Jahns, NF 1982)
- Snakes also seen with other heavy impurities, other plasma regimes

Alcator C-Mod heavy impurity snake: Early stage



SXR: Circular

Displaced core

Rotating kink-like perturbation

Sawtooth-free

Sawtooth-free kink-like state have lifetimes spanning range from few to hundreds of ms

See also:

L. Delgado-Aparicio
C-Mod session
Tues AM

W. Bergerson
Polarimeter data
JP8.96 (Tue. PM)

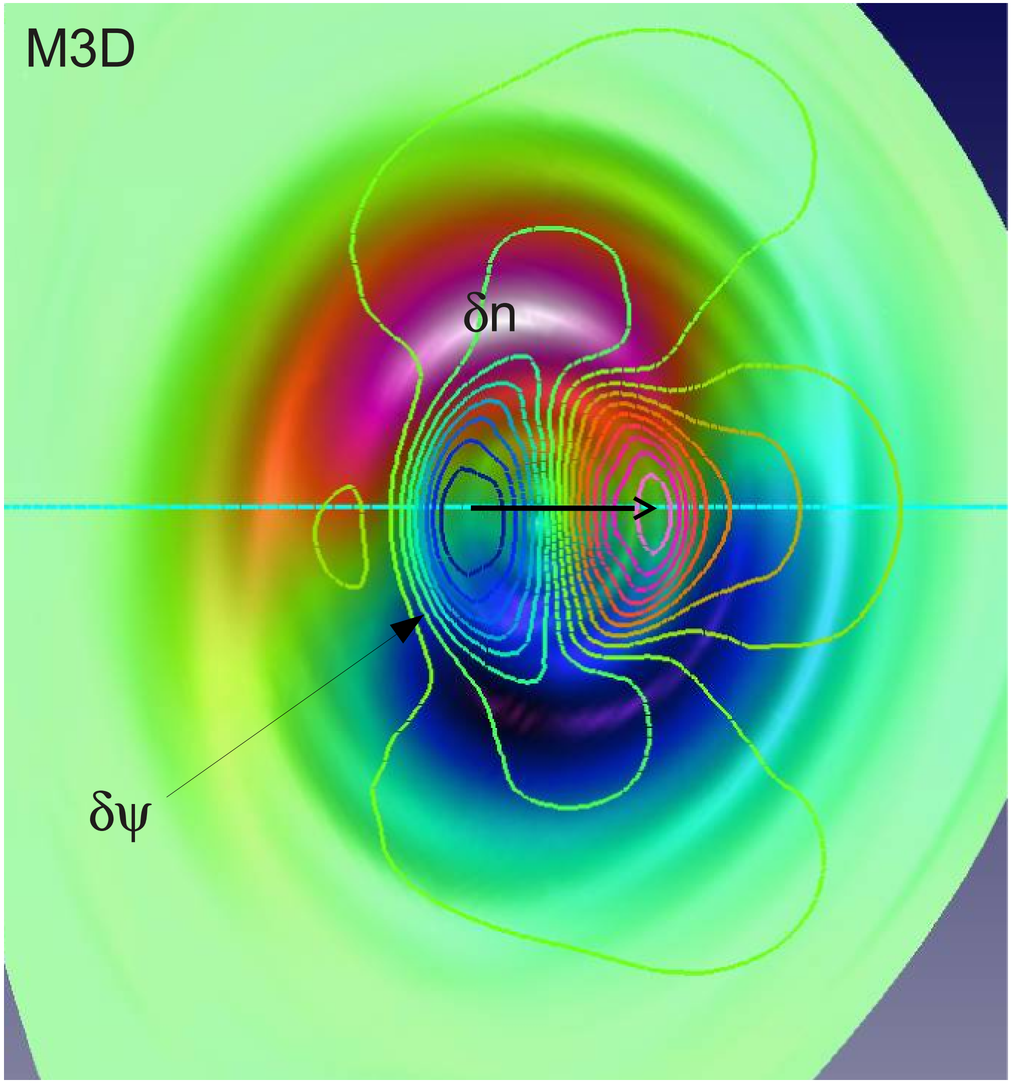
L. Sugiyama
M3D simulations
NP8.115 (Wed. AM)

1/1 density snakes

- First results with dn/dt, dT/dt MHD simulations of Alcator C-Mod heavy impurity snakes (submitted to PoP)
- Density evolution is crucial. Snake is a nonlinear density state.
  - Toroidal rotation of background (small) appears important to match C-Mod results.
  - Helical quasi-steady state density concentration around q=1 layer (including outside) with small helical pressure, since T is perturbed to compensate. Both helicities ($\theta \pm \varphi$). Moderate local q-shear?
  - Helical density drives (or mostly suppresses) a 1/1 internal kink-like perturbation over q<1. Kink motion is perpendicular to density concentration at q=1, so a sawtooth would not destroy the snake.
    - 1/1 mode also has convective motion aligned with the kink p,ψ. Here, driven by v_φ rotation.
- Best fit to early C-Mod snake has density helicity aligned with B, toroidal rotation of magnitude similar to experiment.

Dynamic MHD snake with density and temperature resembles early C-Mod impurity snake

- Nonlinear MHD simulations with toroidal rotation and separate temperature and density evolution find a new nonlinear m/n=1/1 kink-like mode over $q \lesssim 1$ compatible with C-Mod early snake.
- Quasi-steady-state helical density perturbation peaks near q=1 and extends outside $q \gtrsim 1$.
- The helical temperature tends to minimize the perturbed pressure gradient (and free energy) for both q~1 and q<1 perturbations
- No magnetic island
- 1/1 kink motion is perpendicular to main density near q=1 so density snake would not be affected by the sawtooth crash.



[1] L. Delgado-Aparicio, *et. al.*, MIT-PSFC/JA-11-29 Report. submitted to PRL (2012).
[2] L. Sugiyama, submitted to PoP, (2012).
[3] L. Delgado-Aparicio, *et al.*, IAEA/NF (2012)

Summary

- MHD compressibility in large aspect ratio approximation

$$\mathbf{v} = \epsilon R \nabla_\perp U \times \hat{\boldsymbol{\phi}} + \nabla_\perp \chi + v_\phi \hat{\boldsymbol{\phi}}$$

  allows layers with steep radial gradients
- Narrow singular layers (eg, magnetic reconnection) are not included in standard RMHD small-parameter ordering
- 1/1 sawtooth fast crash, fast onset at low resistivity in MHD
  - Compressible effects are m=2, initially n=1. Thus nonlinear crash is not RMHD, not Sweet-Parker, but much faster.
  - Compressible → dn/dt. Sawtooth crash is not Kadomtsev
  - No need for non-MHD effects like two-fluid
- 1/1 snakes – new nonlinear 1/1 mode fits early C-Mod impurity snakes
  - First consistent analysis showing dynamic interaction of 1/1 kink and snake helical density
  - New experimental results show interplay between 1/1 kink instability and 1/1 snake density is consistent with simulation assumptions – to be studied further