Quiet Start

A. H. Glasser, PSI Center, University of Washington
V. S. Lukin, Naval Research Laboratory

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Key Concepts

- Initial conditions: unstable equilibrium + weakly-growing perturbation, exponential growth.
- If the initial perturbation is far from an eigenfunction, it produces large-amplitude, weakly-damped waves. Like initializing weather simulation with hurricanes and tornadoes. Waste of computational effort and/or excess dissipation.
- Eigenvalues and eigenfunctions can be determined efficiently and accurately with complex PETSc + SLEPc.
- Eigenpairs provide information about spatial resolution and growth rate.
- Quiet Start: initialization with eigenfunction allows larger time steps, better spatial resolution.
- Limited by linear system (KSP) convergence. Can be improved with physics-based preconditioning and algebraic multigrid.
- Test case: GEM challenge problem, magnetic reconnection, 1D equilibrium. Include all spatial variation, dissipative terms.

Visco-Resistive MHD Equations

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v} + \pi) = \mathbf{J} \times \mathbf{B} - \nabla p$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = (\gamma - 1)\left(\eta J^2 + \pi : \mathbf{v}\mathbf{v} - \nabla \cdot \mathbf{q}\right)$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}$$

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} - \eta \mathbf{J}$$

$$\mathbf{q} = -\kappa \nabla T, \quad \pi = -\mu \nabla \mathbf{v}$$

3D Magnetic Reconnection Problem

Equilibrium

$$x \in \frac{1}{2}(-l_x, l_x), \quad y \in \frac{1}{2}(-l_y, l_y), \quad z \in \frac{1}{2}(-l_z, l_z)$$

Periodic in x and z, conducting wall in y

$$A_x = -B_0 y, \quad A_y = 0, \quad A_z = -\lambda \ln\cosh\left(\frac{y}{\lambda}\right)$$

$$B_x = \tanh\left(\frac{y}{\lambda}\right), \quad B_z = B_0$$

$$p = nT = p_0 + \mathrm{sech}^2\left(\frac{y}{\lambda}\right), \quad T = \frac{1}{2} \quad \rho v_x = \rho v_y = \rho v_z = 0$$

Parameters

$$l_x = 25.6, \quad l_y = 12.8, \quad l_z = 6.4, \quad \lambda = \frac{1}{2}, \quad p_0 = .2, \quad B_0 = 0$$

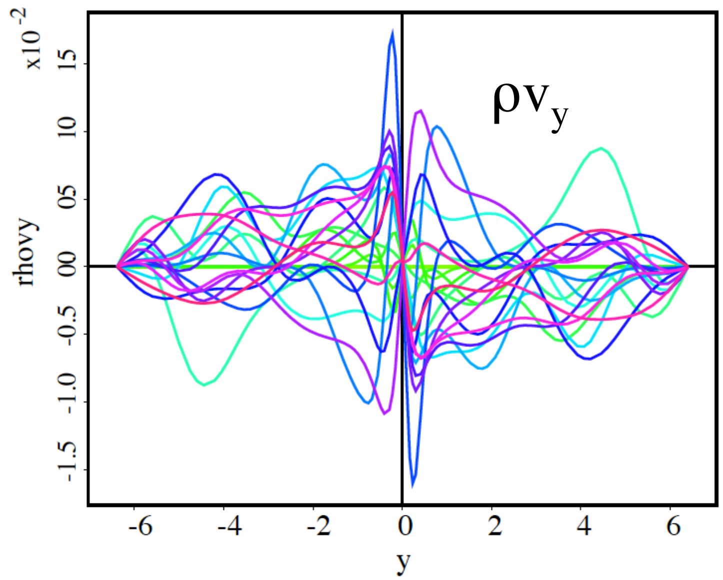
$$\eta = 10^{-3}, \quad \mu = \kappa = 10^{-2}$$

Noisy Initial Conditions

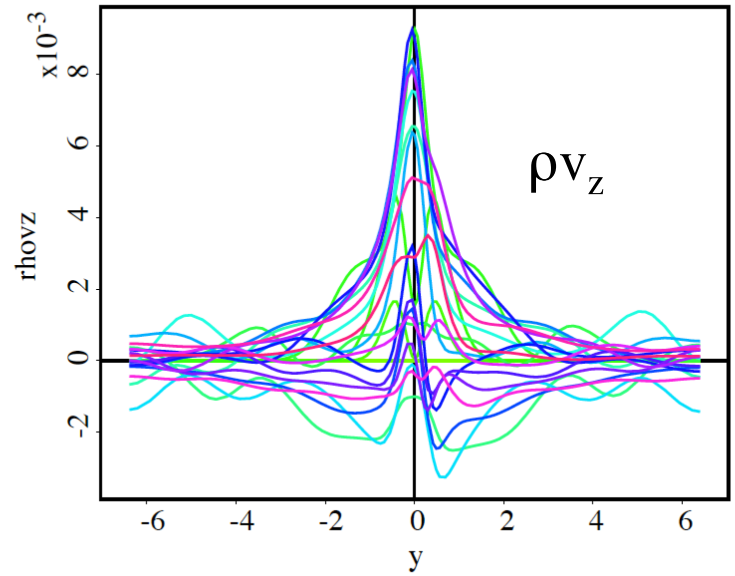
$$\tilde{A}_z = \delta \cos(k_x x)\cos(k_z z)\exp(-y^2/\lambda^2)$$

Smooth perturbation, but out of balance.

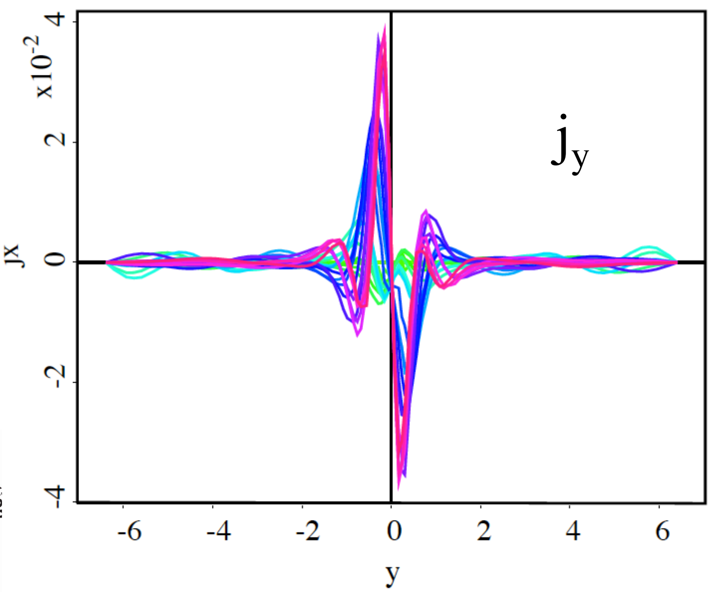
Noisy Start



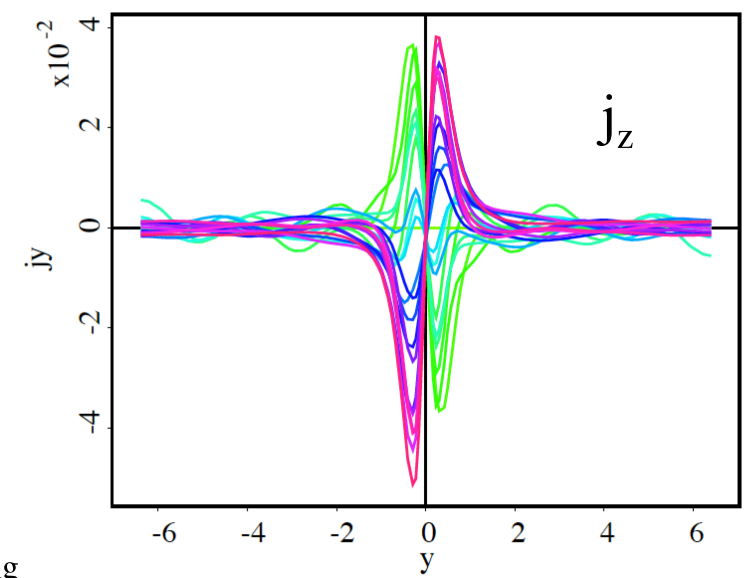
ρv_y
rhovy
x10^-2
-1.5 -1.0 -0.5 00 05 10 15
-6 -4 -2 0 2 4 6
y



ρv_z
rhovz
x10^-3
-2 0 2 4 6 8
-6 -4 -2 0 2 4 6
y



j_y
jx
x10^-2
-4 -2 0 2 4
-6 -4 -2 0 2 4 6
y



j_z
jy
x10^-2
-4 -2 0 2 4
-6 -4 -2 0 2 4 6
y

1D Generalized Eigenvalue Problem

Flux-Source Form

$$\frac{\partial u_i}{\partial t} + \nabla \cdot \mathbf{F}_i = S_i, \quad \mathbf{F}_i = \mathbf{F}_i(t, \mathbf{x}, u_j, \nabla u_j), \quad S_i = S_i(t, \mathbf{x}, u_j, \nabla u_j)$$

Galerkin Expansion, Spatial Discretization

$$u_i(\mathbf{x}, t) = u_{ij}(t)\alpha_j(\mathbf{x})$$

$$(\alpha_i, \alpha_j)\dot{u}_j = \int_\Omega d\mathbf{x}\,(S\alpha_i + \mathbf{F} \cdot \nabla\alpha_i) - \int_{\partial\Omega} \mathbf{n} \cdot \mathbf{F}\alpha_i$$

$$\mathsf{M}\dot{\mathbf{u}} = \mathbf{r}(\mathbf{u})$$

1D Static Equilibrium + Linearization

$$u_i(x, y, z, t) = u_{i0}(y) + u_{i1}(y) \exp\left[i(k_x + k_z z) + st\right]$$

$$\frac{\partial}{\partial t} \to s, \quad \nabla \to \left(ik_x, \frac{\partial}{\partial y}, ik_z\right), \quad J_{ij} = \left.\frac{\partial r_j}{\partial u_i}\right|_{u=u0}$$

Expand in 1D spectral elememts in y

Generalized 1D Eigenvalue Problem

$$\mathsf{A}\mathbf{u} = s\mathsf{B}\mathbf{u}$$

SLEPc

- Scalable Library for Eigenvalue Problem Computations
  http://www.grycap.upv.es/slepc/documentation/manual.htm

- Developed as an extension of PETSc
  by Jose Román *et al* at the University of Valencia, Spain

- Solution of large sparse eigenproblems on parallel computers.

- Advanced iterative solution procedures.

- Allows selection of a portion of the spectrum
  e.g. largest real eigenvalues

- Accurate solution of 1D complex eigenvalue problem in a few seconds
  on one processor.

Grid Packing: Equations

Grid Packing Function

$$y(\xi, \lambda) = \ln\left(\frac{1+\lambda\xi}{1-\lambda\xi}\right) \Big/ \ln\left(\frac{1+\lambda}{1-\lambda}\right)$$

$$\lim_{\lambda \to 0} y(\xi, \lambda) = \xi$$

Center and Edge Grid Densities

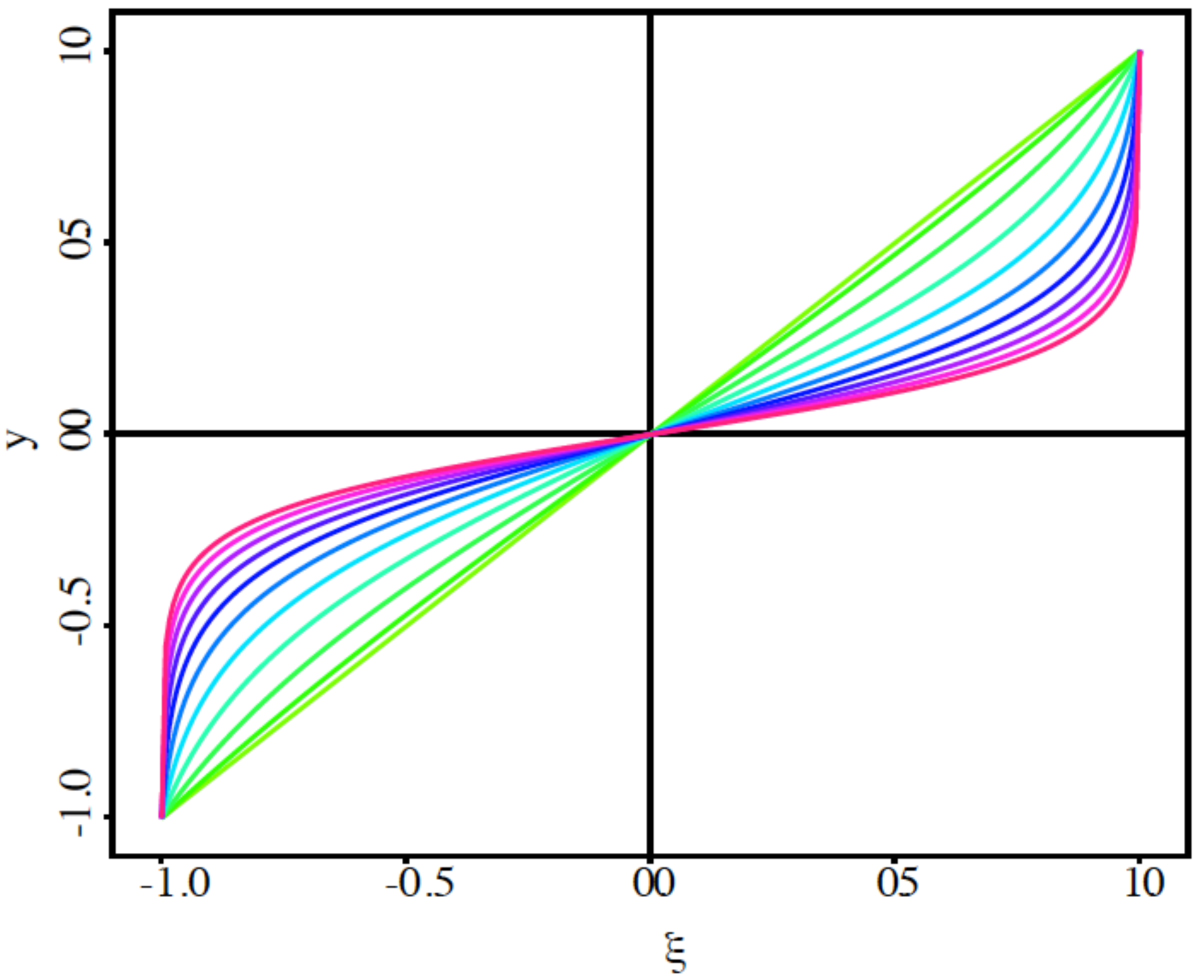
$$\frac{\partial y}{\partial \xi} = \frac{2\lambda}{1-\lambda^2\xi^2}, \quad \left.\frac{\partial y}{\partial \xi}\right|_{\xi=0} = 2\lambda, \quad \left.\frac{\partial y}{\partial \xi}\right|_{\xi=\pm 1} = \frac{2\lambda}{1-\lambda^2}$$

Packing Ratio

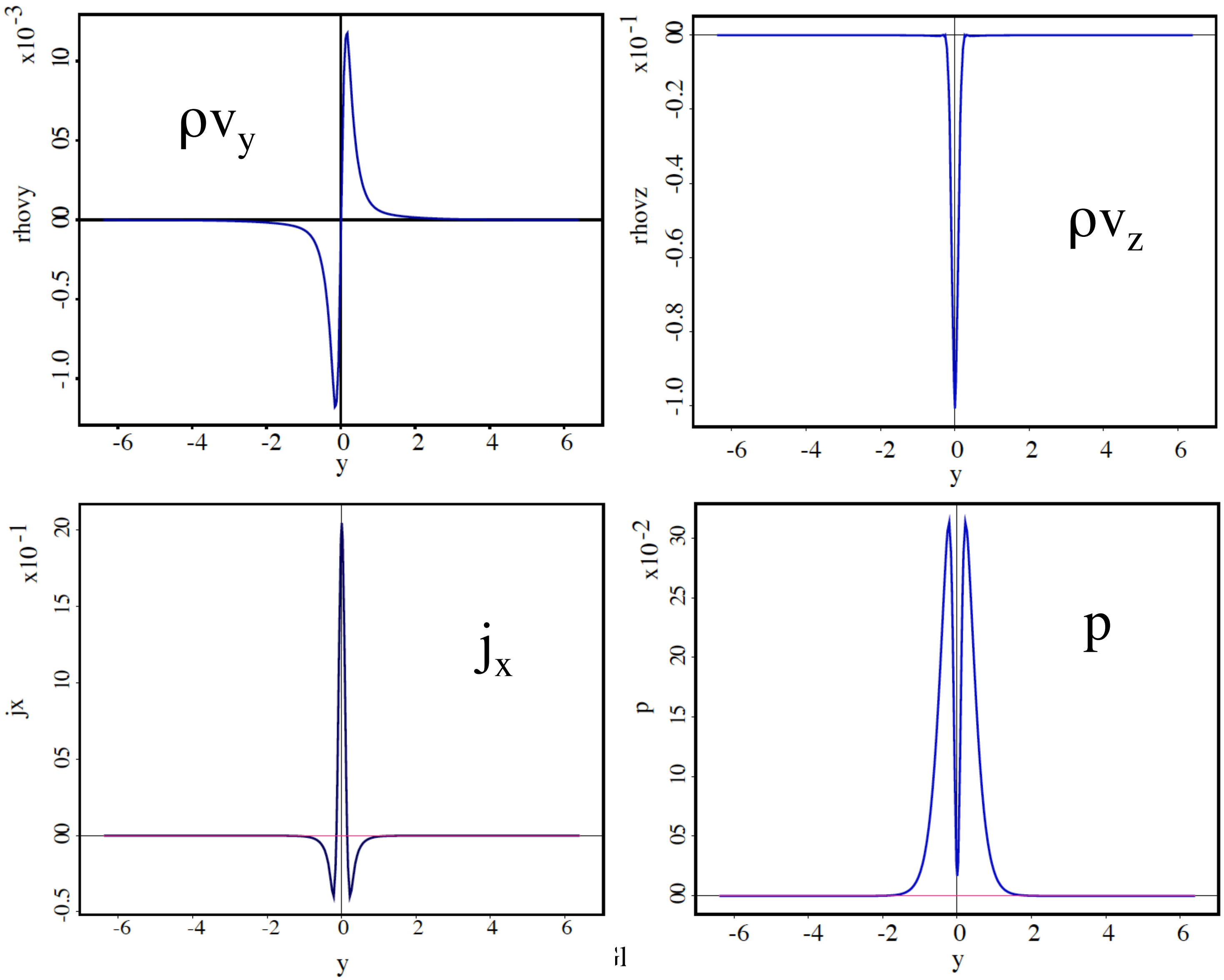
$$P(\lambda) \equiv \frac{\partial y/\partial \xi|_{\xi=0}}{\partial y/\partial \xi|_{\xi=\pm 1}} = 1 - \lambda^2$$

$$\lambda = (1-P)^{1/2}$$

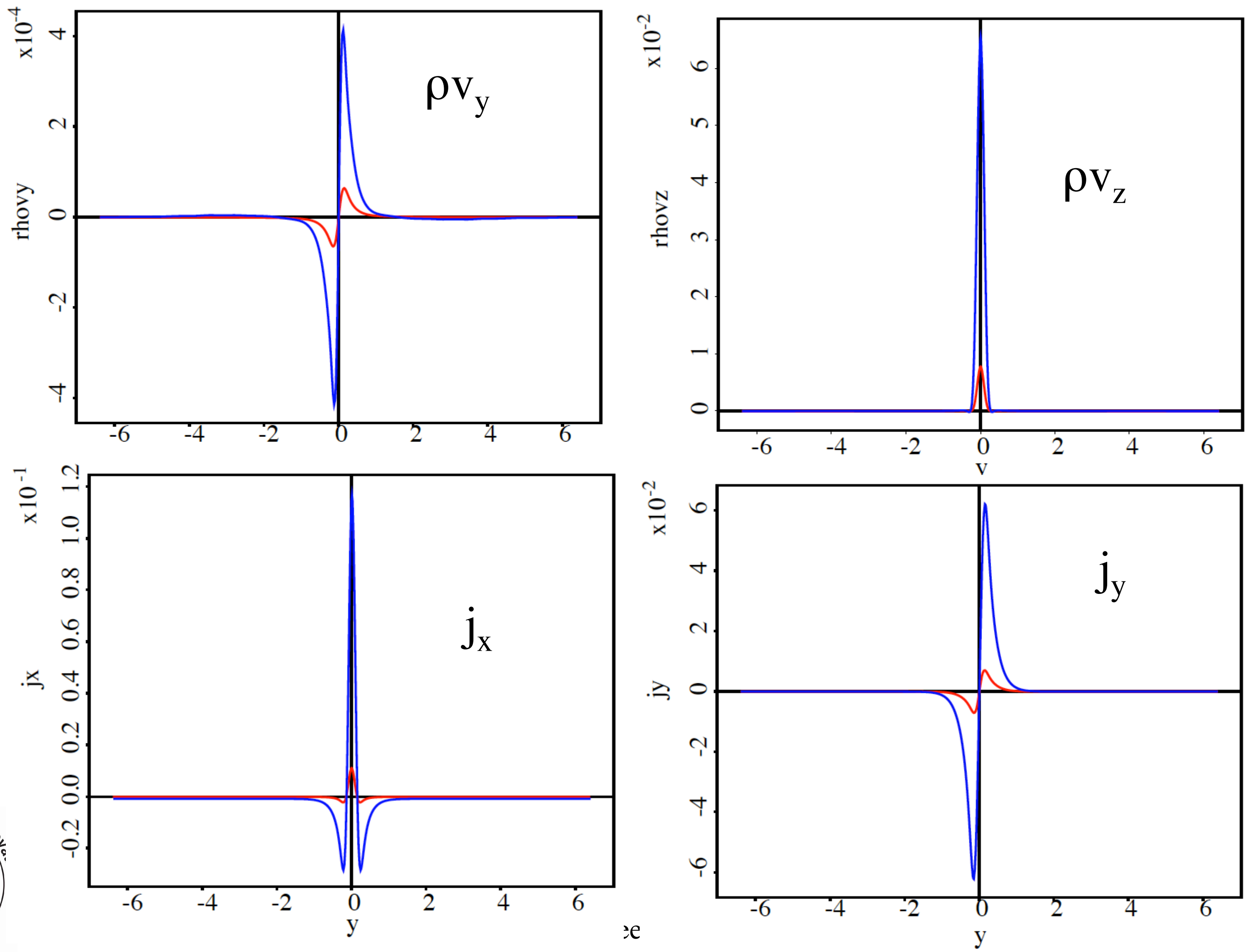
Grid Packing: Graphs



Eigenfunctions



Quiet Start



Details and Conclusions

- $\eta = 1\text{e-}3$, $\mu = \kappa = 0$
- nx = nz = 4, ny = 64, np = 4, ypack = 32: marginally resolved.
- HiFi growth rate agrees well with SLEPc value of s = 1.545e-2.
- Krylov iterations increase rapidly for time step dt > 0.2, s dt > 3e-3, too small, still in the linear regime.
- Physics-based preconditioning and algebraic multigrid should improve Krylov convergence, but there are remaining inaccuracies in the approximate Schur complement which inhibit Newton convergence. Ongoing effort.
- SLEPc is very fast and efficient for 1D equilibrium. Probably satisfactory for 2D equilibrium, probably not 3D equilibrium.