Disruption Current Asymmetry and Rotation

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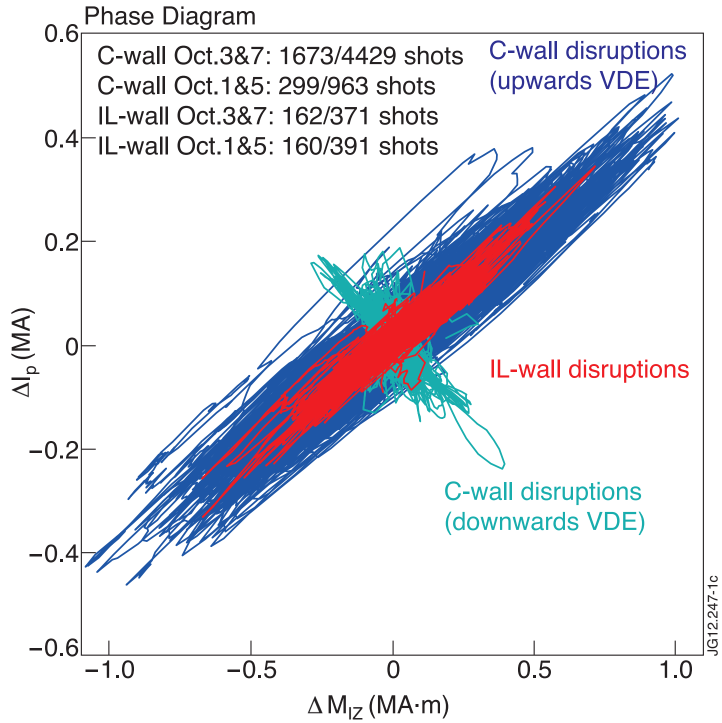
Outline

- Disruption Current Asymmetry
  - theory
  - numerical simulations
- Rotation
  - fit of numerical data and analytic scaling
  - theory of angular momentum generation

Toroidal variation of toroidal current in JET



Toroidal current variation $\Delta I_\phi = \int \tilde{J}_\phi dRdZ$ vs. the vertical moment $\Delta M_{IZ} = \int Z\tilde{J}_\phi dRdZ$ of the current variations. [Gerasimov *et al.* N.F. 2014]

This was interpreted by the Hiro current model [Zakharov *et al.* 2012]. It was shown analytically [Strauss *et al.* 2010] that the slope is proportional to VDE displacement. This is verified by M3D simulations [Strauss, Phys. Plasmas **21**, 102509 (2014)].

Theory of current asymmetry and vertical current moment

The relationship of the toroidal current variations $\tilde{I}_\phi$ to the toroidally varying part of the vertical moment of the current $\tilde{M}_{IZ}$ is essentially a kinematic effect of displacing a current perturbation by a VDE.

The toroidally varying poloidal magnetic field is approximately

$$\mathbf{B} = \nabla\tilde{\psi} \times \widehat{\phi}, \tag{1}$$

and the perturbed toroidal current density in polar coordinates is

$$\tilde{J}_\phi = -\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\tilde{\psi}}{\partial r} - \frac{1}{r^2}\frac{\partial^2\tilde{\psi}}{\partial\theta^2}. \tag{2}$$

with $\tilde{\psi} = \psi - \oint \psi d\phi/(2\pi)$. The toroidal current is

$$\tilde{I}_\phi = \oint \int_0^a \tilde{J}_\phi r dr d\theta = -\oint \frac{\partial\tilde{\psi}}{\partial r} a d\theta \tag{3}$$

in a circular cross section where the boundary is $r = a$. The vertical current moment is

$$\tilde{M}_{IZ} = \oint \int_0^a \tilde{J}_\phi r^2 \sin\theta dr d\theta = -\oint \frac{\partial\tilde{\psi}}{\partial r} a^2 \sin\theta d\theta \tag{4}$$

where it was assumed that the wall is a good conductor, so that $\tilde{\psi} \approx 0$ at $r = a$.

The poloidal flux change $\delta\tilde{\psi}$ produced by an axisymmetric displacement potential Φ satisfies

$$\delta\tilde{\psi} = \nabla\Phi \times \nabla\tilde{\psi} \cdot \widehat{\phi} \tag{5}$$

The VDE displacement potential has the form $\Phi = \xi_{VDE}(r)\cos\theta$. Iterating in ξ_{VDE},

$$\tilde{\psi}_{k+1} = \frac{1}{r}\left(\xi'_{VDE}\frac{\partial\tilde{\psi}_k}{\partial\theta}\cos\theta + \xi_{VDE}\tilde{\psi}'_k\sin\theta\right) \tag{6}$$

where the prime denotes a radial derivative and $\tilde{\psi} = \tilde{\psi}_1 + \tilde{\psi}_2 + \tilde{\psi}_3 + \ldots$ where $\tilde{\psi}_{k+1} \propto \xi^k_{VDE}$. The boundary conditions are $\tilde{\psi}_k(a) = \xi_{VDE}(a) = 0$. This yields, at the wall,

$$\tilde{\psi}'_{k+1} = \frac{\xi'_{VDE}}{a}\left(\frac{\partial}{\partial\theta}(\tilde{\psi}'_k\cos\theta) + 2\tilde{\psi}'_k\sin\theta\right) \tag{7}$$

Summing (7) over k and integrating over θ gives

$$\sum_{k=1}^{K}\oint\tilde{\psi}'_{k+1}d\theta = 2\frac{\xi'_{VDE}}{a}\sum_{k=1}^{K}\oint\tilde{\psi}'_k\sin\theta d\theta \tag{8}$$

Using (3),(4), and (8) gives

$$\tilde{I}_\phi = 2\frac{\xi'_{VDE}}{a^2}\tilde{M}_{IZ}. \tag{9}$$

The ratio $\tilde{I}_\phi/\tilde{M}_{IZ}$ is proportional to the VDE displacement. For an upward VDE, $\xi'_{VDE}(a) > 0$.

M3D simulations of current asymmetry and vertical current moment

ITER FEAT15MA equilibrium was modified by setting toroidal current and pressure to zero outside the $q = 2$ surface, keeping the total toroidal current constant (MGI model) [Izzo *et al.* 2008]. Plasma was evolved in 2D to an initial VDE displacement, then evolved in 3D.

An additional set of states was made by setting current and pressure equal to zero outside the $q = 1.5$ surface. These states were unstable to downward VDEs.

The perturbed current and vertical displacement were measured as

$$\Delta I_\phi = \frac{1}{V}\left(\oint \frac{d\phi}{2\pi} < \tilde{J}_\phi >^2\right)^{1/2} \qquad (10)$$

$$\Delta M_{IZ} = \frac{1}{V}\left(\oint \frac{d\phi}{2\pi} < Z\tilde{J}_\phi >^2\right)^{1/2} \qquad (11)$$

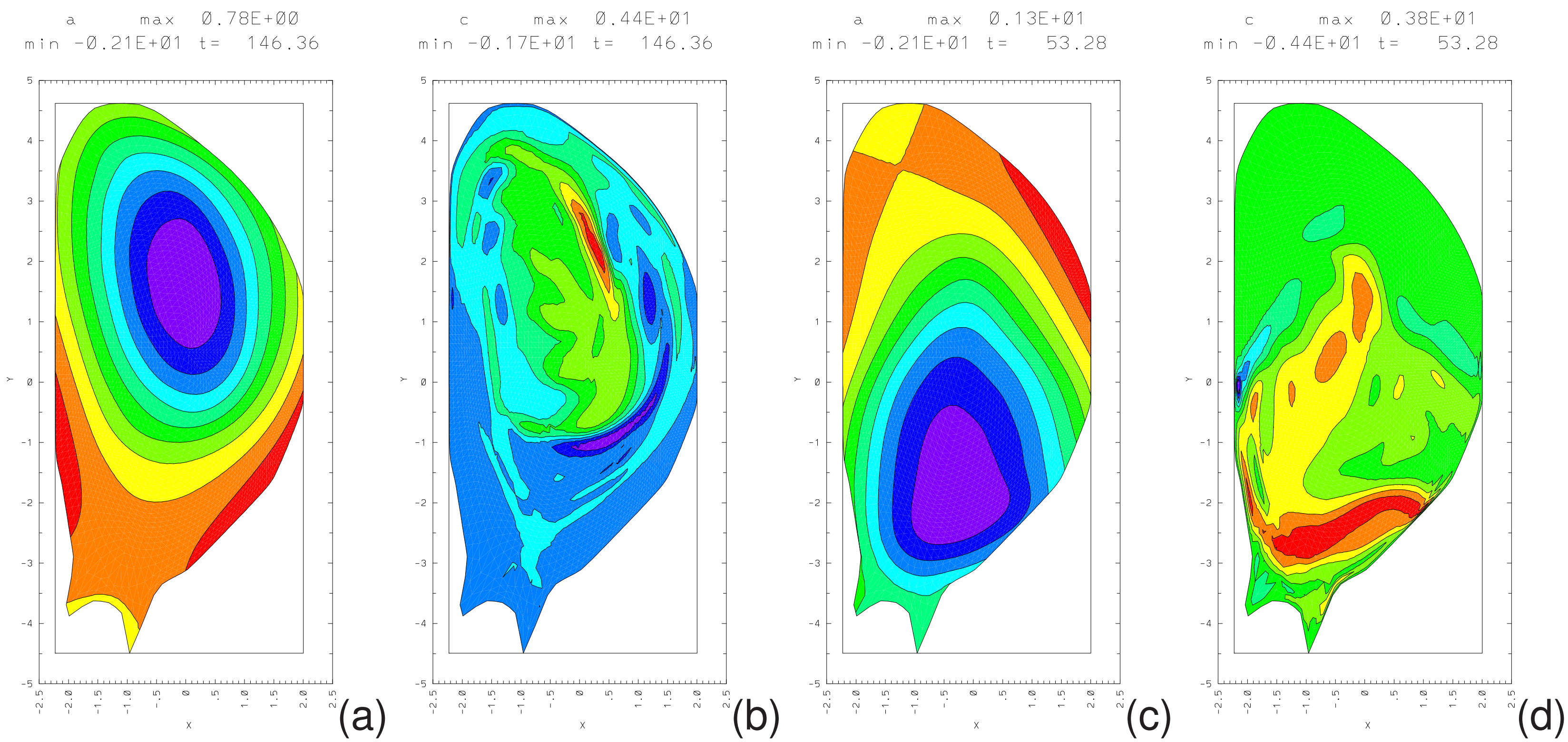
$$(12)$$

where

$$V = \int dRdZ$$

$$< \tilde{J}_\phi > = \int dRdZ\tilde{J}_\phi$$

magnetic flux and toroidal current



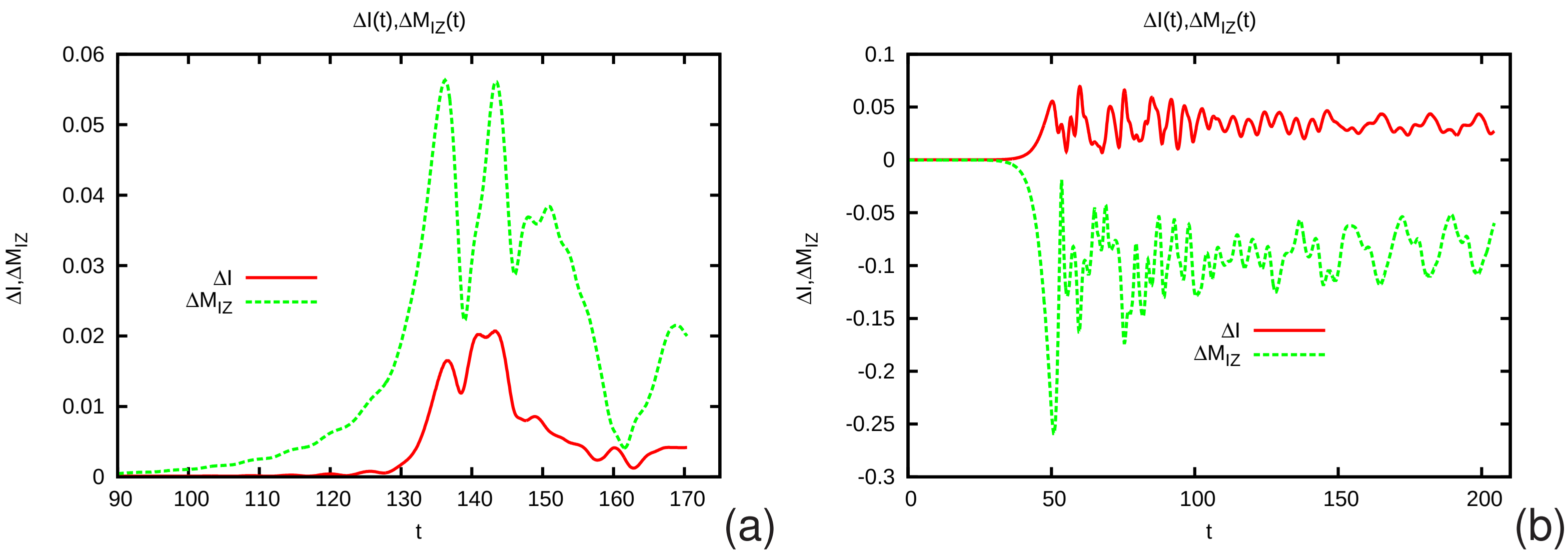
Upward VDE: (a) ψ (b) J_ϕ with $\xi = 0.72a$, time $t = 146\tau_A$, toroidal angle $\phi = 0$.

Downward VDE: (c) ψ (d) J_ϕ with $\xi = -0.71a$, time $t = 53\tau_A$, and $\phi = 0$.

Plasma is turbulent, not an equilibrium with surface current.

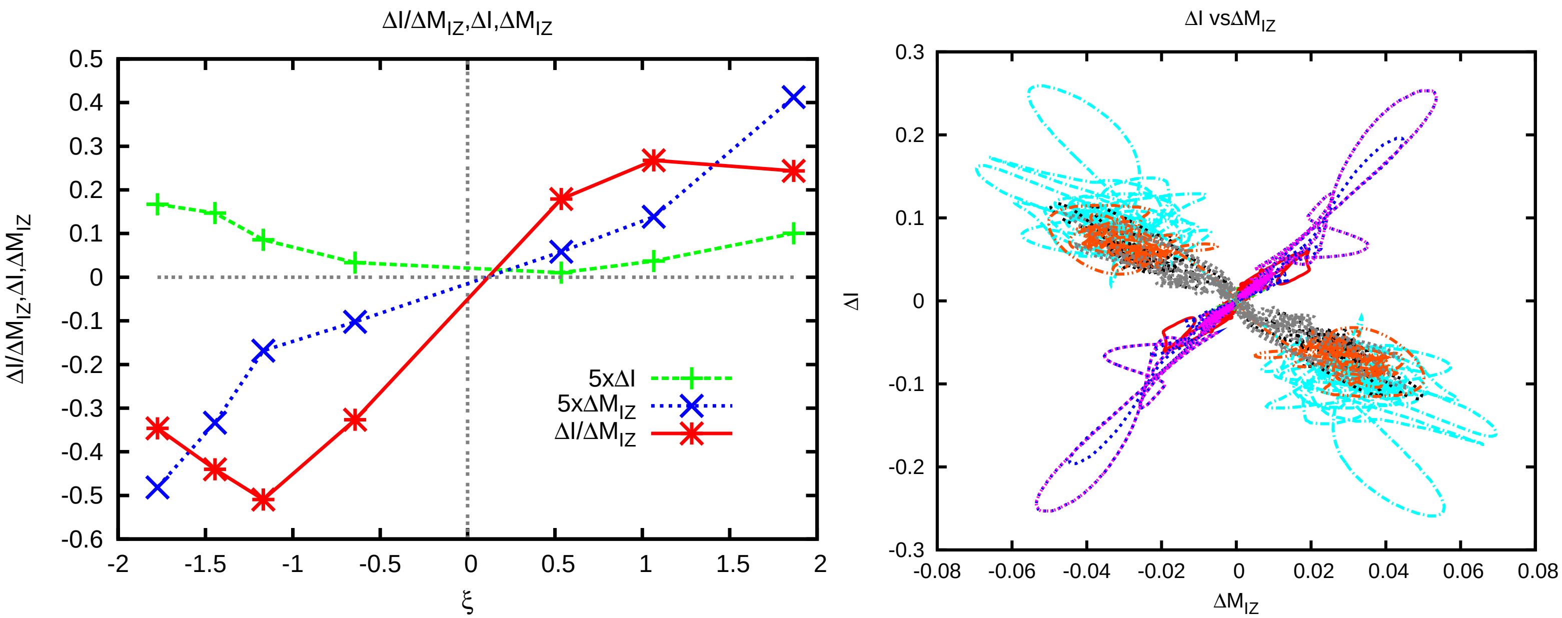
Time history of perturbed current and vertical current moment

M3D simulations were done with $S = 10^6$, wall penetration time $\tau_{wall} = 10^4 \tau_A$. Velocity boundary condition $v_n = 0$.



Time history of $\Delta I_\phi, \Delta M_{IZ}$. (a) upward VDE with $\xi = 0.72a$. (b) downward VDE with $\xi = -0.71a$.

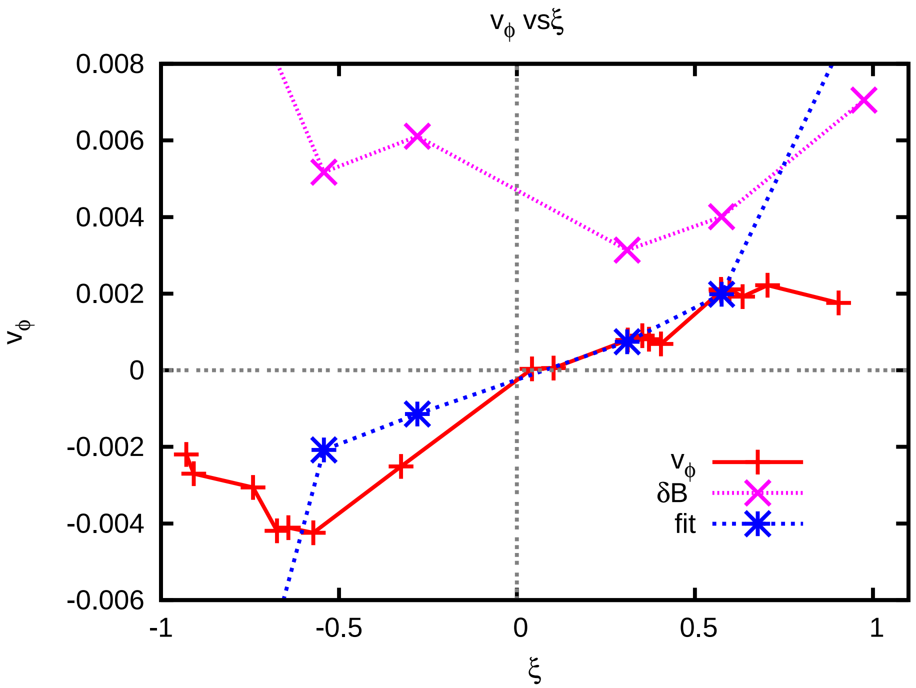
Time averaged $\Delta I_\phi/\Delta M_{IZ}$ and time histories $\Delta I_\phi, \Delta M_{IZ}$



(a) Time averages of $\Delta I_\phi, \Delta M_{IZ}$. Showing $\Delta I_\phi/\Delta M_{IZ} \propto \xi$, for $|\xi| \lesssim 1$, when plasma current channel reaches the wall. (b) Time histories of $\Delta I_\phi, \Delta M_{IZ}$ for the cases in (a). **This is similar to JET data. It does not depend on Hiro current model.**

Dependence of toroidal velocity on vertical displacement

In [Strauss *et al.* 2014] the toroidal rotation caused by disruptions was calculated. The previous set of states was used to calculate V_ϕ, the maximum in time of the volume average of the toroidal velocity.



V_ϕ as a function of vertical displacement. The plasma was evolved in a 2D VDE to height ξ, then evolved in 3D. Also shown is the magnetic perturbation δB, which is calculated from ΔM. Simulational points and V_ϕ fit to (13) are shown. The fit has $A_1 = 12$, $A_2 = 8$ in the function

$$\frac{V_\phi}{v_A} = A_1 \frac{\xi}{r}\left[1 + A_2\left(\frac{\xi}{r}\right)^2\right]\left(\frac{\delta B}{B}\right)^2. \tag{13}$$

Estimating δB from ΔM

In a circular cross section with large aspect ratio, let

$$\tilde{B}_\theta = B_{11}\sin(\theta+\phi) + B_{21}\cos(2\theta+\phi).$$

Displacing

$$B_{21}(r - \xi\sin\theta) \approx B_{21} - B'_{21}\xi\sin\theta$$

by a VDE, then

$$\Delta M_{IZ} = (1/2)B_{11} + (\xi/4)B'_{21}.$$

Note that $\Delta M_{IZ} \approx 0$ for $\xi = 0$, which implies $B_{11} \approx 0$, because it is an internal mode. In the plot

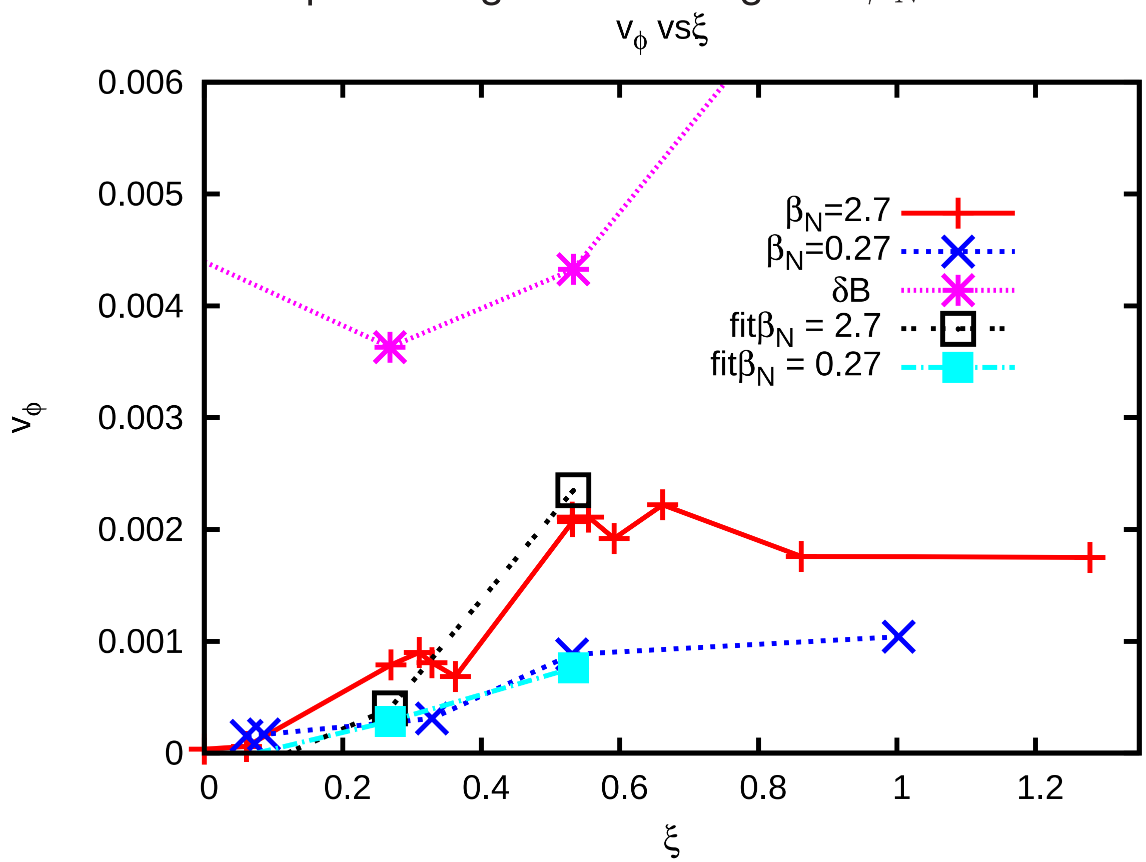
$$\delta B = (r/\xi)\Delta M_{IZ} \approx (1/2)|B_{21}|.$$

For nonzero ξ, B_{11} can be nonzero. The data available at present does not distinguish between $(1,1)$ and $(2,1)$ magnetic perturbations.

Future work will use a better measure of δB.

Scaling of toroidal velocity with ξ and β_N

Two sets of cases were compared to get the scaling with β_N.



These cases compare equilibria with $\beta_N = 2.7$ and $\beta_N = 0.27$. In the low β_N case, $A_2 \approx 0$. The fit is very good for $\xi/r < 0.6$. There is only low β_N data for $\xi > 0$. The $\beta_N = 2.7$ and δB data is the same as on the previous slide, for $\xi > 0$.

Theory: Conservation of toroidal angular momentum

$$\frac{\partial}{\partial t} L_\phi = \oint (R B_\phi B_n - \rho R v_\phi v_n) R dl d\phi \tag{14}$$

where the total toroidal angular momentum is

$$L_\phi = \int \rho R^2 v_\phi dR dZ d\phi \tag{15}$$

and the integral in (14) is over the boundary. Using the M3D magnetic field representation,

$$\mathbf{B} = \nabla\psi \times \nabla\phi + \frac{1}{R}\nabla_\perp F + G\nabla\phi \tag{16}$$

in (14) yields

$$\frac{\partial}{\partial t} L_\phi = \oint G \frac{\partial \psi}{\partial l} dl d\phi \tag{17}$$

where $\partial F/\partial n = 0$ at the boundary. We have assumed that $v_\phi = 0$ at the boundary, but not $v_n = 0$ at the boundary, although we have done so in simulations with M3D.

If $G = G(\psi)$, then toroidal angular momentum L_ϕ is conserved. This is the case in an equilibrium satisfying the Grad - Shafranov equation. If the plasma is not in equilibrium, such as during a disruption or ELM, then net flow can be generated.

Analytic model of rotation source - 1

To express $\dot{L}_\phi$ in terms of magnetic perturbations, the magnetic fluxes ψ and G can be split into equilibrium and toroidally varying parts, $\psi = \psi_0 + \psi_1, G = G_0 + G_1 + G_2$. For simplicity we assume circular equilibrium cross sections, $dl = rd\theta$. To obtain a tractable equation for G, assume radial force balance,

$$\frac{G^2}{R^2} + B_\theta^2 + 2p \approx 0 \tag{18}$$

and assume large aspect ratio so that $R \approx R_0 =$ constant. Then $\dot{L}_\phi$ can be split into two parts, $\dot{L}_\phi = \dot{L}_{\phi B} + \dot{L}_{\phi p}$ where

$$\dot{L}_{\phi B} = -\frac{R}{2B_{\phi 0}} \oint \frac{\partial \psi_0}{\partial \theta} B_{\theta 1}^2 d\theta d\phi \tag{19}$$

$$\dot{L}_{\phi p} = -\frac{R}{B_{\phi 0}} \oint \frac{\partial \psi_0}{\partial \theta} p d\theta d\phi \tag{20}$$

The plasma is displaced by a VDE with $(m, n) = (1, 0)$, $\psi_0 = \psi_0(r - \xi_{10} \sin\theta)$. Hence

$$\frac{\partial \psi_0}{\partial \theta} = \xi_{10} \cos\theta B_{\theta 0} \tag{21}$$

where $B_\theta = -\partial\psi/\partial r$. Then $\dot{L}_{\phi B} = \xi r R/(2q) \oint B_{\theta 1}^2 \cos\theta d\theta d\phi$.

Analytic model of rotation source - 2

There must be at least two modes $(m, n), (m+1, n)$ contributing to $B_{\theta 1}$ which beat together to give a $\cos\theta$ term. Expanding $B_{\theta 1} = \sum_{mn} B_{\theta mn} \cos(m\theta - n\phi)$ gives

$$\dot{L}_{\phi B} = \frac{\pi^2 \xi r R}{2q} \sum_{mn} B_{\theta mn} B_{\theta(m+1)n} \tag{22}$$

To compare with the scaling (13), let $\dot{v}_\phi = \gamma v_\phi$, in (14). Then (22) yields

$$\boldsymbol{A_1 = \frac{1}{4\gamma\tau_A q}} \tag{23}$$

and taking $\gamma\tau_A = 0.01$ gives agreement with $A_1 = 12$ in (13). The calculation of (20), is given in [Strauss *et al.* 2014].

$$\frac{dL_\phi}{dt} = \frac{\pi^2}{2} r q p_0' \xi_{10}^3 \frac{R}{B^3} \sum_{mn} \frac{\partial}{\partial r} \left[\frac{m(m+1) B_{\theta mn} B_{\theta(m+1)n}}{(m-nq)(m+1-nq)} \right] \tag{24}$$

Setting the denominators in (24) equal to unity gives the ratio $\dot{L}_{\phi p}/\dot{L}_{\phi B} = A_2 (\xi/r)^2$, with

$$\boldsymbol{A_2 = \frac{q}{2}[1 + m(m+1)] \beta_N' (\ln \delta B)' r^2} \tag{25}$$

Taking $m = 1$, $q = 2$, $(\ln \delta B)' r = 1$, $\beta_N = \beta_N' r = 2.7$, gives $A_2 = 8$ in agreement with (13).

Conclusions

- Relation of ΔI to ΔM .
  - $\Delta I \propto \xi \Delta M$ where ξ is VDE displacement
  - Simulations include upward and downward VDEs
  - Does not require Hiro current model
- Scaling of V_ϕ with $\xi, \delta B, \beta_N$.
  - used same data set as above
  - estimated δB from ΔM
  - new term independent of β_N.