Landau-Fluid Closures and Continuum Kinetic Algorithms

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A Simple Phase-Mixing Paradigm

Consider simple 1-D kinetic Eq. for $f(z,v_{||},t)$

(Carl Oberman reminded me of this view of Landau damping)

$$\frac{\partial f}{\partial t} + \mathrm{v}_{||}\frac{\partial f}{\partial z} = 0$$

(solutions of this simple equation are Green's functions for more complicated problems that could include E fields etc. on RHS)

Exact solution: $f\left(z,\mathrm{v}_{||},t\right) = f_0\left(z-\mathrm{v}_{||}t,\mathrm{v}_{||}\right)$

Start with Maxwellian with spatial density perturbation: $f_0 = e^{ik_{||}z} f_M(\mathrm{v}_{||})$

$$f \propto e^{ik_{||}(z-\mathrm{v}_{||}t)} n_0 e^{-\mathrm{v}_{||}^2/(2\mathrm{v}_t^2)}$$

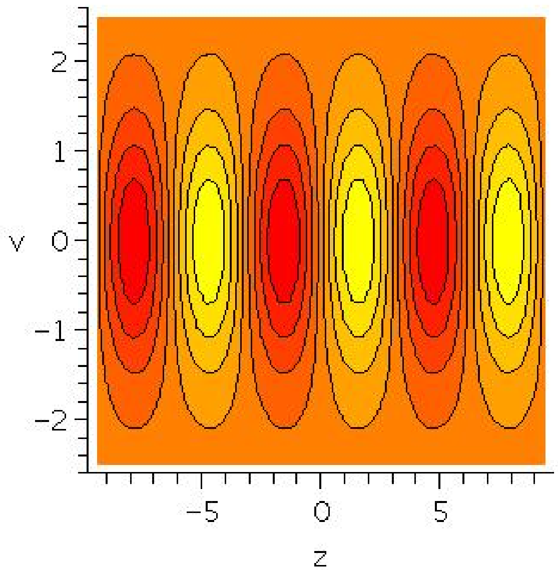
At any fixed $\mathrm{v}_{||}$, f oscillates in time with $\omega = k\,\mathrm{v}_{||}$ and no damping. However, any v-integral of f will exponentially decay in time:

$$n(z,t) = \int d\mathrm{v}_{||} f \quad \propto \; n_0 e^{ik_{||}z} \underbrace{\int d\mathrm{v}_{||}}_{mixing} \underbrace{e^{-ik_{||}\mathrm{v}_{||}t}}_{phases} e^{-\mathrm{v}_{||}^2/(2\mathrm{v}_t^2)}$$

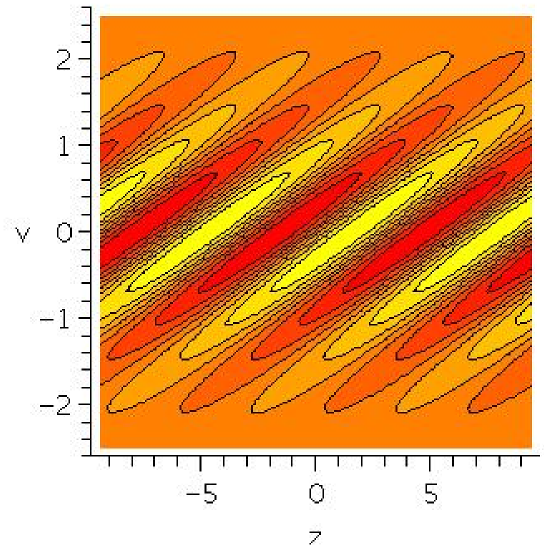
$$\boxed{n(z,t) = n_0 e^{ik_{||}z} e^{-k_{||}^2\mathrm{v}_t^2 t^2/2}}$$

Phase-mixing -> fluid moments of *f* decay in time

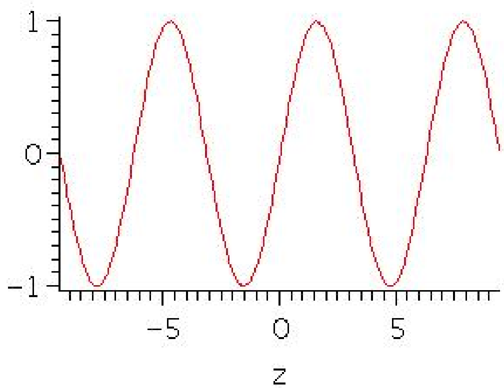
f(z,v)
t=0



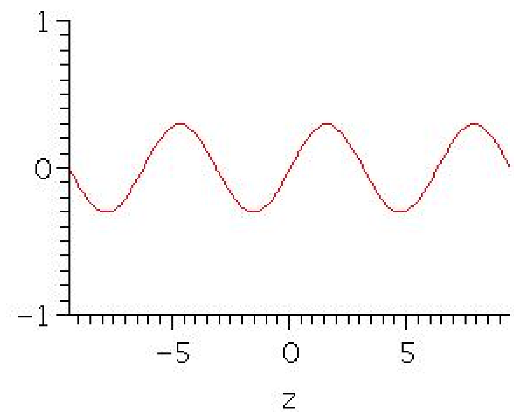
f(z,v)
t=5



n(z)
t=0



n(z)
t=1.5



Fluid closure approximations need to introduce damping at rate ~ $|k_{||}|\, v_t$

$$n(z,t) = n_0 e^{ik_{\|} z} e^{-k_{\|}^2 v_t^2 t^2/2}$$

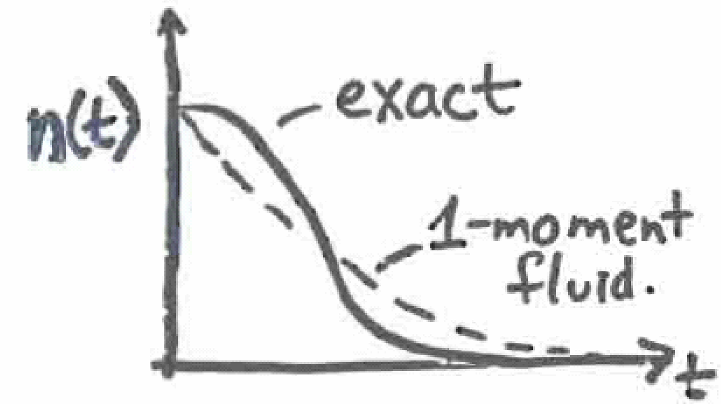
Dissipative Closure Needed to Model Collisionless Phase-Mixing

Simplest possible 1-moment fluid model (too simple for most purposes). Start with exact density conservation Eq.:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nu) = 0$$

Closure approximation for higher moment (*nu*) in terms of lower moment (*n*), with diffusion to model phase-mixing damping

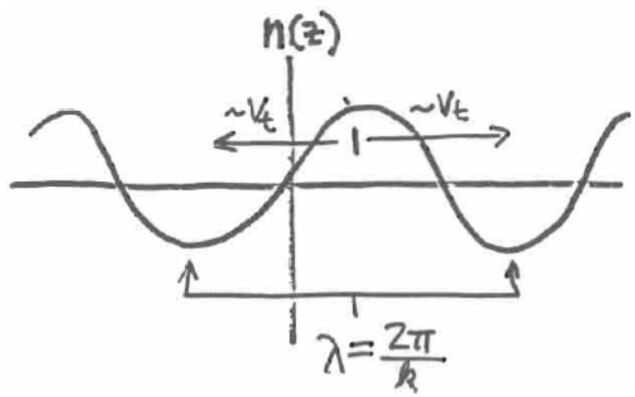
$$nu \approx -D\frac{\partial n}{\partial z} \quad \rightarrow \quad \frac{\partial n_k}{\partial t} + Dk^2 n_k = 0$$



Fluid model: $n_k(t) = \exp\left(-Dk^2 t\right)$

Exact kinetic: $= \exp\left(-k^2 \mathrm{v}_t^2 t^2 / 2\right)$

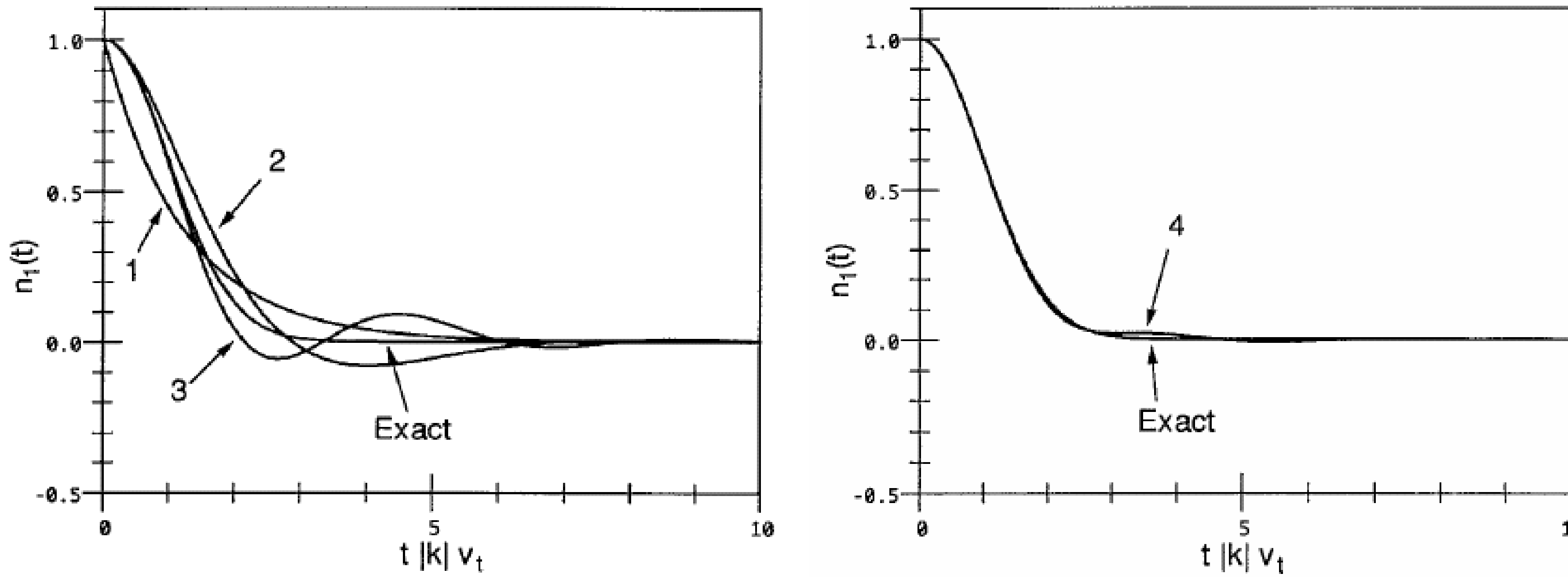
$$\nu \sim |k_\parallel| \mathrm{v}_t \sim Dk^2$$



$$D \sim \frac{\mathrm{v}_t}{|k|}$$

D is an integral operator in real space

Approximation improves by keeping more fluid moments before making closure



Closure approximation introduced for $\int dv\ f(v)\ v^m$, for m = 1-4.

Hammett, Dorland, Perkins, Phys. Fluids B4, 2052 (1992)

Real-space form of heat conduction integral

Closure in k space incl. collisions: $$q_{\|} = -n\mathrm{v}_t \frac{\mathrm{v}_t^2}{\mathrm{v}_t |\mathrm{k}_{\|}| + \nu} ik_{\|} T_{\|}$$

(recovers Braginskii
In collisional limit)

Fourier-transform, get a non-local heat conduction integral along magnetic field lines:

$$q_{\|}(z) = -n_0 \mathrm{v}_t \int_0^{\infty} dz' \frac{T(z+z') - T(z-z')}{z'} \frac{1}{1 + z'^2 / \lambda_{mfp}^2}$$

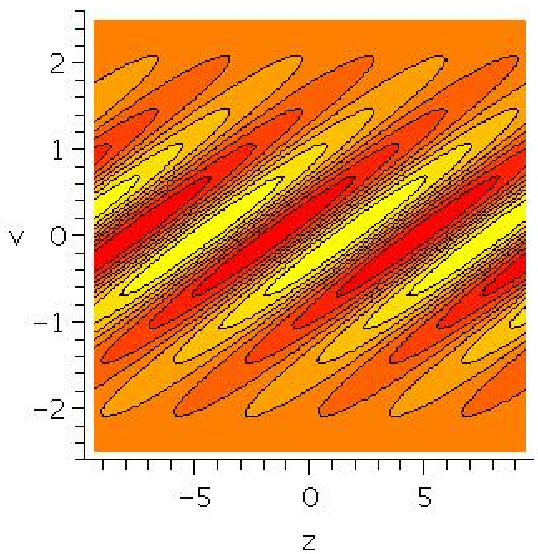
(incl. collisions, in Snyder, Hammett, Dorland, Phys. Plasmas 1997)

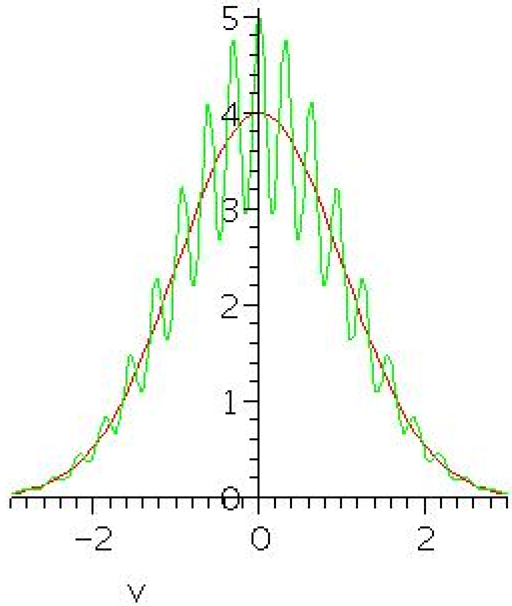
Non-locality means $-q_{\|}(z)dT/dz>0$ not guaranteed everywhere,
but can show that total entropy S satisfies $dS/dt>0$

Landau-fluid closure approximations originally derived for small-amplitude turbulence in core of fusion devices with stiff magnetic field: fast evaluation using FFTs. For edge turbulence, and for astrophysical applications, could benefit from nonlinear extensions (work in progress, at least some nonlinear improvements look feasible...), & need to integrate along fluctuating magnetic fields.

Phase-mixing -> very fine scales in velocity easily wiped out by a small amount of collisions

f(z,v)
t=5





At late times, $f = exp(- i\, k_{||}\, v\, t\,)\, f_M(v)$ is very oscillatory in v

Collisions dominate at time $\tau \sim (\, 3\, /\, \nu\, v_t^2\, k_{||}^2)^{1/3}$

$$C(f) \approx \nu\, \mathrm{v}_\mathrm{t}^2 \frac{\partial^2 f}{\partial \mathrm{v}^2} \approx -\nu\, \mathrm{v}_\mathrm{t}^2 k_{||}^2 t^2 f$$

Full resolution in velocity requires:

$$\Delta \mathrm{v}_{||} / \mathrm{v}_\mathrm{t} \sim \left(\nu / 3k_{||}\mathrm{v}_\mathrm{t}\right)^{1/3}$$
$$\sim \left(\nu_* / 3\right)^{1/3} (a / R)^{1/2} \sim 0.08$$

($k_{||} \sim 1/(qR)$ ITER $\nu_* \sim 0.008$)

Low collisionality dynamics can be simulated on an even coarser velocity grid using hypercollisions & hyperdiffusion, to damp small velocity and spatial scales

Caveats/Motivation Re: Landau-Fluid Closures

Can be very useful to extend fluid codes to model kinetic effects fairly quickly, but important to remember that:

- Landau-fluid closure approximations are ***approximations***…

  There are some cases where Landau-fluid approximations don't work well (or would require keeping many fluid moments before introducing closures in order to converge). (plasma echoes, quasilinear flattening near narrow resonances, Mattor et al. 92)

- But works well as a model of rate at which *f* phase-mixes to small velocity scales which can then be ignored. Usually there are many processes that quickly wipe out small velocity scales (weak collisions) or cascade them to small spatial scales and make them incoherent (turbulence, classicial diffusion).

  (Philisophically similar to subgrid turbulence models or hyperviscosity used in CFD, or to the EDQNM simplification of the DIA.)

- Fluid moment Eqs. express important nonlinear conservation laws (particles, momentum, magnetic moment, parallel energy), closures allow coupling to Landau damped modes

Caveats Re: Landau-Fluid Closures

- 2 main challenges:
  - If small scale features in f(v) become important (weak turbulence, particle trapping wave, Mattor et al.)
  - Multidimensional extensions in the presence of recurring orbits

- Difficult to find a single set of closure approximations that work well both for $\omega > v_t/qR,\ \nu$ (high k ITG/drift microturbulence) and $\omega < v_t/qR$ (low k NTM regime, where bounce-averaging can be done). Nonlinear interaction betweenNTM's and microturbulence modifying pressure gradients across NTM island is critical…

1998 Landau-fluid closure improves treatment of Rosenbluth-Hinton flows

- Closures in 2-D harder than 1-D: even when

$$v_{||}\, df/dz \neq 0 \text{ and } v_{drift} \cdot \nabla f \neq 0$$

may have

$$v_{||}\, df/dz + v_{drift} \cdot \nabla f = 0$$

and no more damping.

- Improved Landau-fluid closure (Beer and Hammett, Varenna Conf. Proc. 98, http://w3.pppl.gov/~hammett/papers/) models this by replacing damping terms of the form

$$-|k_{||}| v_t\, q_{||} \;\text{--->}\; -|k_{||}| v_t\, (q_{||} - q_{||0})$$

where

$$q_{||0} \propto dT/dr$$

is the expected parallel heat flow in equilibrium (the heat-flow equivalent to the Pfirsch-Schluter flow $u_{||0} \propto dp/dr$).

- Improves gyrofluid treatment of the Rosenbluth-Hinton undamped component of the zonal flow, accounts for about half of GK/GF diffs.

Kulsrud / Kruskal-Oberman/ Chew-Goldberger-Low Kinetic MHD

Kulsrud, Handbook of Plasma Physics (1983)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \left(\mathbf{V} \cdot \nabla \right) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F_g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}),$$

$$\mathbf{P} = p_\perp \mathbf{I} + \left(p_\| - p_\perp \right) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

$$\frac{\partial f}{\partial t} + \left(v_\| \hat{\mathbf{b}} + \mathbf{v}_E \right) \cdot \nabla f + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{v}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \nabla B + \frac{e}{m}(E_\| + F_{g\|}/e) \right) \frac{\partial f}{\partial v_\|} = C(f),$$

Drift-kinetic equation: similar to gyro-kinetic equation but without FLR, and includes compressional Alfven wave / fast-wave

Mirror force hidden in CGL pressure tensor

$$\mathbf{P} = p_{\parallel}\hat{b}\hat{b} + p_{\perp}\left(\mathbf{1} - \hat{b}\hat{b}\right)$$

- Parallel component of force balance (stationary u=0 equil.):

$$0 = -\left[\nabla \cdot \mathbf{P}\right] \cdot \hat{b} = -\nabla_{\parallel} p_{\parallel} - (p_{\perp} - p_{\parallel}) \frac{\nabla_{\parallel} B}{B}$$

$p_{\perp} - p_{\parallel} > 0$ corresponds to particles trapped in magnetic well, $\nabla_{\parallel} p_{\parallel} \neq 0$

i.d.:
$$\begin{aligned} \nabla \cdot \left(\hat{b}\hat{b}\right) \cdot \hat{b} &= \nabla \cdot \hat{b} + \left(\hat{b} \cdot \nabla \hat{b}\right) \cdot \hat{b} \\ &= \nabla \cdot \hat{b} + \hat{b} \cdot \nabla\left(\tfrac{1}{2}\left|\hat{b}\right|^{2}\right) \\ &= -\frac{\hat{b} \cdot \nabla B}{B} \end{aligned}$$

- Note: mirror force independent of magnitude of B, important for arbitrarily weak B!

Evolution of the Pressure Tensor

(Snyder, Hammett, Dorland, Phys. Plasmas 1997)

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\perp) - q_\perp \nabla\cdot\hat{\mathbf{b}}$$

adiabatic invariance of $\mu \propto m v_\perp^2/B \sim T_\perp/B$

$$\frac{\rho^3}{B^2}\frac{d}{dt}\left(\frac{p_\| B^2}{\rho^3}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\|) + 2 q_\perp \nabla\cdot\hat{\mathbf{b}},$$

$$\frac{p_\| B^2}{\rho^3} \propto \frac{T_\| B^2}{\rho^2}$$

Only parallel compression affects $T_\|$

$q_\perp = q_\| = 0$ CGL or Double Adiabatic Theory

$$q_\| = -n \mathrm{v}_t \frac{\mathrm{v}_t^2}{\mathrm{v}_t |\mathrm{k}_\|| + \nu} \nabla_\| T_\|$$

Closure Models for heat flux (temp. gradients wiped out on ~ a crossing time) => multipole approx. to Landau damping.

recovers Braginskii/Chapman-Enskog in large collision frequency ν limit

Similarities w/ Chang-Callen Closure Approach

- Both use nonlinear fluid equations for lower-order moments of f (insures preservation of key conservation laws), with linear closure approximations for higher-order moments.
- Chang-Callen derive exact linear closure relations for higher-order moments, which depend on Z-functions. In the low-frequency limit their closure reduces to ours (if the same fluid moments were kept before closing).
- We have usually kept more moments before closing, keeping $(n, u_{||}, p_{||}, p_{\perp}, q_{||}, q_{\perp})$, important for higher-frequency drift-wave dynamics, $\omega > v_t/qR,\ \nu$
- while Callen, Held et al. typically keep (n, u, p) and have a closure approx. for the anisotropic pressure tensor, etc. For NTM ordering, $\omega < \nu < v_t/qR$, can bounce-average.

Fully kinetic/gyrokinetic simulations more rigorous than fluid approach & becoming very powerful, but continued interest in using Landau-fluid closure approximations

In fusion research, edge turbulence is high priority and very challenging. Critical problem needs multiple codes to attack it and cross-check each other.

Must span both collisional and moderately collisionless plasmas, and wide range of time and space scales.

Extended fluid approach would allow higher resolution and/or faster simulations. [However, apparent speed advantage over kinetic simulations reduced by need to evaluate non-local heat integral, and the fact that kinetic simulations of core turbulence have found they can converge with relatively few velocity grid points (~10 energies, ~20 pitch angles) using high-order velocity integration and other advanced algorithms.]

Caveats: Landau-fluid closures are approximations & are inaccurate in some regimes unless many fluid moments are kept. best for strong-turbulence regimes where instabilities are basically in a fluid-like regime & nonlinearly couple to Landau-damped modes. Weak-turbulence regimes harder. Several papers on limitations of Landau-fluid approx. and extensions, e.g. to neoclassical effects…

Form of pressure equations used (avoid divide by B):

$$\frac{\partial p_{\parallel_s}}{\partial t} + \nabla \cdot (\mathbf{U} p_{\parallel_s}) + \nabla \cdot (\hat{\mathbf{b}} q_{\parallel_s}) + 2 p_{\parallel_s} \hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}} - 2 q_{\perp_s} \nabla \cdot \hat{\mathbf{b}}$$

$$= -\tfrac{2}{3} \nu_s (p_{\parallel_s} - p_{\perp_s}),$$

$$\frac{\partial p_{\perp_s}}{\partial t} + \nabla \cdot (\mathbf{U} p_{\perp_s}) + \nabla \cdot (\hat{\mathbf{b}} q_{\perp_s}) + p_{\perp_s} \nabla \cdot \mathbf{U} - p_{\perp_s} \hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}}$$

$$+ q_{\perp_s} \nabla \cdot \hat{\mathbf{b}} = -\tfrac{1}{3} \nu_s (p_{\perp_s} - p_{\parallel_s}),$$

Alt. form: average $p=(2p_\perp + p_\parallel)/3$ and
Pressure difference $\delta p = p_\parallel - p_\perp$

$$\mathbf{P} = p\mathbf{1} + \mathbf{\Pi} = p\mathbf{1} + \delta p(3\hat{b}\hat{b} - \mathbf{1})/3$$

$$\frac{dp_s}{dt} + \frac{5}{3}p_s \nabla \cdot \mathbf{U} = -\frac{2}{3}\nabla \cdot (\hat{\mathbf{b}} q_s) - \frac{2}{3}\mathbf{\Pi_s} : \nabla \mathbf{U},$$

$$\frac{d\,\delta p_s}{dt} + \frac{5}{3}\delta p_s \nabla \cdot \mathbf{U} + \mathbf{\Pi_s} : \nabla \mathbf{U} + 3p_s \hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}} - p_s \nabla \cdot \mathbf{U}$$

$$-3q_{\perp_s} \nabla \cdot \mathbf{U} + \nabla \cdot [\hat{\mathbf{b}}(q_{\parallel_s} - q_{\perp_s})] = -\nu_s \delta p_s\,,$$

High collision frequency limit:

$$\delta p_{1s} = -\frac{p_{0s}}{\nu_s}(3\hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}} - \nabla \cdot \mathbf{U}).$$

Agrees well with Braginskii's anisotropic viscosity (5-25% diffs).

Scaling of PIC & Continuum Integration Errors

Rosenbluth & Metropolis Monte Carlo method ideal for high-dimensional integration:

$$\text{Error} \sim \frac{C}{N^{1/2}}$$

independent of dimensionality (*N* = number of samples). (Caution: *C* highly dependent on problem/algorithm. Otherwise could simulate whole universe with 1% accuracy with only 10,000 particles.)

Continuum / Vlasov / Eulerian algorithm with $N = N_1^d$ total grid points and piece-wise parabolic / WENO 3rd order accuracy:

$$\text{Error} \sim \frac{1}{N_1^3} \sim \frac{1}{N^{3/d}}$$

Continuum competitive for dimensionality *d* <= *6*

Scaling of PIC & Continuum Integration Errors (II)

For nonlinear plasma problems, spatial resolution required to resolve structures set by physics and must be comparable for PIC and continuum algorithms.
2-D velocity integrals needed to calc currents and fields:

$$j_{\|}(\vec{x}) \propto \int d\,\mathrm{v}_{\|} \int d\mu\, B\, \mathrm{v}_{\|} f(\vec{x}, \mathrm{v}_{\|}, \mu)$$

$$\text{3rd order continuum error} \sim \frac{1}{(\#\text{ vel. space grid points})^{3/2}}$$

$$\text{PIC error} \sim \frac{1}{(\#\text{ particles per smoothing volume})^{1/2}}$$

PIC & Continuum converge in different ways: Velocity space sampling improves for long wavelength modes with PIC, independent of k for continuum. PIC algorithms easier to write for complicated systems.

GS2 & GYRO use high-order Gauss-Legendre integration, spectral-like convergence:
N_1 grid points in 1 dimension --> error ~ $1/N_1^{N_1}$ (theoretical scaling only for smooth solutions, not achieved in practice).

Continuum/Eulerian Approach to Electromagnetic Gyrokinetic Turbulence

GS2 (Dorland & Kotschenreuther), GENE (Jenko), and GYRO (Candy & Waltz) have demonstrated that direct Eulerian simulations of microturbulence using the 5-D electromagnetic gyrokinetic equations can be effective, by

(1) Using modern massively parallel supercomputers and clusters, and

(2) Using modern advanced algorithms, including

- implicit / semi-implicit methods (or carefully designed explicit methods)

- pseudo-spectral and/or Arakawa treatment of nonlinearities (preserves all 3 conservation properties of Poisson bracket nonlinearities)

- pseudo-spectral and/or high-order upwind advection algorithms: very low dissipation at long wavelengths, effective sink at small scales.

- high-order velocity-space integration algorithms,

- efficient field-aligned coordinate systems, …

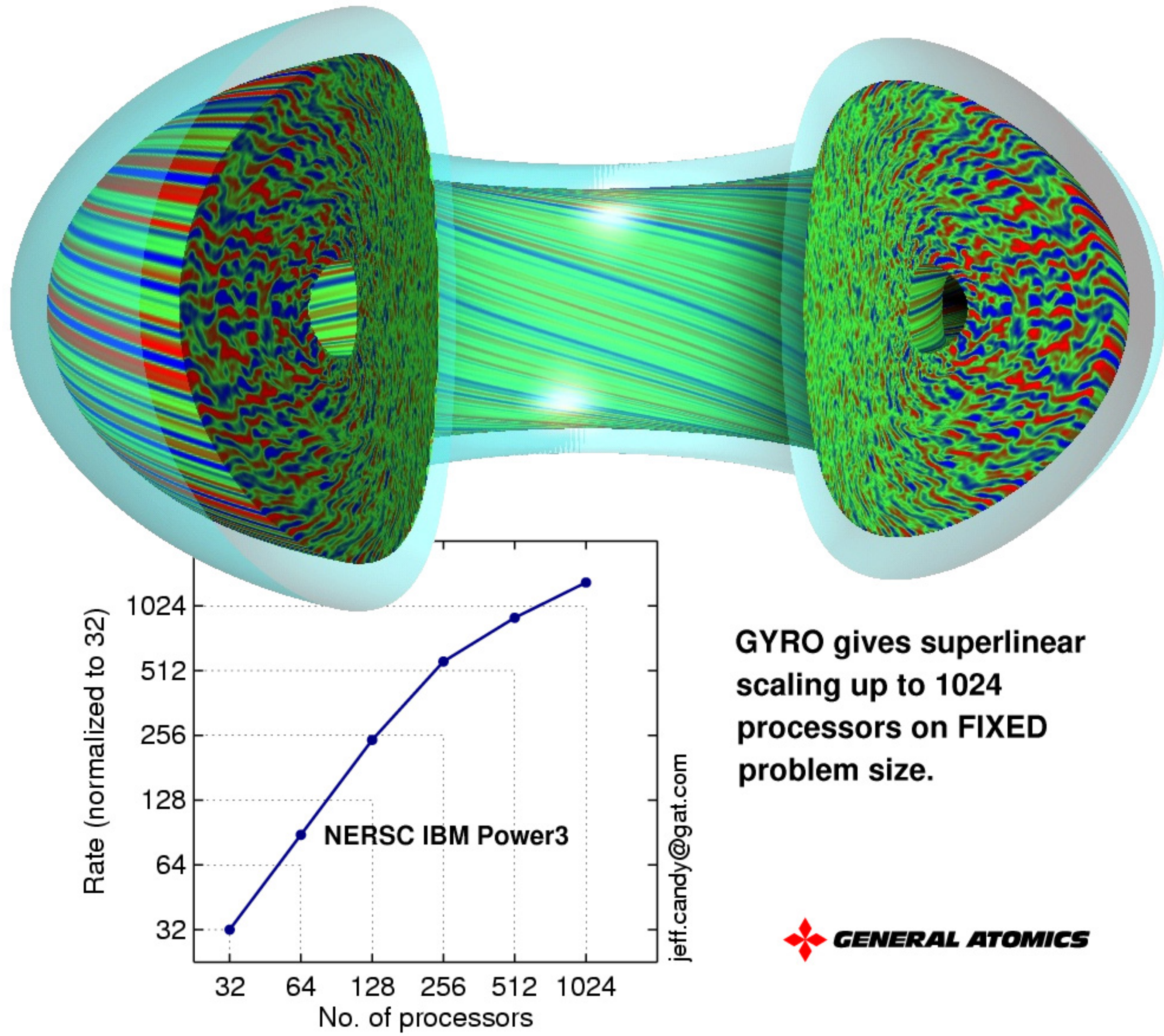
Typical Resolution for Continuum Gyrokinetics

Typical moderate-resolution parameters for GYRO:

$$
\begin{array}{cccccc}
f\,(r, & n_\phi, & \tau_{\|}, & E, & \lambda\,) & \\
140\ \mathrm{x} & 32\ \mathrm{x} & 12\ \mathrm{x} & 8\ \mathrm{x} & 16 & \mathrm{x}\ 2\ \text{species}
\end{array}
$$

Much larger simulations also done.

Extend continuum approach some day to full-torus kinetic-MHD?



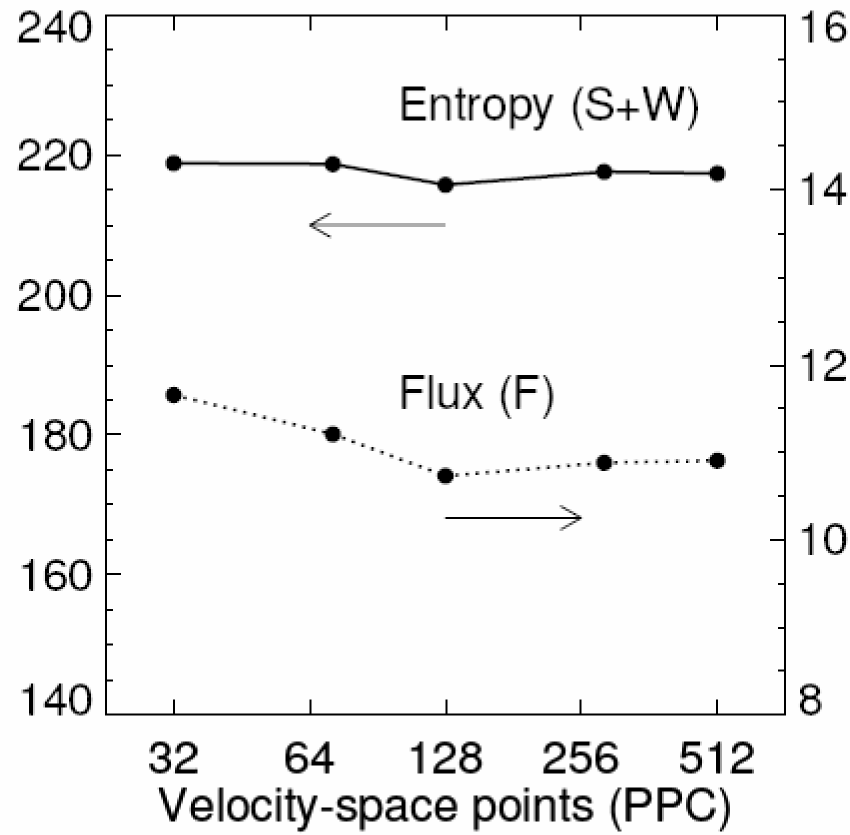
Comprehensive 5-D computer simulations of core plasma turbulence developed by Plasma Microturbulence Project. Candy & Waltz (GA) movies shown: d3d.n16.2x_0.6_fly.mpg & supercyclone.mpg, from http://fusion.gat.com/comp/parallel/gyro_gallery.html (also at http://w3.pppl.gov/~hammett/refs/2004).

GYRO well-converged w/ velocity resolution

Fluctuations in *f* cascade to small spatial scales where dissipation needed for steady state is provided by hyperdiffusion from with high-order upwind algorithms.

Fluctuations in *f* also cascade to small velocity scales where collisions (or hypercollisions) are also a sink.

(Hyperdiffusion was more important in this low ν case.)



Note suppressed zeros!

Candy & Waltz, 2005, GA-A25149 “Velocity-space resolution, entropy production & upwind dissipation in Eulerian gyrokinetic simulations”
http://fusion.gat.com/comp/parallel/gyro_publications.html

Phase-mixing -> very fine scales in velocity easily wiped out by a small amount of collisions

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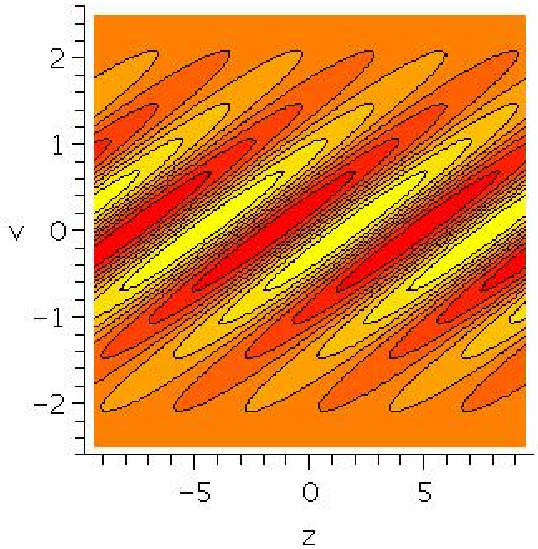

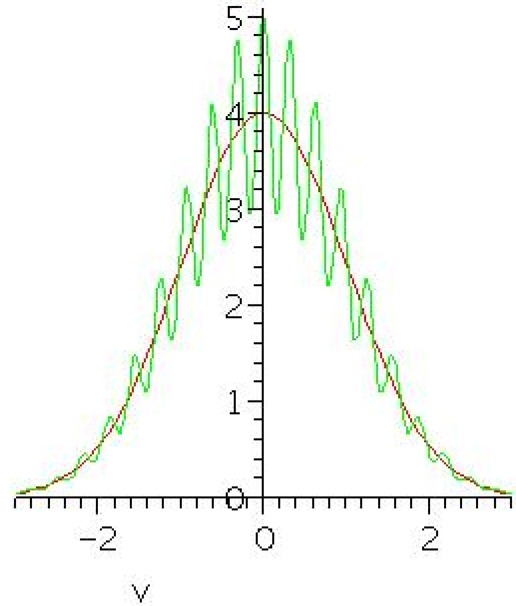

At late times, $f = exp(- i\, k_{||}\, v\, t\,)\, f_M(v)$ is very oscillatory in v

Collisions dominate at time $\tau \sim (3 / \nu\, v_t^2\, k_{||}^2)^{1/3}$

$$C(f) \approx \nu\, \mathrm{v}_\mathrm{t}^2 \frac{\partial^2 f}{\partial \mathrm{v}^2} \approx -\nu\, \mathrm{v}_\mathrm{t}^2 k_{||}^2 t^2 f$$

Full resolution in velocity appears to require:

$$\Delta \mathrm{v}_{||} / \mathrm{v}_\mathrm{t} \sim \left(\nu / 3 k_{||} \mathrm{v}_\mathrm{t}\right)^{1/3}$$
$$\sim \left(\nu_* / 3\right)^{1/3} (a / R)^{1/2} \sim 0.08$$

($k_{||} \sim 1/(qR)$ ITER $\nu_* \sim 0.008$)

Low collisionality dynamics can be simulated on an even coarser velocity grid using hypercollisions & hyperdiffusion, to damp small velocity and spatial scales

Gyro-Fluid vs. Fully Gyro-Kinetic

- I began work on Landau-fluid closure approximations in 1989. In 2010, computers will be 16,000 times more powerful. Equivalent to increasing resolution by x 5 in each of 6 dimensions (incl. time).

- Gyro-Landau-fluid equations involves 6 pieces of velocity space information per species (n, $u_{||}$, $p_{||}$, $p_{\perp}$, $q_{||}$, $q_{\perp}$) and requires a nonlocal heat-flux integral along perturbed magnetic field lines for full accuracy.
  (early electrostatic gyrofluids did this quickly with FFTs along fixed field lines)

- Direct gyrokinetic simulation is a local calculation. With 64 velocity grid points it isn't that much more expensive. Hypercollisions/hyperdiffusion gets reasonable answers on a coarse mesh. Gyrofluid-like speed on coarse velocity grids, and can check convergence by occasionally increasing resolution.

- Issues for a fully global code: For sufficient accuracy with coarse mesh, need energy grid scaled to radially (& time?) varying temperature? Other issues ??

- There are some problems of interest for which we need extremely high spatial resolution, for which the reduced velocity-space treatment of gyrofluid equations is useful

- These are very hard problems: always useful to have a range of algorithms to cross-compare with each other. Different algorithms have different strengths and weaknesses, converge in different ways.

References

General Landau-fluid closure approximation technique: Hammett & Perkins (1990), PRL 64, 3019. Hammett, Dorland, & Perkins (1992) Phys. Fluids B4, 2052 (much more tutorial than short PRL).

Independent derivation: Chang & Callen (1992), Phys. Fluids B4, 1167.

Hammett, Beer, Dorland, Cowley, & Smith (1993) Plasma Phys. & Contr. Fus. 35, 973

Mattor (1992) Phys. Fluids B4, 3952

Kinetic MHD / Landau-MHD: Snyder, Hammett, & Dorland (1997), Phys. Plasmas 4, 3974

Parker & Carati (1995) PRL

Dimits et al. (2000) Phys. Fluids 7, 969.