Hybrid Kinetic-MHD simulations with NIMROD

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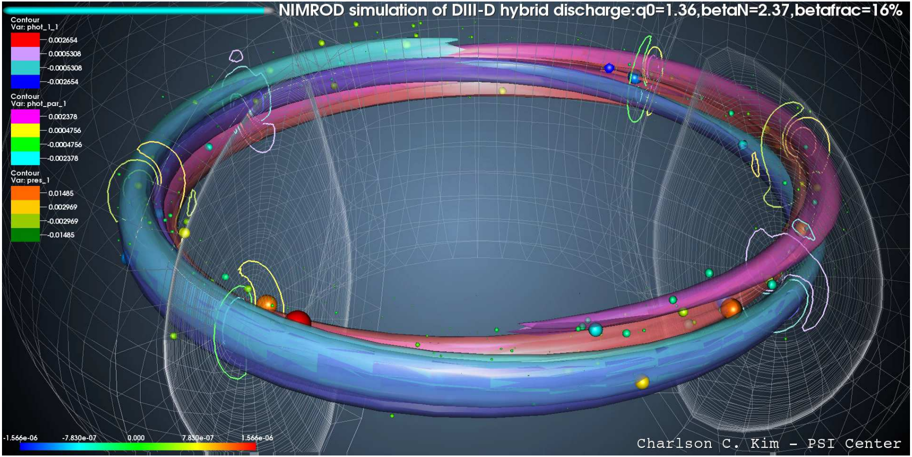
Outline

1. ITPA TAE benchmark
   - Velocity Space Diagnostics
2. $n = 1$ D3D simulations
3. KSTAR simulations

Summary of Capabilities

- tracers, linear, nonlinear
- two equations of motion
  - drift kinetic $(v_{\parallel}, \mu)$
  - Lorentz force $(\vec{\mathbf{v}})$
- multiple spatial profiles - loading in **x**
  - proportional to MHD profile, uniform, peaked gaussian
- multiple distribution functions - loading in **v**
  - slowing down distribution, Maxwellian, monoenergetic
- multispecies capability
  - HKMHD visualization uses self-consistent+tracers
  - e.g. - Maxwellian+slowing down, drift+Lorentz
- multiple coupling models
  - pressure coupling
  - current coupling

more on YouTube, search:charlsonification

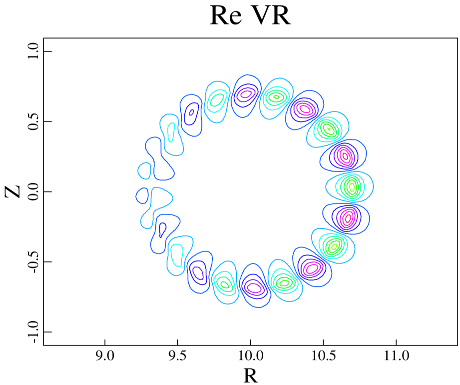


visualizations generated with **VisIt**

ITPA TAE benchmark
A. Mishchenko, PoP (2009)

- Toroidicity induced shear Alfven Eigenmodes (TAE)
  - toroidicity produces ‘gaps’ in Alfven continuum
  - discrete modes can be excited in these ‘gaps’
  - energetic particles with $\omega_{gap} \simeq k_{||} v$ can excite TAEs
- circular cross section $R = 10,\ a = 1,\ B_0 = 3$
- $q = 1.71 + .16(r/a)^2$ single ‘gap’
- bulk ions are H^+ at $n = 2 \times 10^{19}\ \ T = 1KeV$
- energetic particles are $D^+\ \ T_h = (100KeV, 800KeV)$
- $n_h = n_0 \exp\left(-\frac{\Delta}{L} \tanh\left(\frac{\rho - \rho_0}{\Delta}\right)\right)$
- $n_0 = 7.5 \times 10^{16},\ \Delta = .2,\ L = .2,\ \rho_0 = .5$
- $\omega_{gap} \simeq 4 \times 10^5$

$n = 6$ eigenmode shows good agreement



- current coupling model
- δV_{normal} shows clear $m = 10, m = 11$
- $\omega \simeq 4.1 \times 10^5$ good agreement with gap
- γ small by $\times 2$

Slowing Down Distribution for Hot Particles

- slowing down distribution function $f_{eq} = \dfrac{P_0 \exp(\frac{P_\zeta}{\psi_0})}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$
- $P_\zeta = g\rho_\parallel - \psi$ canonical toroidal momentum, ε energy, ψ_p poloidal flux, ψ_0 gradient scale length, ε_c critical energy

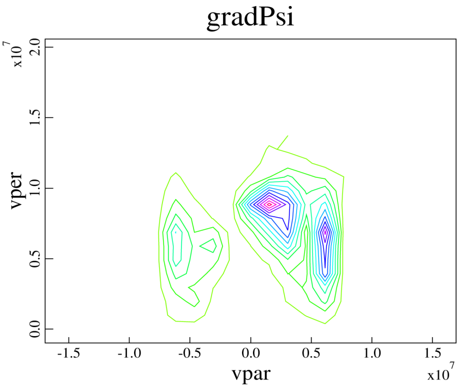
$$
\begin{aligned}
\dot{\delta f} &= f_{eq} \left\{ \frac{mg}{e\psi_0 B^3} \left[\left(v_\parallel^2 + \frac{v_\perp^2}{2} \right) \delta\mathbf{B} \cdot \nabla B - \mu_0 v_\parallel \mathbf{J} \cdot \delta\mathbf{E} \right] \right. \\
&\quad \left. + \frac{\delta\mathbf{v} \cdot \nabla \psi_p}{\psi_0} + \frac{3}{2} \frac{e\varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_c^{3/2}} \mathbf{v}_D \cdot \delta\mathbf{E} \right\} \\
\delta\mathbf{v} &= \frac{\delta\mathbf{E} \times \mathbf{B}}{B^2} + \mathbf{v}_\parallel \cdot \frac{\delta\mathbf{B}}{B} \\
\mathbf{v}_D &= \frac{m}{eB^3} \left(v_\parallel^2 + \frac{v_\perp^2}{2} \right) (\mathbf{B} \times \nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2} \mathbf{J}_\perp
\end{aligned}
$$

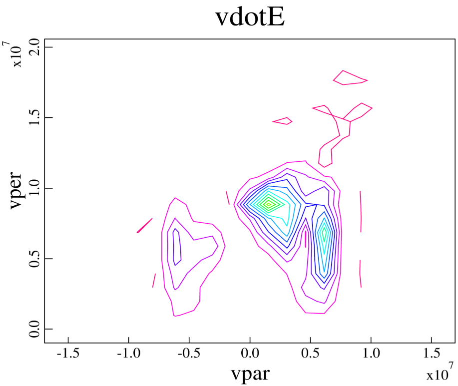
3 Components of $\dot{\delta f}$ for phase space analysis

$$
\begin{aligned}
\frac{1}{f_{eq}}\dot{\delta f} &= \frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}\cdot\delta\mathbf{E}\right] (1) \\
&\quad + \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0} (2) + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_c^{3/2}}\mathbf{v}_D\cdot\delta\mathbf{E} (3) \\
\delta\mathbf{v} &= \frac{\delta\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\cdot\frac{\delta\mathbf{B}}{B} \\
\mathbf{v}_D &= \frac{m}{eB^3}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp
\end{aligned}
$$

- $\dot{\delta f}$ has 3 components
  1. $g\rho_\parallel$ - kinetic correction term
  2. $\delta v_{E\times B}\cdot\nabla\psi$ - radial particle flux
  3. $v_D\cdot\delta\mathbf{E}$ - energy exchange
- examine δf moments of components, e.g. $\int(v_D\cdot\delta\mathbf{E})\delta f d\mathbf{v}$
- convolution is axisymmetric, i.e. $n = 0$ and static

Components of $\dot{\delta f}$ show symmetry

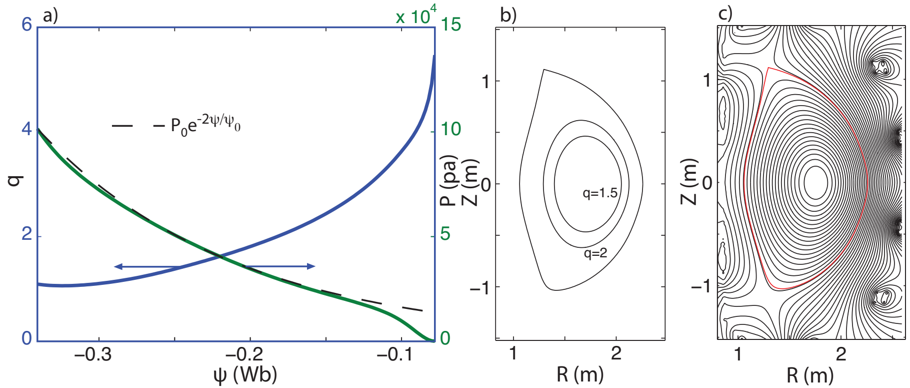




- clear $v_{||}$ resonance
- symmetry between spatial and velocity term
- dominantly passing particles!

$n = 1$ DIII-D linear simulations

D. P. Brennan, C. C. Kim, and R. J. LaHay, Nuclear Fusion **52**, 033004, (2012)

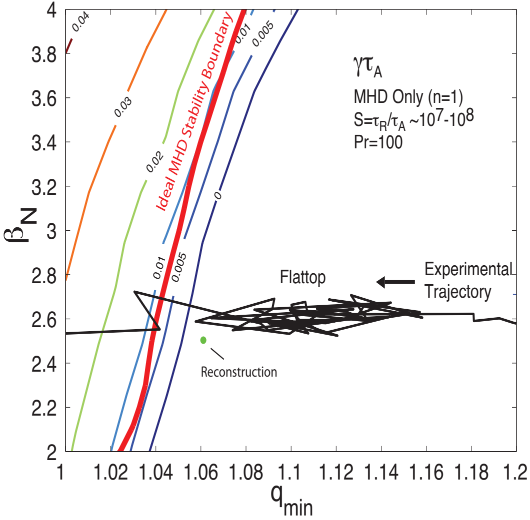


- understand the stability properties of D3D 'hybrid' discharge
- 'hybrid' discharge sensitive to small changes in β_N
- explore range of $q = [.9, 1.4]$ $\beta_N = [2, 4]$

DIII-D experiments indicate sensitivity to increases in $\delta\beta_N$

La Haye - *Physics of Plasmas*, **17** (2010), *Nuclear Fusion*, **51** (2011)

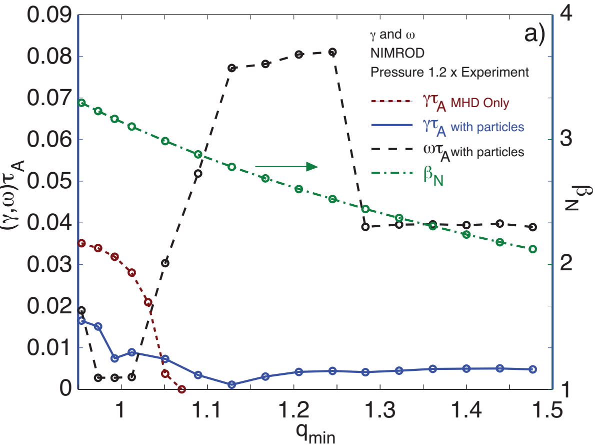
n = 1 MHD Stability Map in (q_{min}, β_N)



- contours of MHD growth rate (NIMROD)
  - $\gamma\tau_A \Uparrow$ as $q_{min} \Downarrow$
- Ideal MHD Stability Boundary in red (PEST)
  - stable to right
- Experimental Trajectory primarily in stable region of MHD stability map
- $\Uparrow \beta_N$ **trigger** $2/1$ **in experiments**

q scan at fixed q-profile highlights abrupt transitions

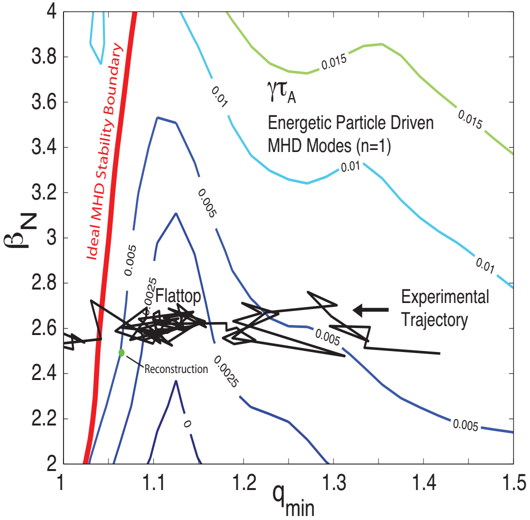
increase in q accompanied by decrease in β_N



- energetic particles stabilizing in ideal region
- map divides into 3 regions
  - *ideal MHD unstable*
  - *high frequency*
  - *low frequency*
- abrupt transition at $q \sim 1.25$
  - $m = 1 \rightarrow m = 2$

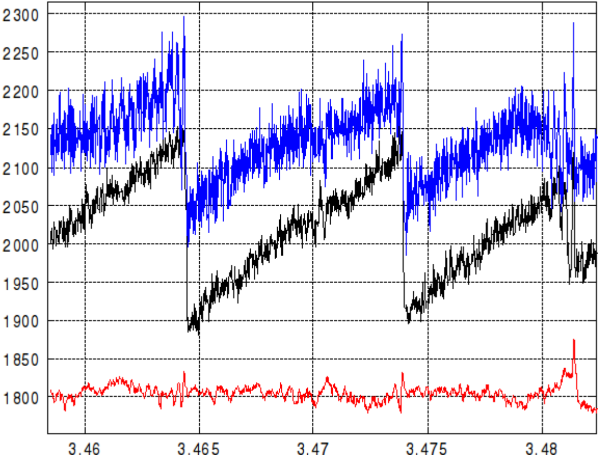
Energetic particles may help explain sensitivity to $\delta\beta_N$

Kinetic-MHD Stability Map in (q_{min}, β_N)



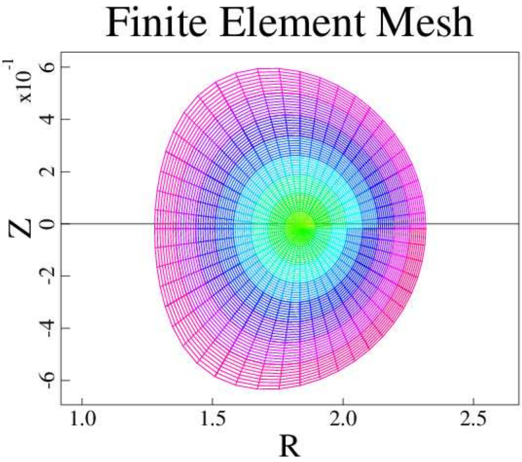
- contours of $\gamma\tau_A$ have dependence on β_N
- *Ideal MHD Stability Boundary in red*
- **EP**'s drive instabilities in MHD stable region

Typical sawtooth activity in L-mode KSTAR
K.I. You et al.



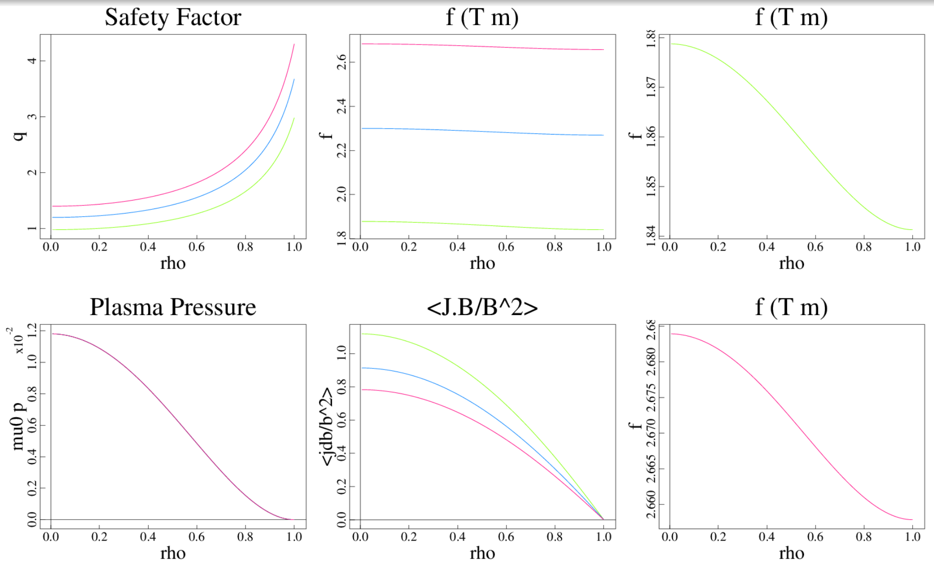
- moderate frequency sub-signal, $T \sim \mathcal{O}(10^{-4})sec$
- ECI indicates a coherent mode
- undetermined poloidal mode number $m = 1, 2?$
- EP driven nonresonant mode?

Linear $n = 1$ HKMHD simulation of KSTAR



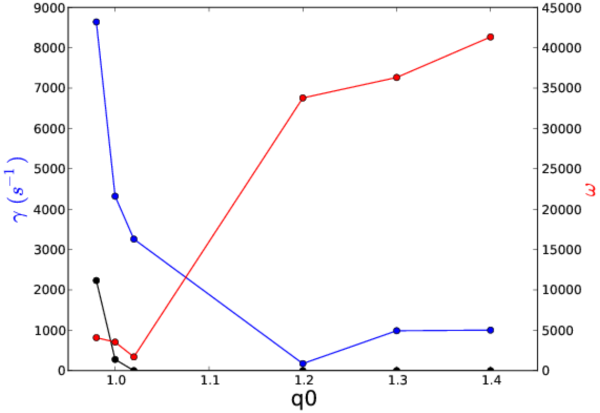
- EFIT equilibrium **shot**#6017 at $t = 3502$
  - $B_0 = 1.96\,T$, $\beta \sim 10^{-3}$
  - $q_0 = 1.85$, $q_{99} = 5.7$
- NIMROD - scale q-profile to perform HKMHD scan
- energetic D^{+1} 10% of β
- $f_0(\mathbf{v}) \propto 1/(v^3 + v_c^3)$, $v_{max} = 1 \times 10^6 m/s$
- should be considered *Proof of Concept*

Scaled profiles for **g006017.003502**, $q_0 = [.98, 1.2, 1.4]$



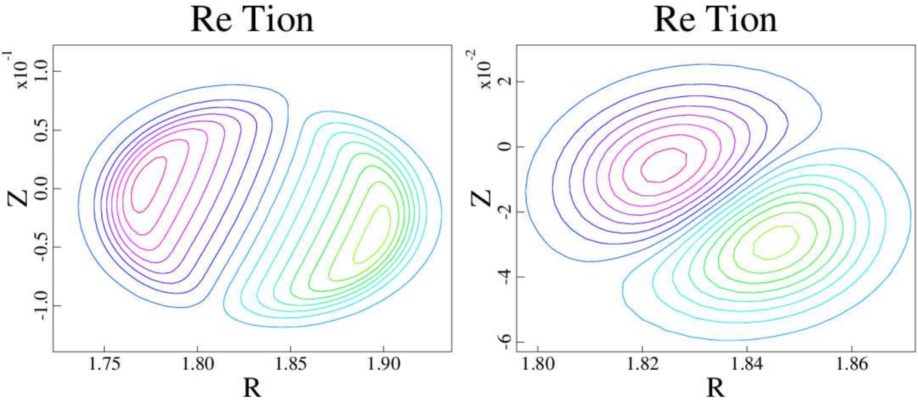
γ and ω vs q_0 **shot**#6017 shows abrupt transition

- energetic particles destabilizing for $q < 1$!
- abrupt transition in γ and ω
- differs from D3D hybrid shot
- **results not necessarily converged**



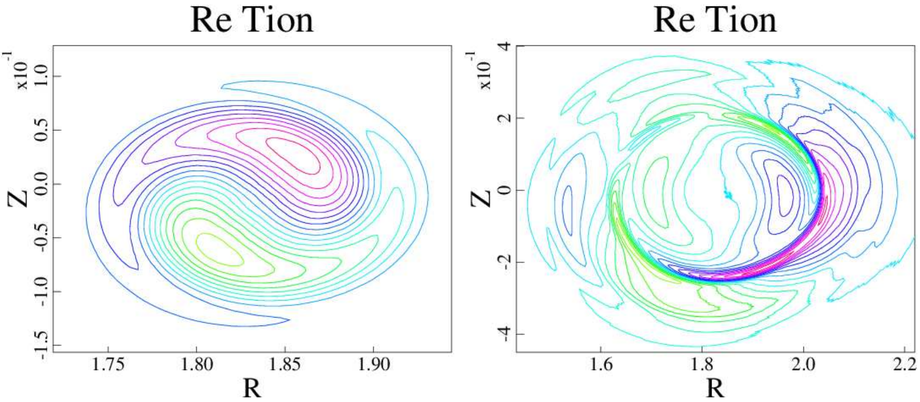
Ideal $n = 1$ contours of T_i shows dominant $m = 1$

$q_0 = [.98, 1.0]$ - difference in radius and profile



HKMHD $n = 1$ contours show $m = 1 \rightarrow 2$ transition

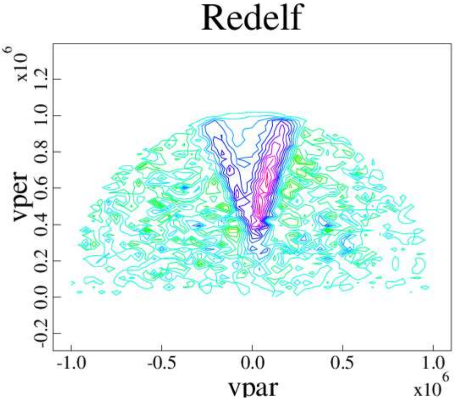
$q_0 = [.98, 1.3]$ - topology differs from ideal mode

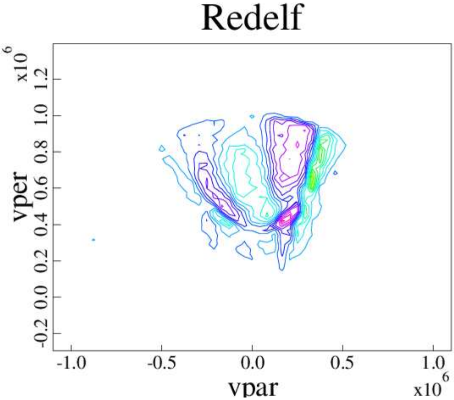


$r_{mode} \simeq 10cm$

Velocity space contours are an orthogonal view in $(\mathbf{x}, \mathbf{v})$

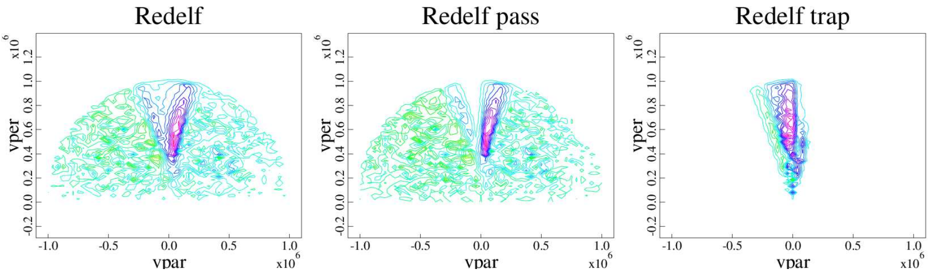
$q_0 = [.98, 1.3]$ - more activity in $q_0 = 1.3$





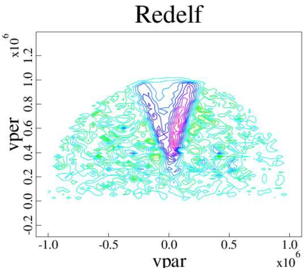
broader “cone” suggests interactions at larger radii

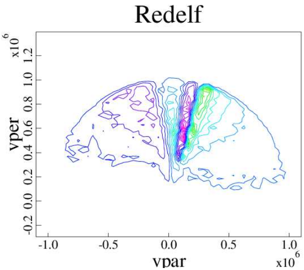
$q_0 = .98$ shows dominantly passing particles

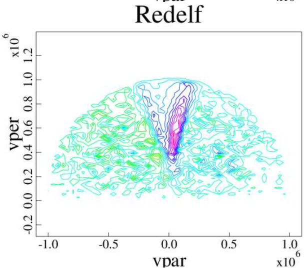


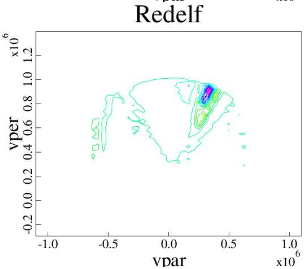
$+v_{\parallel}$ "barely" passing particles (interacting on inboard side) have strongest contribution

$q_0 = .98$ radial subslice shows $q < 1$ dominant

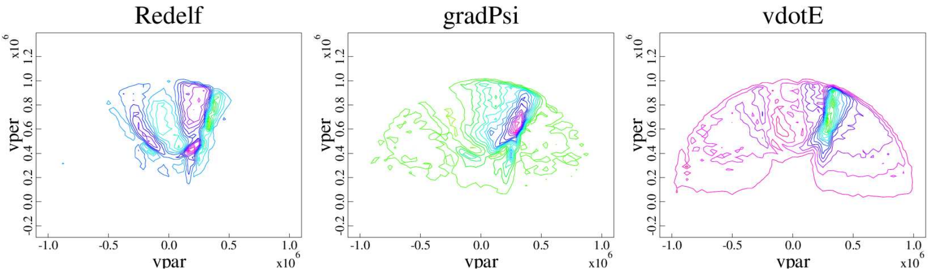






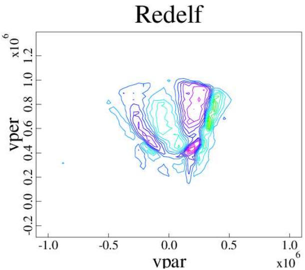


$q_0 = 1.3$ shows Alfven-like signature

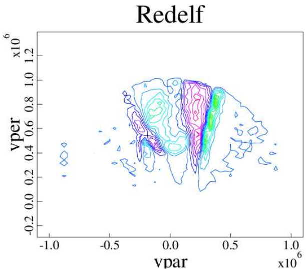


- symmetric $\delta v_{E\times B} \cdot \nabla\psi$ and $v_D \cdot \delta \mathbf{E}$ is Alfven-like
- ω too small for TAE, maybe BAE?

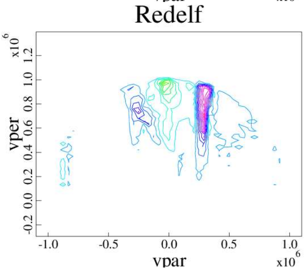
$q_0 = 1.3$ radial subslice shows $q > 1$ dominant



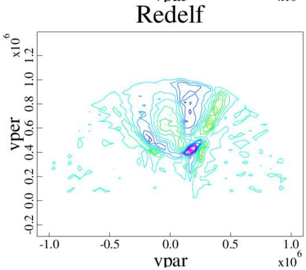
total



$q > 1$
$r < .2m$

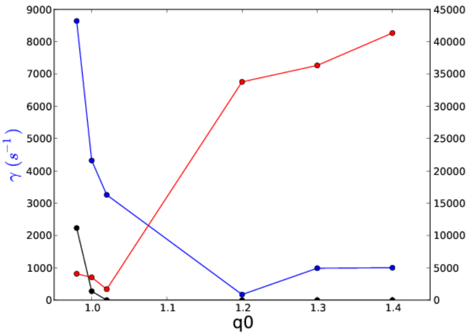


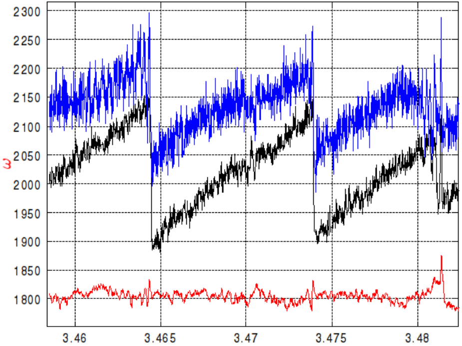
$q < 1$



$r > .2m$

$n = 1$ HKMHD offers plausible explanation





1. core heats, moves left in q_0
2. start high ω, low γ plateau
3. at $q_0 \sim 1$ high $\gamma \rightarrow (1,1)$ kink
4. kink drives crash

Conclusion

- ITPA TAE benchmark shows reasonable agreement
  - more model work required
- DIII-D hybrid discharge published - **NF**(2012)
- preliminary KSTAR simulations promising
  - nonresonant $n = 1$ EP driven activity similar to observations
  - topology and features reminiscent ECI and inversion radius
- **BUT** results are preliminary and not necessarily converged
- velocity space diagnostics useful in inferring HKMHD interaction

Drift Kinetic Equation of Motion

- follows gyrocenter in limit of zero Larmour radius
- reduces $6D$ to $4D+1$ $\left[\mathbf{x}(t), v_{\|}(t), \mu = \frac{\frac{1}{2}mv_{\perp}^2}{\|\mathbf{B}\|}\right]$
- drift kinetic equations of motion

$$
\begin{aligned}
\dot{\mathbf{x}} &= v_{\|}\hat{\mathbf{b}} + \mathbf{v}_D + \mathbf{v}_{E\times B} \\
& \quad \mathbf{v}_D = \frac{m}{eB^4}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)\left(\mathbf{B}\times\nabla\frac{B^2}{2}\right) + \frac{\mu_0 m v_{\|}^2}{eB^2}\mathbf{J}_{\perp} \\
& \quad \mathbf{v}_{E\times B} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} \\
m\dot{v}_{\|} &= -\hat{\mathbf{b}}\cdot(\mu\nabla B - e\mathbf{E})
\end{aligned}
$$

Drift Kinetic requires CGL-like $\delta \underline{\mathbf{p}}_h$ PIC moment

- for drift kinetic equations, assume CGL-like

$$\delta \underline{\mathbf{p}}_h = \begin{pmatrix} \delta p_\perp & 0 & 0 \\ 0 & \delta p_\perp & 0 \\ 0 & 0 & \delta p_\parallel \end{pmatrix}$$

- evaluate pressure moment at $\mathbf{x}$

$$\delta \underline{\mathbf{p}}(\mathbf{x}) = \int m \langle \mathbf{v} - \mathbf{V} \rangle \langle \mathbf{v} - \mathbf{V} \rangle \delta f(\mathbf{x}, \mathbf{v}) d^3 v$$

δf is perturbed phase space density, m mass of particle, and V is COM velocity, usually substitute COM of particles

- implemented as $\delta \underline{\mathbf{p}} = \delta p_\perp \mathbf{I} + \delta \Delta p \mathbf{bb}$, $\delta \Delta p = \delta p_\parallel - \delta p_\perp$

δf PIC reduces $1/\sqrt{N}$ noise

S. E. Parker and W. W. Lee, *PFB* **5**, (1993)

- δf PIC reduces the discrete $1/\sqrt{N}$ particle noise
- Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

- split phase space $f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$
- δf evolves along the characteristics $\dot{\mathbf{z}}$ (equations of motion)

$$\dot{\delta f} = -\dot{\delta \mathbf{z}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

$$using \ \dot{\mathbf{z}} = \dot{\mathbf{z}_{eq}} + \dot{\delta \mathbf{z}} \ and \ \dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$$

- moments of distribution function are $A_n = \displaystyle\int v^n \, \delta f \, d\mathbf{v}$