Kinetic MHD Equations for Simulating the Edge Pedestal

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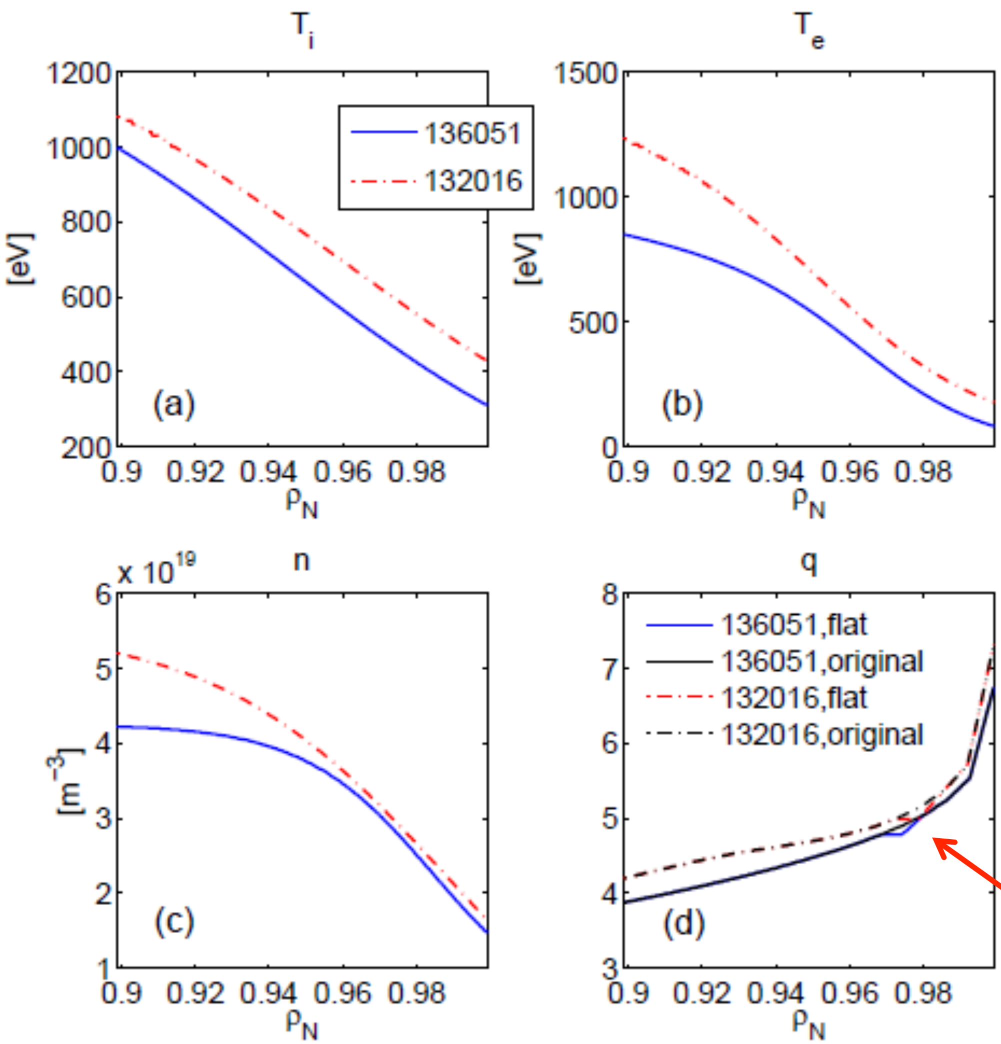
Outline

- Global GK simulations of pedestal
- 2nd order implicit kinetic MHD model
- Tearing mode results
- ITG and ETG results
- Kinetic electron extensions

Two DIII-D discharges are used

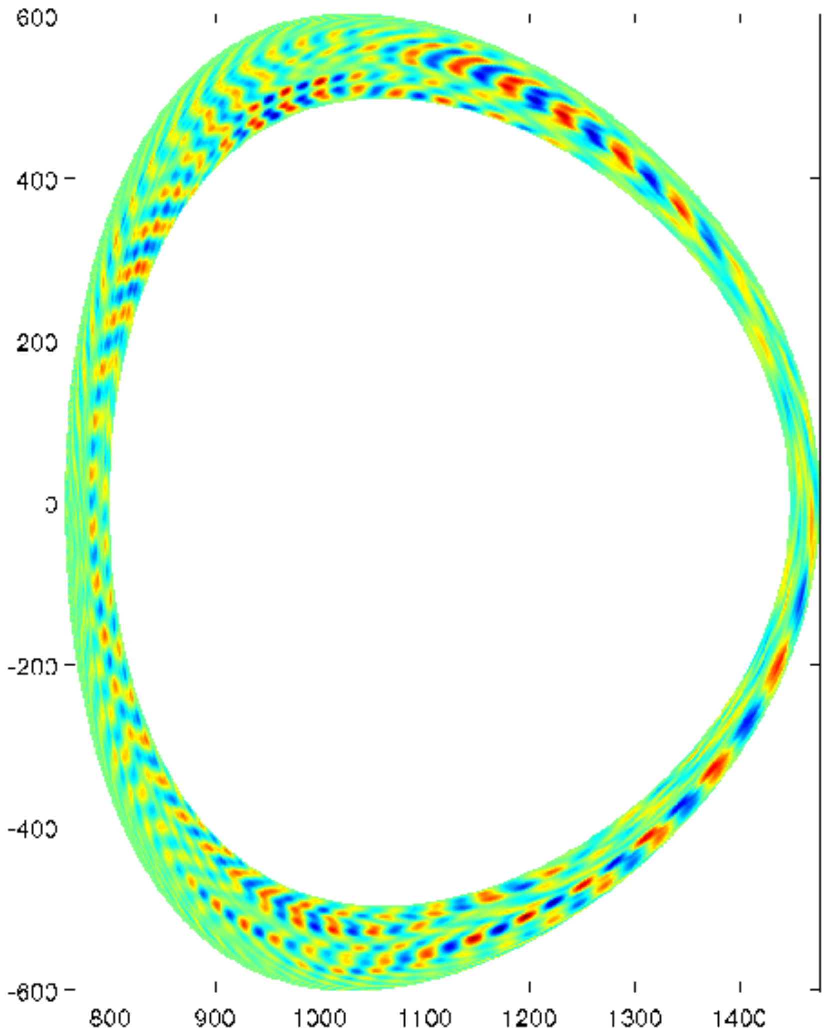


- Profiles at the end of an ELM cycle.
- 132016: kinetic EFIT.
- 136051: previously reported to show characteristics of KBM: Yan et al., PoP 18, 056117 (2011) .
- The experimental g-file is read-in by the code Fluxgrid and Miller Eq parameters are generated and then used by GEM.

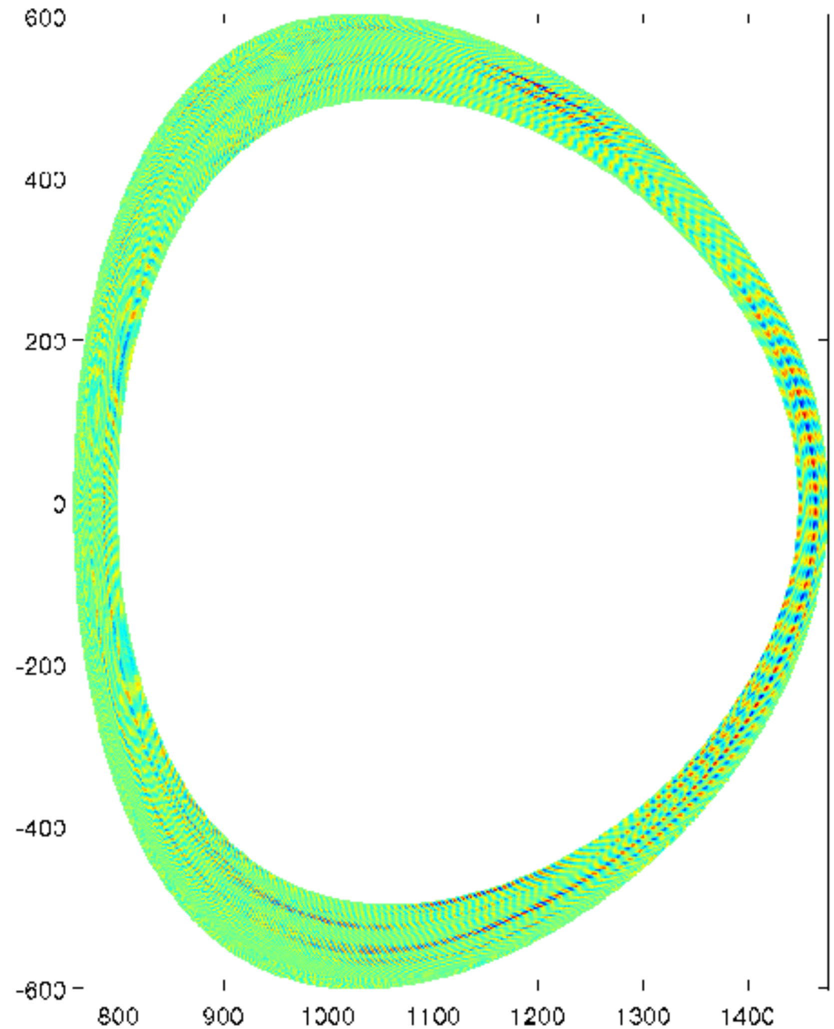
Flatten the magnetic shear at the steep gradient region to see KBM

Global GEM simulations of DIII-D: Kinetic PBM and KBM

n=10



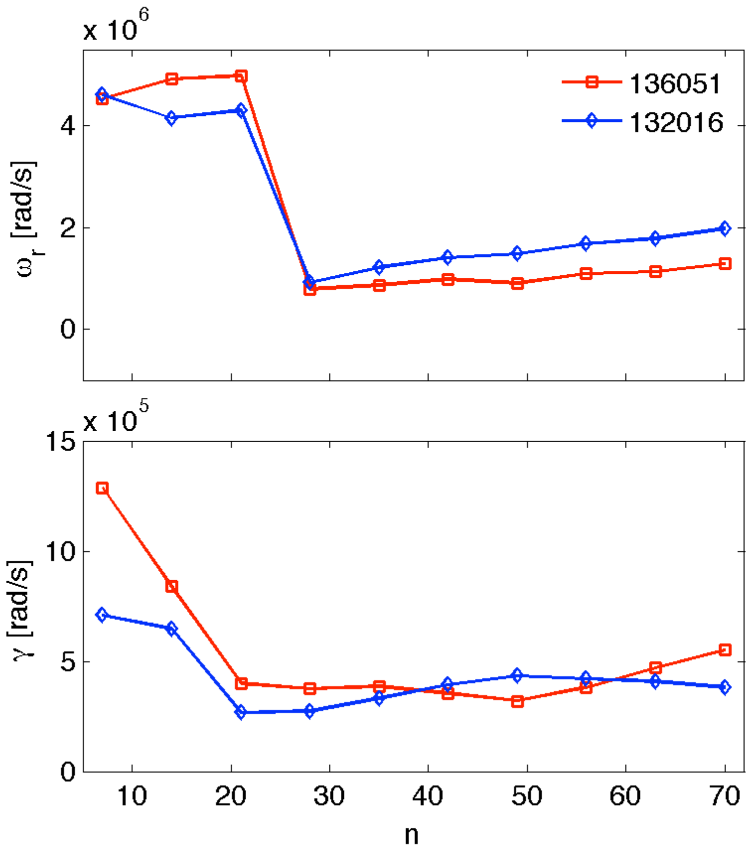
n=63



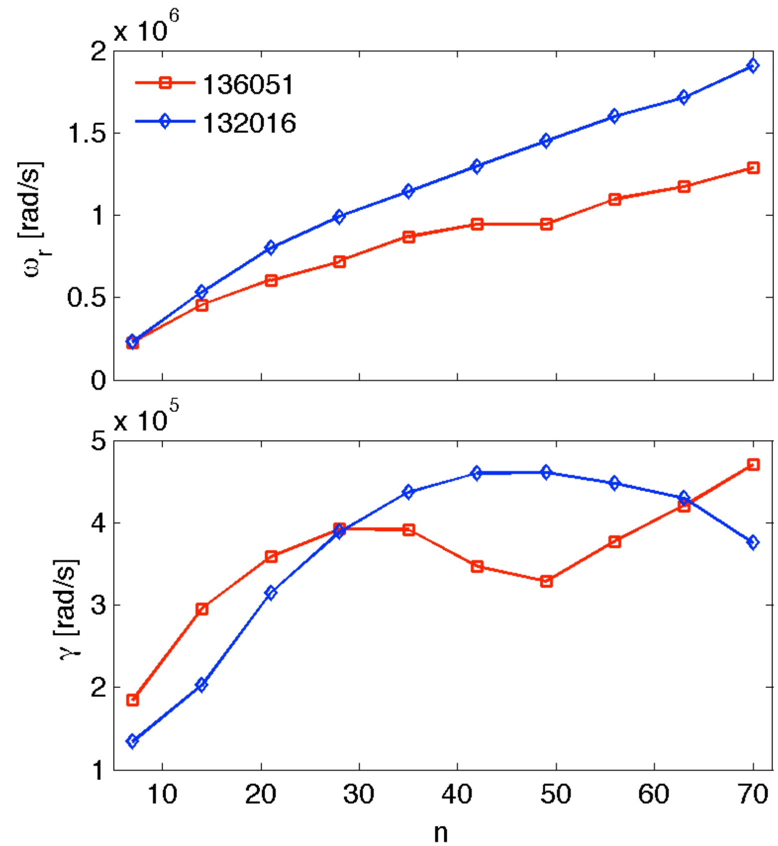
Wan, et al. Submitted to PRL, 2012, Wan et al. (2012) arXiv.org

Effects of the flattened q-profiles

Original *q*

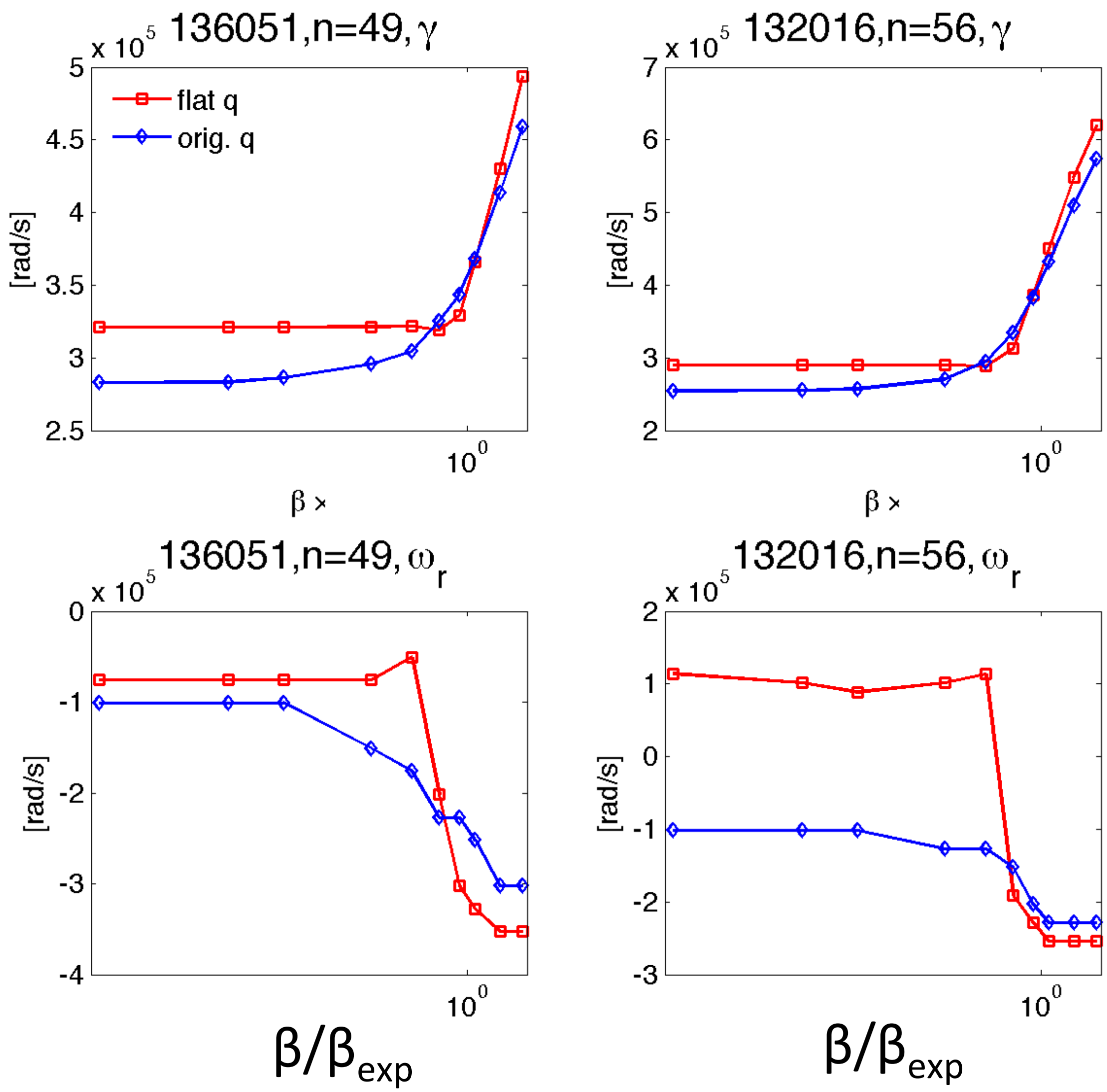


Flattened *q*



- KPBM significantly stabilized.
- KBM becomes dominant instability.
- Collisions increase KBM's growth rate.

It is a KBM indeed!



- Sudden jump of real frequency at critical β
- The growth rate jumps up near experimental β
- Collisions are destabilizing
  - Consistent with KBM
  - **In**consistent with TEM

GK model needs verification

- GK ordering may not be valid: $\rho/L \ll 1$

  L_{Te}, $L_n < L_{Ti}$
- Even if valid what terms to keep?

  (Hahm, Qin, Dimits)
- $L/\rho \approx$ 15-20 for DIII-D
- $L/\rho \approx$ 2-3 for NSTX!
- $\Omega_i \Delta t \approx 1$ anyway ($\approx$2 for DIII-D, $\approx$0.2 for NSTX!)

Ion equations of motion and field equations

- Lorentz force ions

$$\frac{d\boldsymbol{v}_i}{dt} = \frac{q_i}{m_i}(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B})$$
$$\frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i$$

- Ampere's equation

$$\nabla \times \boldsymbol{B} = \mu_0 \left(n_i q_i \boldsymbol{u}_i - n_e e \boldsymbol{u}_e\right)$$

- Faraday's law

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Generalized Ohm's law

- Using quasi-neutrality $n_i = n_e$, the electron density and flow can be calculated directly from particle ions

$$
\begin{aligned}
& en_i(1 + \frac{m_e}{m_i}\frac{q_i^2}{e^2})\boldsymbol{E} + \frac{m_e}{\mu_0 e} \bigtriangledown \times(\bigtriangledown \times \boldsymbol{E}) \\
= & -(1 + \frac{m_e}{m_i}\frac{q_i}{e})\, \boldsymbol{j}_i \times \boldsymbol{B} + \frac{1}{\mu_0}(\bigtriangledown \times \boldsymbol{B}) \times \boldsymbol{B} \\
& + \eta\frac{en_i}{\mu_0}(1 + \frac{m_e}{m_i}\frac{q_i^2}{e^2}) \bigtriangledown \times \boldsymbol{B} - \nabla \cdot \boldsymbol{\Pi}_e + \frac{m_e}{m_i}\frac{q_i}{e} \bigtriangledown \cdot \boldsymbol{\Pi}_i,
\end{aligned}
$$

- In general, we need an electron model to calculate Π_e. Here we assume the electrons are isothermal and Π_e reduces to

$$P_e = n_e T_e = n_i T_e$$

Future plans include drift-kinetic and gyro-kinetic electron models.

Second order semi-implicit scheme

- The velocity, length and time are normalized to $c_s^2 = T_e/m_i$, $\rho_s = m_i c_s/eB_0$ and $\Omega_{ci}^{-1} = m_i/eB_0$. $\beta_e = \mu_0 n_0 T_e/B_0^2$ is defined upon the uniform background plasma.

- The equations of motion are

$$
\begin{aligned}
\frac{\boldsymbol{x}^{n+1} - \boldsymbol{x}^n}{\Delta t} &= (1-\theta)\ \boldsymbol{v}^n + \theta\ \boldsymbol{v}^{n+1},\\
\frac{\boldsymbol{v}^{n+1} - \boldsymbol{v}^n}{\Delta t} &= (1-\theta)\ \boldsymbol{a}^n + \theta\ \boldsymbol{a}^{n+1},\\
\frac{w^{n+1} - w^n}{\Delta t} &= -(1-\theta)\ (\boldsymbol{v}^n \cdot \bigtriangledown + \boldsymbol{a}^n \cdot \partial_{\boldsymbol{v}}) \ln f_0(\boldsymbol{x}^n, \boldsymbol{v}^n)\\
&\quad -\theta\ (\boldsymbol{v}^{n+1} \cdot \bigtriangledown + \boldsymbol{a}^{n+1} \cdot \partial_{\boldsymbol{v}}) \ln f_0(\boldsymbol{x}^{n+1}, \boldsymbol{v}^{n+1}),
\end{aligned}
$$

  where $\boldsymbol{a} = \frac{q_i}{m_i}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$.

- Generalized Ohm's law:

$$
\begin{aligned}
&(n_{i0} + \delta n_i^{n+1})(1 + \frac{m_e}{m_i} q_i^2)\boldsymbol{E}^{n+1} + \frac{m_e}{m_i}\frac{1}{\beta_e} \bigtriangledown \times(\bigtriangledown \times \boldsymbol{E}^{n+1})\\
= &-(1 + \frac{m_e}{m_i} q_i)\delta \boldsymbol{j}_i^{n+1} \times (\boldsymbol{B}_0 + \delta \boldsymbol{B}^{n+1}) + \frac{1}{\beta_e}(\bigtriangledown \times \delta \boldsymbol{B}^{n+1}) \times \boldsymbol{B}_0\\
&+\frac{1}{\beta_e}(\bigtriangledown \times (\boldsymbol{B}_0 + \delta \boldsymbol{B}^{n+1})) \times \delta \boldsymbol{B}^{n+1} + \frac{\eta}{\beta_e}(1 + \frac{m_e}{m_i} q_i^2)(n_{i0} + \delta n_i^{n+1}) \bigtriangledown \times \delta \boldsymbol{B}^{n+1}\\
&-\nabla \delta n_i^{n+1} + \frac{m_e}{m_i} q_i\ \bigtriangledown \cdot P_i^{n+1},
\end{aligned}
$$

Ion current and nonlinear terms

- The first term on the right hand side of the generalized Ohm's law involves the future ion current density

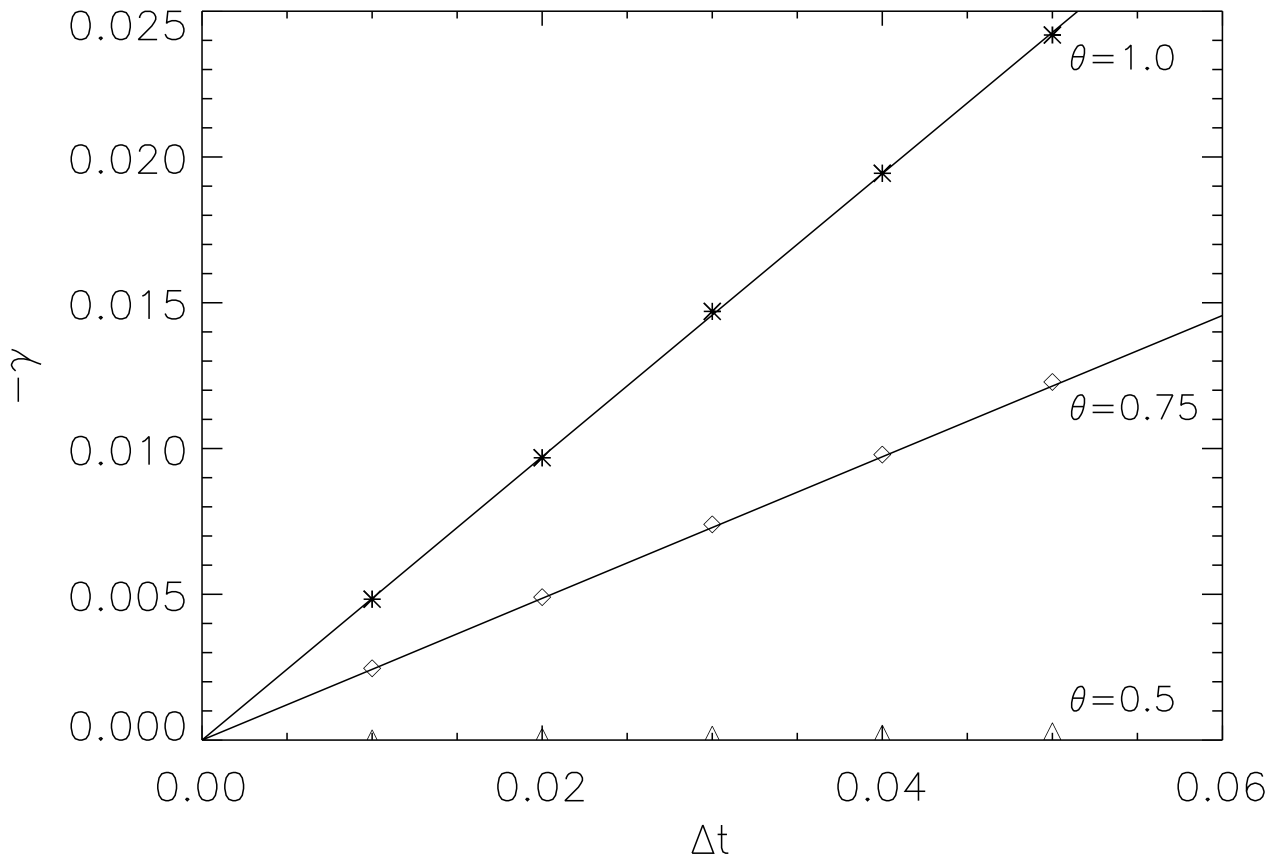
$$e n_i\left(1+\frac{m_e}{m_i}\frac{q_i^2}{e^2}\right)\boldsymbol{E}^{n+1}+\cdots=-\left(1+\frac{m_e}{m_i}\frac{q_i}{e}\right)\,\delta\boldsymbol{j}_i^{n+1}\times\boldsymbol{B}^{n+1}+\cdots$$

we approximate $\delta\boldsymbol{j}_i^{n+1}$ as follows

$$\begin{aligned}\delta\boldsymbol{j}_i^{n+1}&=q_i\,\sum_j w_j^{n+1}\boldsymbol{v}_j^{n+1}\\&=\delta\boldsymbol{j}_i^{\star}+q_i\,\theta\,\Delta t\,\sum_j\frac{q_i}{T_i}\boldsymbol{E}^{n+1}(\boldsymbol{x}_j^{n+1})\cdot\boldsymbol{v}_j^{n+1}\boldsymbol{v}_j^{n+1}\\&\simeq\delta\boldsymbol{j}_i^{\star}+\theta\,\Delta t\,\frac{q_i^2}{m_i}\,\boldsymbol{E}^{n+1}\equiv\boldsymbol{J}_i'.\end{aligned}$$

- For accuracy issues, we iterate on the differences between $\delta\boldsymbol{j}_i^{n+1}$ and $\boldsymbol{J}_i'$.
- For every k_y and k_z mode, the generalized Ohm's law is solved in x direction using finite difference. The equilibrium part is solved by direct matrix inversion. And the nonlinear terms are treated iteratively.

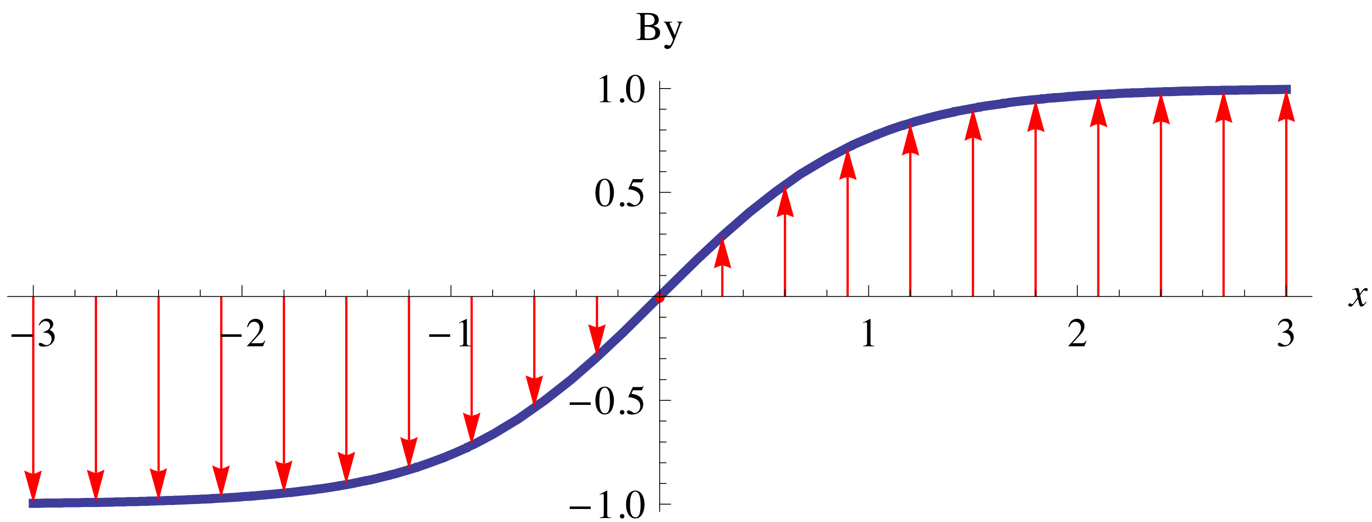
Numerical damping of the whistler wave



$16 \times 16 \times 32$ grids, 131072 particles, $k_\perp = 0, k_\parallel \rho_i = 0.0628, \beta = 0.004.$

Harris sheet equilibrium

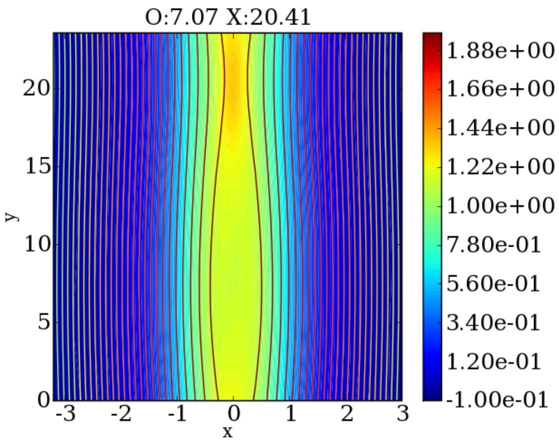
- Zero-order magnetic field $\boldsymbol{B}_0(\boldsymbol{x}) = B_{y0} \tanh(\frac{x}{a})\ \hat{\boldsymbol{y}} + B_G\ \hat{\boldsymbol{z}}$



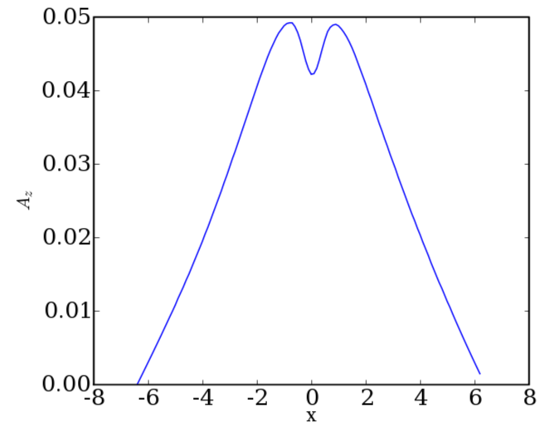
- The equilibrium distribution function is

$$f_{0s} = n_{h0}\ \mathrm{sech}^{\,2}(\frac{x}{a}) \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left[-\frac{m(v_x^2 + v_y^2 + (v_z - v_{ds})^2)}{2T_s}\right]$$
$$+ n_b \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left(-\frac{mv^2}{2T_s}\right),$$

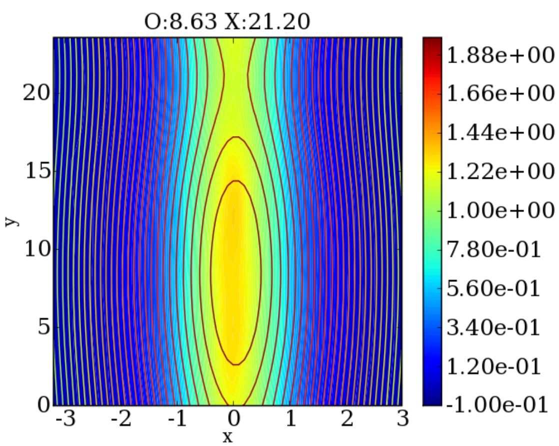
Island and eigenmode structure



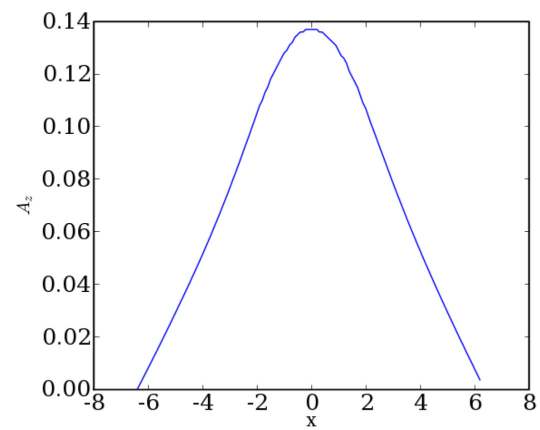
(a)



(b)



(c)



(d)

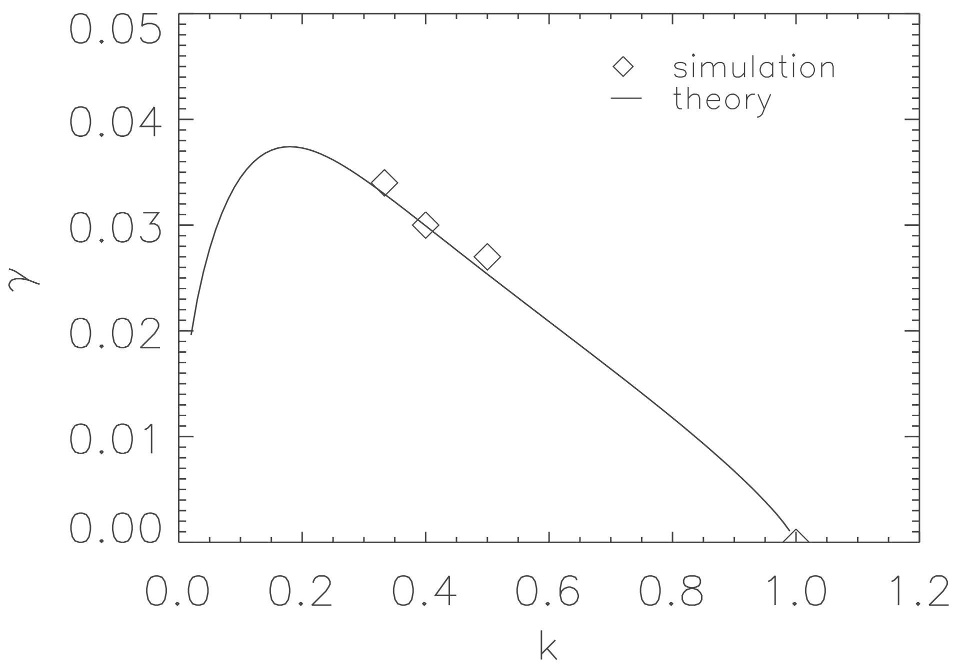
(a)(b) $t = 233\Omega_i^{-1}$, (c)(d) $t = 495\Omega_i^{-1}$ $128 \times 32 \times 64$ grids, 8388608 particles. $\frac{a}{\rho_i} = 1.0, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.5,$ $\eta\frac{en_0}{B_0} = 15 \times 10^{-4}, \frac{\boldsymbol{B}_G}{B_0} = 0, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \frac{l_y}{\rho_i} = 25.12$

The linear growth rate vs k

- Linear Tearing mode theory shows that the growth rate is (scaled)

$$\gamma = 0.55\ (\frac{1}{\beta})^{1/5}\ \Delta'^{4/5}\eta^{3/5}\ (k\ B'_{y0})^{2/5}.$$

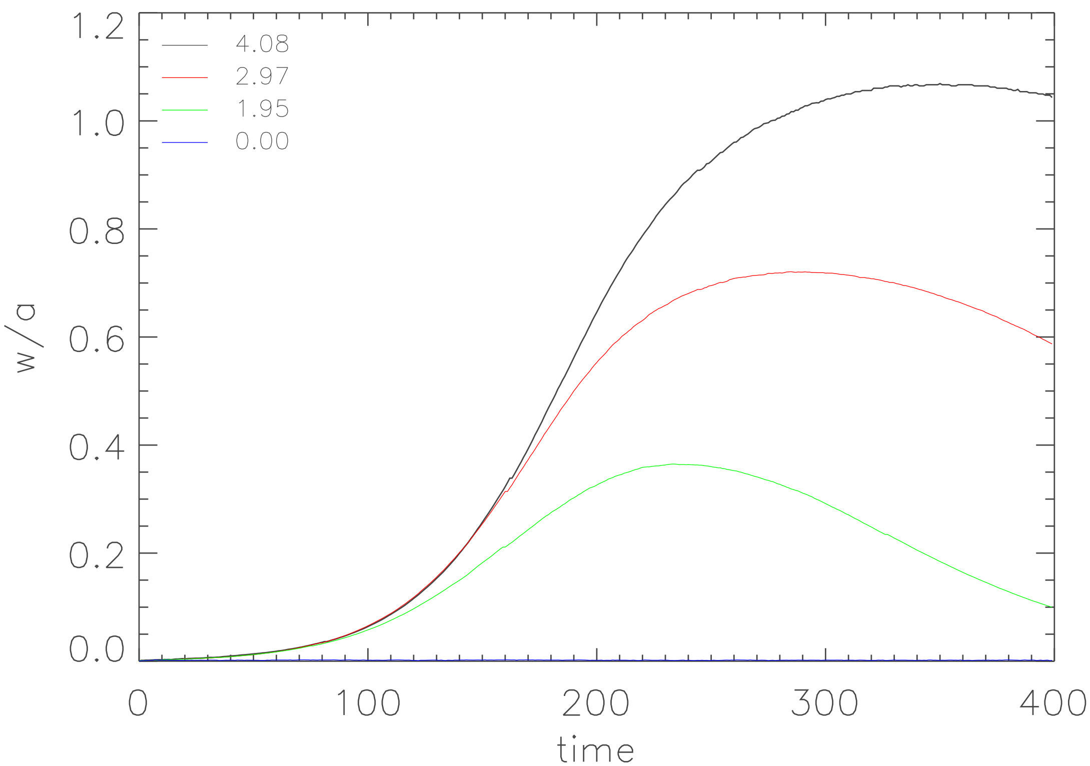
$$\Delta' = \frac{2}{a}(\frac{1}{ka} - ka)\frac{ka - \tanh(l_x/2a)\tanh(kl_x/2)}{ka\tanh(kl_x/2) - \tanh(l_x/2a)}.$$



$128 \times 32 \times 64$ grids, 8388608 particles. $\frac{a}{\rho_i} = 1.0, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.5, \frac{\boldsymbol{B}_G}{B_0} = 0, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \eta = 0.0015$

Full evolution with different Δ'

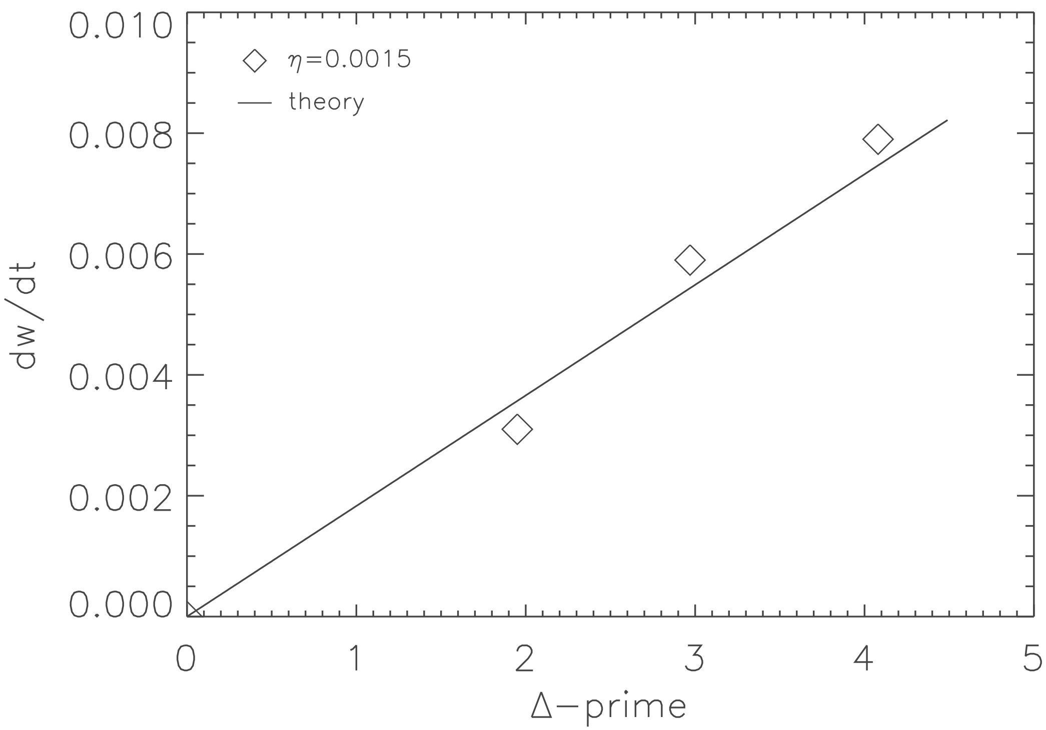
- Tearing mode evolution with different Δ',



$128 \times 32 \times 64$ grids, 8388608 particles. $\frac{a}{\rho_i} = 1.0, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.5, \frac{\boldsymbol{B}_G}{B_0} = 0, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \eta = 0.0015$

Rutherford stage

- Island growth can be described by $\frac{dw}{dt} = 1.22\eta(\Delta' - \alpha' w)$, which reduces to the Rutherford equation when w is small ($\alpha' = 0.82$ in this case).

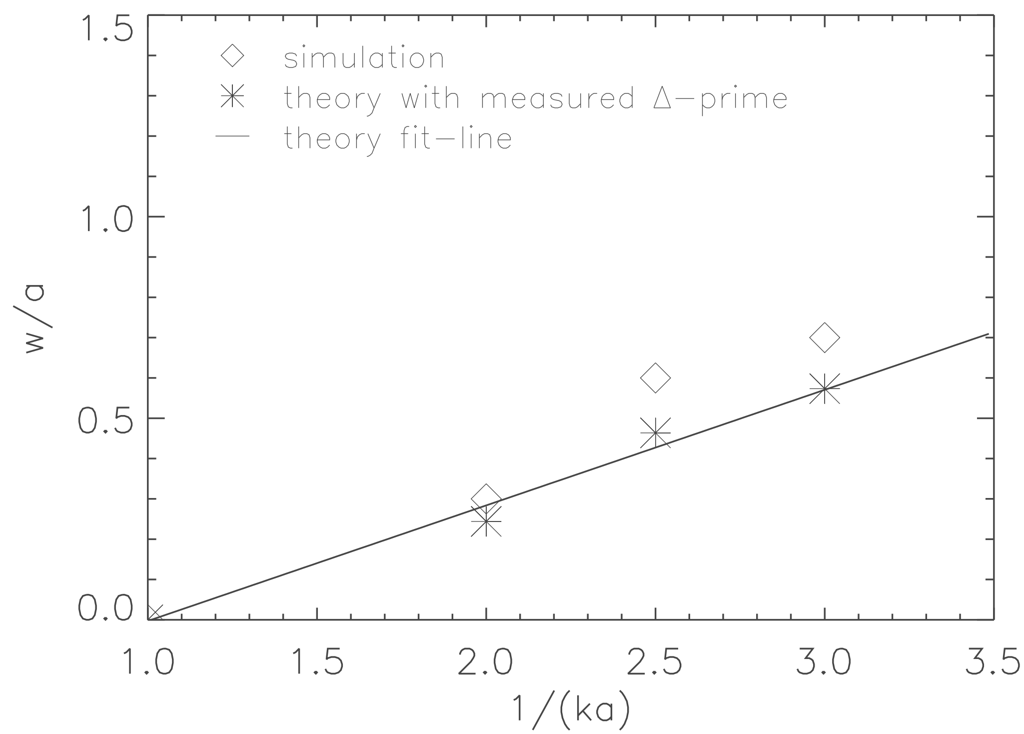


$128 \times 32 \times 64$ grids, 8388608 particles. $\frac{a}{\rho_i} = 1.0, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.5, \frac{\boldsymbol{B}_G}{B_0} = 0, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \eta = 0.0015$

D. F. Escande and M. Ottaviani, Phys. Lett. A **323**, 278-284(2004)

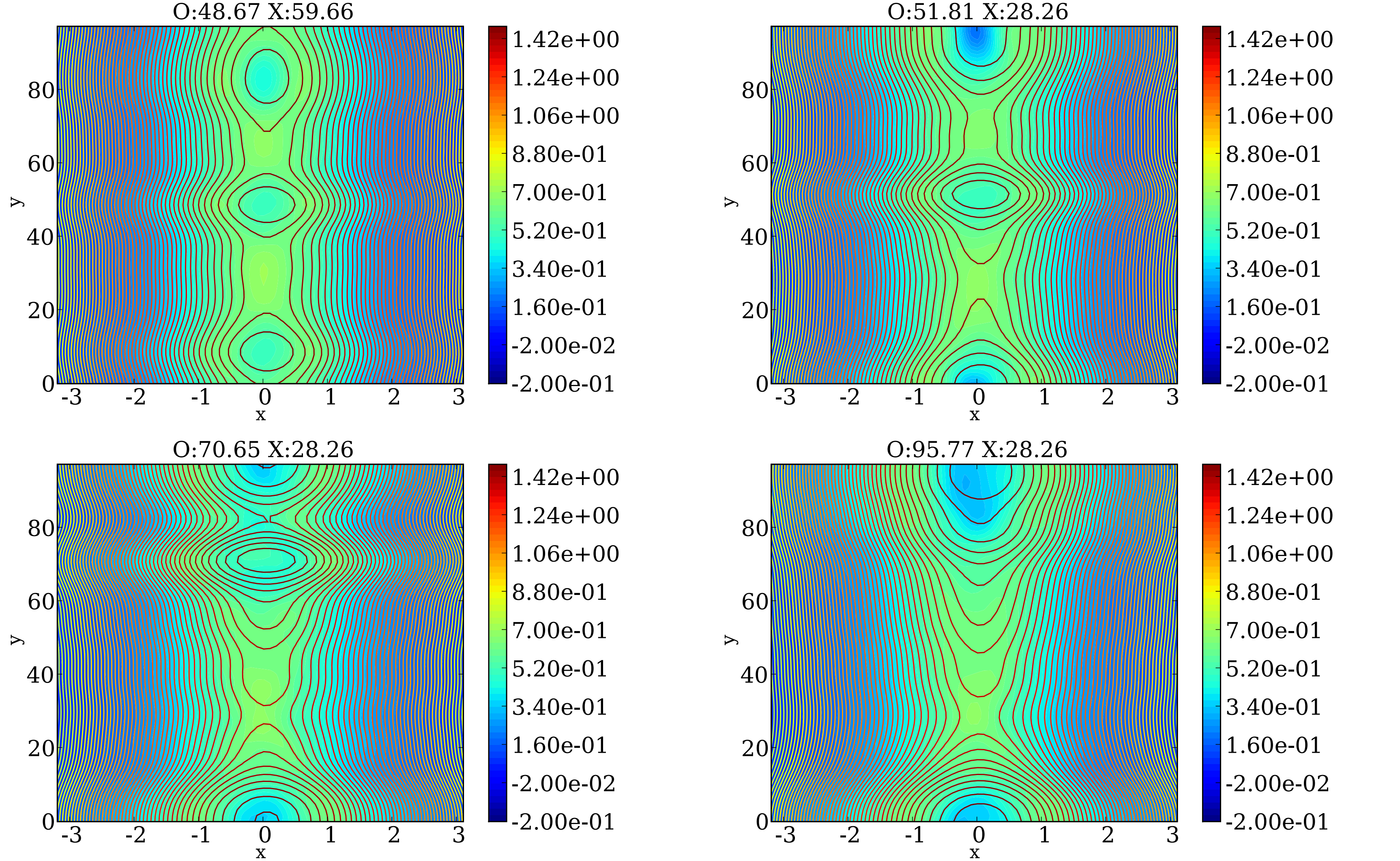
Saturation

- From the equation $\frac{dw}{dt} = 1.22\eta(\Delta' - \alpha' w)$, the island width at saturation is $w_s = 1.22\Delta'$.



$128 \times 32 \times 64$ grids, 8388608 particles. $\frac{a}{\rho_i} = 1.0, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.5, \frac{\boldsymbol{B}_G}{B_0} = 0, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \eta = 0.0015$

Island evolution—$\Delta' = 7.875$



$128 \times 32 \times 64$, 8388608 particles. $\frac{a}{\rho_i} = 2.0, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.5$, $\eta \frac{e n_0}{B_0} = 0.0015$, $\frac{\boldsymbol{B}_G}{B_0} = 0$, $\frac{T_i}{T_e} = 1$, $\frac{l_x}{\rho_i} = 12.8$, $\frac{l_y}{\rho_i} = 100.48$

From left to right, top to bottom: $t = 744, 1064, 1532, 1776\ \Omega_i^{-1}$

The Lorentz ion/Drift kinetic electron model

Lorentz ions:

$$\frac{d\mathbf{v}_i}{dt} = \frac{q}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \qquad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

Drift kinetic electrons: $\varepsilon = \frac{1}{2} m_e v^2$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}_G \equiv v_\parallel \left(\mathbf{b} + \frac{\delta \mathbf{B}_\perp}{B_0} \right) + \mathbf{v}_D + \mathbf{v}_E$$

$$\frac{d\varepsilon}{dt} = -e\mathbf{v}_G \cdot \mathbf{E} + \mu \frac{\partial B}{\partial t}, \qquad \frac{d\mu}{dt} = 0$$

Ampere's equation

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J}_i - en_e(\mathbf{V}_{\mathrm{e}\perp} + u_{\parallel \mathrm{e}} \mathbf{b}))$$

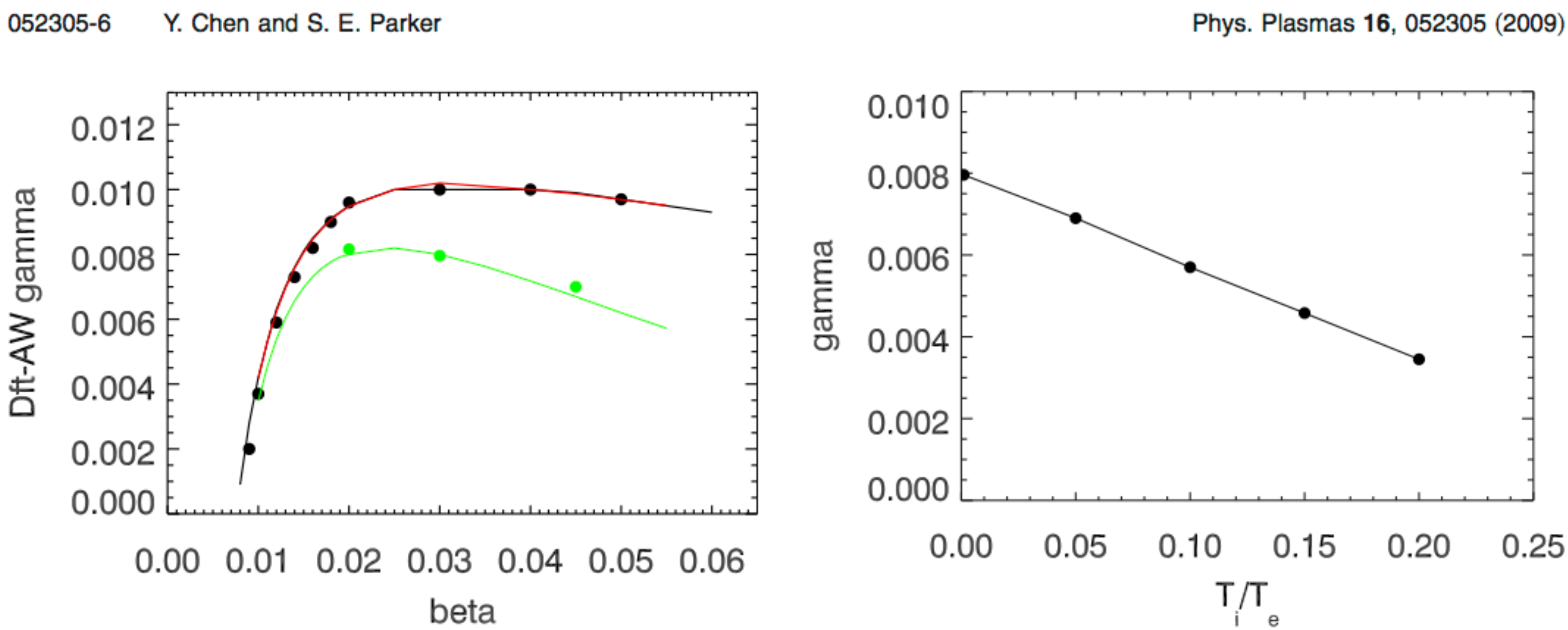
$$\mathbf{V}_{\mathrm{e}\perp} = \frac{1}{B} \mathbf{E} \times \mathbf{b} - \frac{1}{enB} \mathbf{b} \times \nabla P_{\perp \mathrm{e}}$$

$$\mathbf{J}_i = \int f_i \mathbf{v} \, d\mathbf{v}, \quad u_{\parallel \mathrm{e}} = \int f_e \, v_\parallel \, d\mathbf{v}, \quad P_{\perp \mathrm{e}} = \int f_e \frac{1}{2} m_e v^2 \, d\mathbf{v}$$

Faraday's equation,

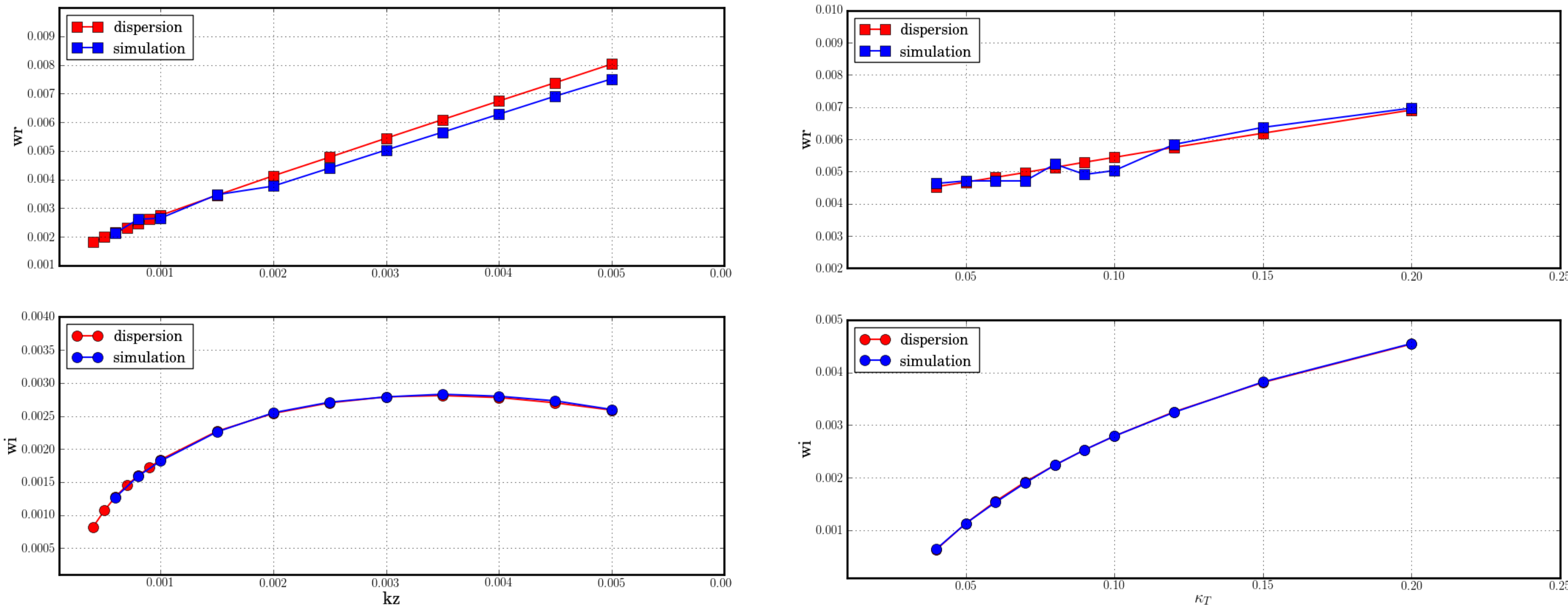
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Alfvenic ETG with DK Electrons Kinetic-MHD vs. Dispersion Relation



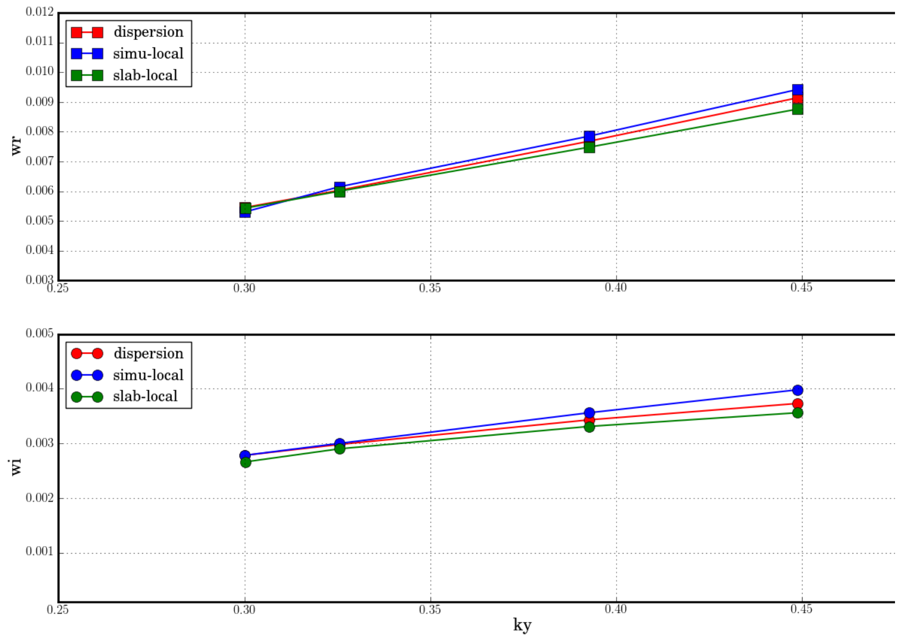
Chen and Parker PoP (2009)

Linear Electrostatic ITG Comparison Kinetic-MHD vs. Dispersion Relation

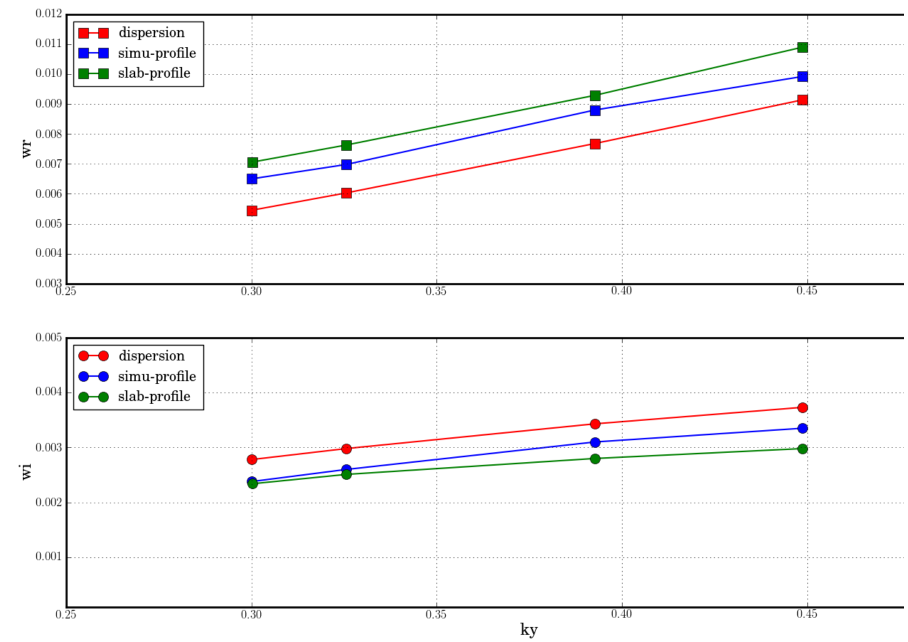


Linear ITG Comparison
Kinetic-MHD vs. GK

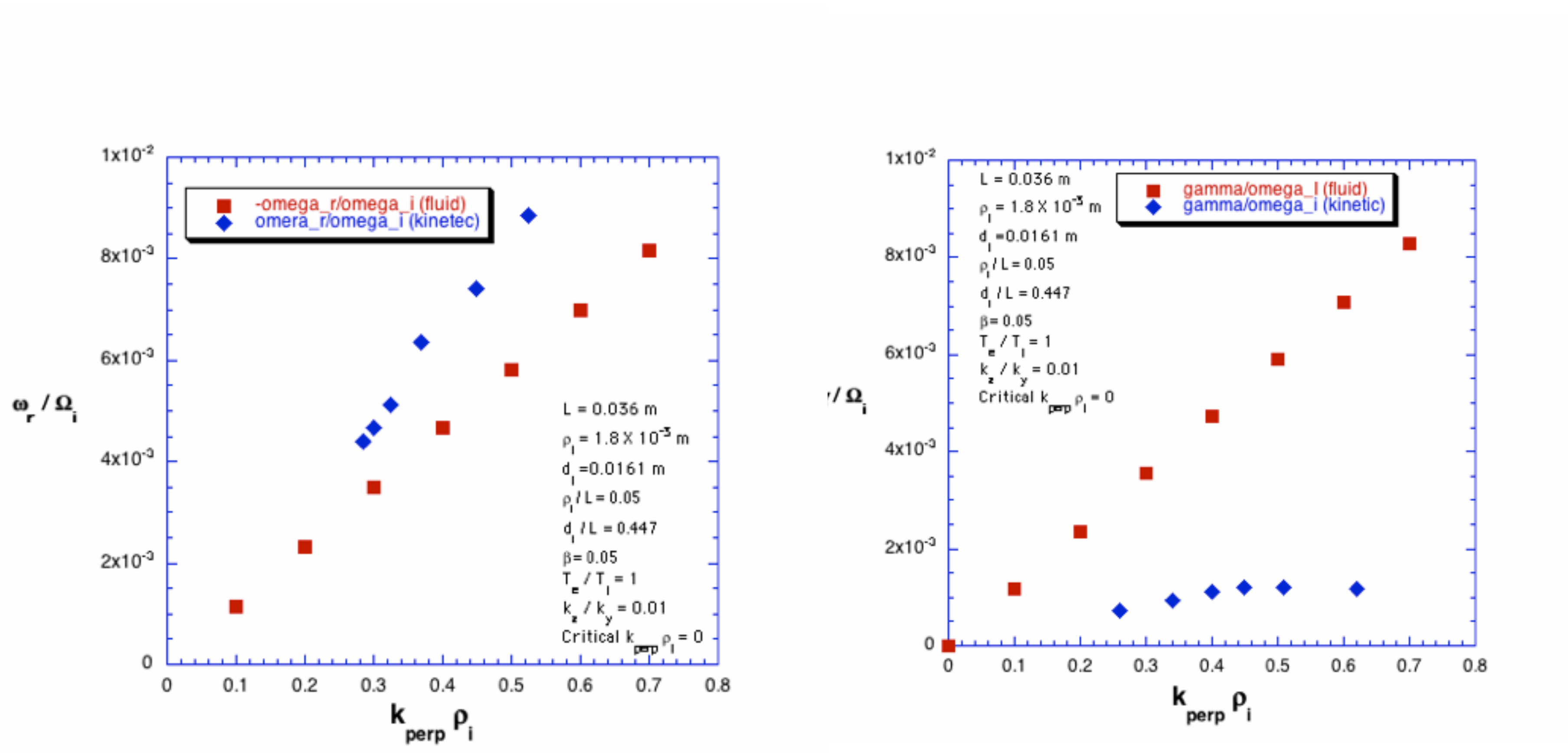
Local



With profile variation



Linear ITG Comparison
Kinetic-MHD vs. NIMROD two-fluid



NIMROD results from D. Schnack

Summary

- GK models need verification
  - Lorentz ion kinetic-MHD model (2nd order implicit)
- Current closure or pressure closure?

  Pressure: Barnes, Cheng, Parker PoP 2008

  Current: Chen, Parker PoP 2009

  - Distinct Perp. and parallel Ohm's law
- Model demonstration for ITG, Alfvenic ETG, and Tearing mode

Possible implementation in NIMROD

- NIMROD equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} + \kappa_{divb} \nabla \nabla \cdot \boldsymbol{B}$$
$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} - \boldsymbol{V}_{ss} \times \boldsymbol{B} - \boldsymbol{V} \times \boldsymbol{B}_{ss} + \frac{\eta}{\mu_0} \nabla \times \boldsymbol{B}$$

- Instead of pushing the particle conservation and momentum equation in NIMROD, we could use the flow and density directly with values coming directly from the particles to advance $\boldsymbol{B}$ and $\boldsymbol{E}$,

$$\delta n_i = \int \delta f_i d^3 \boldsymbol{V} \qquad (\delta n_e = \delta n_i)$$
$$\boldsymbol{V}_i = \frac{1}{ne} \int \boldsymbol{V} \delta f_i d^3 \boldsymbol{V}$$
$$\boldsymbol{V}_e = \boldsymbol{V}_i - \frac{1}{\mu_0 n e} \nabla \times \boldsymbol{B}$$

- The only issue is that we calculate $\boldsymbol{E}$ from Ohm's law and then go to Faraday's law to advance $\boldsymbol{B}$ while NIMROD incoporates the Ohm's law into Faraday's law to directly evolve $\boldsymbol{B}$ instead.

C. R. Sovinec *et al*, J. Comput. Phys **195**, 355-386 (2004)

C. C. Kim *et al*, Kinetic Particles in the NIMROD fluid code, Sherwood poster, 2003