Sawtooth-free states in 3D non-linear M3D-C1 simulations

Isabel Krebs, S. Jardin, S. Günter, K. Lackner, N. Ferraro, M. Hölzl

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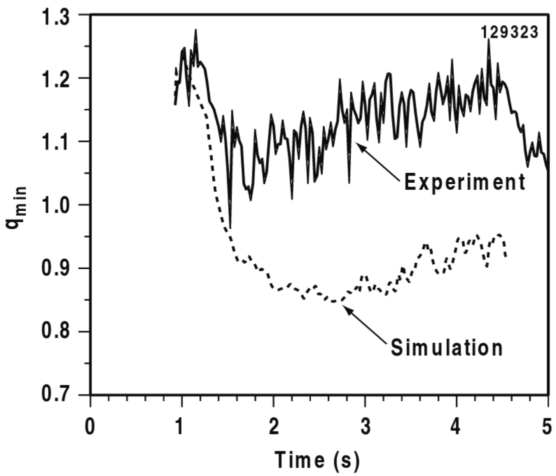
Hybrid discharges (or “Improved H-mode”)

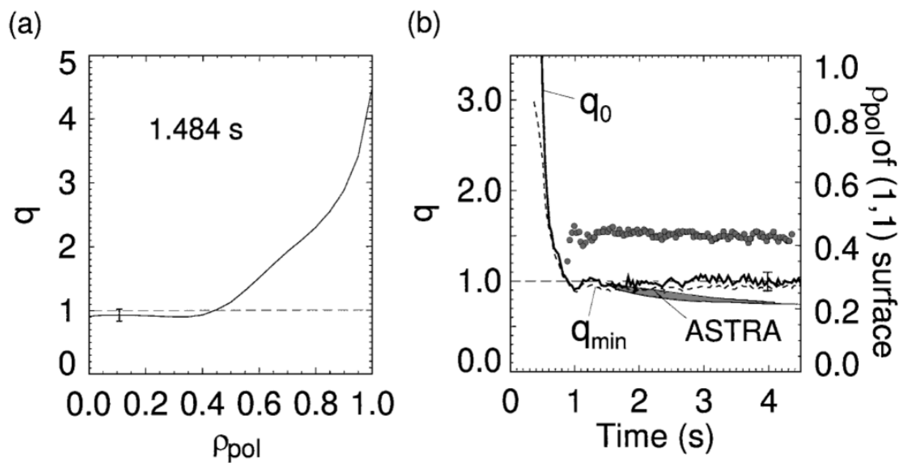
- sawtooth-free discharges
- central q-profile flat and slightly above unity
- generated by additional heating during current-ramp phase
- transport simulations predict q_0 to drop below unity
- current is redistributed by unknown mechanism (“flux pumping”)
- relevant for “advanced tokamak” scenarios

DIII-D

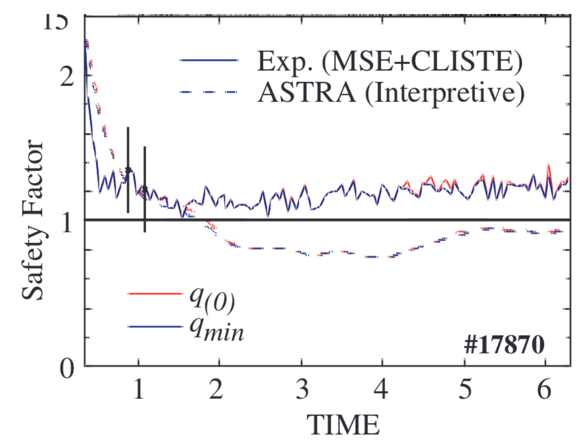


[Petty et al., NF 2015]

ASDEX-Upgrade



[Gruber et al., PRL 1999]

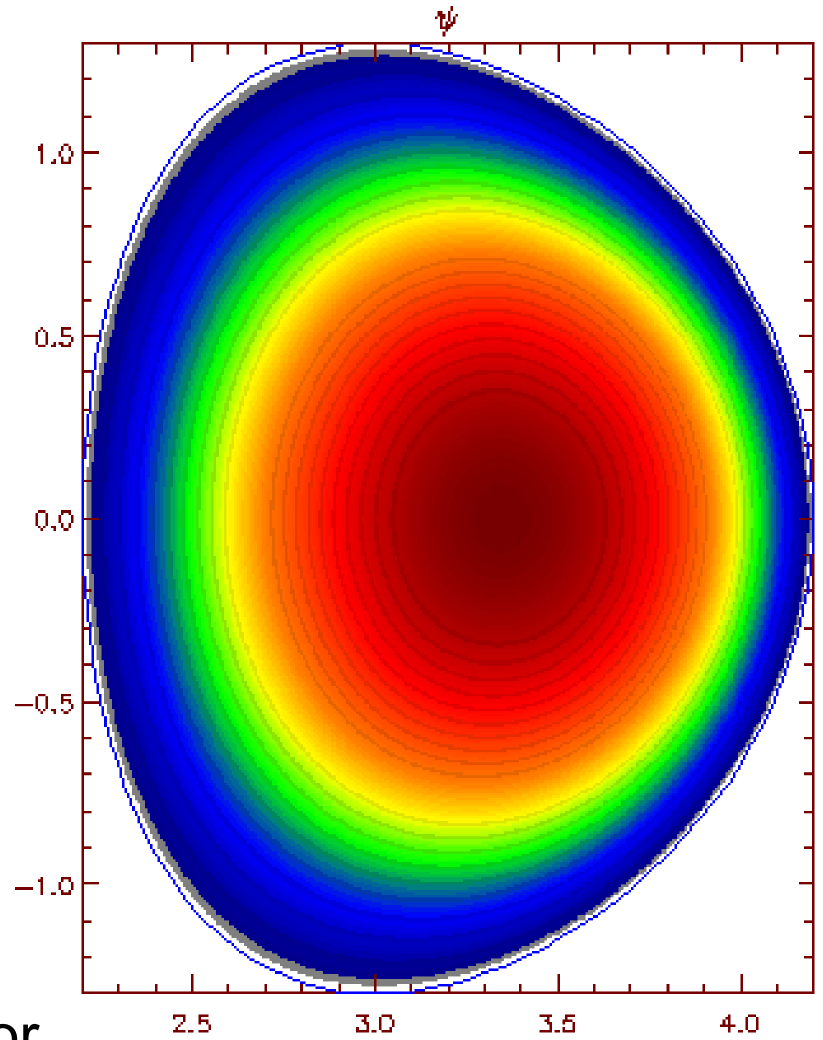


[Staebler et al., NF 2005]

Outline

- The simulations
- What are the mechanisms responsible for “flux-pumping” in simulations?
- Under which conditions are sawteeth avoided?
- Different regimes of sawtooth-free states
- Observations regarding sawteeth in simulations

Simulations



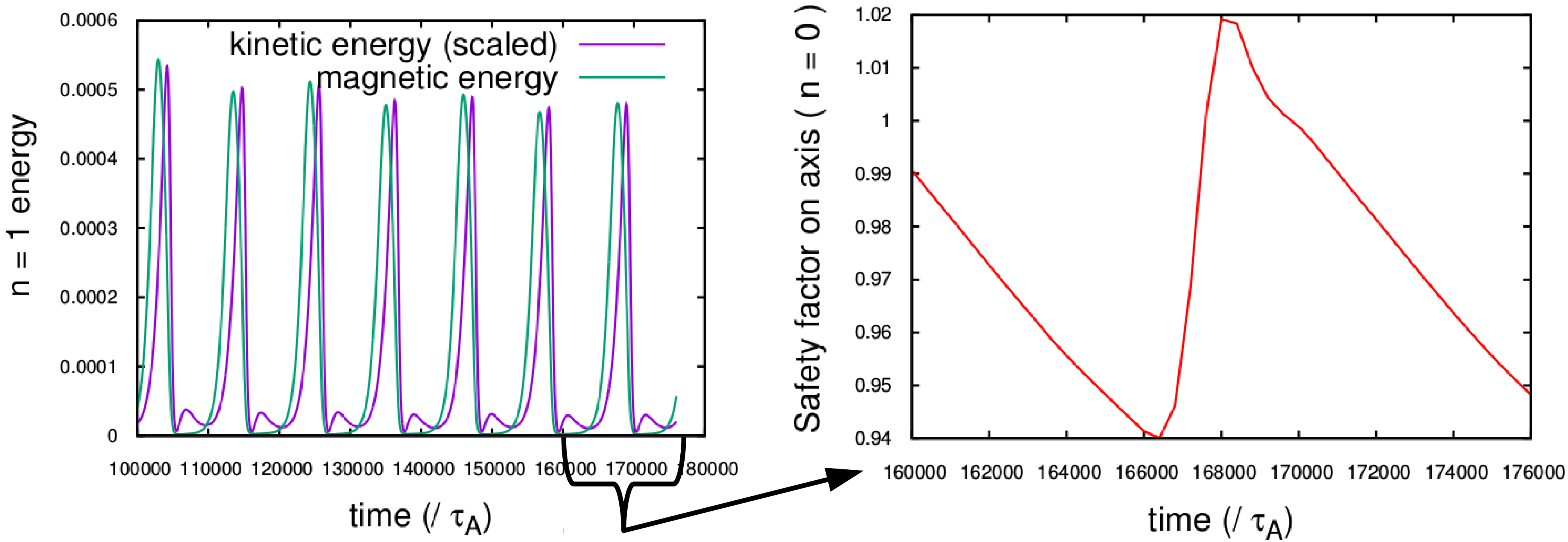
M3D-C1 model

- single-fluid full MHD
- 3D non-linear (and 2D non-linear for comparison)
- toroidal geometry, fixed boundary

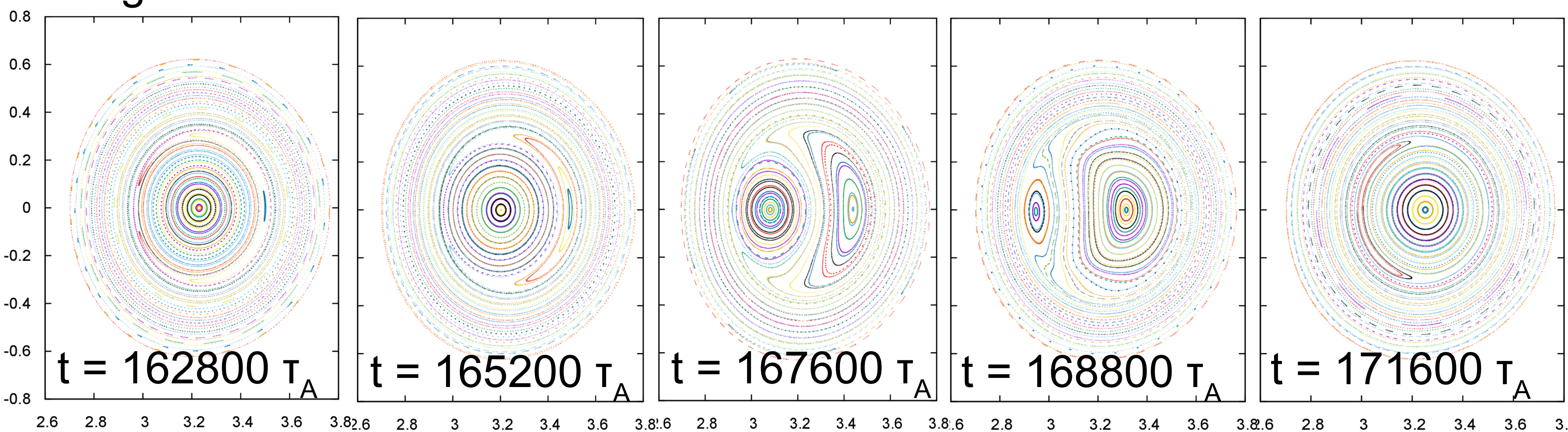
Simulation set-up

- focus on long-term behavior
  - ➔ determined by sources & diffusion coefficients
- varied parameters:
  β, $\kappa_\perp$ & heat source S_T, shape of S_T
- Spitzer resistivity scaled to be similar for all runs
- comparison with experimental parameters:
  $\eta \approx 4 \cdot 10^{-6}\,\Omega\mathrm{m} \approx 10^3 \cdot \eta_\mathrm{exp}$ $\kappa_\perp/\eta \approx \kappa_{\perp,\mathrm{exp}}/\eta_\mathrm{exp}$

Sawtooth-like behavior

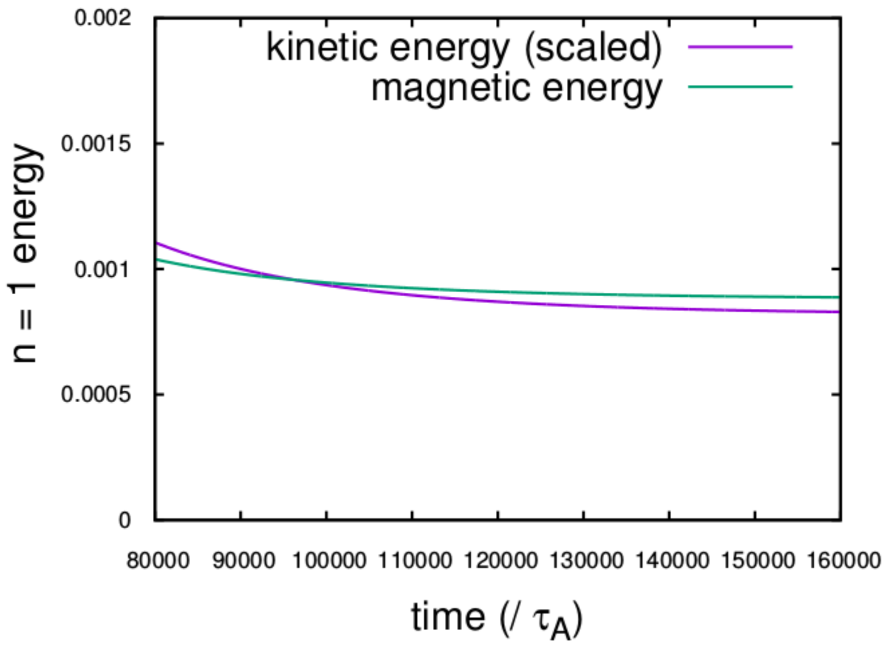


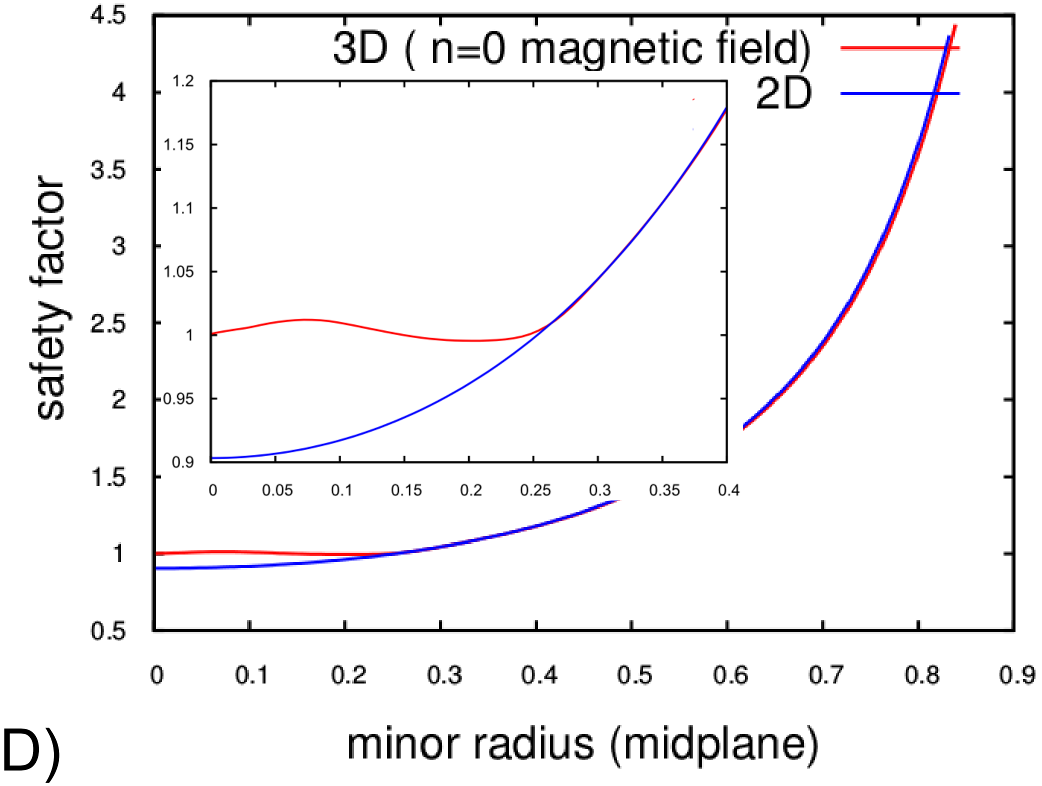
Magnetic field line structure



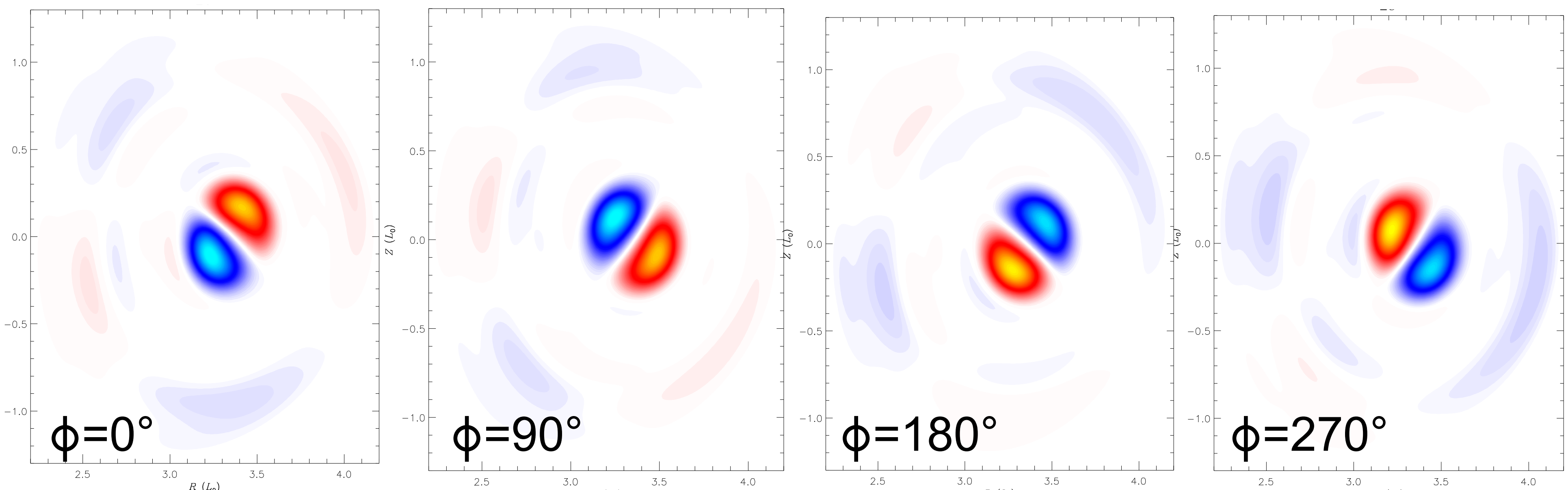
case “m0”

Sawtooth-free helical states





Poloidal velocity stream function (3D-2D)



case "n"

What keeps the central current density profile flat?

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \quad \text{and} \quad \mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \quad \partial_t \mathbf{A} = -\mathbf{E} - \nabla\Phi + \frac{V_L}{2\pi}\nabla\phi$$

insert $\mathbf{A} = R^2\nabla\phi \times \nabla f + \Psi\nabla\phi - F_0 \ln R \hat{Z}$ and $\mathbf{E} = \eta\mathbf{J} - \mathbf{v} \times \mathbf{B}$

take toroidal component $R^2\nabla\phi \cdot [\ \ ]$

$$\Rightarrow \partial_t \Psi = -R\eta J_\phi + R\hat{\phi} \cdot (\mathbf{v} \times \mathbf{B}) - R\hat{\phi} \cdot \nabla\Phi + \frac{V_L}{2\pi}$$

write in terms of $n = 0$ and $n = 1$ components and take toroidal average

$(\mathbf{v}_0 = 0,\ \nabla\Phi_0 = 0)$

$$\Rightarrow \partial_t \Psi_0 = -R\eta_0 J_{\phi,0} - R\eta_1 J_{\phi,1} + R\hat{\phi} \cdot (\mathbf{v}_1 \times \mathbf{B}_1) + \frac{V_L}{2\pi} \qquad \langle 3\mathrm{D} \rangle$$

What keeps the central current density profile flat?

$$\Rightarrow \partial_t \Psi_0 = -R\eta_0 J_{\phi,0} - R\eta_1 J_{\phi,1} + R\hat{\phi}\cdot(\mathbf{v}_1 \times \mathbf{B}_1) + \frac{V_L}{2\pi} \qquad \langle 3\mathrm{D}\rangle$$

linearize the $\langle 3\mathrm{D}\rangle$ equation around the 2D solution:

$$0 = -R\eta_{2D} J_{\phi,2D} + \frac{V_L}{2\pi} \qquad 2\mathrm{D}$$

define $\quad \Delta J_\phi = J_{\phi,0} - J_{\phi,2\mathrm{D}} \quad$ and $\quad \Delta\eta = \eta_0 - \eta_{2\mathrm{D}}$

$$\Rightarrow \partial_t \Psi_0 = -R\Delta\eta J_{\phi,2\mathrm{D}} - R\eta_{2\mathrm{D}}\Delta J_\phi + R\hat{\phi}\cdot(\mathbf{v}_1 \times \mathbf{B}_1) - R\eta_1 J_{\phi,1}$$

vanishes for stationary cases & quasi-stationary cases in time-average

resistivity flattening

resulting current flattening

dynamo effect

small

[Jardin et al. PRL 2015]

What keeps the central current density profile flat?

q ~ 1 and low magnetic shear in center

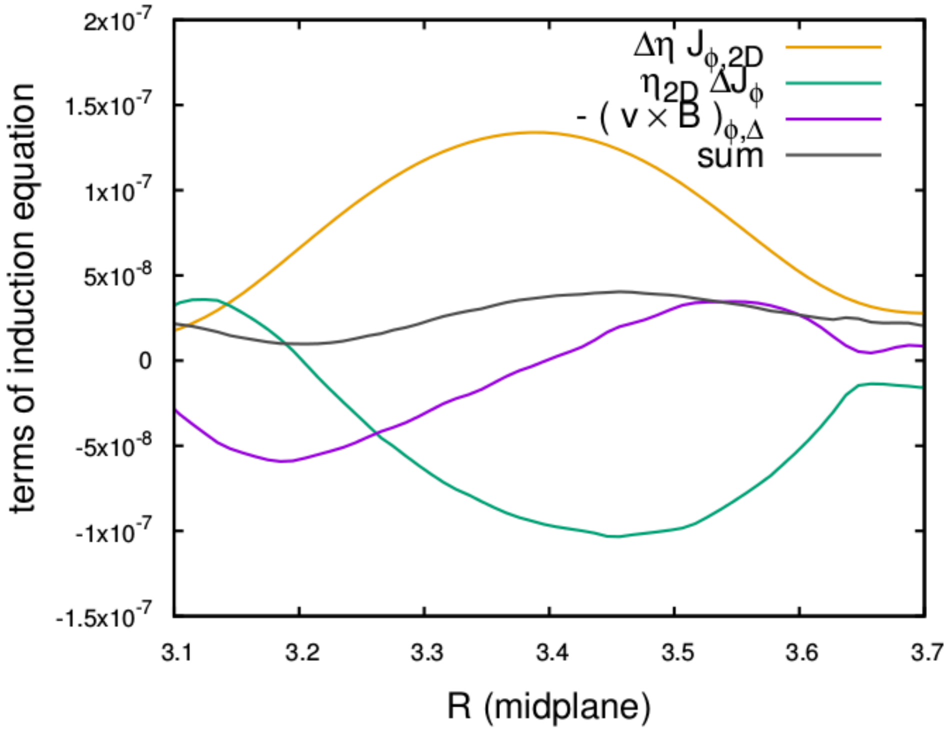
pressure driven instability

[Hastie et al., NF 1988]

(1,1) flow

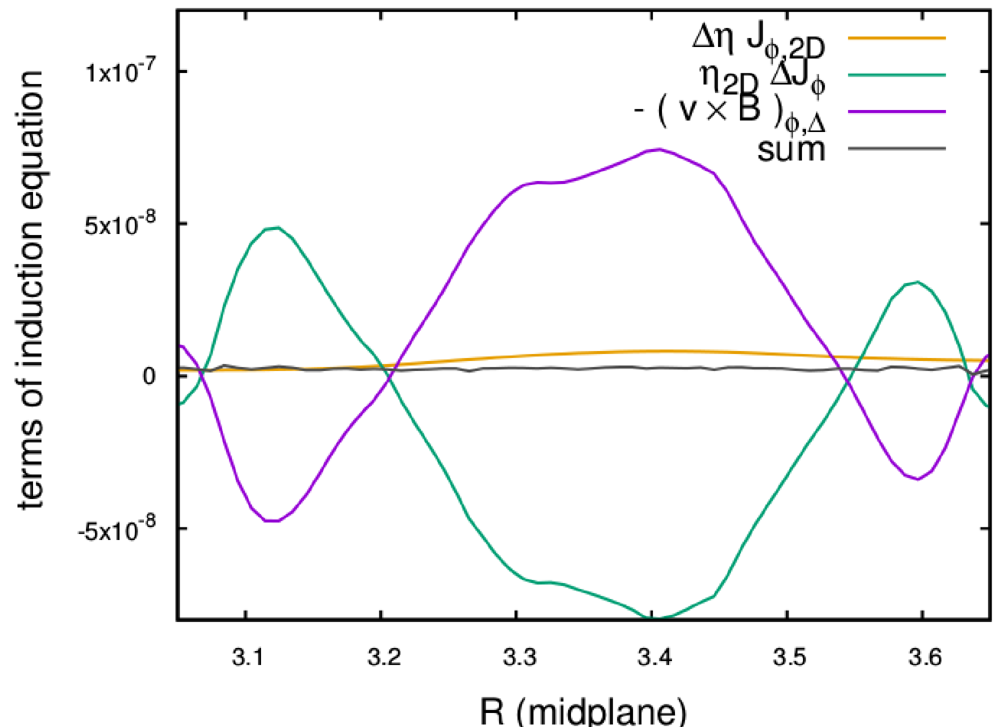
$$\partial_t \Psi_0 = -\boxed{R\Delta\eta J_{\phi,\mathrm{2D}}} - \boxed{R\eta_{\mathrm{2D}}\Delta J_\phi} + \boxed{R\hat{\phi}\cdot(\mathbf{v}_1 \times \mathbf{B}_1)} - R\eta_1 J_{\phi,1}$$

strong resistivity flattening



case “r”

strong dynamo term



case “n”

Under which conditions are sawteeth avoided?

Varied parameters

$$\beta = \frac{2\mu_0}{B_\theta^2} \int_0^{r_1} \left(\frac{r}{r_1}\right)^2 \left(-\frac{dp}{dr}\right) dr$$

→ linear drive of instability that enables current flattening

$\kappa_\perp$ & heat source S_T

→ stiffness of temperature profile

peakedness of S_T

→ how strong the current flattening effect needs to be in order to keep q on axis at unity

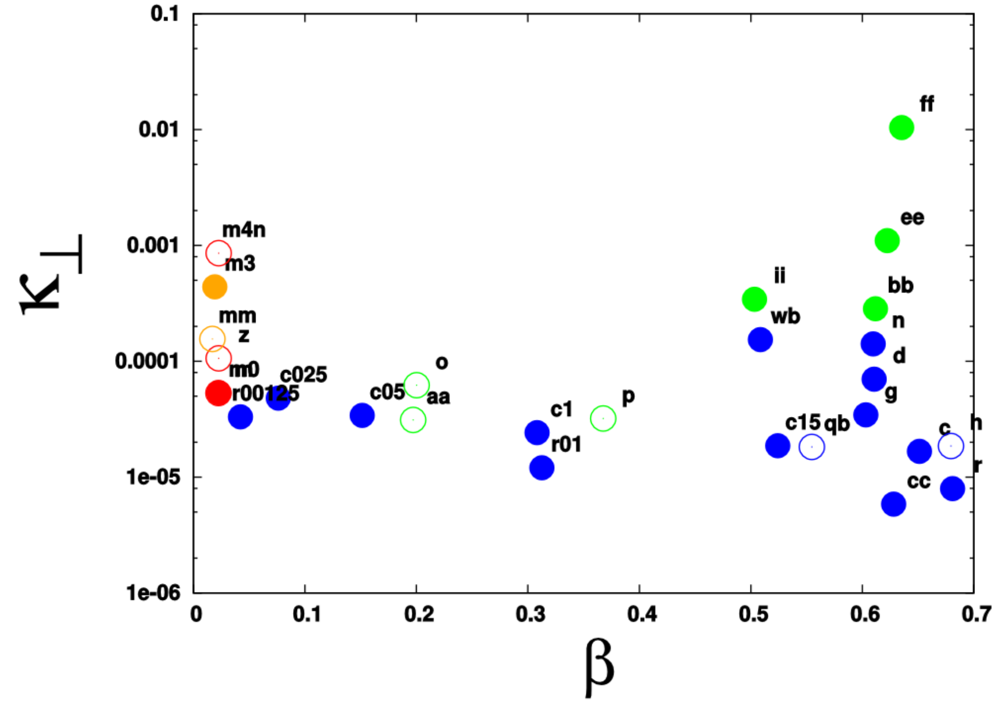
$$\eta J_{\phi,0}(q_0 = 1) - \eta J_{\phi,0}(q_0 = q_{0,\mathrm{2D}})$$

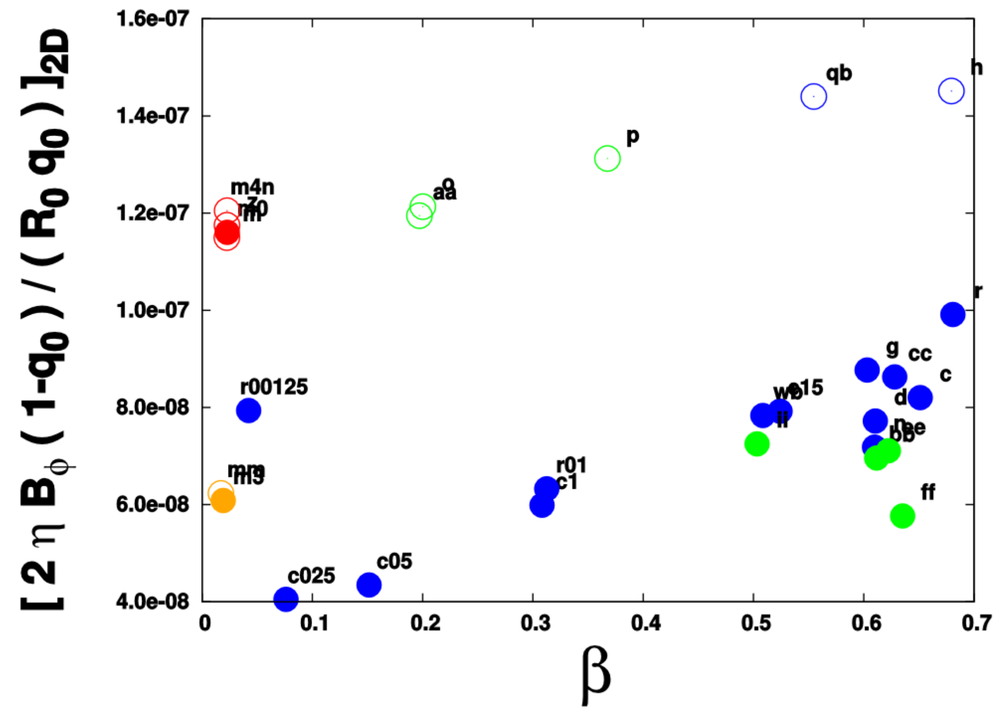
$$\approx -\frac{2\eta B_{\phi,0}}{\mu_0 R_0} \frac{(1 - q_{0,\mathrm{2D}})}{q_{0,\mathrm{2D}}}$$

Under which conditions are sawteeth avoided?

sawtoothing: red & orange

sawtooth-free: green & blue





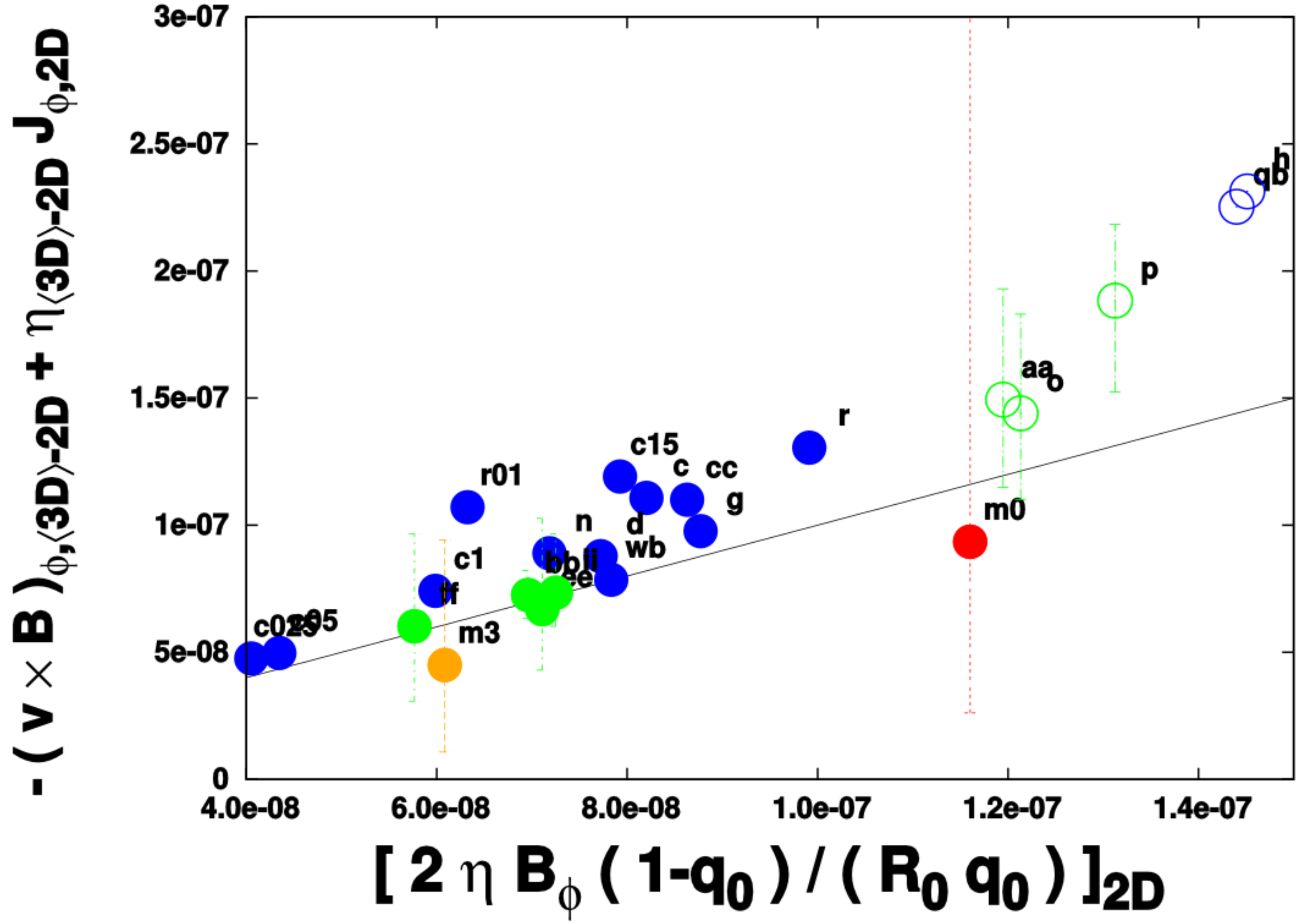
Heat source
strongly peaked
less peaked

Under which conditions are sawteeth avoided?

resistivity flattening

+

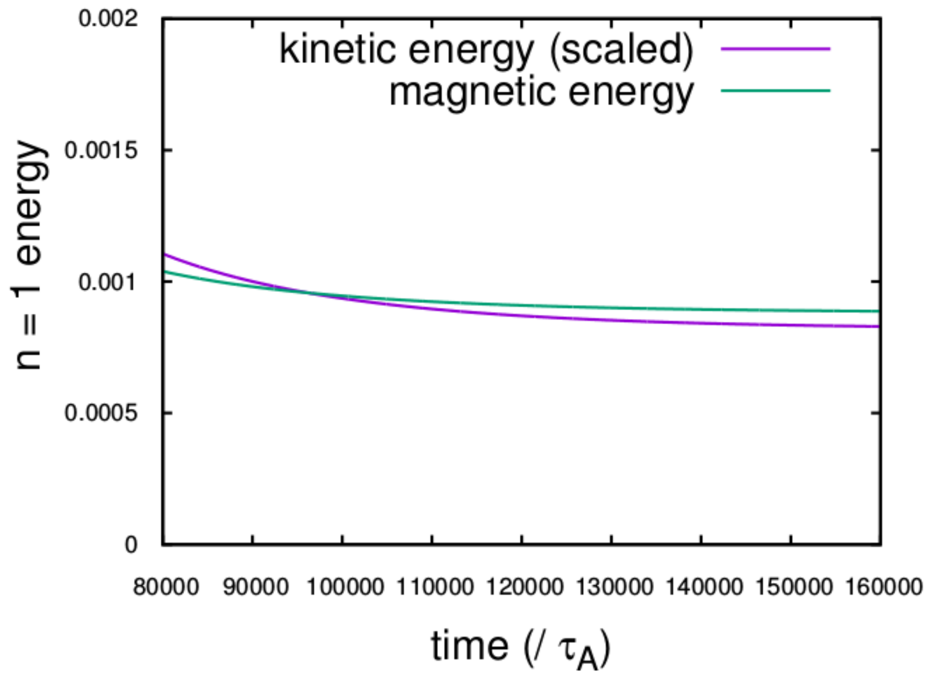
dynamo effect



- At high enough β, the current flattening mechanisms are strong enough to prevent sawtoothing

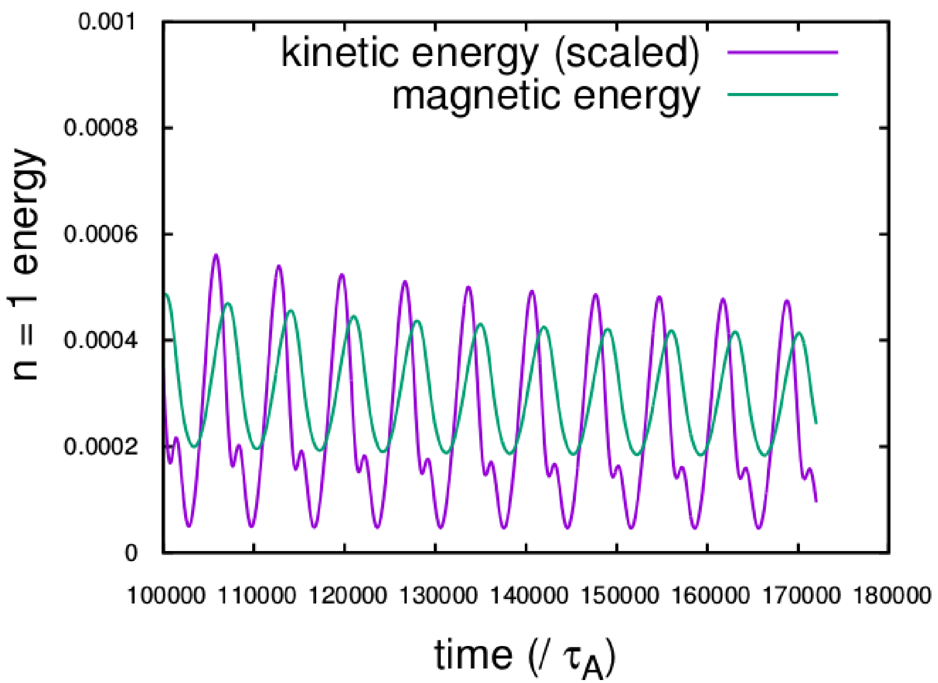
sawtoothing: red & orange
sawtooth-free: green & blue

Two types of sawtooth-free cases



stationary

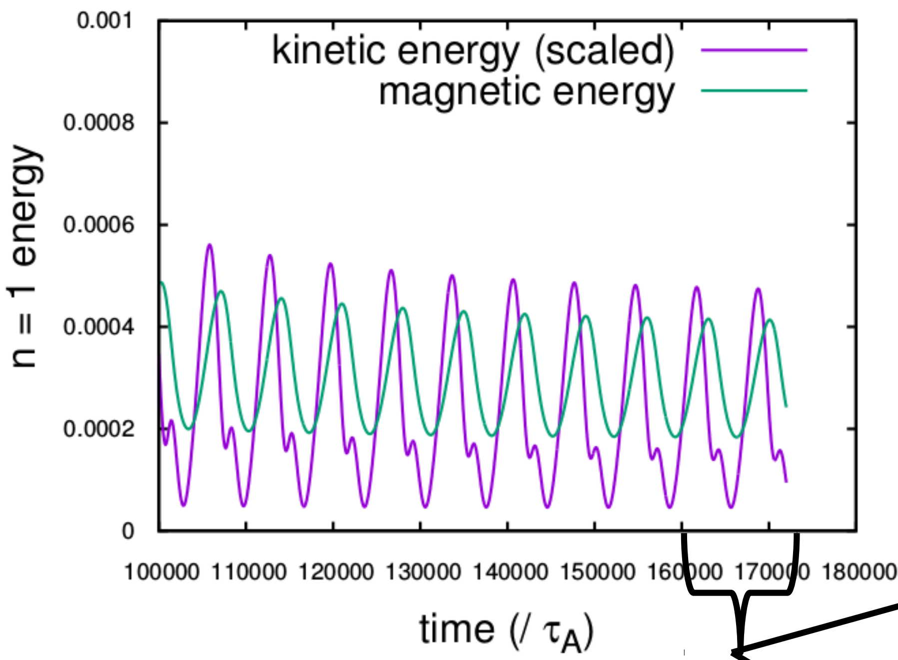
case “n”

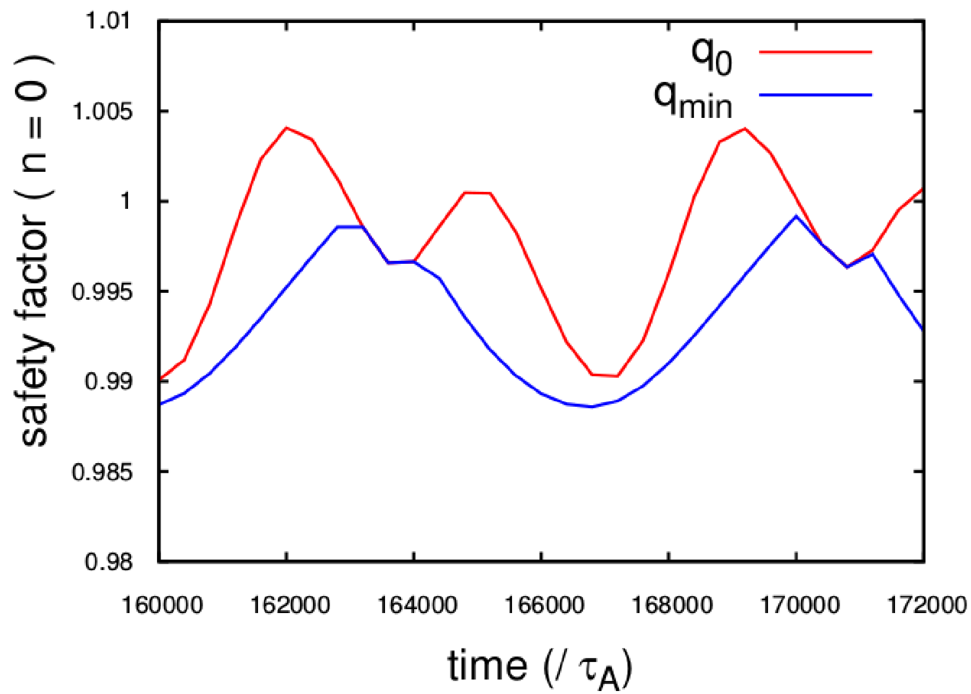


quasi-stationary oscillating

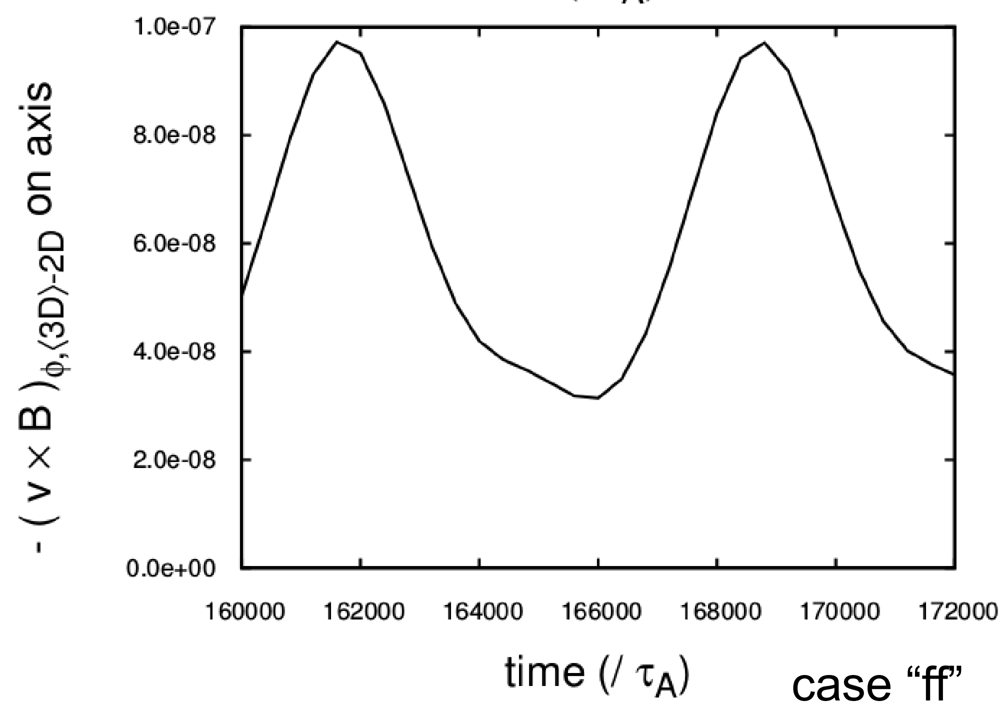
case “ff”

Oscillating states





dynamo
effect



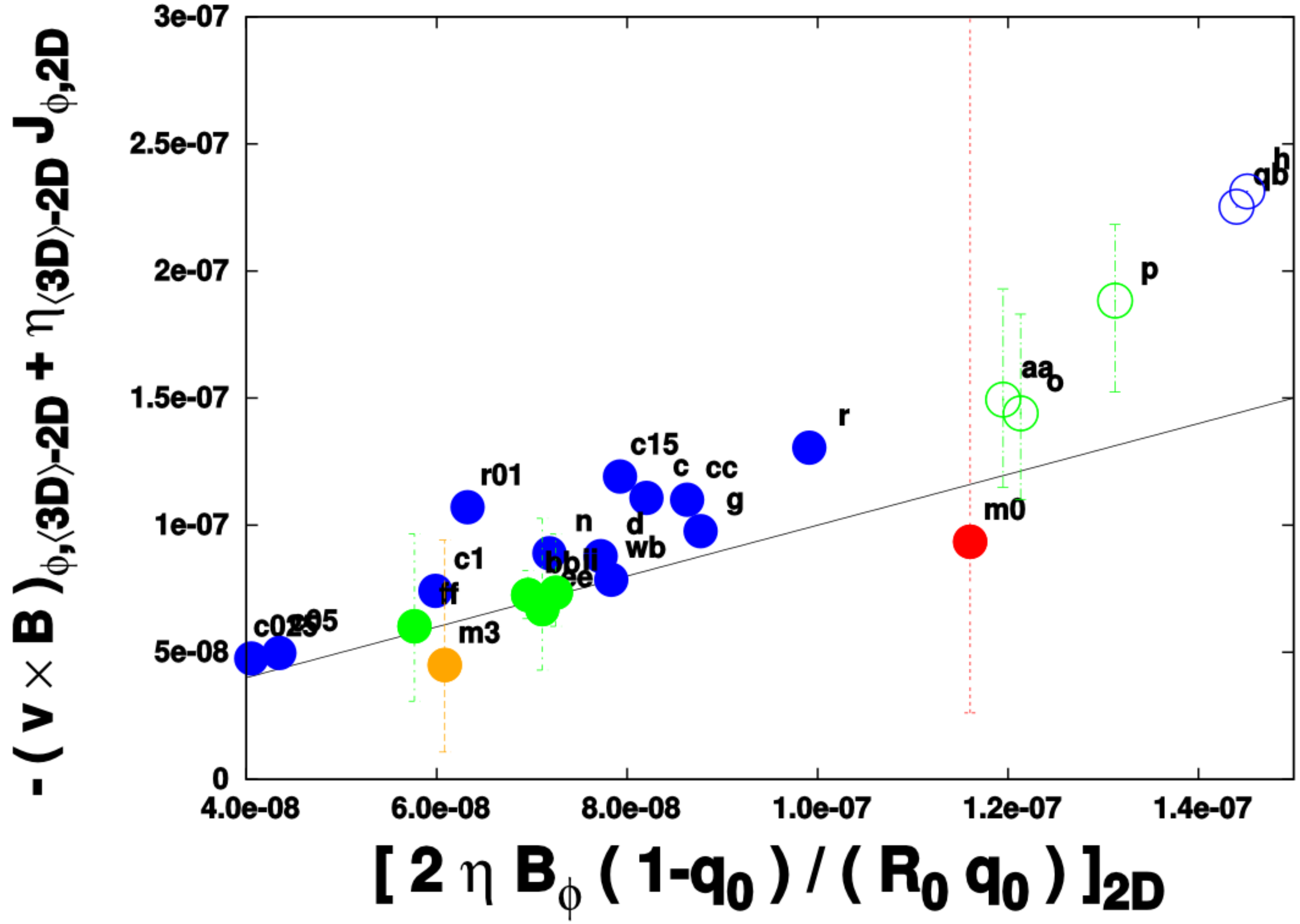
case “ff”

Under which conditions are sawteeth avoided?

resistivity
flattening

+

dynamo
effect

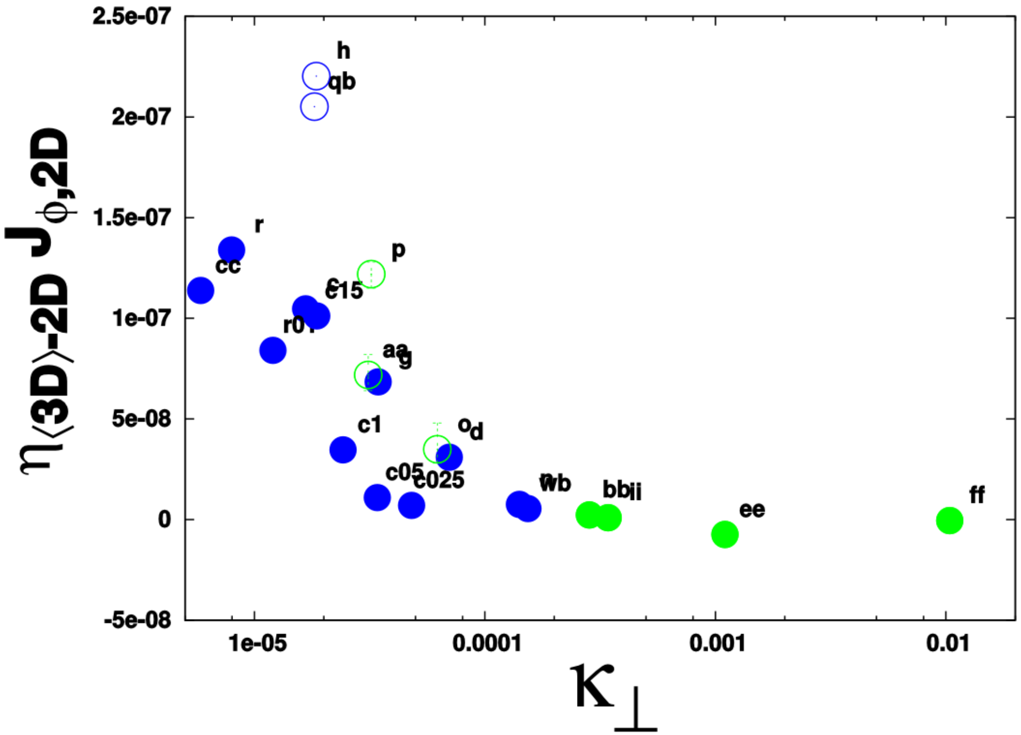


- At high enough β, the current flattening mechanisms are strong enough to prevent sawtoothing

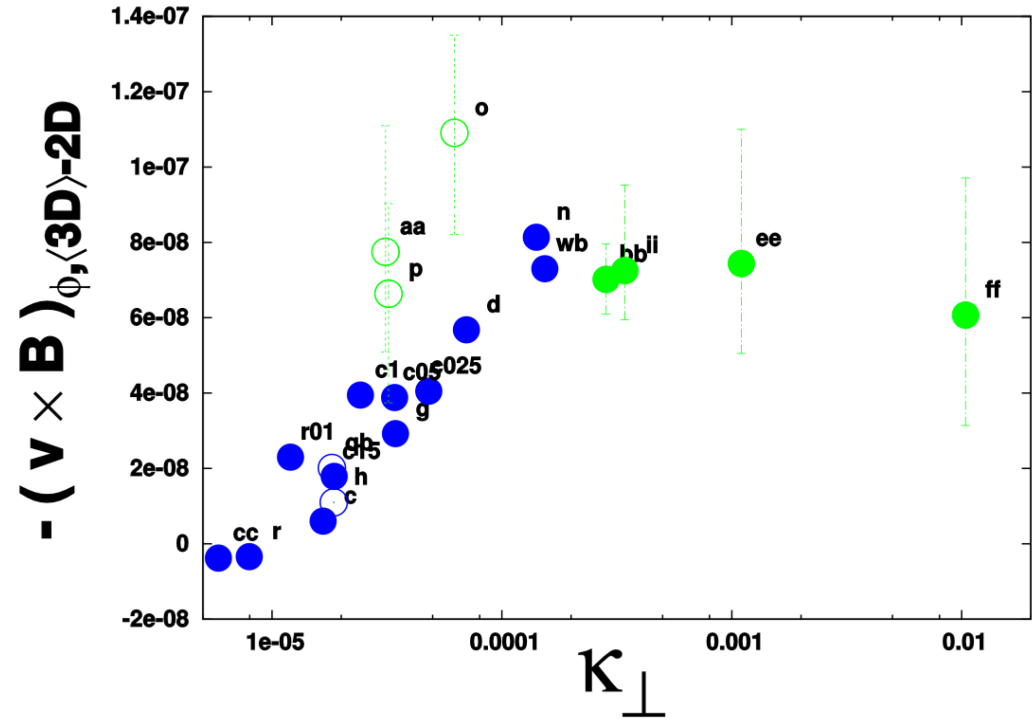
sawtoothing: red & orange
sawtooth-free: green & blue

Which is the dominant current flattening effect?

resistivity flattening



dynamo loop voltage

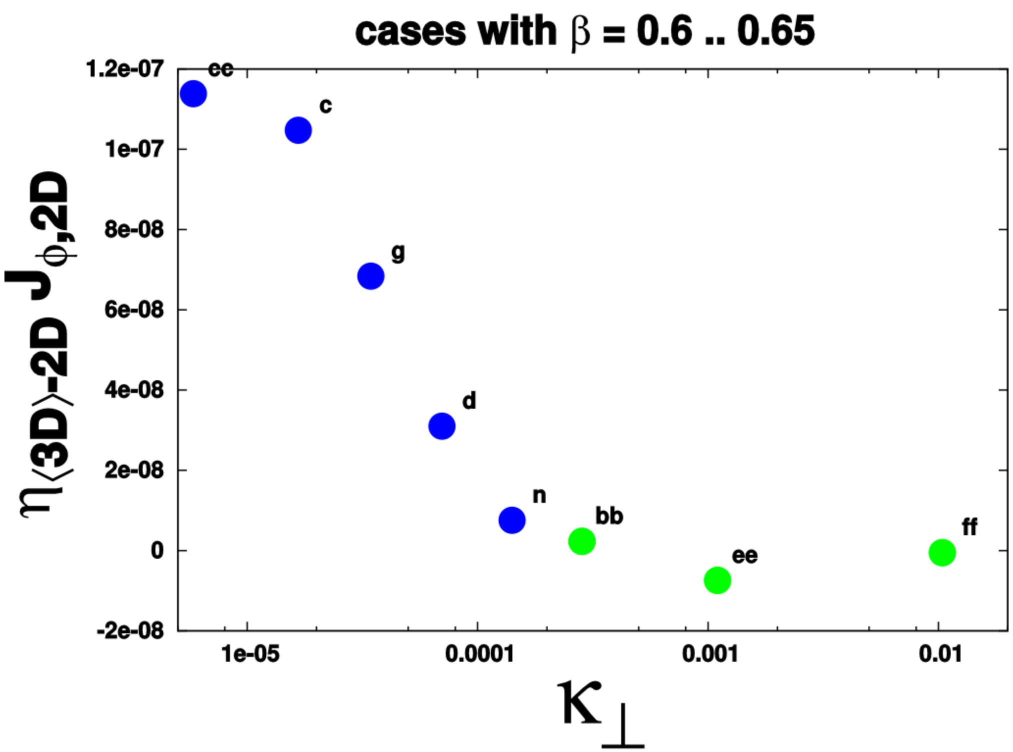


- At high $\kappa_\perp$ & S_T, the convective resistivity flattening becomes less efficient and the dynamo effect more important

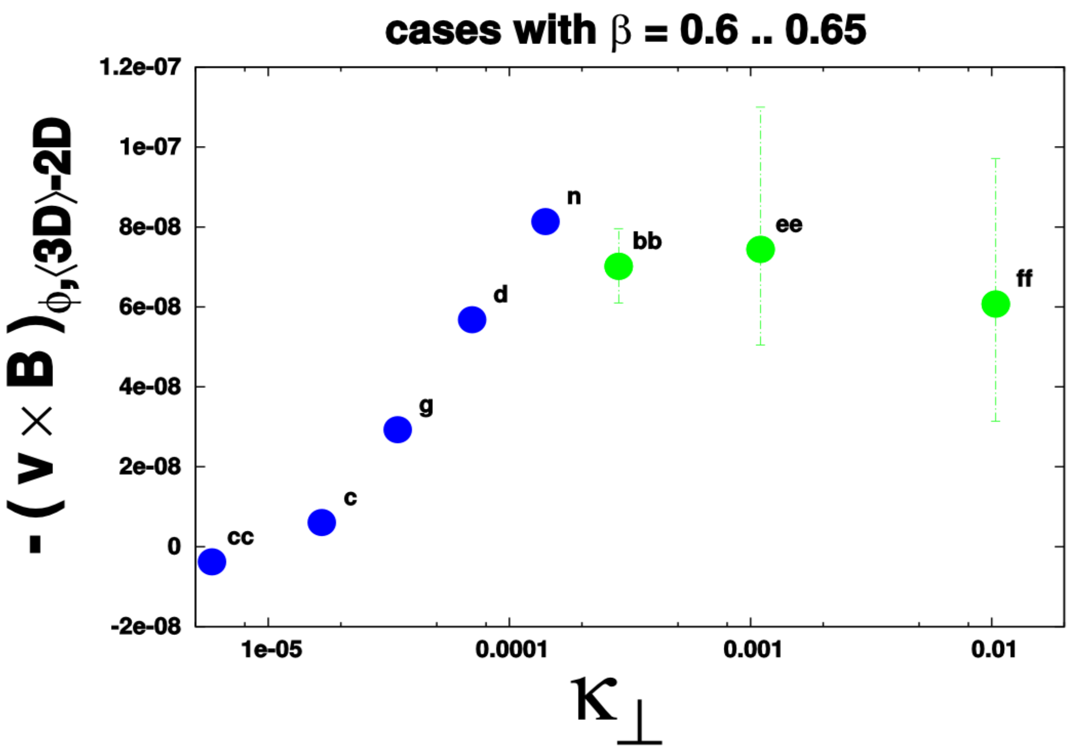
stationary
oscillating

Which is the dominant current flattening effect?

resistivity flattening



dynamo loop voltage

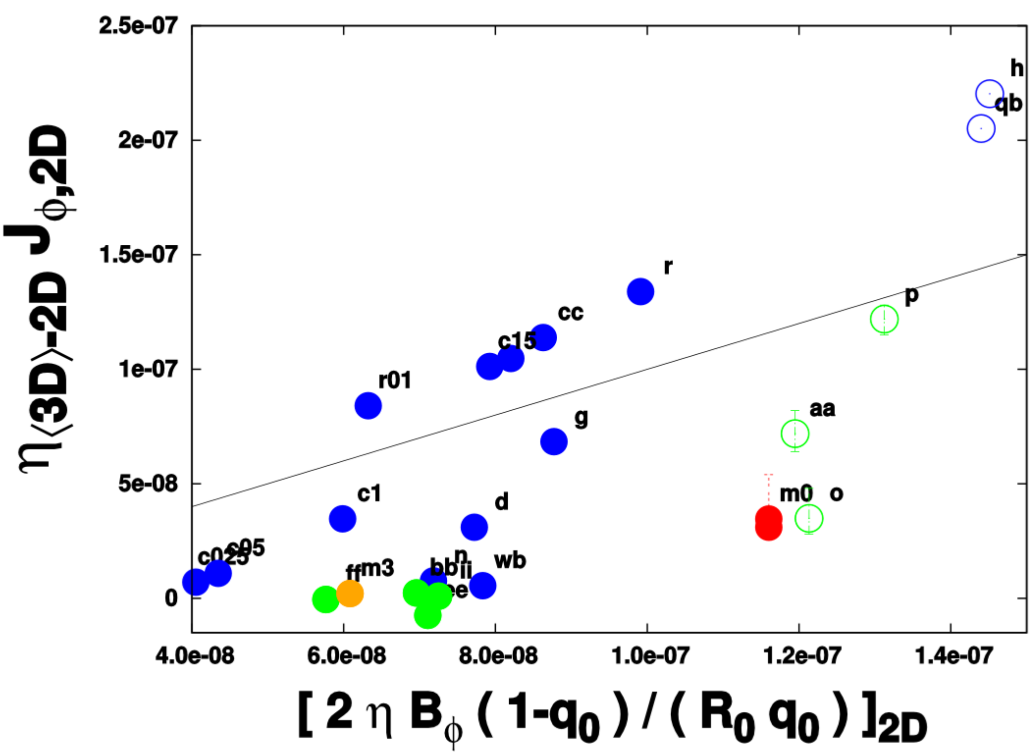


- At high $\kappa_\perp$ & S_T, the convective resistivity flattening becomes less efficient and the dynamo effect more important

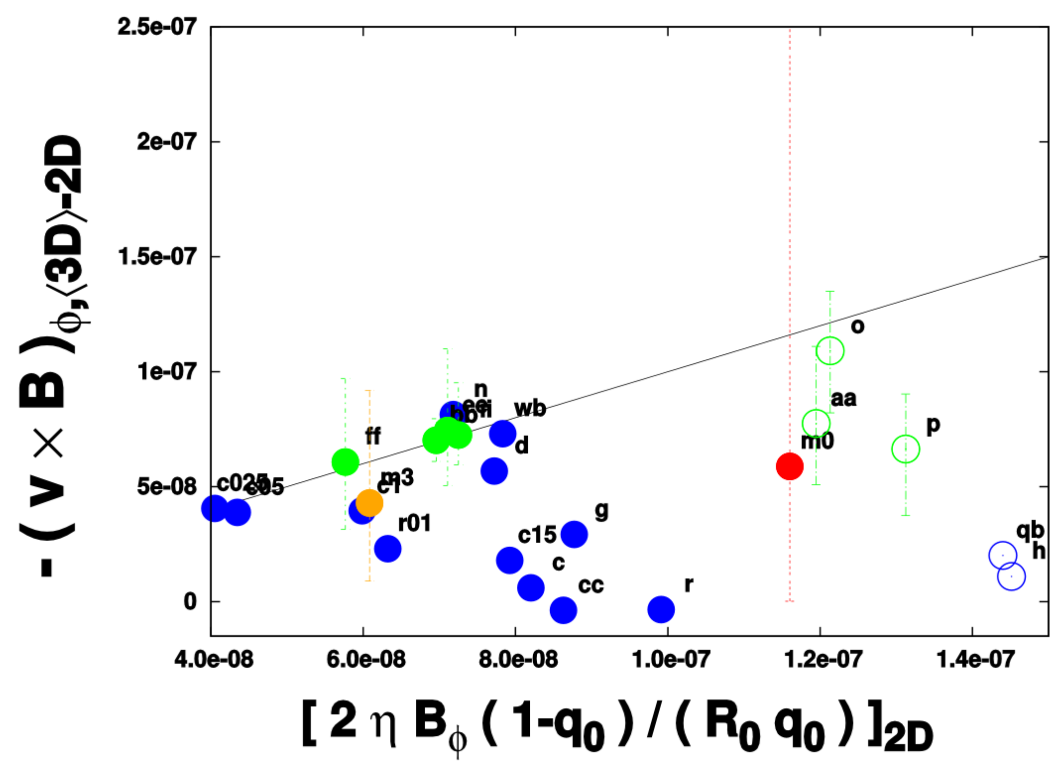
stationary
oscillating

Oscillating behavior tends to occur if dynamo loop voltage effect is important

resistivity flattening



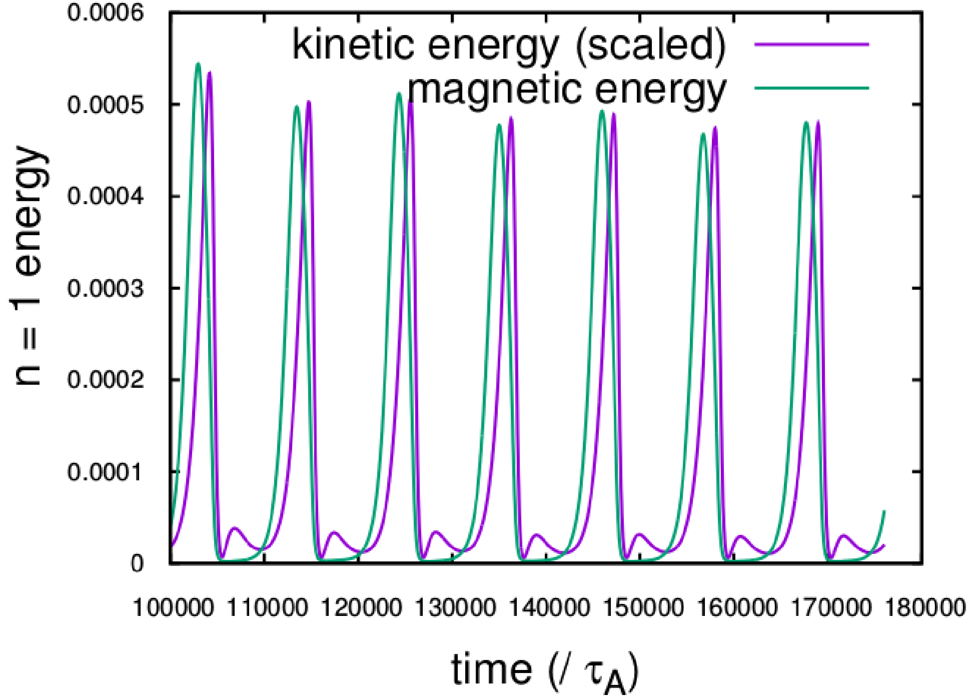
dynamo loop voltage



stationary
oscillating

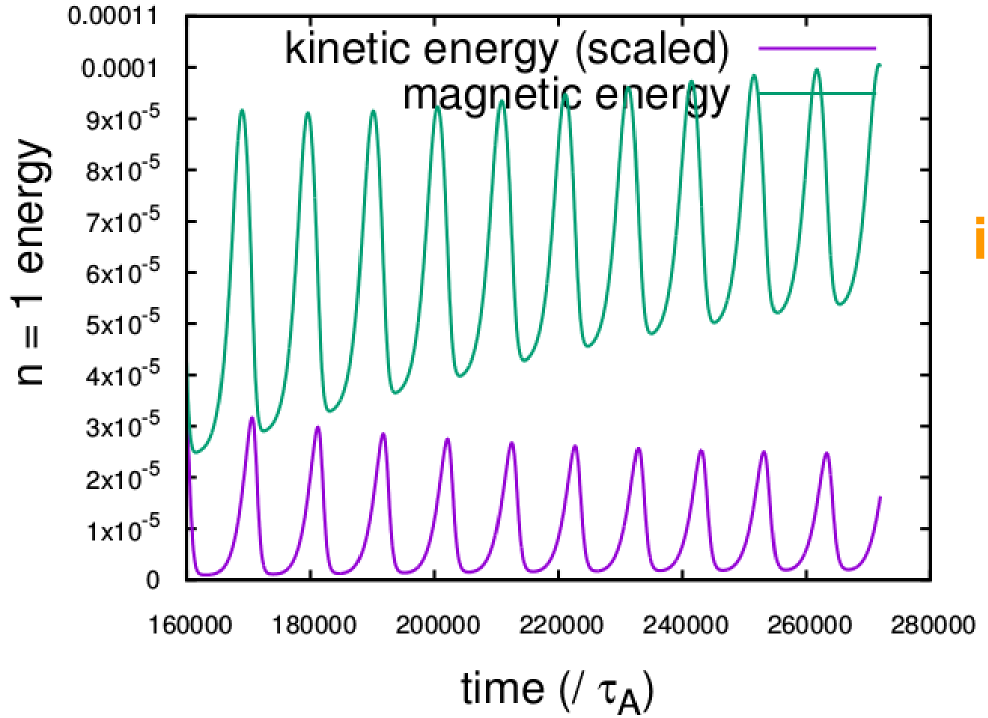
Back to sawteeth...

Incomplete reconnection cycling



complete reconnection cycles

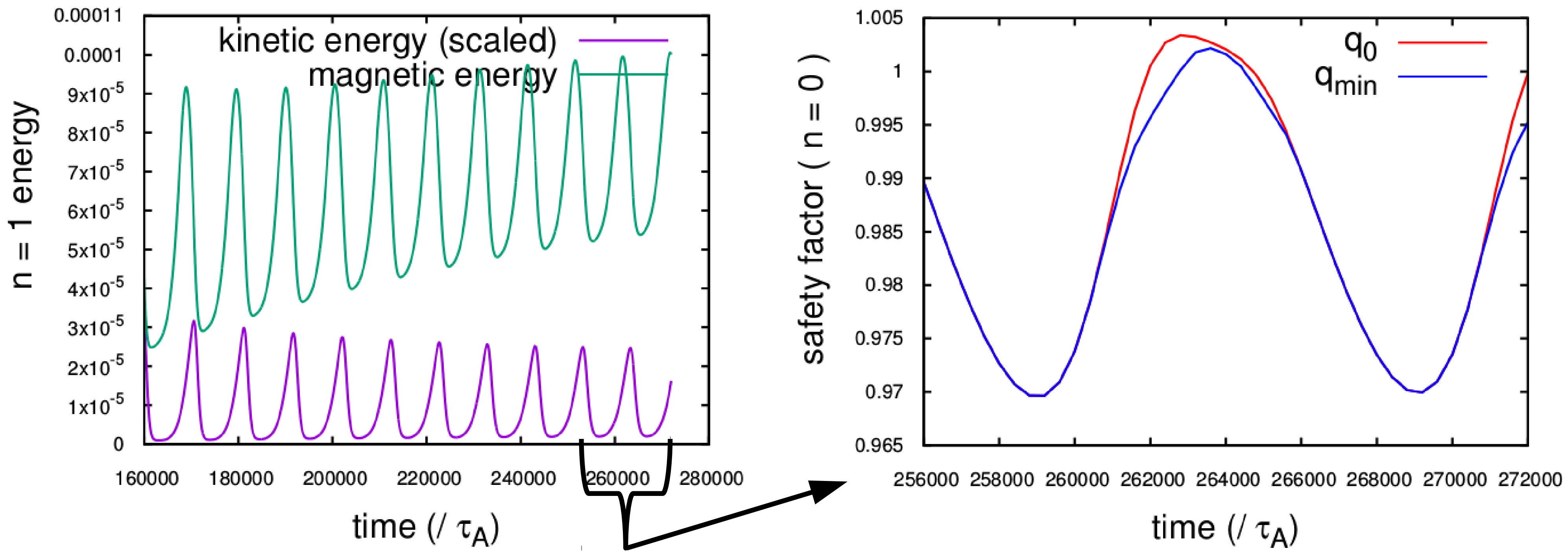
case "m0"



incomplete reconnection cycles

case "m3"

Incomplete reconnection cycling

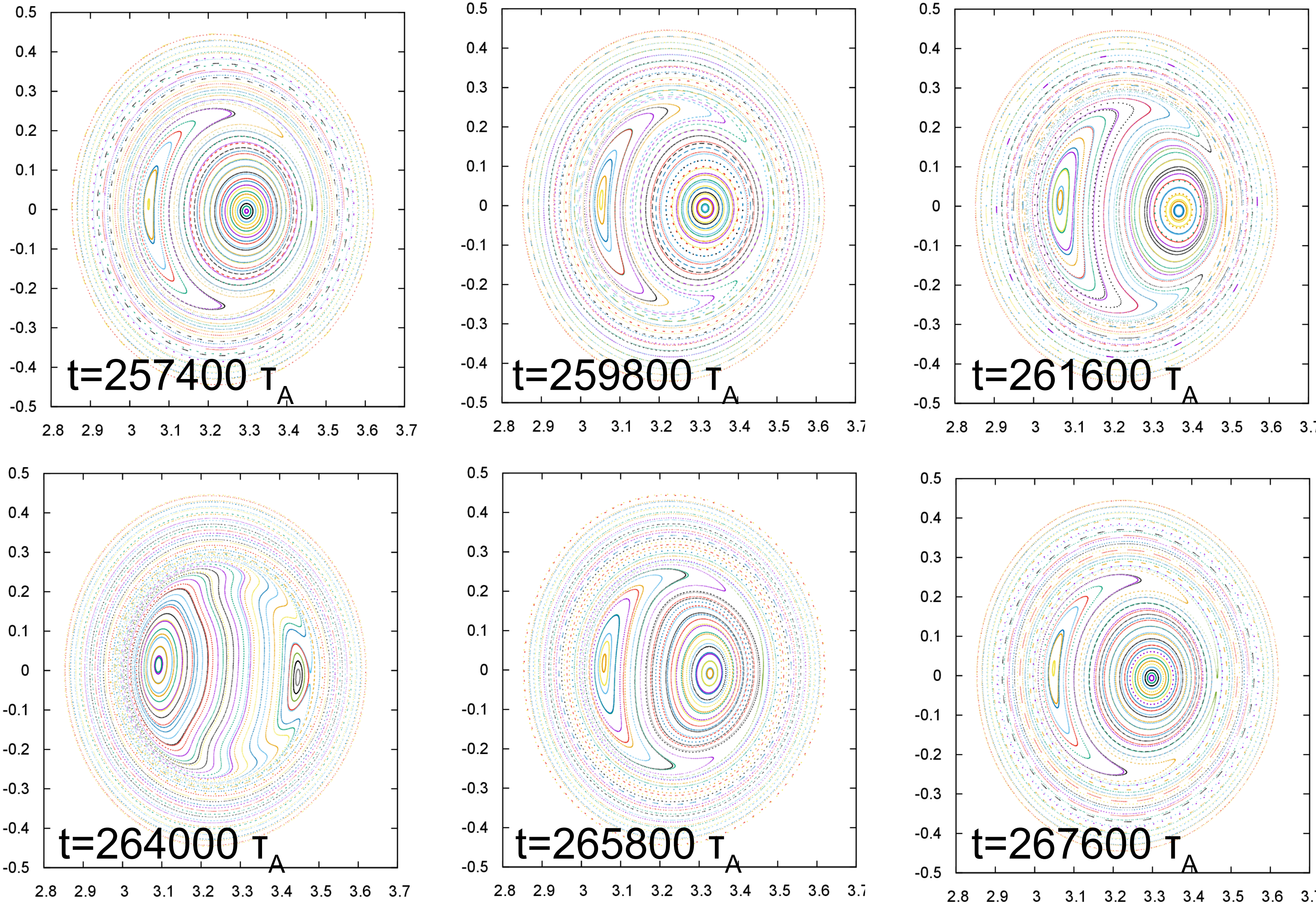


case “m3”

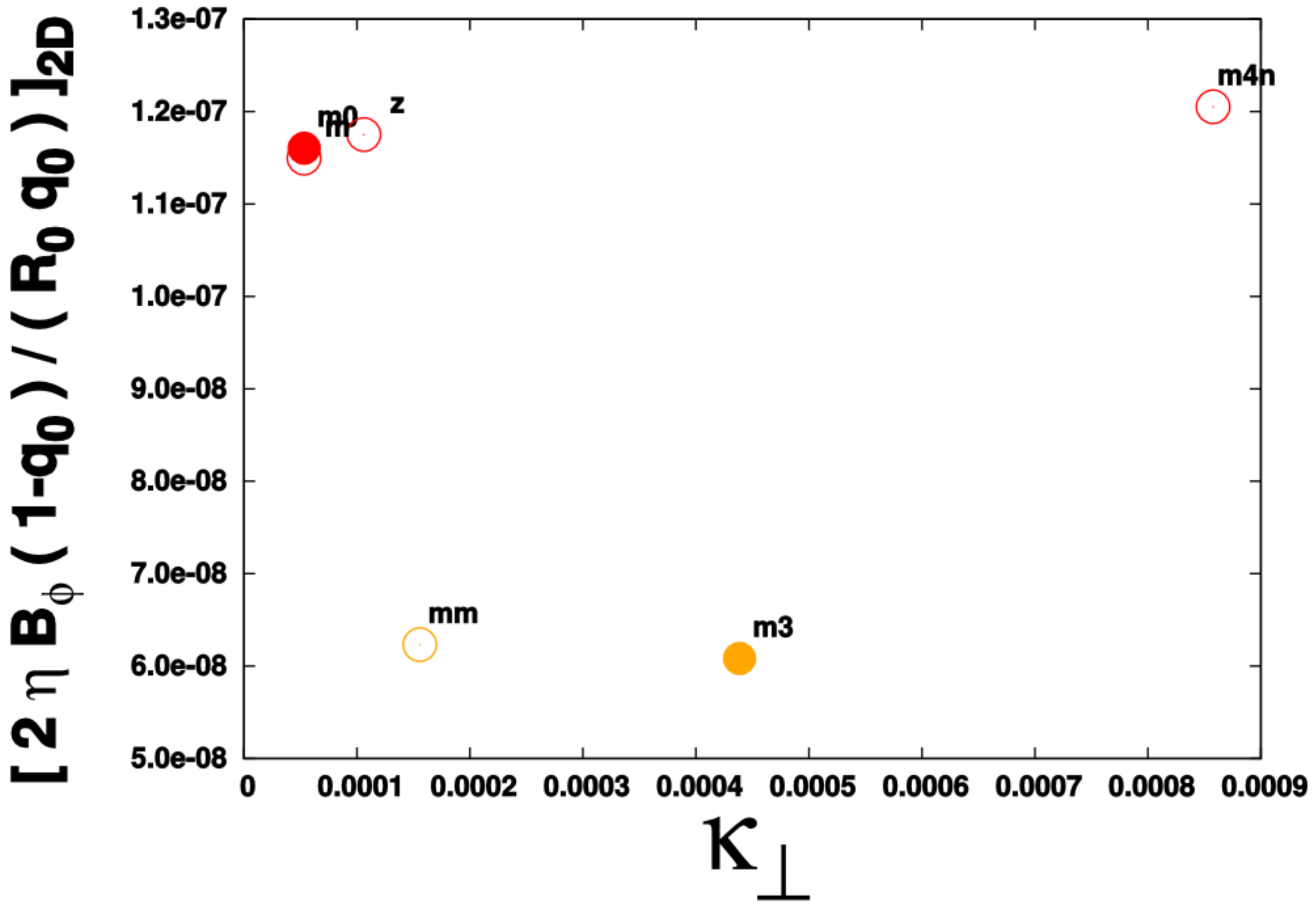
Incomplete reconnection cycling

Magnetic field line structure (one cycle)

case “m3”



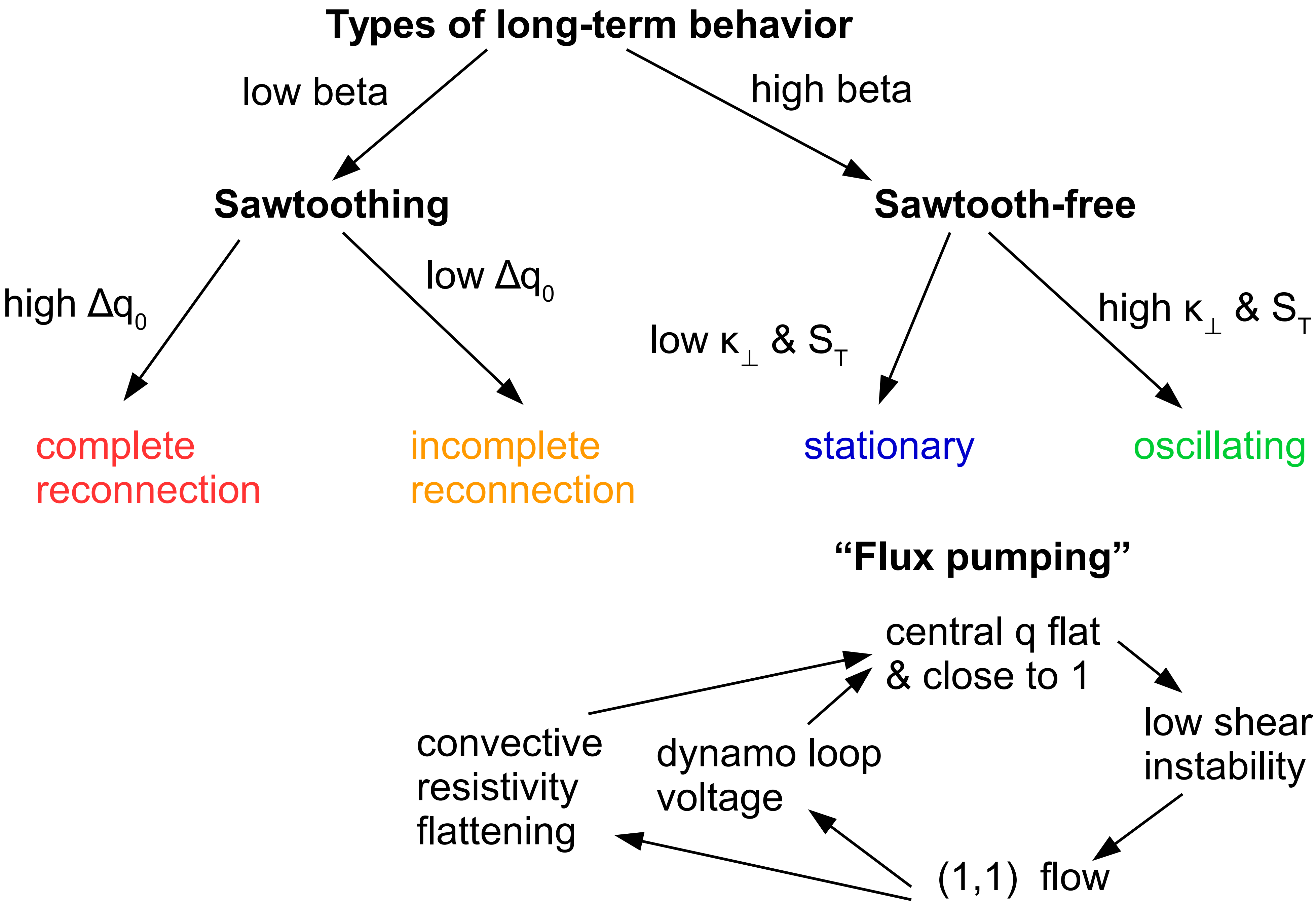
Incomplete reconnection cycling



- only occurs if heat source is weakly peaked,
  mechanism that stabilizes kink is missing in single-fluid MHD

case “m3”

Summary

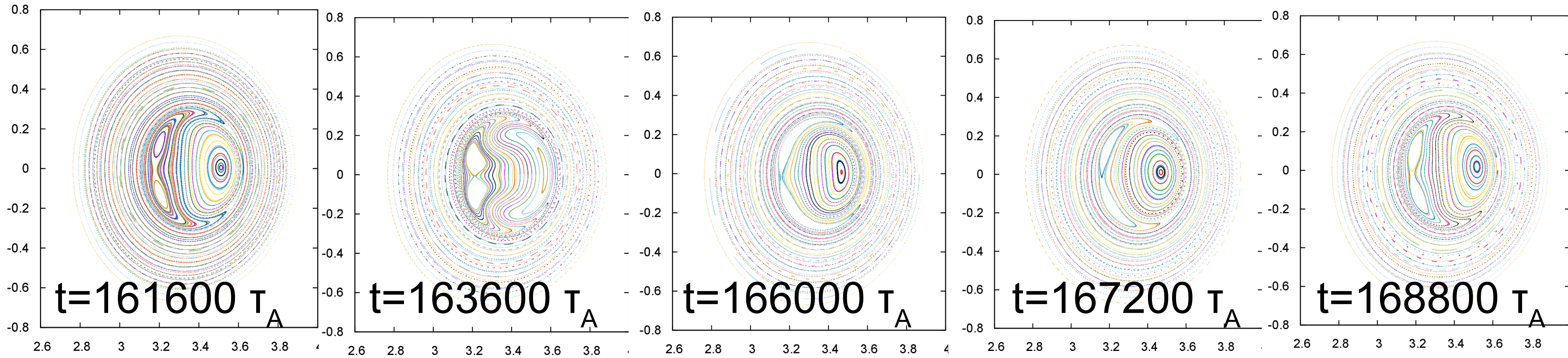


Types of long-term behavior
low beta
high beta
Sawtoothing
Sawtooth-free
high Δq_0
low Δq_0
low $\kappa_\perp$ & S_T
high $\kappa_\perp$ & S_T
complete reconnection
incomplete reconnection
stationary
oscillating
"Flux pumping"
central q flat & close to 1
low shear instability
(1,1) flow
convective resistivity flattening
dynamo loop voltage

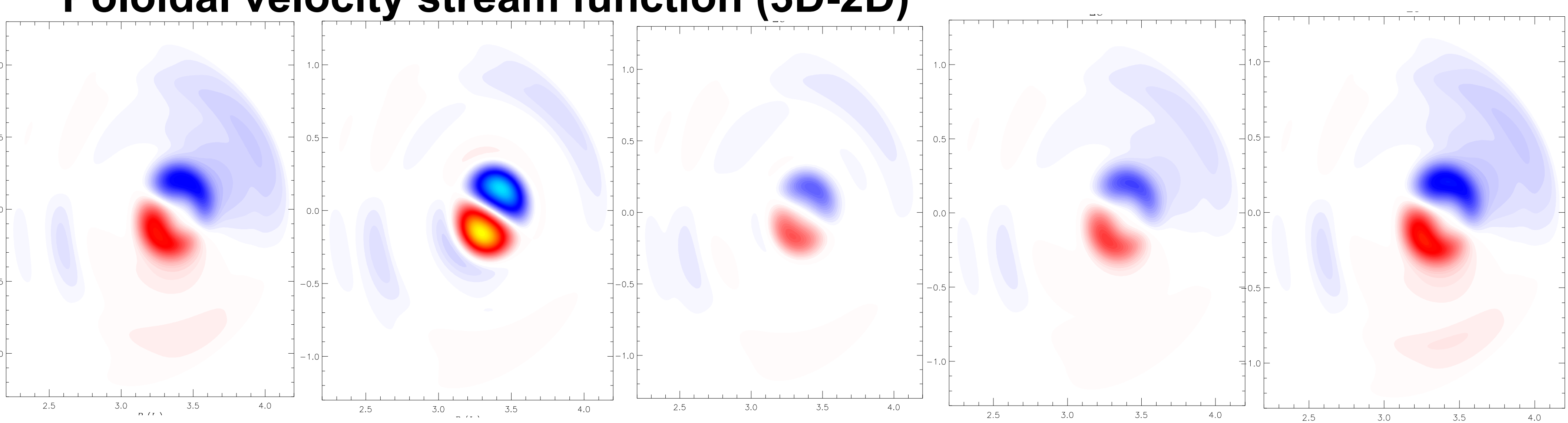
BACKUP SLIDES

Oscillating states

Magnetic field line structure



Poloidal velocity stream function (3D-2D)



case “ff”