Preconditioning of the HiFi Code by Linear Discretization on the Legendre-Gauss-Lobatto Nodes

A. H. Glasser, PSI Center, University of Washington
V. S. Lukin, Naval Research Laboratory

Presented at the CEMM and APS/DPP Meetings
Denver, CO, November 10-15, 2013

Outline

- Properties of the HiFi code.
- Hierarchical parallel solvers.
- Scalable solver performance for ideal MHD waves.
- Magnetic reconnection: failure of physics-based preconditioning.
- The Jed Brown Fix.
- Nodal basis functions and the Legendre-Gauss-Lobatto Grid.

The HiFi Code

- **Spatial Discretization**
  - 2D or 3D unstructured collection of structured grids.
  - Cartesian product of 1D high-order spectral elements, modal or nodal.
  - Logical to physical coordinate mapping.
- **Time Step**
  - Fully implicit Newton-Krylov.
- **Solver**
  - PETSc matrix-free Newton-Krylov iteration applied to full system of preconditioned equations.
  - Physics-Based Preconditioning used to reduce matrix size and condition number.
  - Static condensation used to algebraically eliminate higher-order coefficients.
  - Solution of preconditioning equations uses hierarchy of methods accessible through PETSc, including additive Schwarz and HYPRE/BoomerAMG.
- **Grid Adaptation**
  - 2D harmonic grid generation of mapping from logical to physical coordinates
  - Adaptation to evolving solution.
  - Extension to 3D straightforward but not yet implemented.

Scalable Parallel Solver for Extended MHD

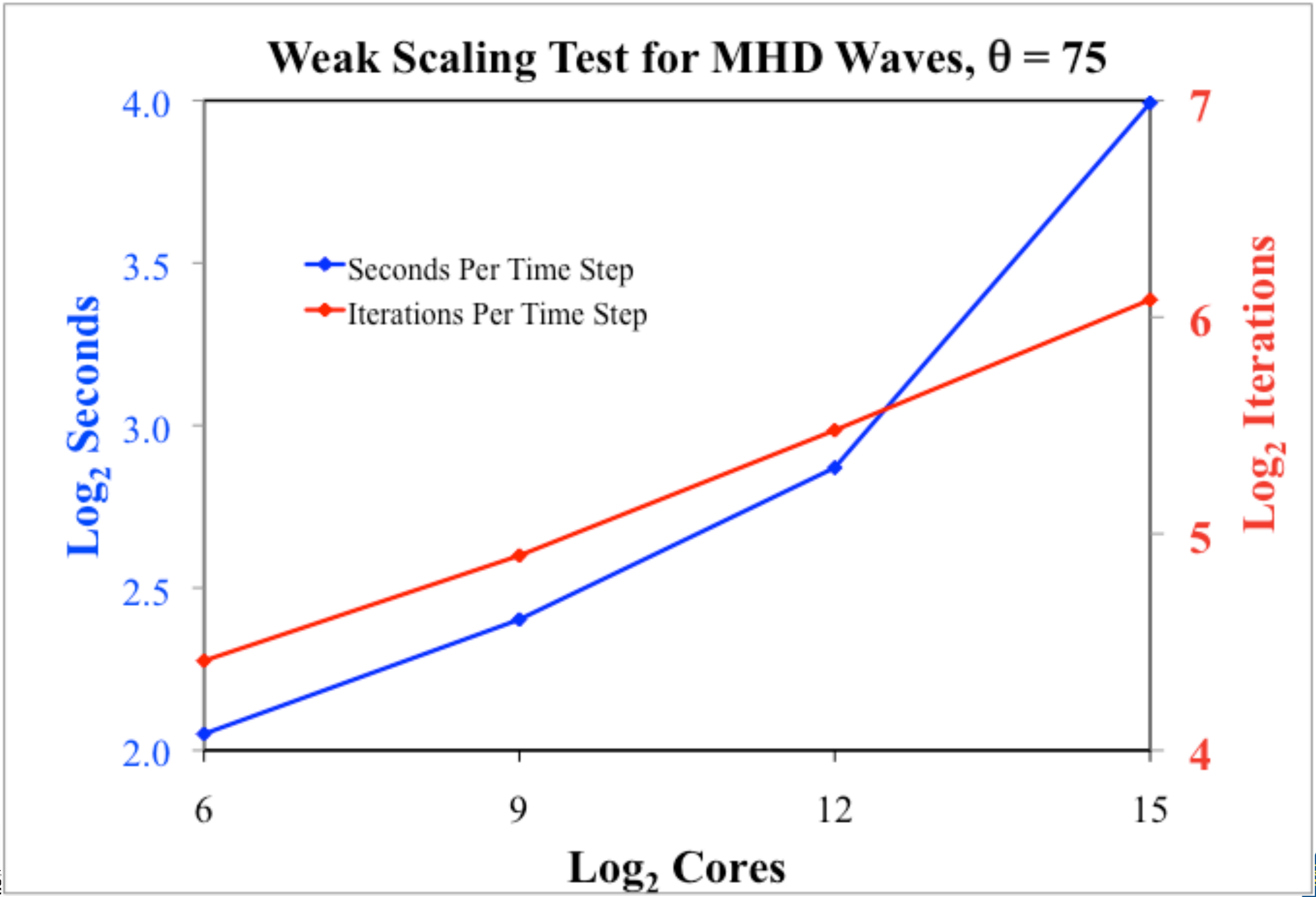
- **Jacobian-Free Newton-Krylov (JFNK)**
  PETSc SNES solver with Physics-Based Preconditioning. Time-centered solution of full nonlinear system of equations.

- **Physics Based Preconditioning (PBP)**
  Chacón. Reduces full hyperbolic linear system to smaller parabolic systems.
  - Partition 1: Mass matrix **M**
    mass density, plasma pressure, magnetic fields, currents
  - Partition 2: Approximate Schur complement matrix **S**
    fluid momenta

- **Static Condensation (SC)**
  Exploits C^0 continuity of spectral element representation. Uses small, local direct solves to eliminates cell interior degrees of freedom in terms of cell boundaries.

- **Solution of Reduced, Condensed Linear Systems**
  - Solver: CG for SPD matrices, GMRES for non-SPD.
  - Preconditioners
    - Schwarz overlap preconditioned by core-wise SuperLU_DIST.
      Fast and efficient but not scalable, increasing number of Krylov iterations
    - Algebraic multigrid, Hypre/BoomerAMG.
      Scalable for limited range of test cases; smoother requires nodal basis.

Scaling for Algebraic Multigrid Schur Solve, np = 6



Magnetic Reconnection, Gem Problem

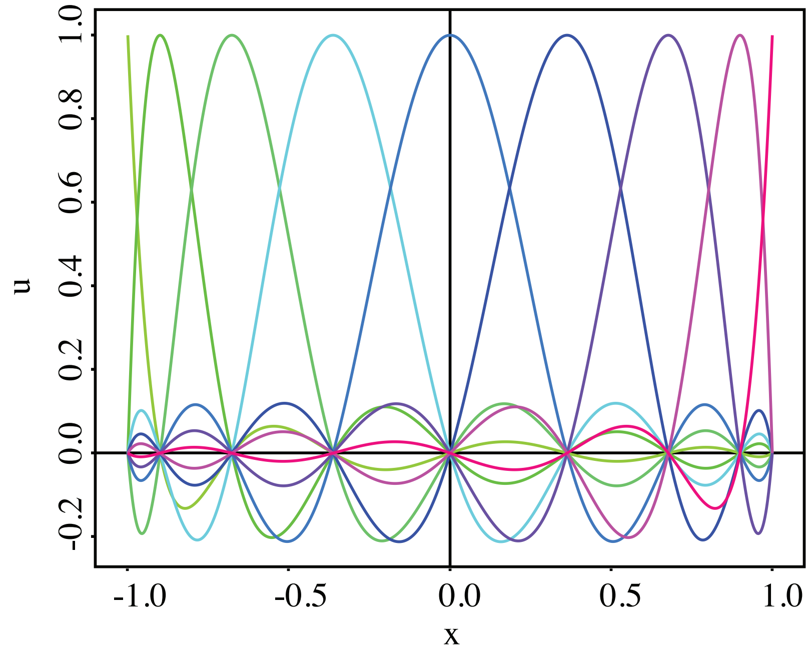
- Strong nonuniformity and nonlinearity.
- Requires improvements in approximate Schur complement.
- Unsuccessful attempts, slower than without Physics Based Preconditioning.
- Rules out use of multigrid.
- Rethink whole approach to scalable solver.

The Jed Brown Fix

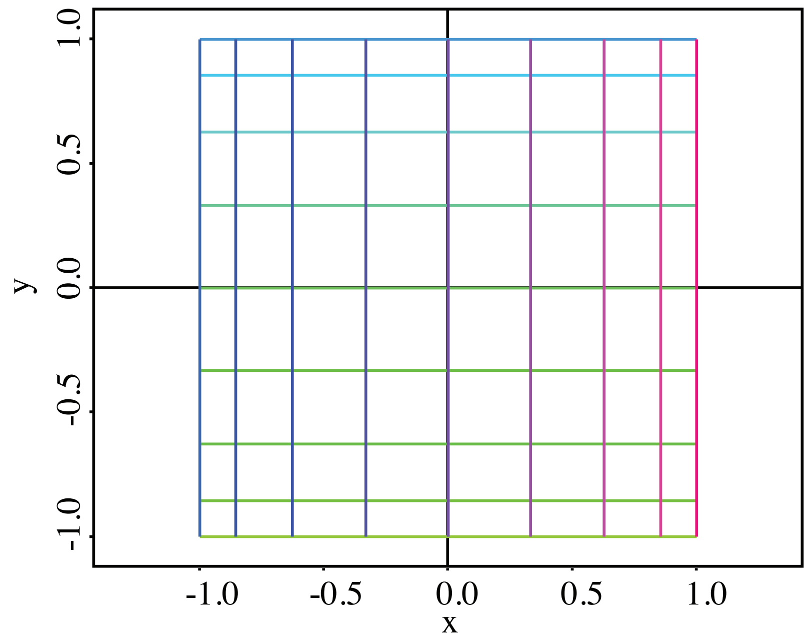
- Jed Brown, "Efficient Nonlinear Solvers for Nodal High-Order Finite Elements in 3D," J. Sci. Comput. **45**, 48-63 (2010).
- For np ≥ 4, matrix formation and partial factorization dominate iterative solution because of loss of sparsity. All physical quantities and basis functions couple within a grid cell.
- Cure: form approximate Jacobian using linear finite element discretization on the grid of LGL nodal points; use as preconditioner for matrix-free solution methods.
- Greatly reduces number of nonzero matrix elements; accelerates matrix formation and matrix-vector multiplication; reduces storage. Replaces static condensation.
- Retains benefits of high-order methods while minimizing cost.
- Status: code written, compiled, run; not yet converging properly, probably due to bugs in the linear discretization.

Nodal Basis Functions and the Legendre-Gauss-Lobatto Grid

Nodal Basis Functions, np = 8



2D LGL Grid, np = 8



LGL Nodes: $(1-x_i^2)P_n'(x_i) = 0$

Full: high-np coupling over each macrocell.

JB: Linear discretization over each microcell.

Large reduction in the number of nonzero matrix elements.

Sufficiently accurate to serve as preconditioner.

No dependence on the form of the equations.