Two-Fluid Modeling of the Sawtooth Instability

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  A. Nonlinear behavior

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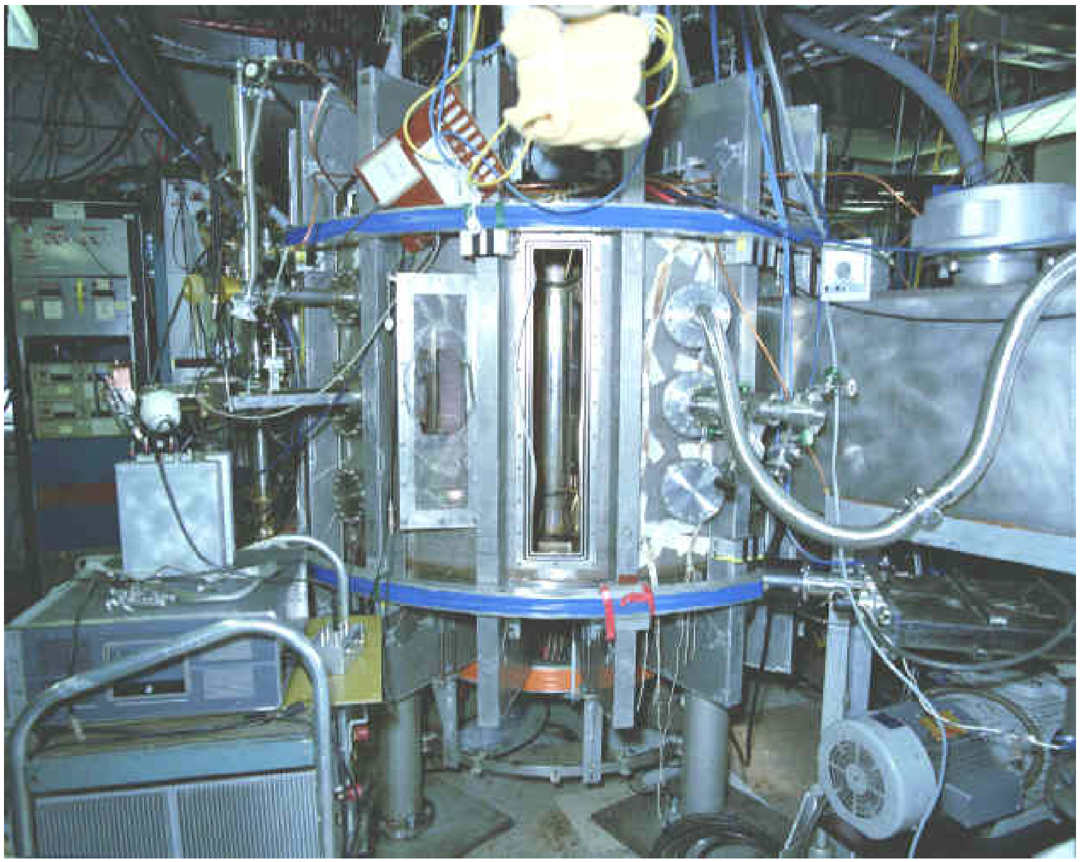
Motivation

- As a nonlinear code benchmark, model sawtooth events in a tokamak plasma with extended MHD models using realistic physical values to make quantitative predictions.
  - Large tokamaks have large disparities in spatial and temporal scales to be resolved.
    - Resistive MHD: Current sheet thickness ~ $S^{-1/2}$
    - Two-fluid MHD: ion skin depth ~ $n^{-1/2}$
  - Small tokamaks operate in regimes accessible to present-day codes.

Characteristics of the Current Drive Experiment Upgrade (CDX-U)

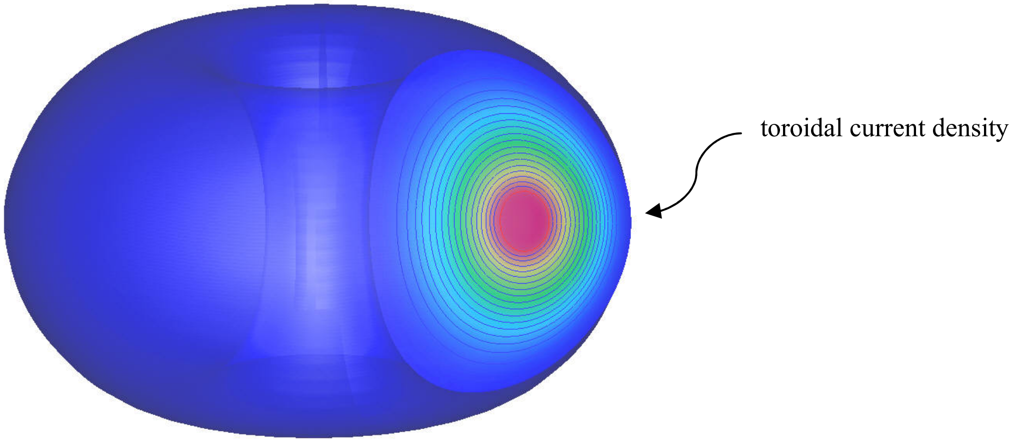


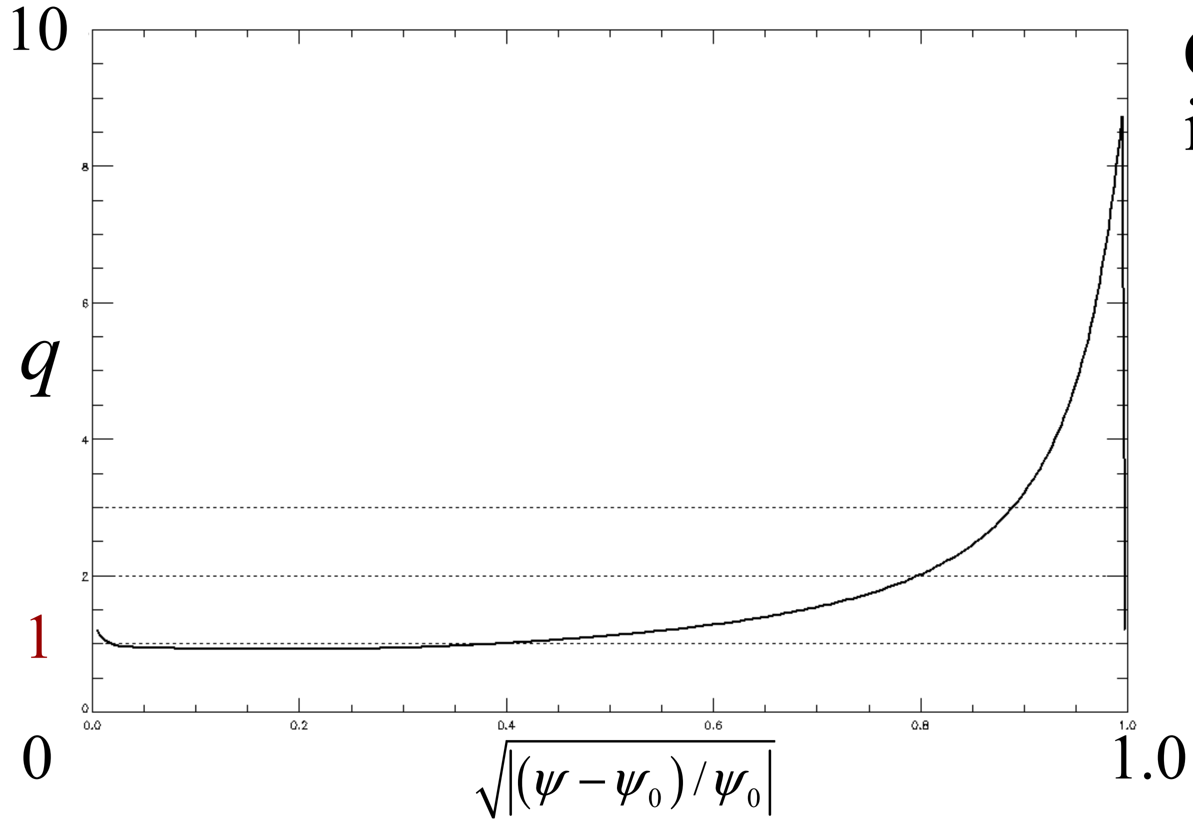
- Low aspect ratio tokamak ($R_0/a = 1.4 - 1.5$)
- Small ($R_0 = 33.5$ cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $I_p \sim 70$ kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV $\rightarrow$ S ~ 10^4
- Discharge time ~ 12 ms

- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  - sawteeth
  - "snakes"

Equilibrium: $q_0 < 1$

- Equilibrium taken from a TSC sequence (Jsolver file).
- $q_{\min} \approx 0.922$
- $q(a) \sim 9$





Questions to investigate:

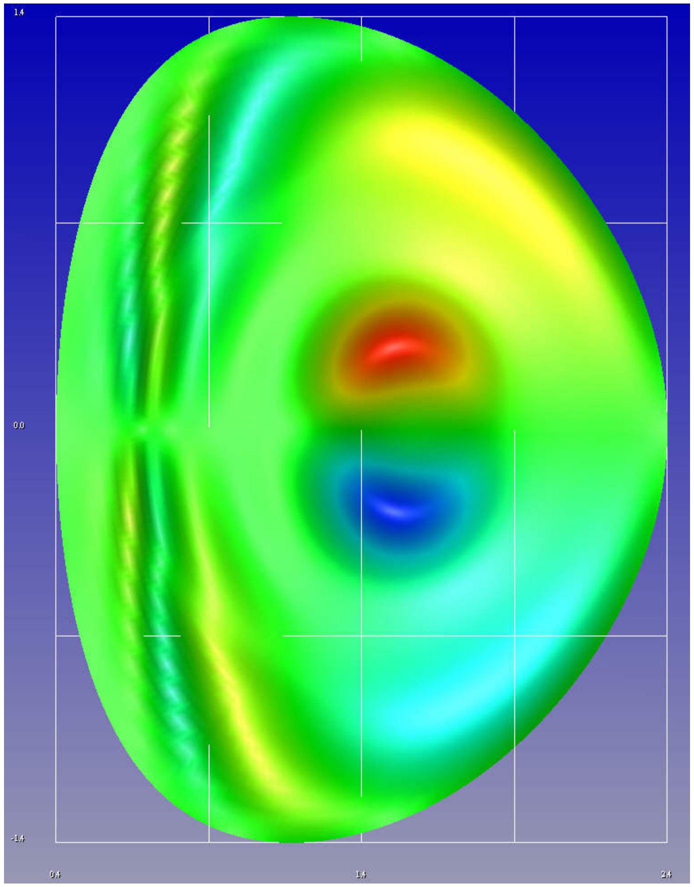
- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Baseline Parameters for CDX

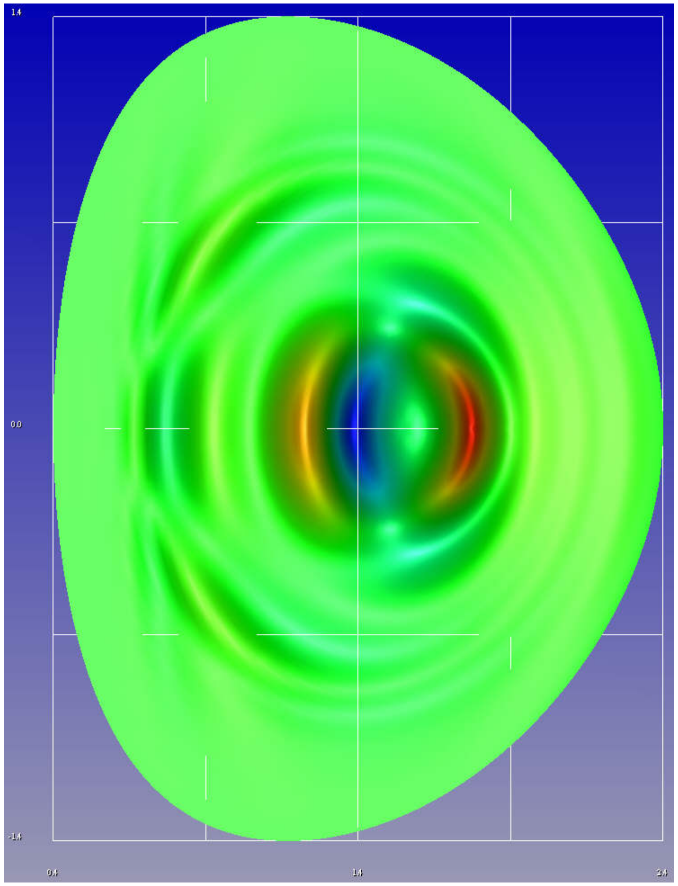
Lundquist Number S	$\sim 2\times 10^4$ on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number Pr	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	200 m^2/s (measured value at edge)
Parallel thermal conduction (sound wave)	$V_{Te} = 6\ V_A$
Peak Plasma β	$\sim 3\times 10^{-2}$ (low-beta).
Density Evolution	Turned on for nonlinear phase.
Nonlinear initialization	Pure n=1 perturbation such that $\frac{\max(B^1_{\text{pol}})}{\max(B^0_\phi)} = 10^{-4}$

n=1 Eigenmode

Incompressible velocity
stream function U

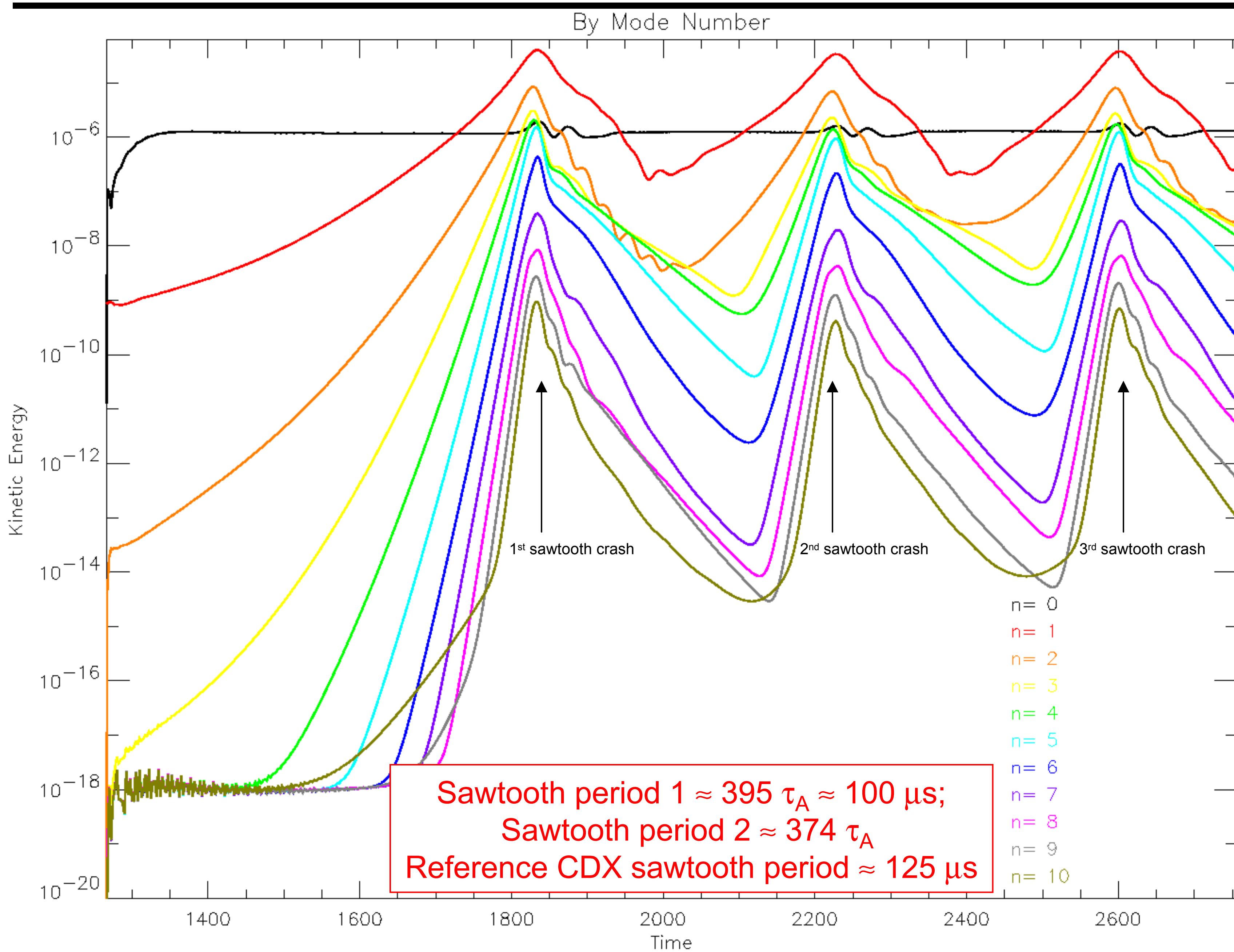


Toroidal current density
J_{ϕ}



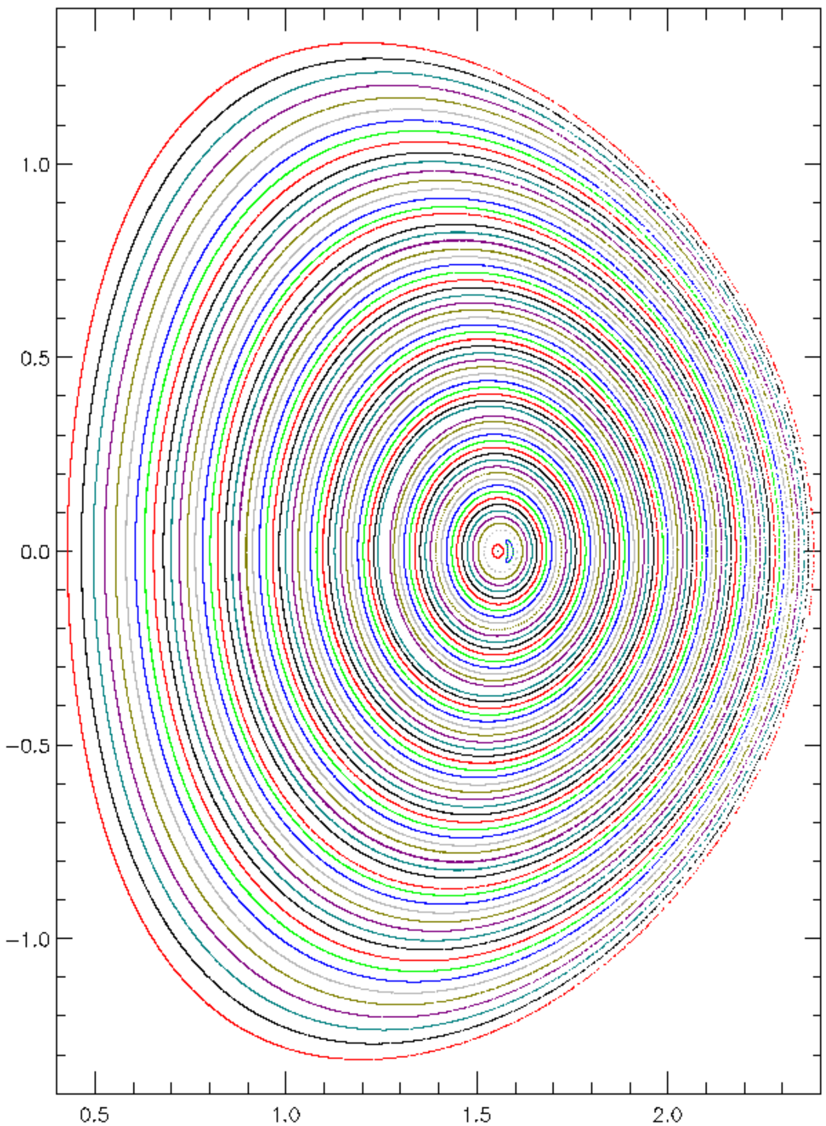
$\gamma\tau_A = 8.61 \times 10^{-3} \rightarrow$ growth time = 116 τ_A

Resistive MHD: Nonlinear History

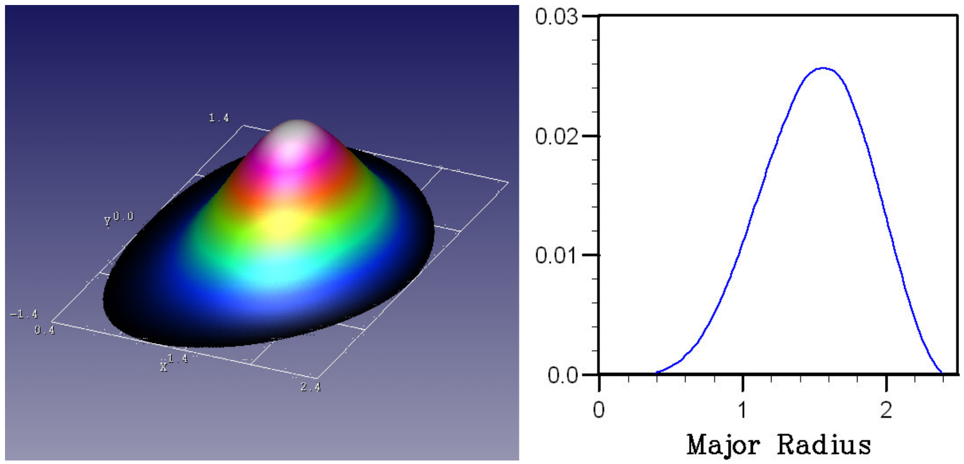


Initial state: t = 1266.17

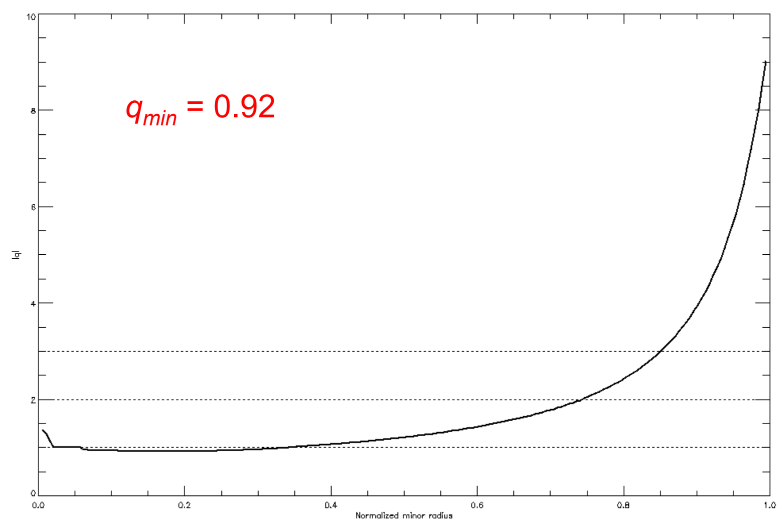
Poincaré plot



Temperature profile

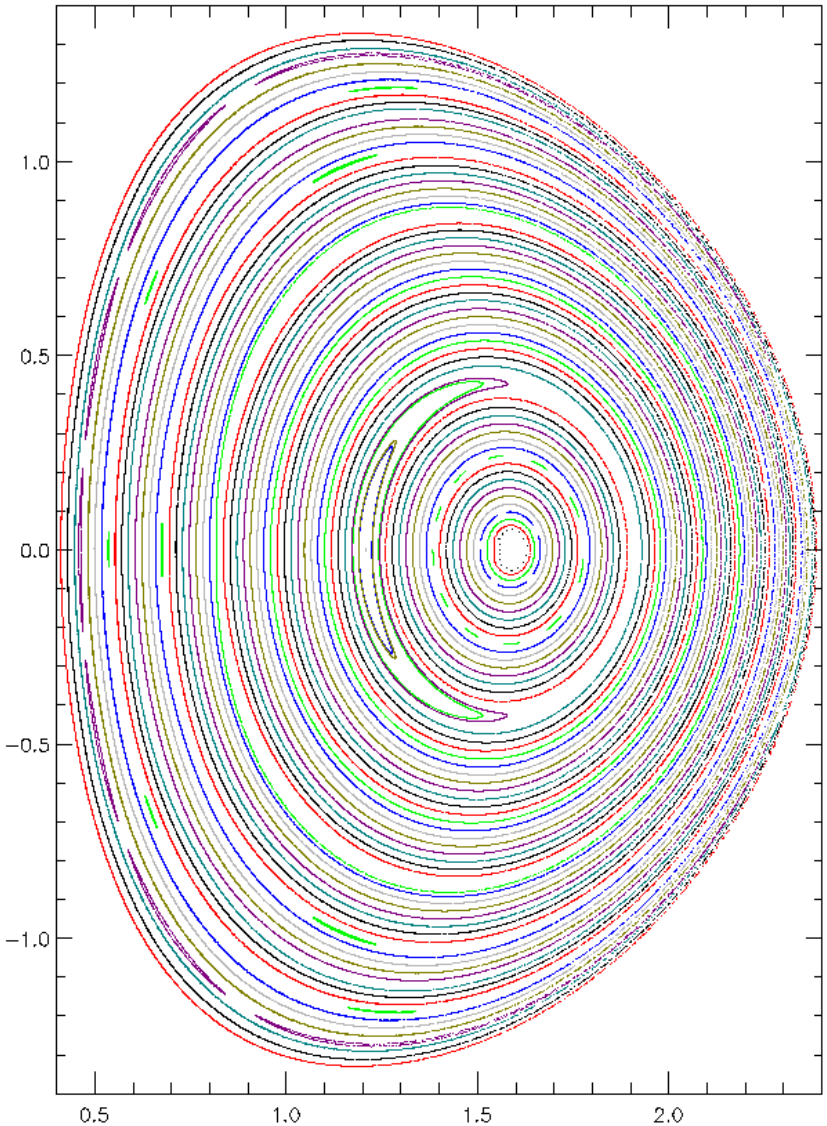


q profile



Island growing: t = 1660.70

Poincaré plot



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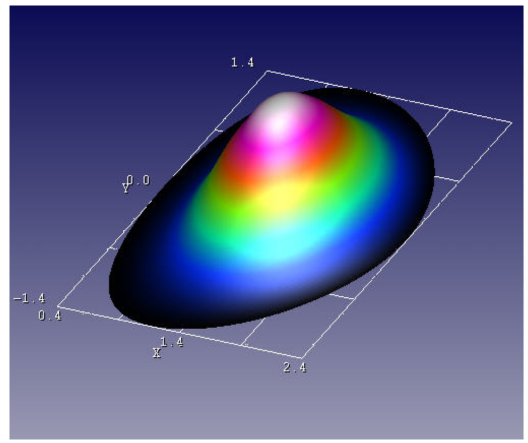

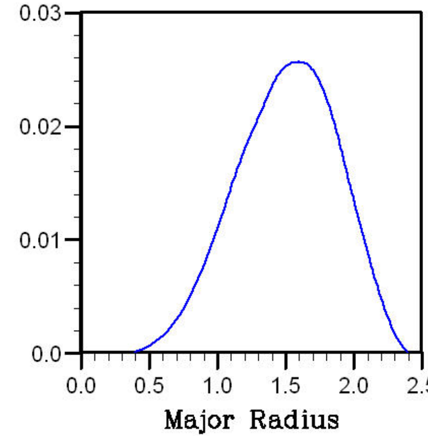

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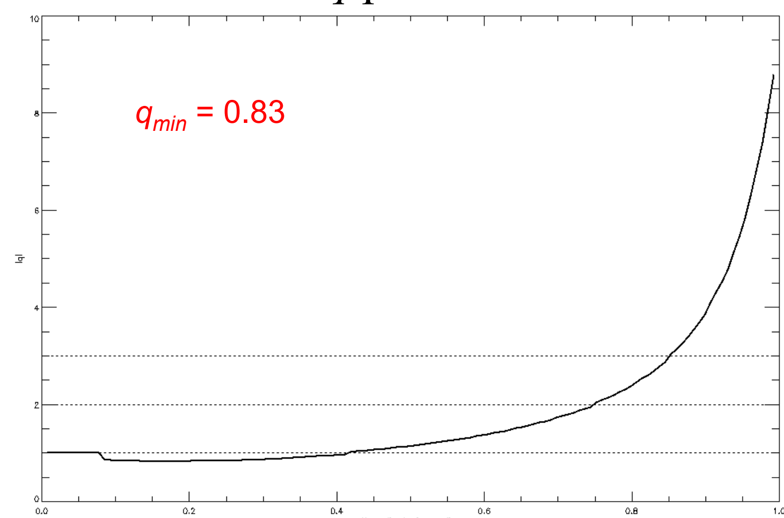
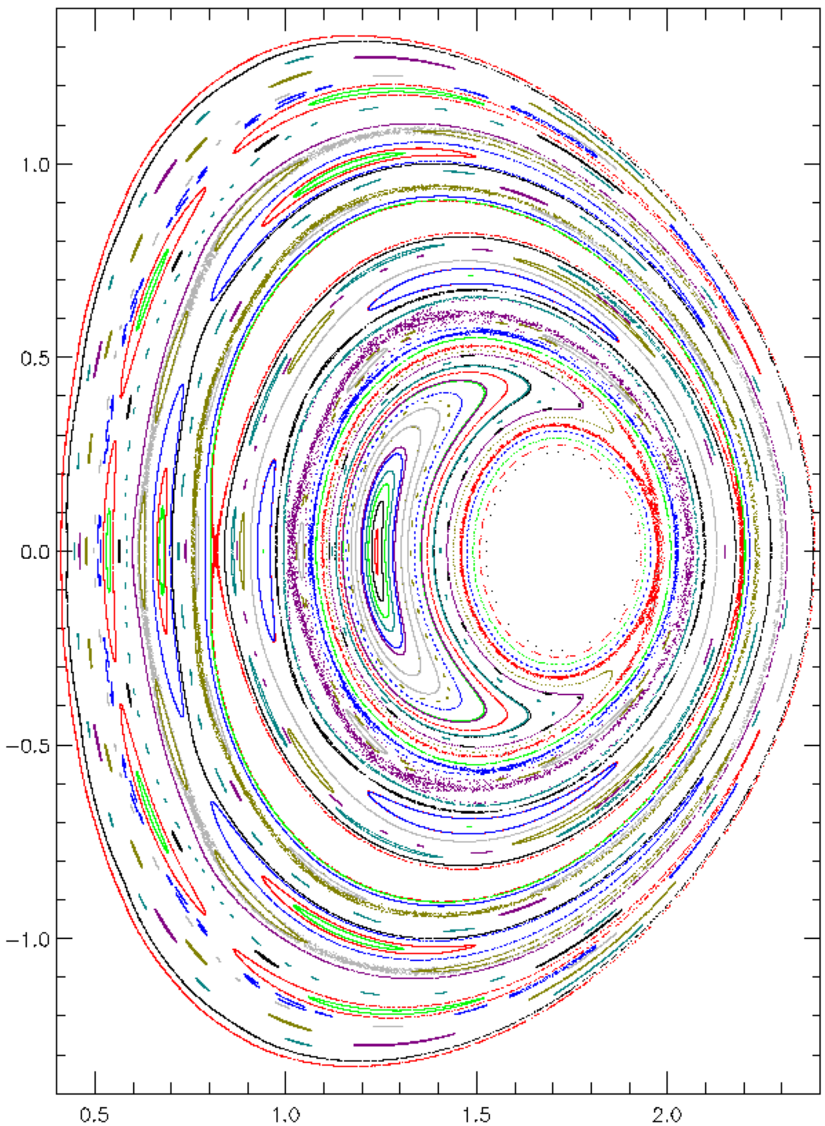


Nonlinear phase: t = 1795.61

Poincaré plot



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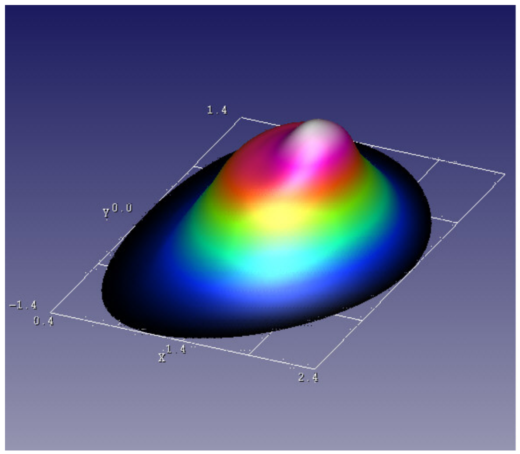
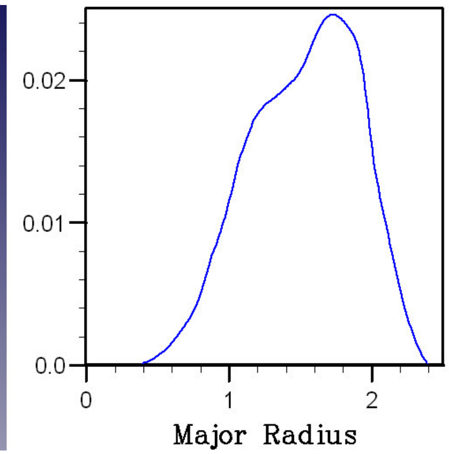
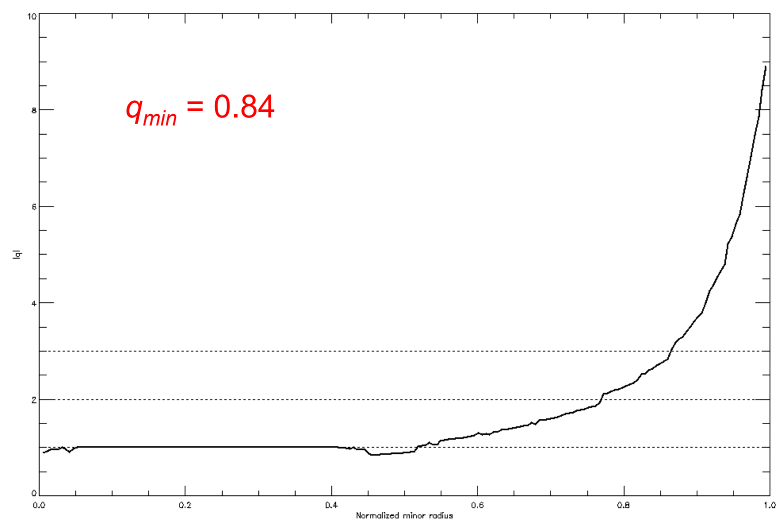


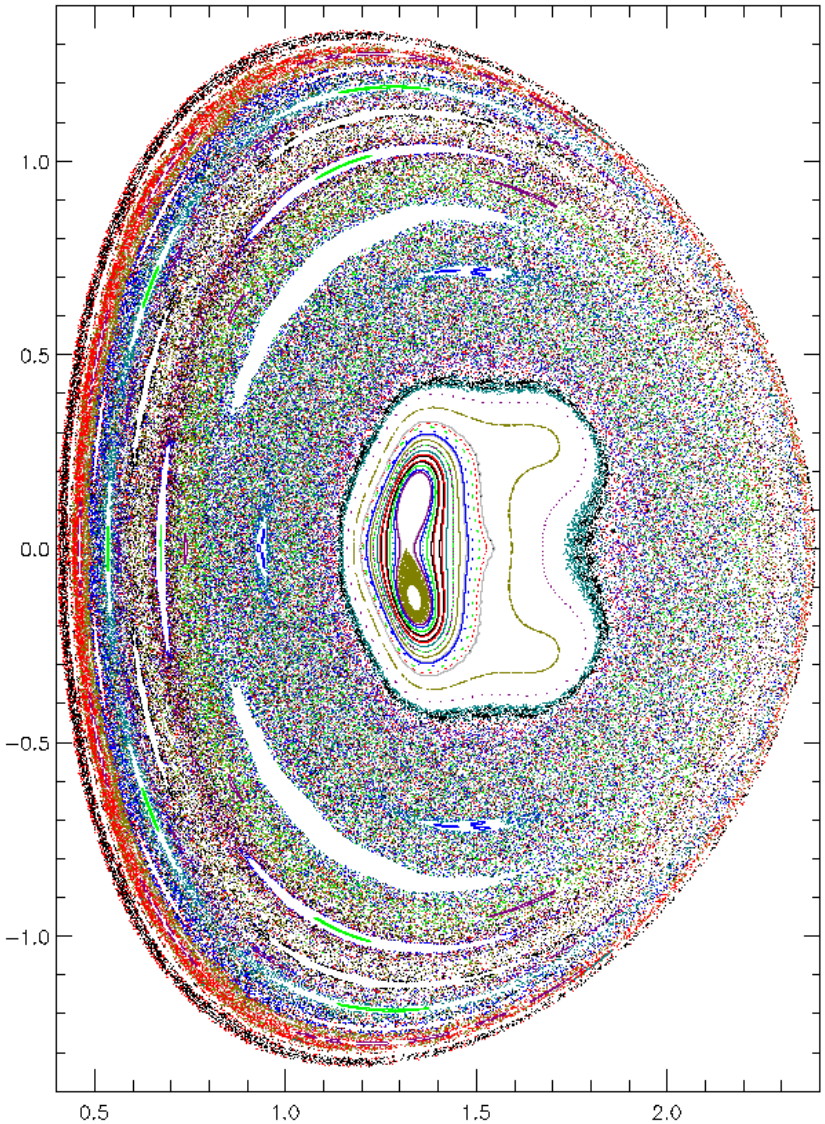


q profile

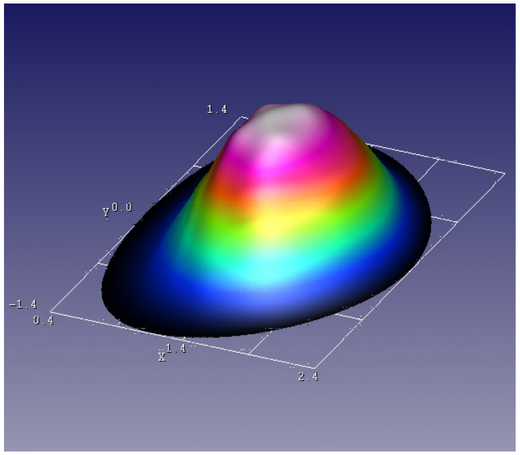


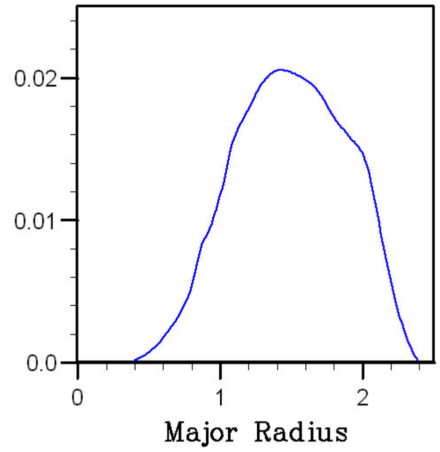
After 1st Crash: *t* = 1839.86

Poincaré plot

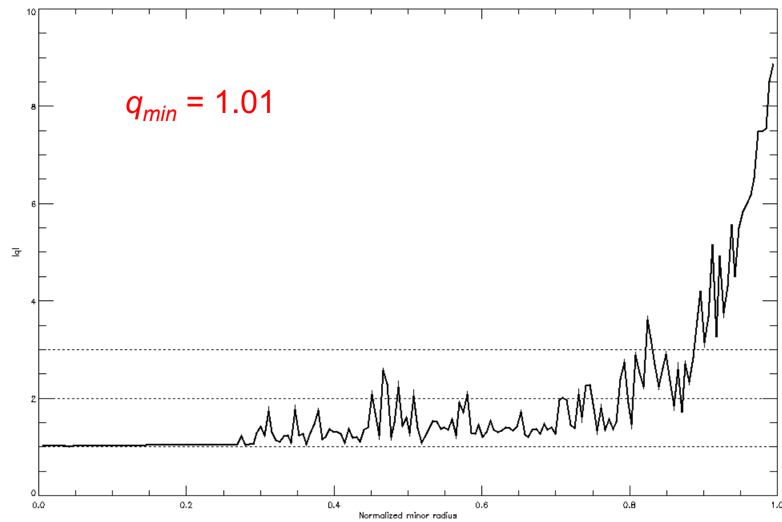


Temperature profile



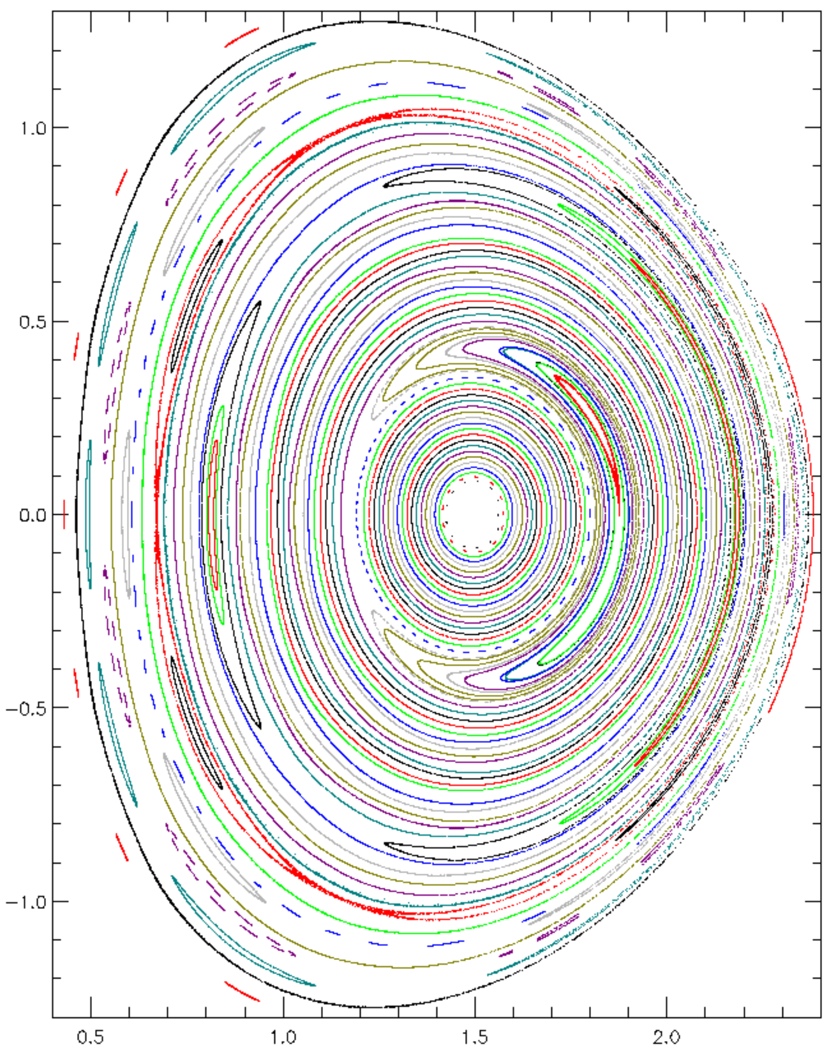


q profile

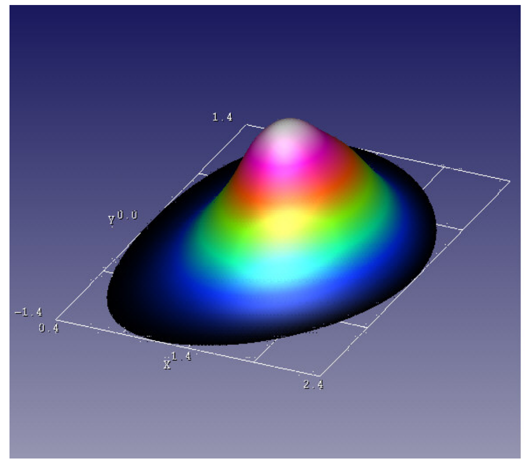


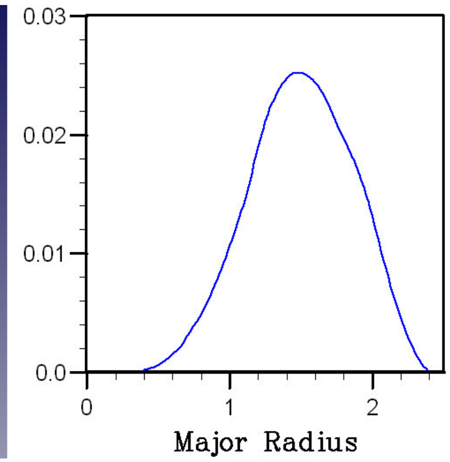
Flux surfaces recovered: t = 2094.08

Poincaré plot



Temperature profile





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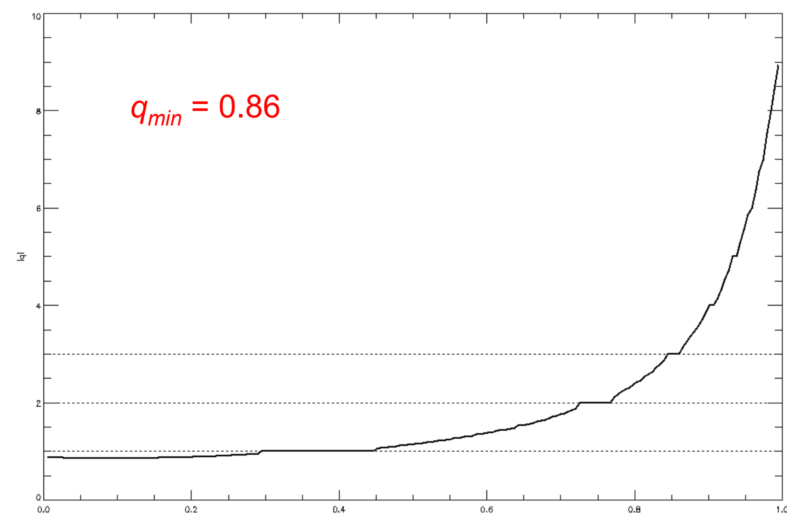


The M3D Two-Fluid Model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_i) = 0$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \left(\mathbf{v}_i^* \cdot \nabla \right) \mathbf{v}_\perp \right] = -\nabla \mathbf{p} + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{v}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} - \frac{\nabla_{||} p_e}{ne}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \kappa_\perp \nabla \left(\frac{p}{\rho} \right) - \mathbf{v}_i^* \cdot \nabla p - \gamma p \nabla \cdot \mathbf{v}_i^* + \frac{\mathbf{J} \cdot \nabla p_e}{ne} + \gamma p_e \mathbf{J} \cdot \nabla \left(\frac{1}{ne} \right)$$

$$\frac{\partial p_e}{\partial t} + \mathbf{v} \cdot \nabla p_e = -\gamma p_e \nabla \cdot \mathbf{v} + \rho \nabla \cdot \kappa_{\perp e} \nabla \left(\frac{p_e}{\rho} \right) + \frac{\mathbf{J}_{||} \cdot \nabla p_e}{ne} - \gamma p_e \nabla \cdot \left(\mathbf{v}_e^* - \frac{\mathbf{J}_{||}}{ne} \right)$$

where

$$\mathbf{v}_i^* \equiv \mathbf{v}_e^* + \frac{\mathbf{J}_\perp}{ne}, \qquad \mathbf{v}_i \equiv \mathbf{v} + \mathbf{v}_i^*,$$

$$\mathbf{v}_e^* \equiv -\frac{\mathbf{B} \times \nabla p_e}{neB^2}, \quad \mathbf{v}_e \equiv \mathbf{v} + \mathbf{v}_e^* - \frac{J_{||}}{ne}$$

Artificial sound wave model for $\kappa_{||}$:

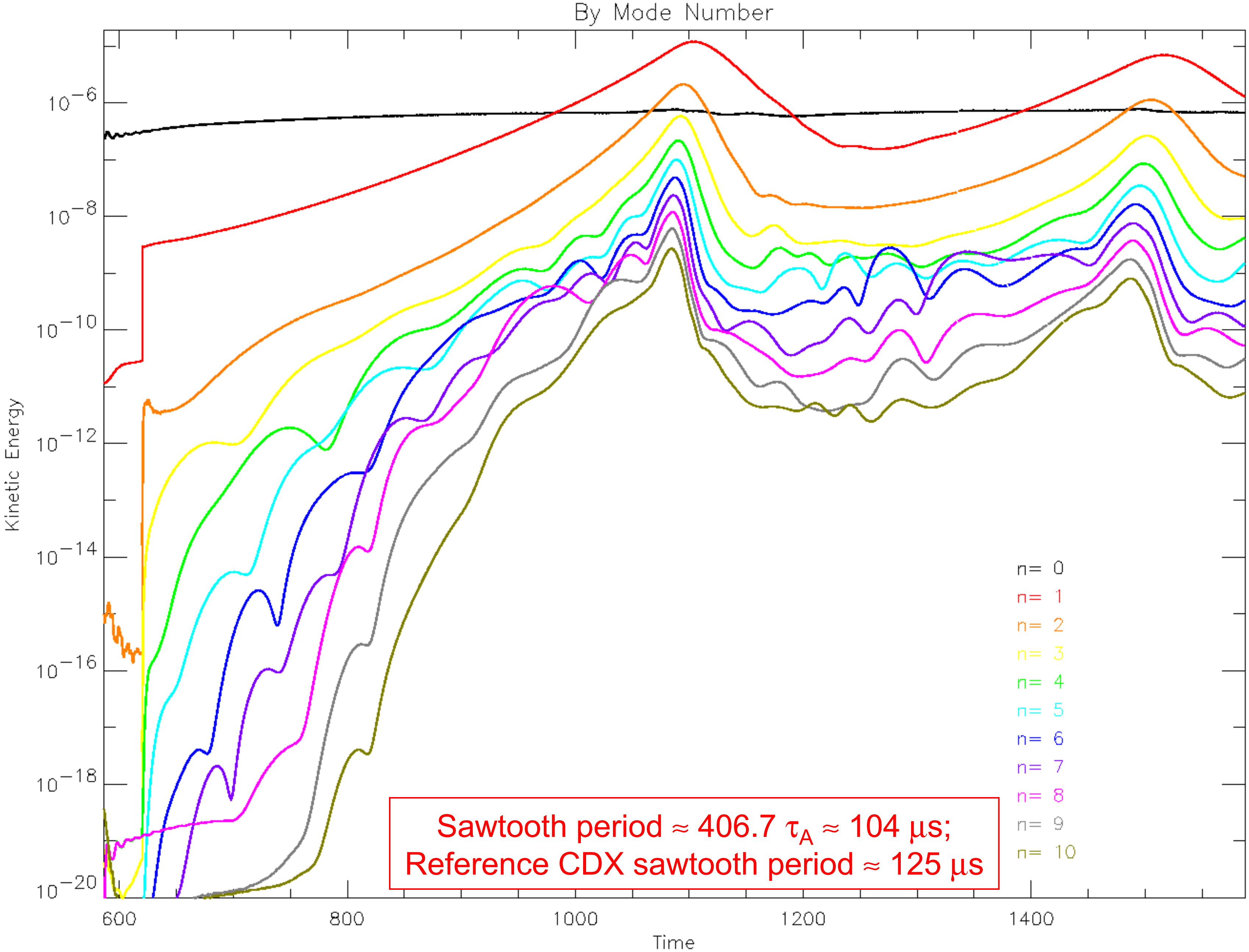
$$\frac{\partial T}{\partial t} = s \frac{\mathbf{B} \cdot \nabla u}{\rho}$$

$$\frac{\partial u}{\partial t} = s \mathbf{B} \cdot \nabla T + \nu \nabla^2 u$$

Two-Fluid Study

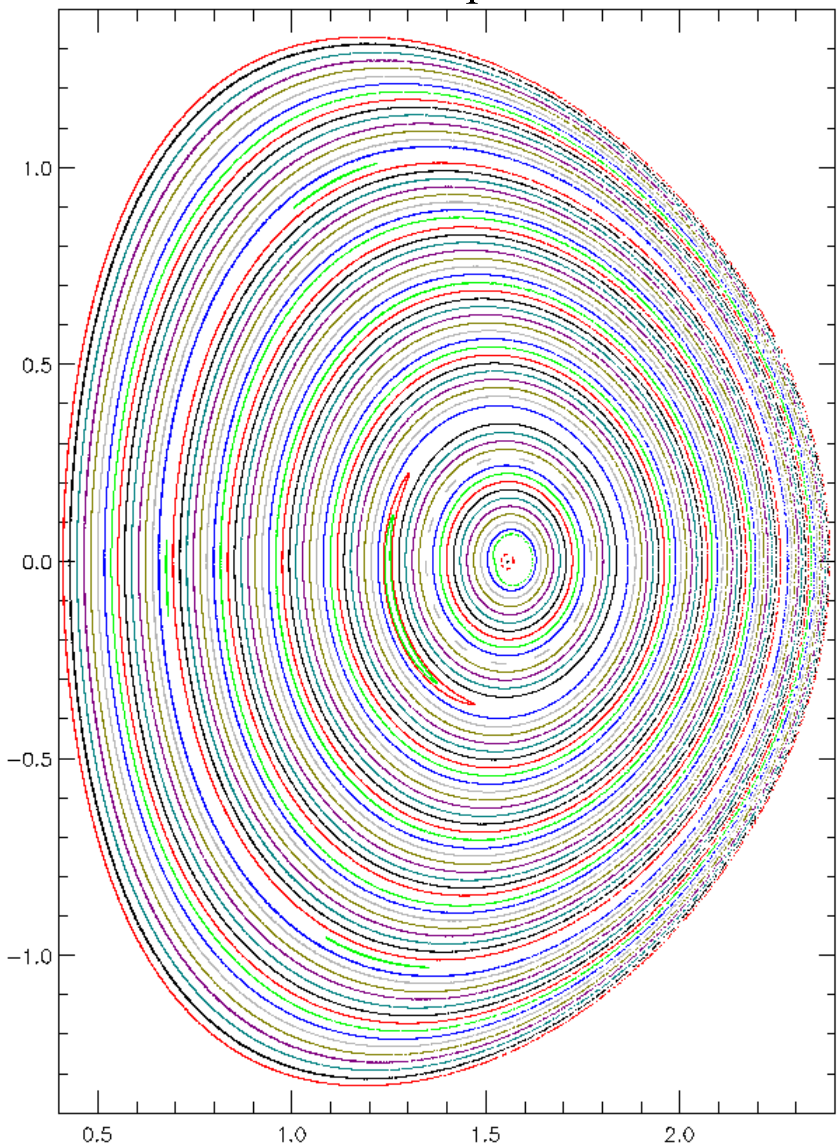
- Same parameters as single-fluid, but ω^*_i term on.
- Ion skin depth = 0.05 minor radii.
- Pressure divided evenly between electrons, ions.
- Modest increase in poloidal resolution relative to 1st single-fluid study (89 *vs*. 79 radial grids); same toroidal resolution (24 planes).
- Start nonlinear run with MHD n=1 (1,1) eigenmode, $\gamma\tau_A=5.1\times10^{-3}$ as small initial perturbation.

Two-Fluid Sawooth Energy History

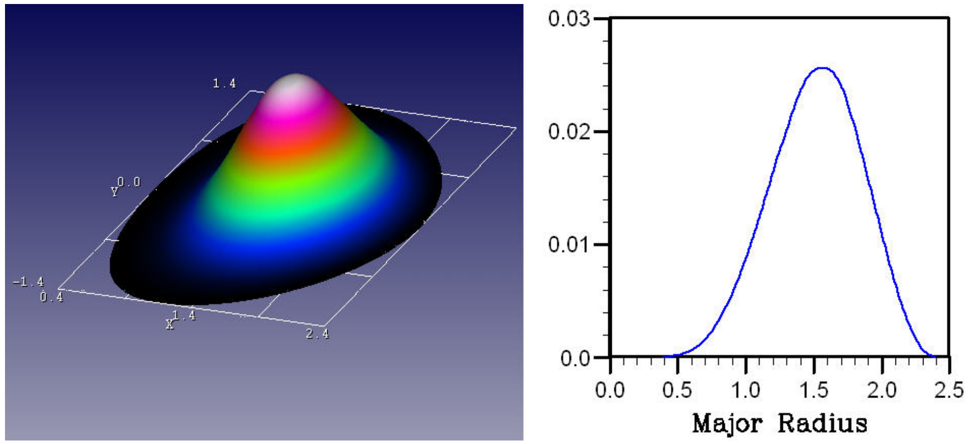


Early state: t = 653.95

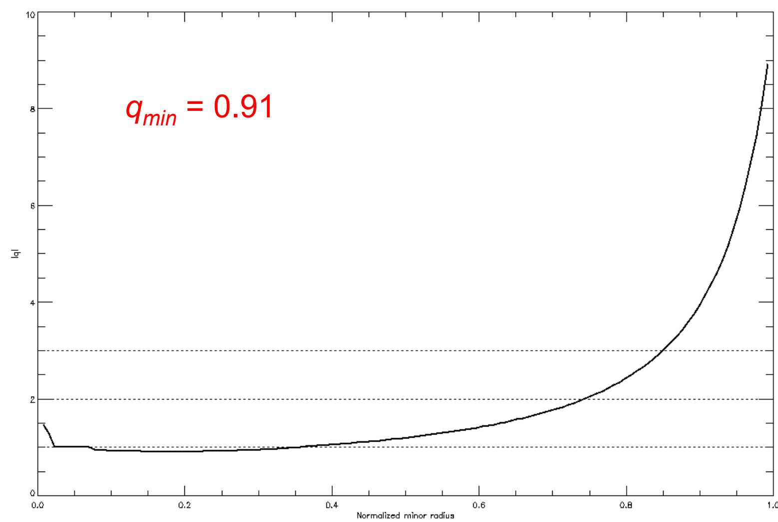
Poincaré plot



Temperature profile

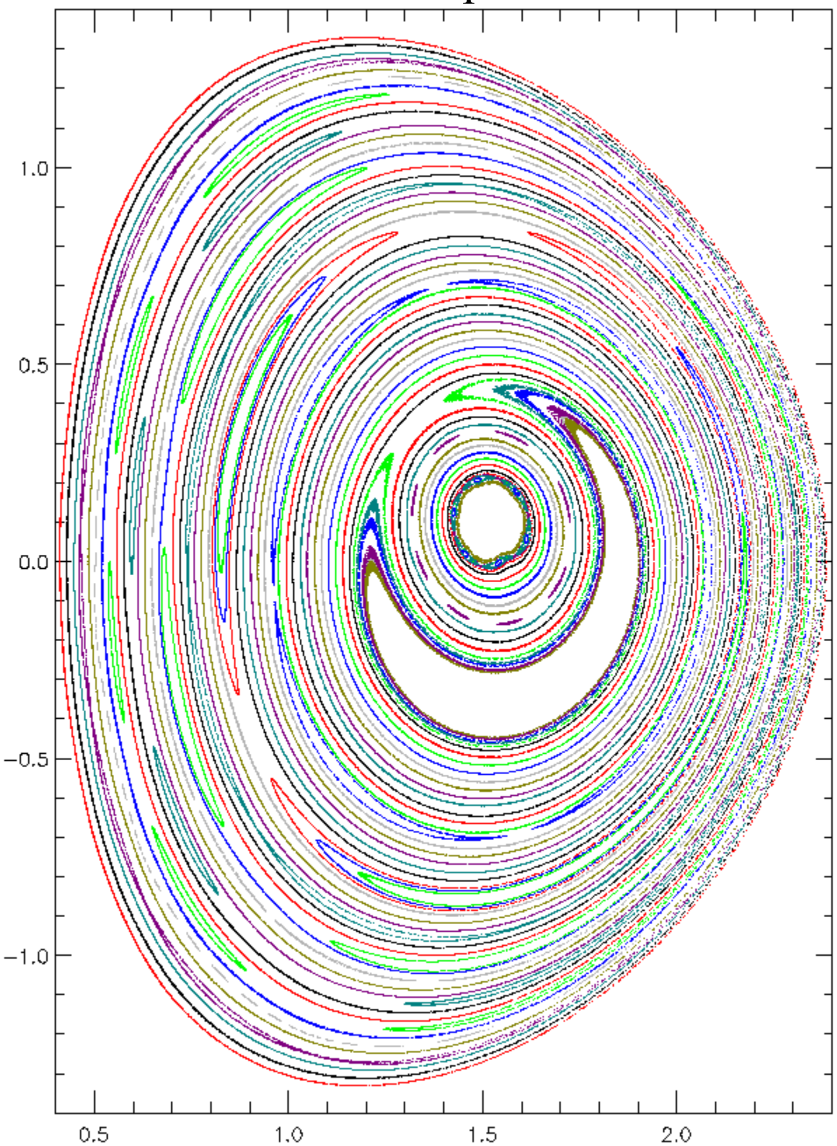


q profile

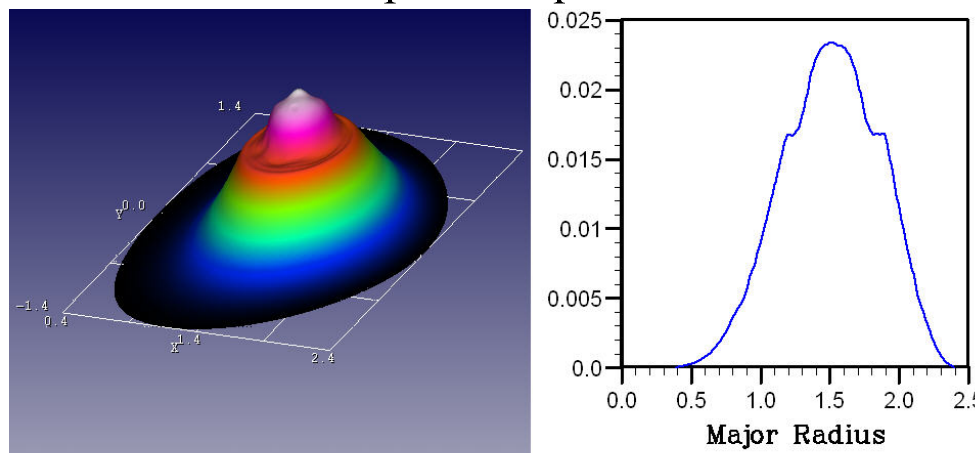


Nonlinear phase: t = 1008.38

Poincaré plot



Temperature profile



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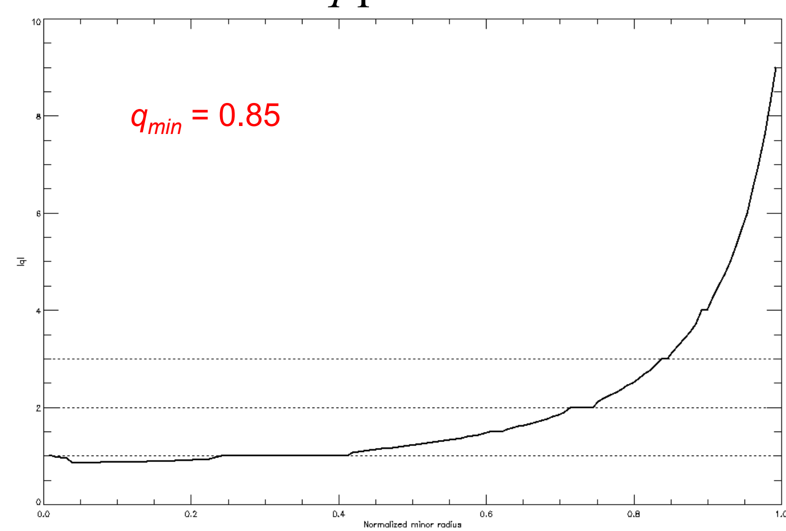
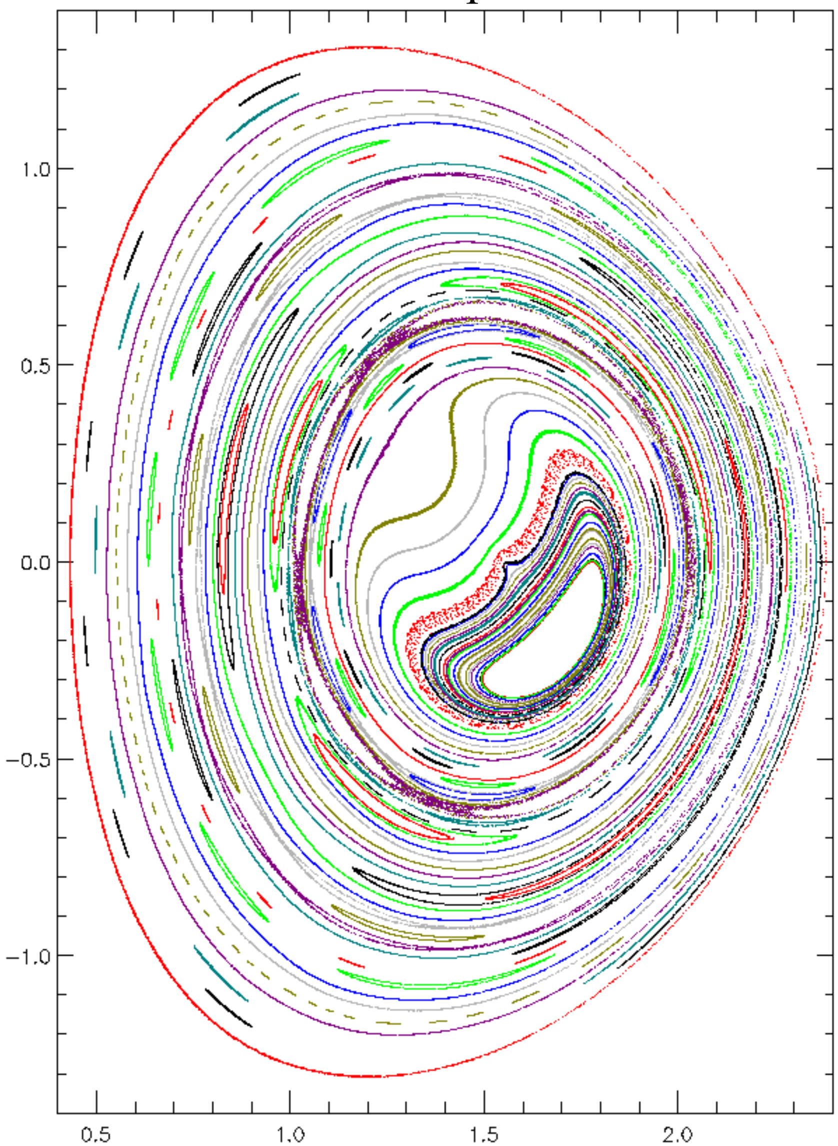
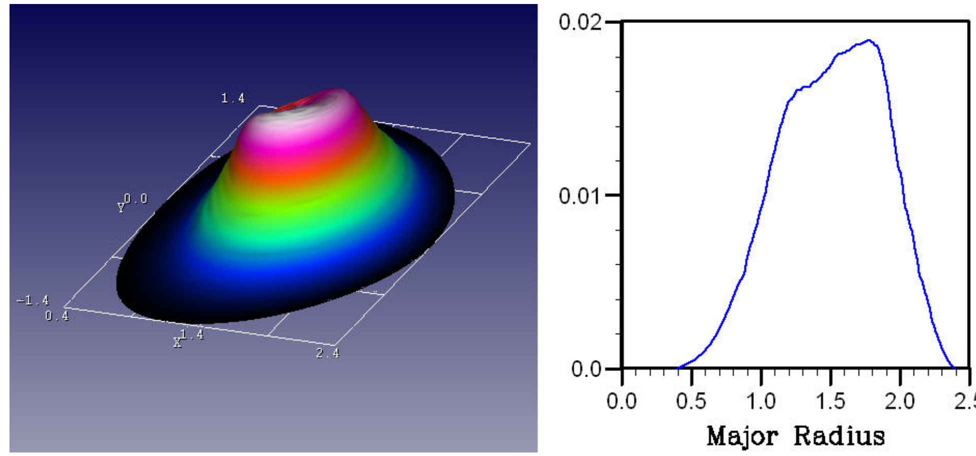


After 1st Crash: t = 1118.44

Poincaré plot



Temperature profile



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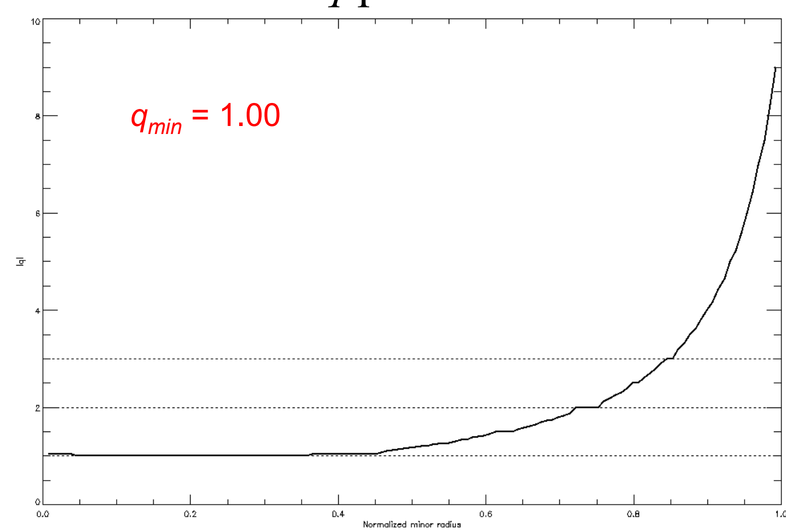
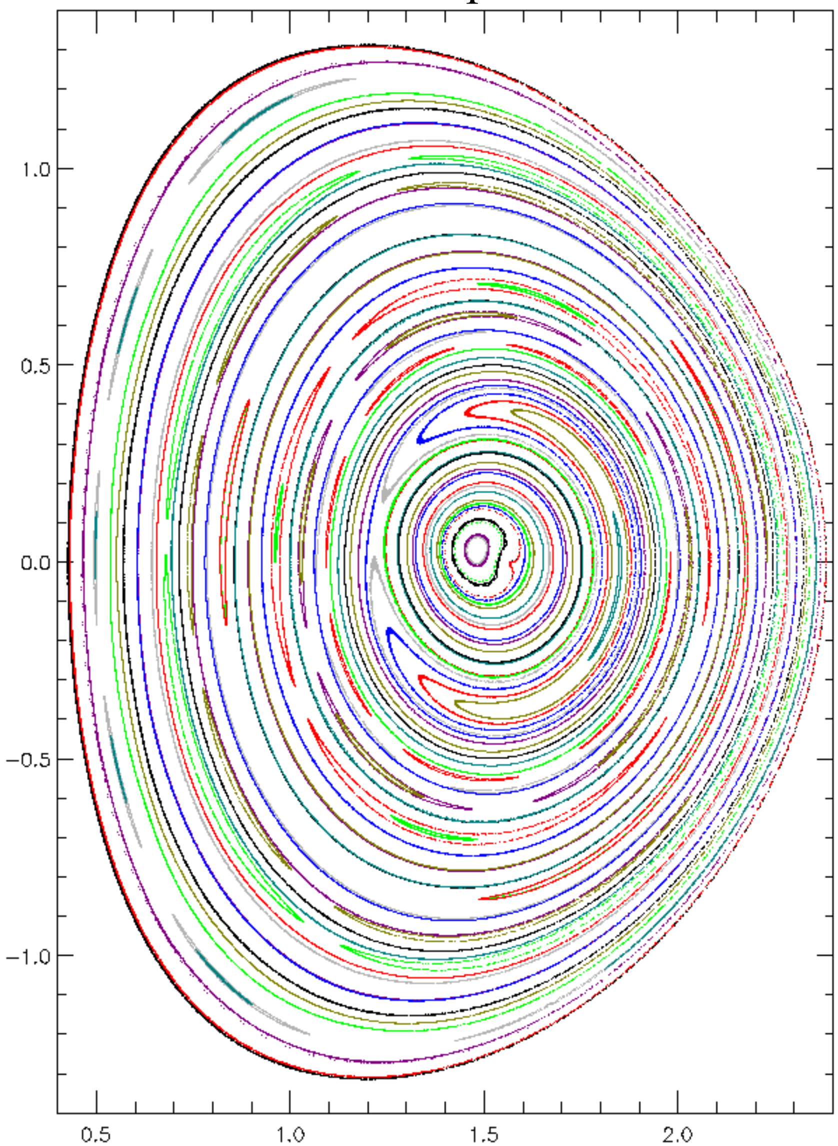
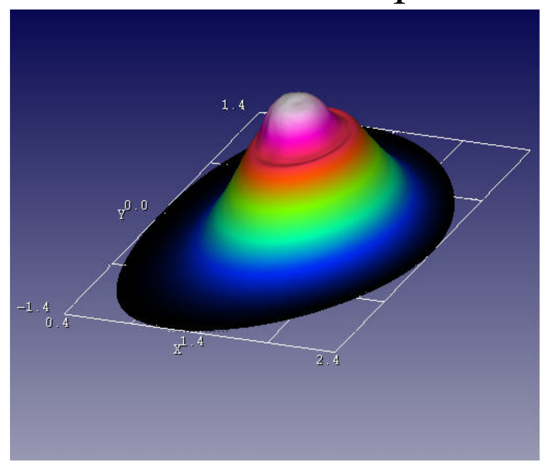


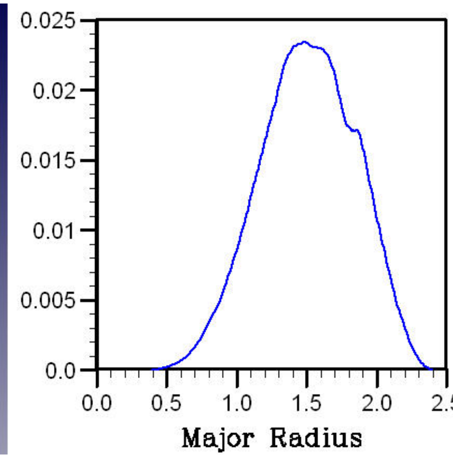
Between Crashes: t = 1316.83

Poincaré plot



Temperature profile





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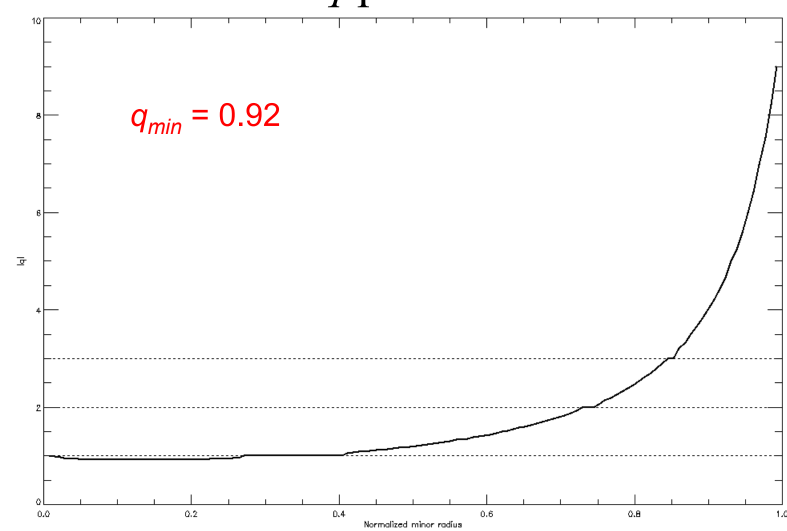
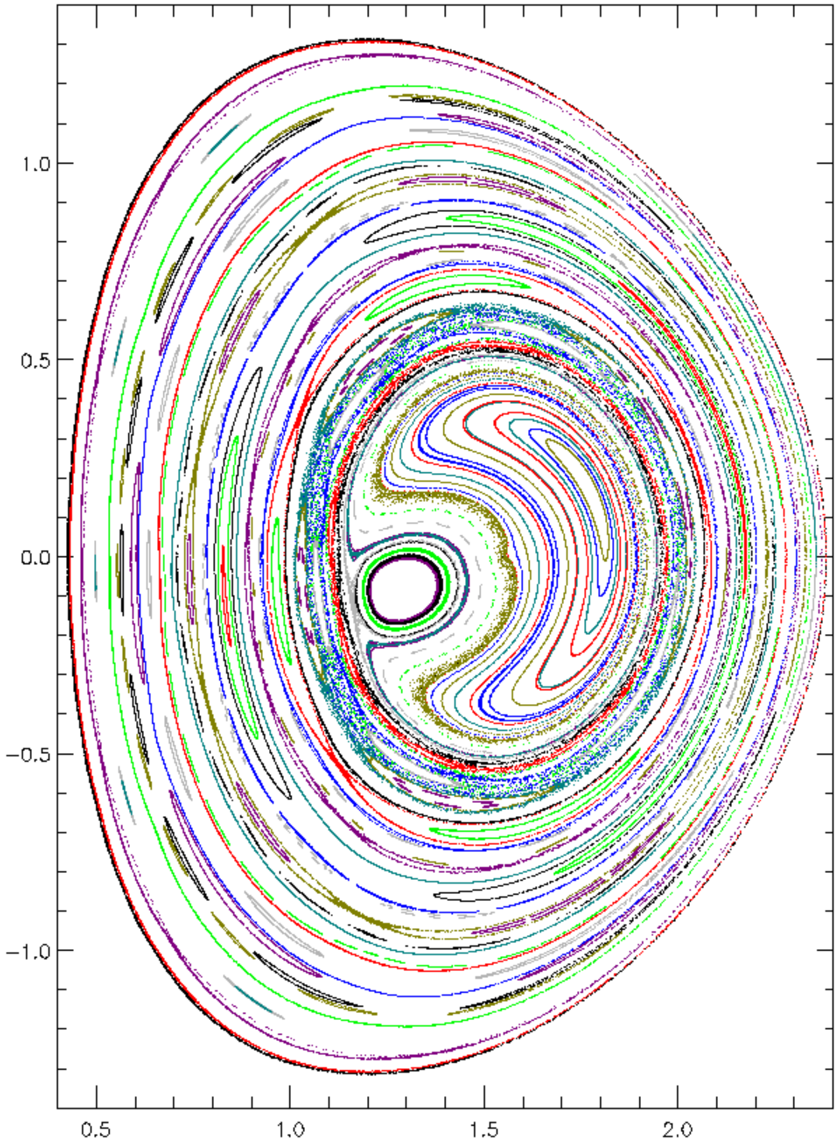
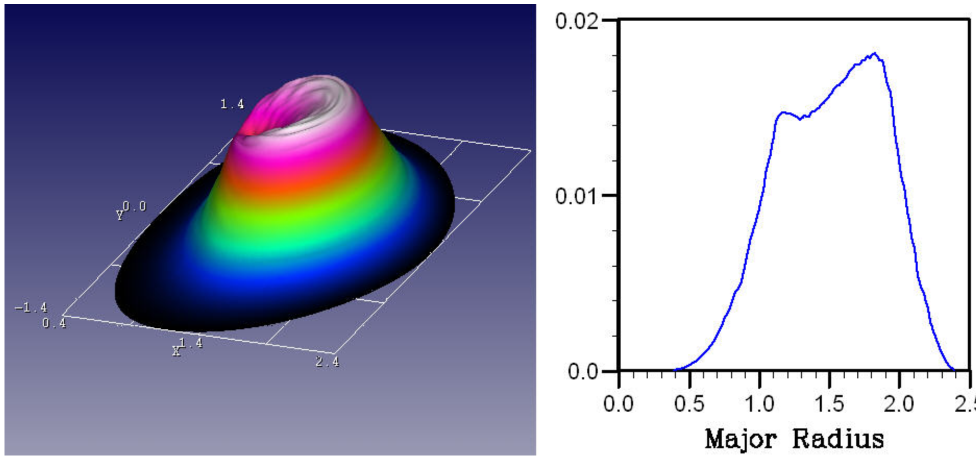


Second Crash: t = 1502.53

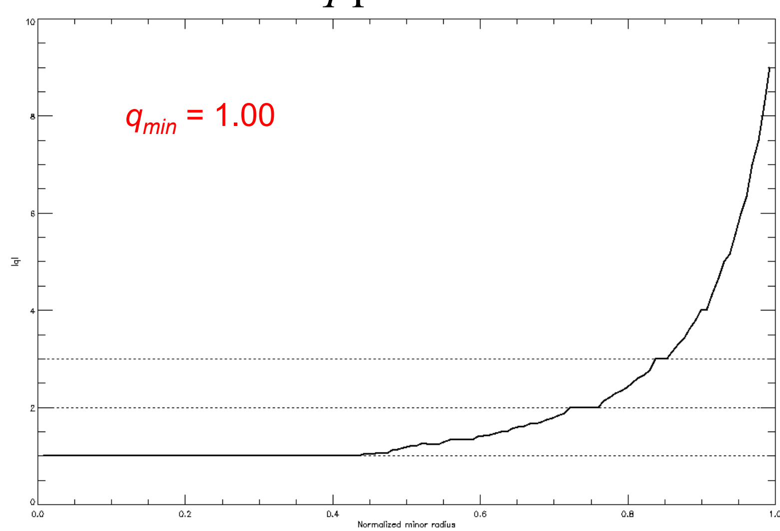
Poincaré plot



Temperature profile



q profile



Summary of Observed Two-Fluid Effects

- Plasma rotation.
- Oscillations in energy of higher-n modes.
- Sawtooth period increases slightly.
- Magnetic field does not become stochastic over most of plasma cross-section.
- Reconnection is incomplete in second crash.

Conclusions

- Nonlinear MHD simulation with actual device parameters is capable of tracking evolution through repeated sawtooth reconnection cycles.

- Two-fluid simulation with ω^*_i term on can now also be applied to the problem.

  - Qualitatively similar to MHD predictions, <u>but</u>

  - Sheared rotation inhibits island growth, reducing stochasticity of field.

  - Incomplete reconnection in second cycle suggests possibility of saturated islands.