Verification of NIMROD with Fluid “ITG-like” Modes

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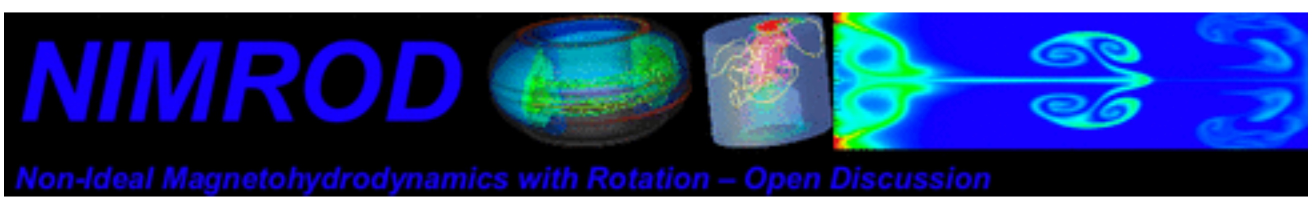

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“ITG mode” = Ion Temperature Gradient Mode

Verification and Validation

- Verification
  - Are the equations being solved correctly?
  - Comparison with known solutions, or benchmarking with independent codes
- Validation
  - Are the right equations being solved?
  - Direct comparison with experiment
- Here we will deal with Verification

Verification of NIMROD in MHD

- NIMROD has been successfully verified in most realms of ideal and resistive MHD
  - Ideal MHD waves and instabilities
  - Resistive instabilities (linear and non-linear) in slab, cylindrical, and toroidal geometry
  - Anisotropic thermal conduction (comparison with theory)
  - Peeling and ballooning edge modes (comparison with ELITE)
  - Saweeth (comparison with M3D)
  - High-β disruption (comparison with theory)

Verification of NIMROD in Extended MHD

- Energetic minority ion species
  - Kink stabilization/TAE destabilization (comparison with NOVA-K and M3D)
- Two-Fluid/FLR
  - Stabilization of *g*-mode in slab geometry (comparison with theory)
  - Drift-tearing modes (King)
  - *De-stabilization of parallel sound wave by FLR effects (ITG-like mode)*
    - *Comparison with theory*
    - *Hope for comparison with kinetic code*

FLR Effects on Fluid Modes

Two-fluid/FLR Equations

- Low order moments for ions and electrons

$$\frac{\partial n}{\partial t} = -\nabla \cdot n\mathbf{V}_i = -\nabla \cdot n\mathbf{V}_e \quad \text{(quasi-neutrality)}$$

$$Mn\left(\frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i\right) = ne\left(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}\right) - \nabla p_i - \underbrace{\nabla \cdot \mathbf{\Pi}_i}_{\text{Ion viscous stress}}$$

$$0 = -ne\left(\mathbf{E} + \mathbf{V}_e \times \mathbf{B}\right) - \nabla p_e \quad (m_e = 0, \text{Ohm's law})$$

$$\frac{\partial p_i}{\partial t} + \mathbf{V}_i \cdot \nabla p_i = -\frac{5}{3} p_i \nabla \mathbf{V}_i - \frac{2}{3} \underbrace{\nabla \cdot \mathbf{q}_i}_{\text{Ion heat flux}} + Q \quad (\Gamma_i = \frac{5}{3})$$

$$\frac{\partial p_e}{\partial t} + \mathbf{V}_e \cdot \nabla p_e = -p_e \nabla \mathbf{V} \quad (\Gamma_e = 1, \text{ isothermal})$$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \ , \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \cdot \mathbf{A} = 0 \ , \quad \mathbf{J} = ne\left(\mathbf{V}_i - \mathbf{V}_e\right) \ , \quad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

Extended MHD

- 2-fluid equations can be combined into "single fluid form" (extended MHD)

$$\frac{\partial n}{\partial t} = -\nabla \cdot n\mathbf{V}$$

$$Mn\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V}\right) = -\nabla\left(p_i + p_e\right) + \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{\Pi}_{\wedge}$$

$$\frac{\partial p_i}{\partial t} + \mathbf{V} \cdot \nabla p_i = -\frac{5}{3} p_i \nabla \mathbf{V} - \frac{2}{3} \nabla \cdot \mathbf{q}_{\wedge} \quad , \quad \frac{\partial p_e}{\partial t} + \mathbf{V}_e \cdot \nabla p_e = -p_e \nabla \mathbf{V}_e$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad , \quad \mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{1}{ne}\left(\mathbf{J} \times \mathbf{B} - \nabla p_e\right) + \eta \mathbf{J}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \quad , \quad \mathbf{V}_e = \mathbf{V} - \frac{\mathbf{J}}{ne}$$

Expressions for Stress Tensor in Magnetized Plasma

- Can be decomposed as

$$\mathbf{\Pi} = \mathbf{\Pi}_{\|} + \mathbf{\Pi}_{\wedge} + \mathbf{\Pi}_{\perp} \text{ orthogonal components}$$

$$\mathbf{\Pi}_{\|} = \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \mathbf{\Pi} = \frac{3}{2}\frac{p}{\nu_c}\left(\hat{\mathbf{b}} \cdot \mathbf{W} \cdot \hat{\mathbf{b}}\right)\left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathbf{I}\right) \sim 1/\nu_c \text{ unphysical as } \nu_c \to 0$$

$$\mathbf{\Pi}_{\wedge} = \left(\hat{\mathbf{b}} \times \mathbf{I}\right) \cdot \mathbf{\Pi} = \frac{p}{4\Omega}\left[\left(\hat{\mathbf{b}} \times \mathbf{W}\right) \cdot \left(\mathbf{I} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) + \left(\mathbf{I} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \left(\mathbf{W} \times \hat{\mathbf{b}}\right)\right] \text{ independent of } \nu_c$$

$$\begin{aligned}\mathbf{\Pi}_{\perp} = \hat{\mathbf{b}} \times \left(\hat{\mathbf{b}} \times \mathbf{I}\right) \cdot \mathbf{\Pi} = \frac{p\nu_c}{\Omega^2}\Big\{&\left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \mathbf{W} \cdot \left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) - \frac{1}{2}\left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) : \mathbf{W} \\ &+4\left[\left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \mathbf{W} \cdot \hat{\mathbf{b}}\hat{\mathbf{b}} + \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \mathbf{W} \cdot \left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right)\right]\Big\} \sim \nu_c \text{ , } \to 0 \text{ as } \nu_c \to 0\end{aligned}$$

- Only $\Pi_{\wedge}$ is independent of collision frequency
- Called the *gyro-viscosity*
- Captures lowest order (in $k_{\text{perp}}\,\varrho_i$ << 1) effect of finite ion Larmor radiius

Properties of Gyro-viscosity

- Independent of collisions
  - Remains in collisionless limit
- Causes no heating or dissipation

$$Q_i \equiv \mathbf{\Pi} : \nabla \mathbf{V} = 0$$

- Completely reversible transport of momentum due to spatial distribution of ion Larmor orbits
  - FLR effect
  - No increase in entropy

Closures: Heat Flux

- Can be decomposed as

$$\mathbf{q}_i = \mathbf{q}_{||} + \mathbf{q}_{\wedge} + \mathbf{q}_{\perp}$$

$$\mathbf{q}_{||} = \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \mathbf{q}_i = -\kappa_{||}\hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \nabla T_i$$

$$\mathbf{q}_{\wedge} = \hat{\mathbf{b}} \times \mathbf{q}_i = +\kappa_{\wedge}\hat{\mathbf{b}} \times \nabla T_i$$

$$\mathbf{q}_{\perp} = \hat{\mathbf{b}} \times \left(\hat{\mathbf{b}} \times \mathbf{q}_i\right) = -\kappa_{\perp}\left(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \nabla T_i$$

- Dependence on collision frequency

$$\kappa_{||} \sim 1/\nu_c \quad , \quad \kappa_{\wedge} \text{ independent of } \nu_c \quad , \quad \kappa_{\perp} \sim \nu_c$$

- $\varkappa_{\wedge}$ survives for collisionless model
- Ion diamagnetic heat flux
- Reversible flux of heat due to spatial distribution of ion Larmor orbits
  - No increase in entropy

Diamagnetic Flows and Fluxes

- Solve ion momentum equation for velocity

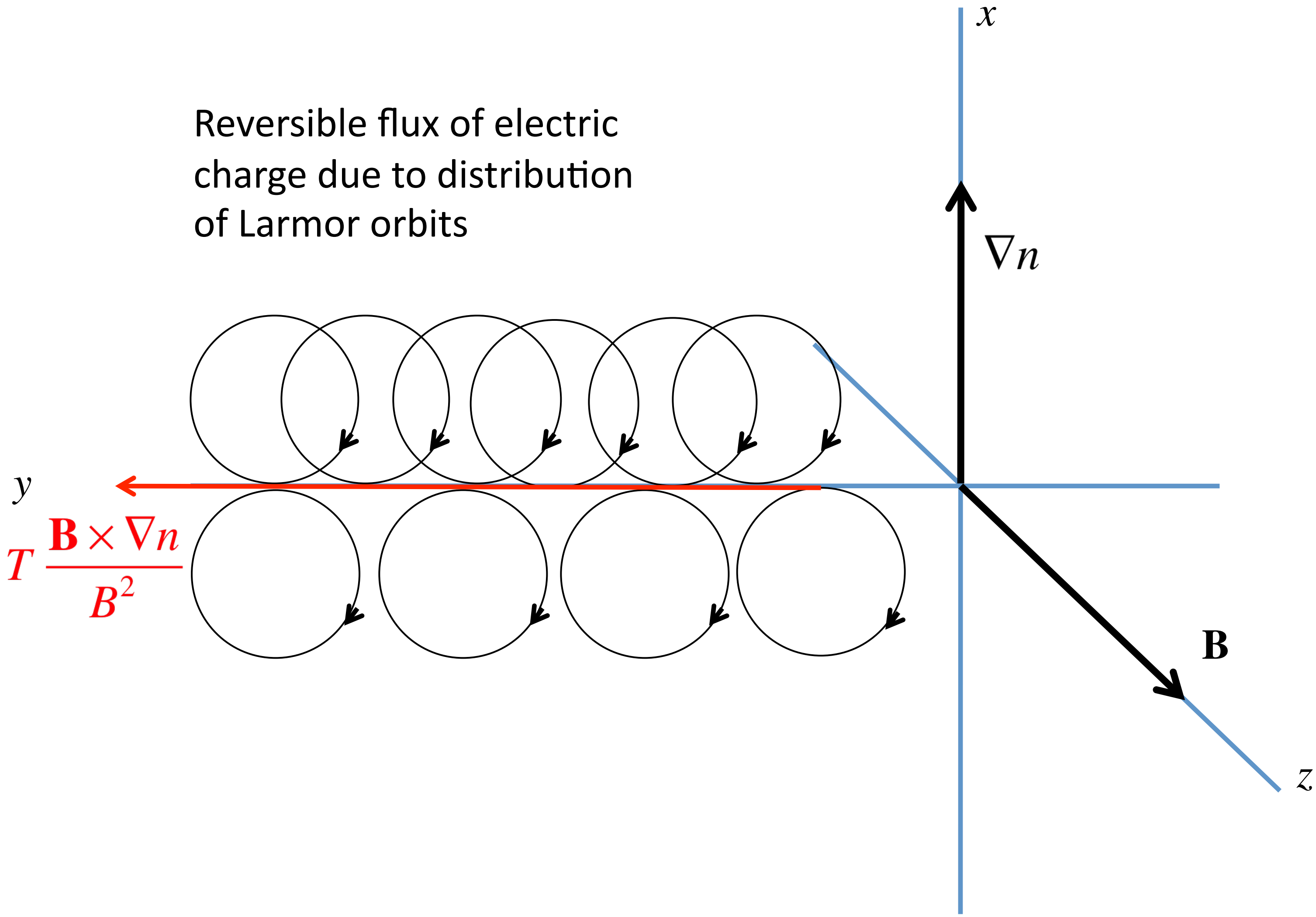
$$\mathbf{V}_{i\perp} = \underbrace{\frac{\mathbf{E}\times\mathbf{B}}{B^2}}_{E\times B\text{ drift}} + \underbrace{\frac{\mathbf{B}\times\nabla p_i}{neB^2}}_{\text{Diamagnetic drift}} + \underbrace{\frac{M}{eB^2}\mathbf{B}\times\frac{d\mathbf{V}_i}{dt}}_{\text{Polarization drift}} + \underbrace{\frac{\mathbf{B}\times\nabla\cdot\mathbf{\Pi}}{neB^2}}_{???}$$

$$= \underbrace{\mathbf{V}_E}_{\text{MHD velocity}} + \underbrace{\mathbf{V}_{*i}}_{\text{Diamagnetic drift velocity}} + \ldots$$

- These can be consider ordered in ϱ_i/L <<1
- These flows can cause ``transport" by fluxes, i.e. $\sim nV_*$
- This is the origin of the FLR closures

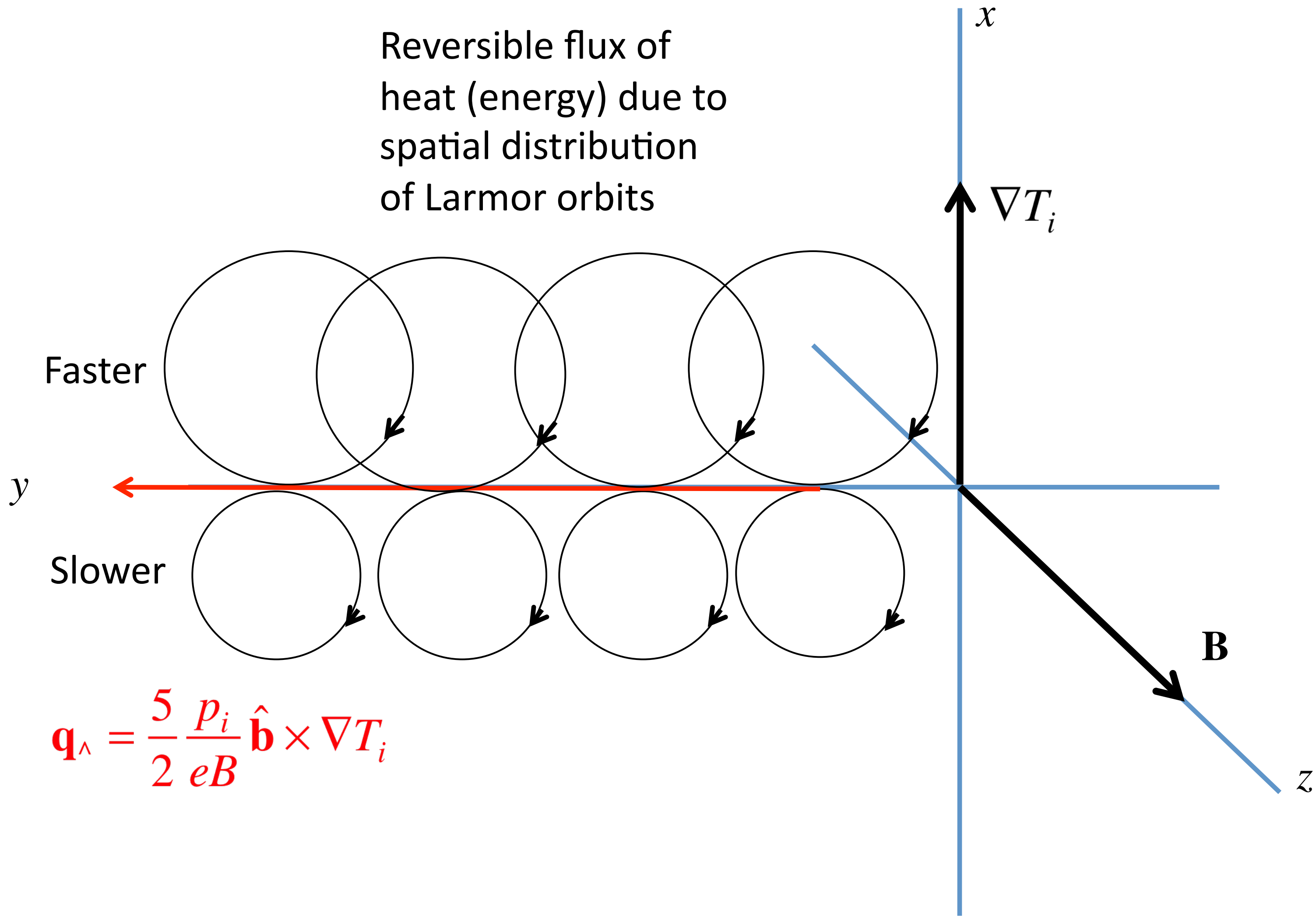
Diamagnetic Current

Reversible flux of electric charge due to distribution of Larmor orbits



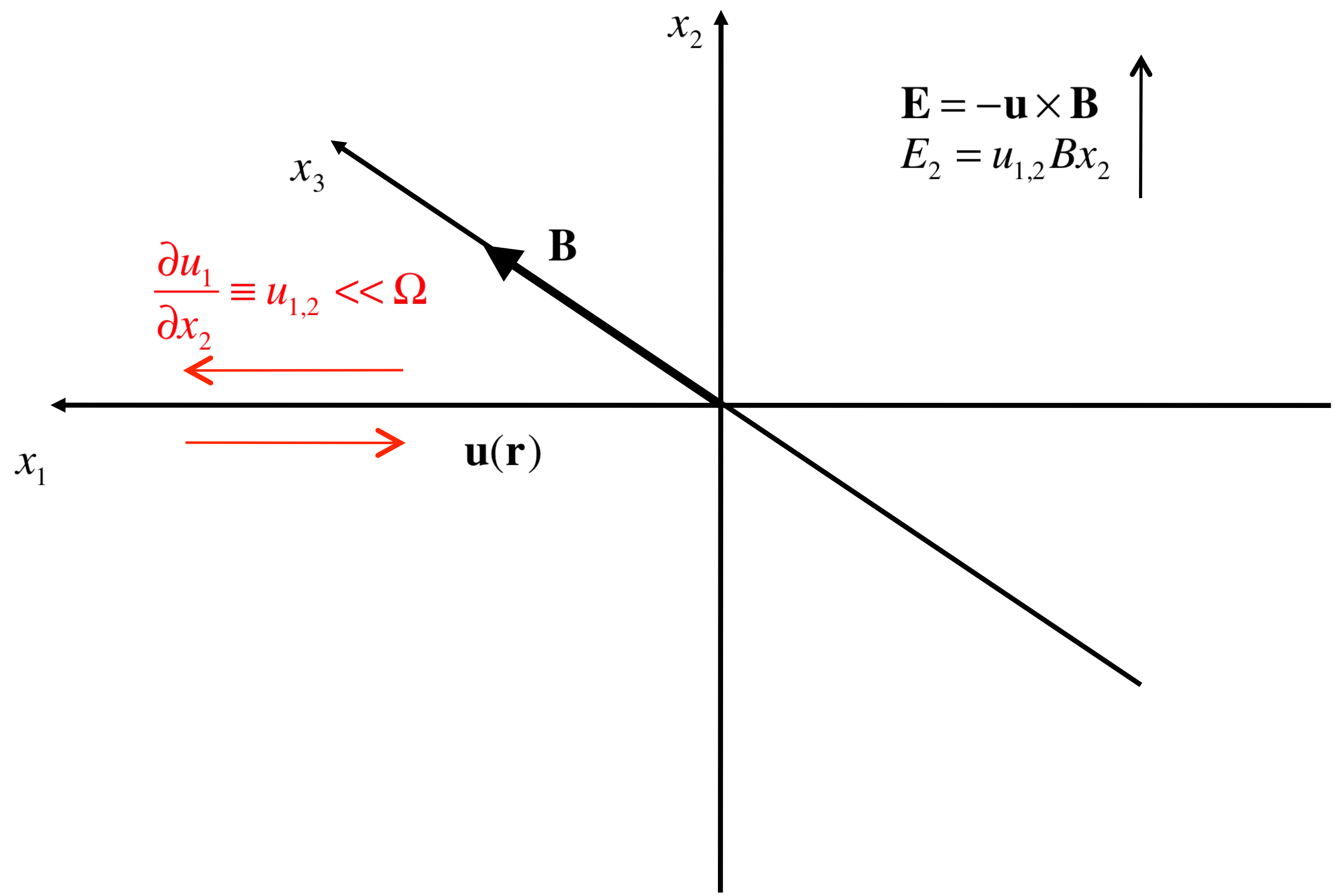
$$\mathbf{J} = T\,\frac{\mathbf{B}\times\nabla n}{B^2}$$

Diamagnetic Heat Flux

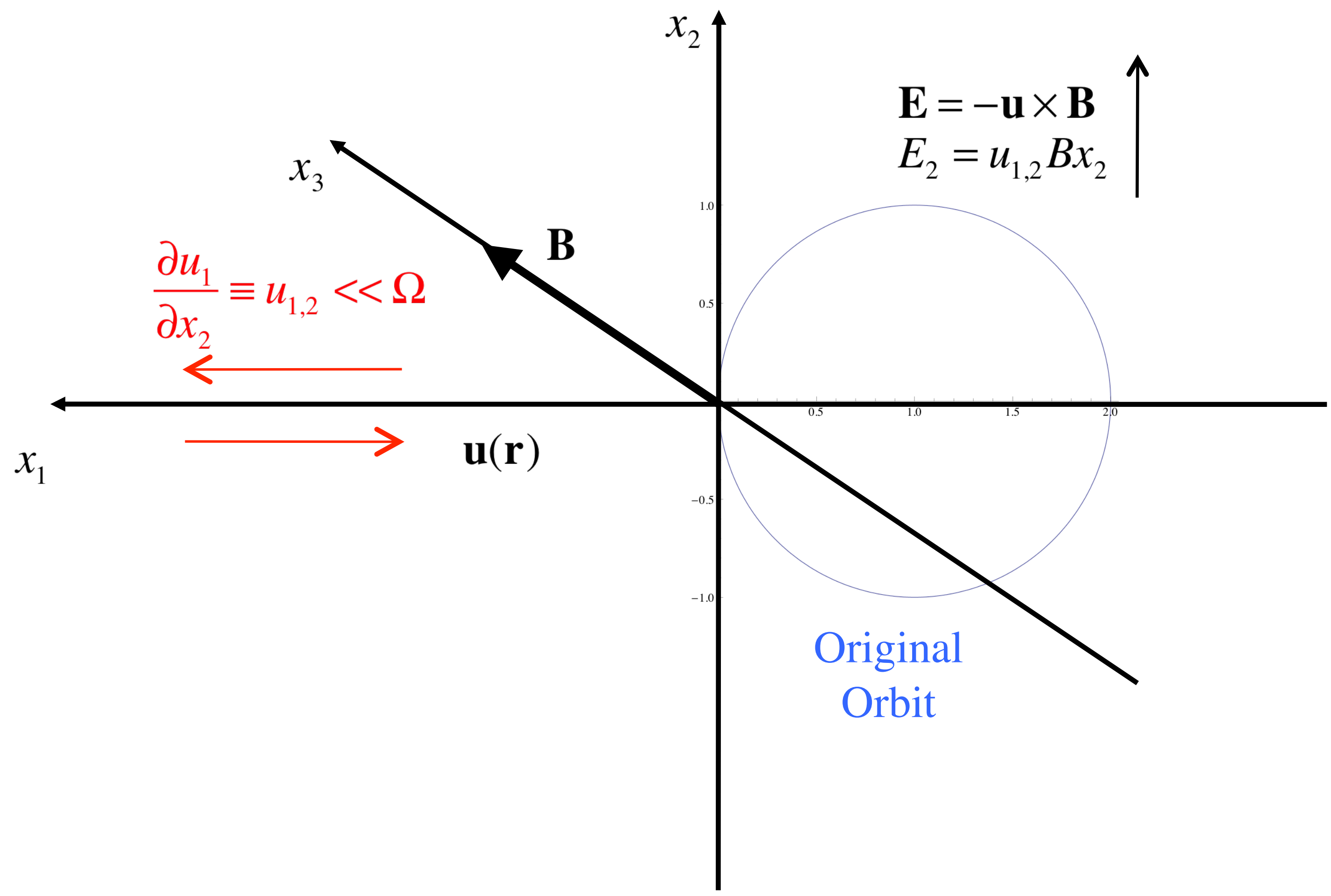


$$\mathbf{q}_{\wedge} = \frac{5}{2}\frac{p_i}{eB}\hat{\mathbf{b}} \times \nabla T_i$$

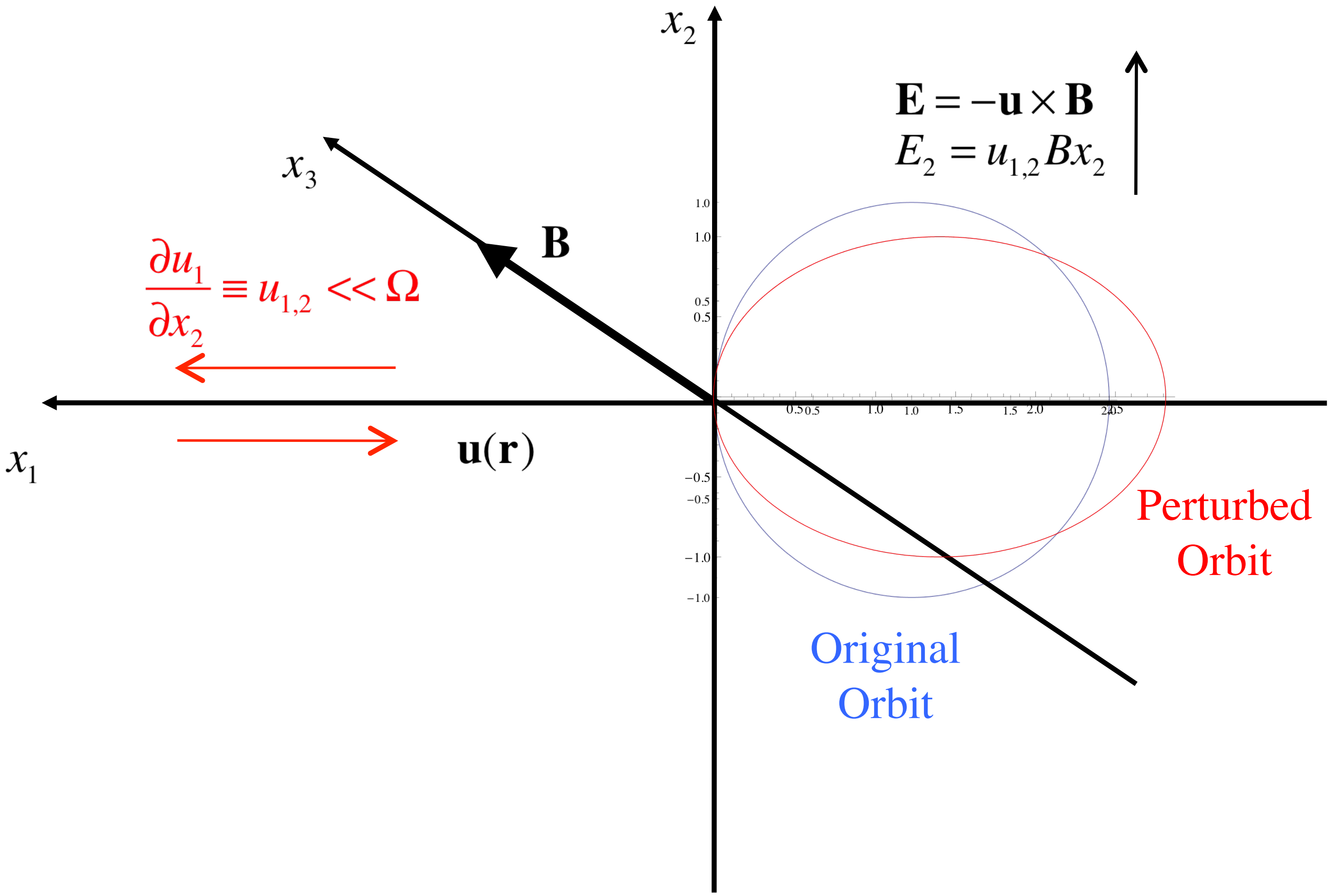
Gyro-viscosity



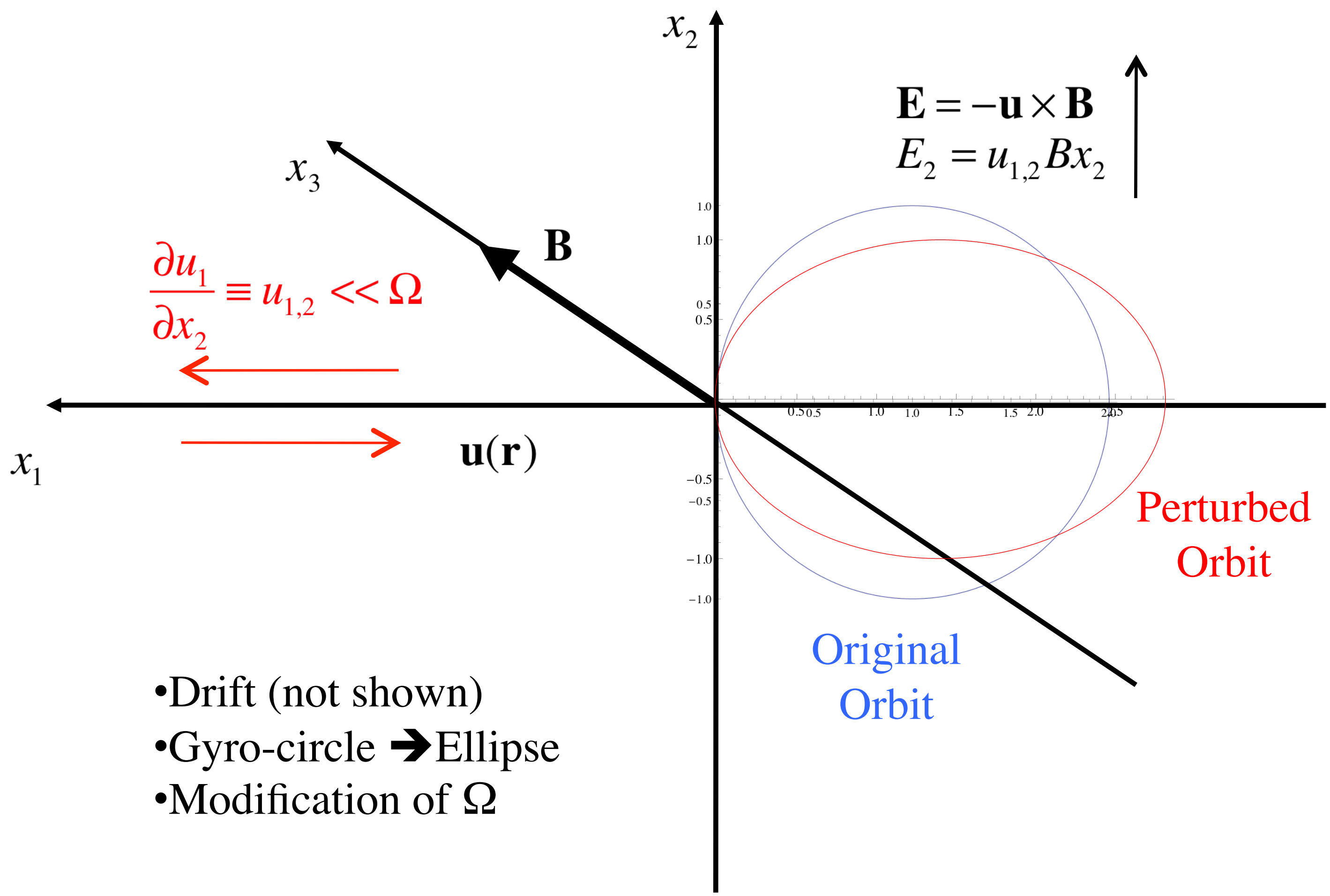
Gyro-viscosity



Gyro-viscosity

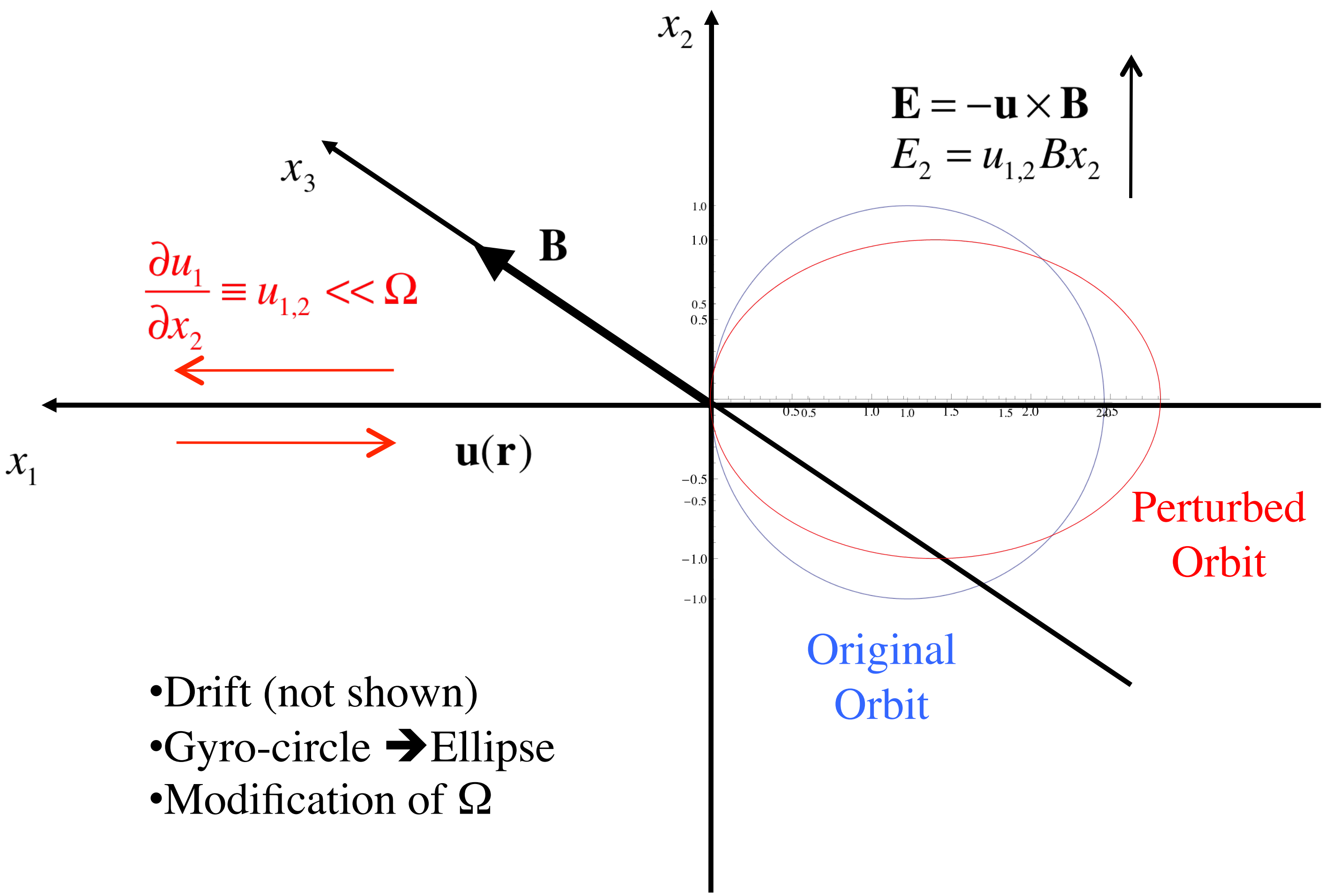


Gyro-viscosity



- Drift (not shown)
- Gyro-circle ➔ Ellipse
- Modification of Ω

Gyro-viscosity



- Drift (not shown)
- Gyro-circle ➔Ellipse
- Modification of Ω

A. N. Kaufman, Phys. Fluids **3**, 610 (1960)

Gyro-viscosity

Stress tensor components:

$$\Pi_{i,j} \sim mn\left\langle v_i v_j \right\rangle = mn\left\langle \dot{x}_i \dot{x}_j \right\rangle$$

Perturbed orbits:

$$x_1 = x_1^{(0)} + u_{1,2} x_2^{(0)} t + \rho\sqrt{\frac{\Omega}{\Omega - u_{1,2}}}\cos\left[\sqrt{\Omega\left(\Omega - u_{1,2}\right)}t + \alpha\right]$$

$$x_2 = x_2^{(0)} + \rho\sin\left[\sqrt{\Omega\left(\Omega - u_{1,2}\right)}t + \alpha\right]$$

Particles passing through origin at $x_1 = x_2 = t = 0$:

$$\dot{x}_1 = \rho\left(\Omega - u_{1,2}\right)\sin\alpha, \qquad \dot{x}_2 = \rho\sqrt{\Omega\left(\Omega - u_{1,2}\right)}\cos\alpha$$

To first order in $u_{1,2}$:

$$\left\langle \dot{x}_1^2 \right\rangle = \frac{1}{2}\left\langle \rho^2 \right\rangle\left(\Omega^2 - 2\Omega u_{1,2}\right), \qquad \left\langle \dot{x}_2^2 \right\rangle = \frac{1}{2}\left\langle \rho^2 \right\rangle\left(\Omega^2 - \Omega u_{1,2}\right)$$

Gyro-viscosity

Stress tensor component:

$$\Pi_{1,1} - \Pi_{2,2} = mn\left\langle \dot{x}_1^2 - \dot{x}_2^2 \right\rangle = -\frac{1}{2}mn\left\langle \rho^2 \right\rangle \Omega u_{1,2} = -\frac{1}{2}mn\left\langle v_\perp^2 \right\rangle u_{1,2} / \Omega$$

$$= -\left(p_\perp / \Omega\right) u_{1,2}$$

Do same calculation for $u_{2,1} \neq 0$:

$$\Pi_{1,1} - \Pi_{2,2} = -\left(p_\perp / \Omega\right) u_{2,1}$$

Combine:

$$\Pi_{1,1} - \Pi_{2,2} = -\frac{2p_\perp}{\Omega} U_{1,2} \equiv -\frac{p_\perp}{\Omega}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

Off-diagonal components found by considering $\left(x_1, x_3\right)$ orbits

Gyro-viscous Cancellation

- In Drift MHD (small deviations fom equilibrium), acceleration ~ stress

$$n\frac{d\mathbf{V}_i}{dt} \equiv n\left[\frac{\partial \mathbf{V}_i}{\partial t} + \left(\mathbf{V}_E + \mathbf{V}_* + \mathbf{V}_{||}\right)\cdot\nabla\mathbf{V}_i\right] \sim -\nabla\cdot\Pi_\wedge$$

- Since $\mathbf{\Pi}_\wedge$ arises from drifts, there is a partial cancellation between the gyro-viscous force and advection by $\mathbf{V}_*$: $n\mathbf{V}_*\cdot\nabla\mathbf{V}_i + \nabla\cdot\Pi_\wedge \approx 0$
- This is *the gyro-viscous cancellation*
- It is often *assumed* to be complete:

$$n\mathbf{V}_*\cdot\nabla\mathbf{V}_i + \nabla\cdot\Pi_\wedge = 0$$

~ GV Cancellation can be seen from Form of GV Stress Tensor

- For unsheared slab equilibrium with $p_i = p_i(x)$:

$$\mathbf{V}_{*i} = \frac{\mathbf{B}\times\nabla p}{neB^2} = \frac{1}{neB}\frac{dp}{dx}\hat{\mathbf{e}}_y \quad , \qquad \Pi_{xx} = -\frac{p}{2\Omega}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)$$

$$Mn\left(\mathbf{V}_{*i}\cdot\nabla\mathbf{V}_i\right)_y + \nabla\cdot\Pi_\wedge \approx \frac{1}{\Omega}\frac{dp}{dx}\frac{\partial V_x}{\partial y} - \frac{1}{2}\frac{1}{\Omega}\frac{dp}{dx}\frac{\partial V_x}{\partial y} + \cdots$$

- Gyro-viscous cancellation is incomplete, *but* …
- Assuming it is exact is often a good approximation, and simplifies the algebra

Diamagnetic Heat Flux Cancellation

- Diamagnetic heat flux in ion energy cancels advection by diamagnetic drift in continuity equation
- Electrostatic: $\nabla \cdot \mathbf{q}_\wedge = \frac{5}{2}\frac{1}{B}\hat{\mathbf{b}} \cdot \nabla T_i \times \nabla n$
- Solve continuity for $\nabla \cdot \mathbf{V}_i$, substitute into ion energy:

$$\frac{\partial n}{\partial t} = -n\nabla \cdot \mathbf{V}_i - \left(\mathbf{V}_i + \mathbf{V}_{*i}\right) \cdot \nabla n \quad , \qquad \left(\nabla \cdot \mathbf{V}_{*i} = 0\right)$$

$$\frac{\partial p_i}{\partial t} + \left(\mathbf{V}_i + \mathbf{V}_{*i}\right) \cdot \nabla p_i = -\frac{5}{3}\nabla \cdot \mathbf{V}_i - \frac{2}{3}\nabla \cdot \mathbf{q}_\wedge$$

$$= \frac{5}{3}\frac{p_i}{n}\frac{\partial n}{\partial t} + \frac{5}{3}\frac{p_i}{n}\mathbf{V}_i \cdot \nabla n + \underbrace{\frac{5}{3}\frac{p_i}{n}\mathbf{V}_{*i} \cdot \nabla n - \frac{2}{3}\nabla \cdot \mathbf{q}_\wedge}$$

- Then

$$\mathbf{V}_{*i} \cdot \nabla n = \frac{1}{neB}\hat{\mathbf{b}} \times \nabla p_i \cdot \nabla n = \frac{1}{eB}\hat{\mathbf{b}} \cdot \nabla T_i \times \nabla n$$

$$\frac{5}{3}\frac{p_i}{n}\mathbf{V}_{*i} \cdot \nabla n - \frac{2}{3}\nabla \cdot \mathbf{q}_\wedge = \frac{5}{3}\frac{T_i}{eB}\hat{\mathbf{b}} \cdot \nabla T_i \times \nabla n - \left(\frac{2}{3}\right)\left(\frac{5}{2}\right)\frac{T_i}{eB}\hat{\mathbf{b}} \cdot \nabla T_i \times \nabla n = 0$$

- Diamagnetic heat flux cancellation is complete in electrostatics

FLR Effects on Modes

- Interchange type modes
  - g-mode: $\omega(\omega\text{-}\omega_*)+\gamma^2_{\text{MHD}} = 0, \quad \gamma^2_{\text{MHD}}=g/L_{n0}$
    - *Unstable* in MHD ($\omega_* = 0$)
    - *Stable* if $\omega_* > 2\,\gamma_{\text{MHD}}$
  - MRI, driven by plasma rotation (Ferraro)
    - Gyro-viscosity completely stabilizing if $\beta >> 1$
- Parallel sound waves
  - *Stable* in MHD and Hall MHD
  - *Destabilized* by FLR (GV *and* IDHF)
  - ITG-like fluid modes

ITG-like Fluid Mode

- Consider modes driven by ion temperature gradient in slab geometry
  - No density gradient, $n_0(x) = n_0$
  - Constant electron temperture, $T_{e0}(x) = T_{e0}$
  - No magnetic shear, $\mathbf{B}_0 = B_{z0}(x)\, \mathbf{e}_z$
  - Ion temperature gradient, $T_{i0}(x) = T_{i0}\, e^{x/L}$
  - Parallel sound wave driven *unstable* by FLR effects (compare with g-mode)
- Originally derived from kinetic theory
  - L. I. Rudakov and R. Z. Segdeev, Sov. Phys. – Doklady **6**, 415 (1961)
  - B. Coppi, M. N. Rosenbluth, and R. Z. Segdeev, Phys. Fluids **10**, 582 (1967) (First fluid derivation)
- Very important mode in tokamak transport (toroidal effects, magnetic shear, etc.)….one of the most studied modes in plasma physics
- *Stable in ideal, resistive, and Hall MHD!*
- Requires FLR effects for *instability*
  - Ion gyro-viscous stress
  - Ion diamagnetic heat flux
- Good mode for verification of extended fluid model

Fluid Dispersion Relation

$$\omega^3 - \frac{1}{2}k_z^2\left(\beta_e + \frac{5}{3}\beta_i\right)\omega - \frac{1}{4}k_y k_z^2 \beta_i \left(\beta_e + \frac{5}{3}\beta_i\right)\left(1 + \frac{1}{6}\beta_i\right)\frac{d_i}{L_{Ti0}} = 0$$

Fluid Dispersion Relation

$$\boxed{\omega^3 - \frac{1}{2}k_z^2\left(\beta_e + \frac{5}{3}\beta_i\right)\omega} - \frac{1}{4}k_y k_z^2 \beta_i \left(\beta_e + \frac{5}{3}\beta_i\right)\left(1 + \frac{1}{6}\beta_i\right)\frac{d_i}{L_{Ti0}} = 0$$

$$\omega^2 = \frac{1}{2}C_s^2 k_z^2$$

Sound Wave

Fluid Dispersion Relation

$$\boxed{\omega^3} - \boxed{\frac{1}{2}k_z^2\left(\beta_e + \frac{5}{3}\beta_i\right)\omega - \frac{1}{4}k_y k_z^2 \beta_i\left(\beta_e + \frac{5}{3}\beta_i\right)\left(1 + \frac{1}{6}\beta_i\right)\frac{d_i}{L_{Ti0}}} = 0$$

$$\omega^2 = \frac{1}{2}C_s^2 k_z^2$$

Sound Wave

$$\omega = -2k_y\beta_i\left(1 + \frac{1}{6}\beta_i\right)\frac{d_i}{L_{Ti0}}$$

Low freq. “drift” mode

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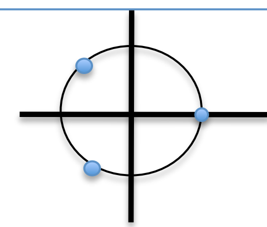
“ITG”



Fluid Dispersion Relation

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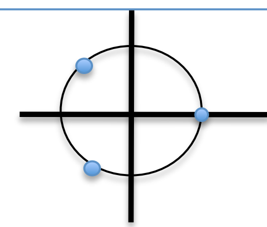
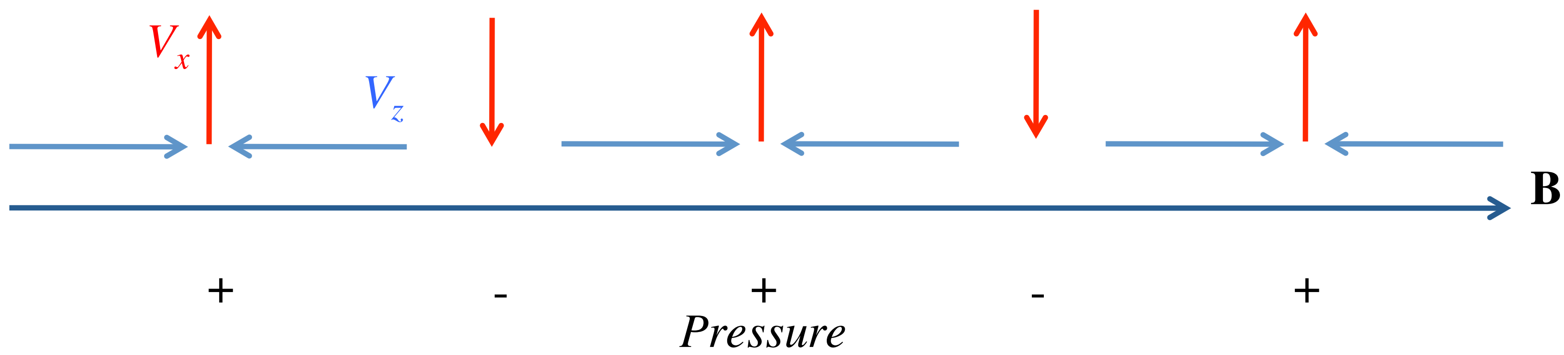
"ITG"



- Electro-static
- Ballooning ordering: $k_z \sim 1$, $k_y \sim 1/\varepsilon^2$, $d_i/L \sim \varepsilon^2$
- Local approximation: $f \sim e^{i(k\_y\, y\, +\, k\_z\, z)}$
- No gyro-viscous or diamagnetic cancellations assumed

Physical Picture(?)

Parallel sound wave, no pressure gradient:



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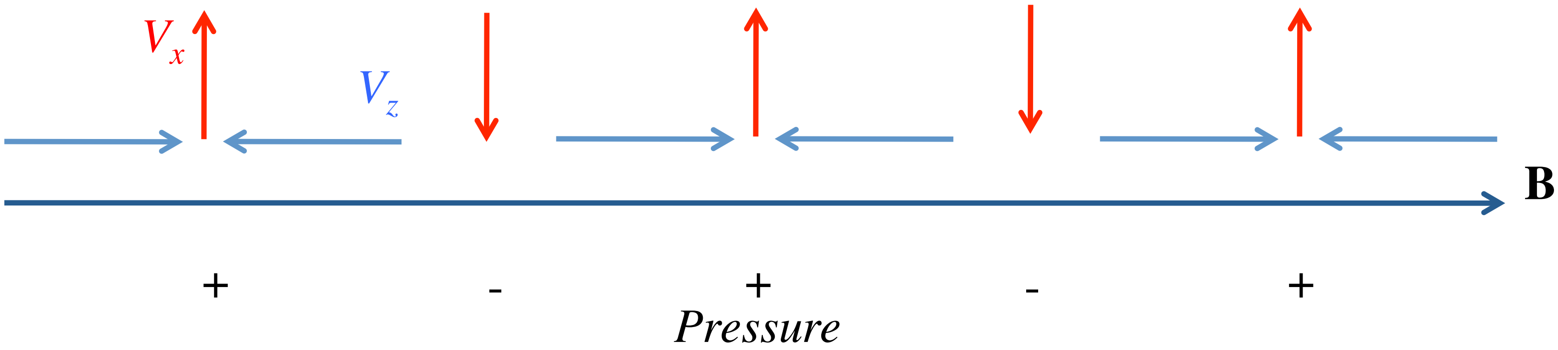

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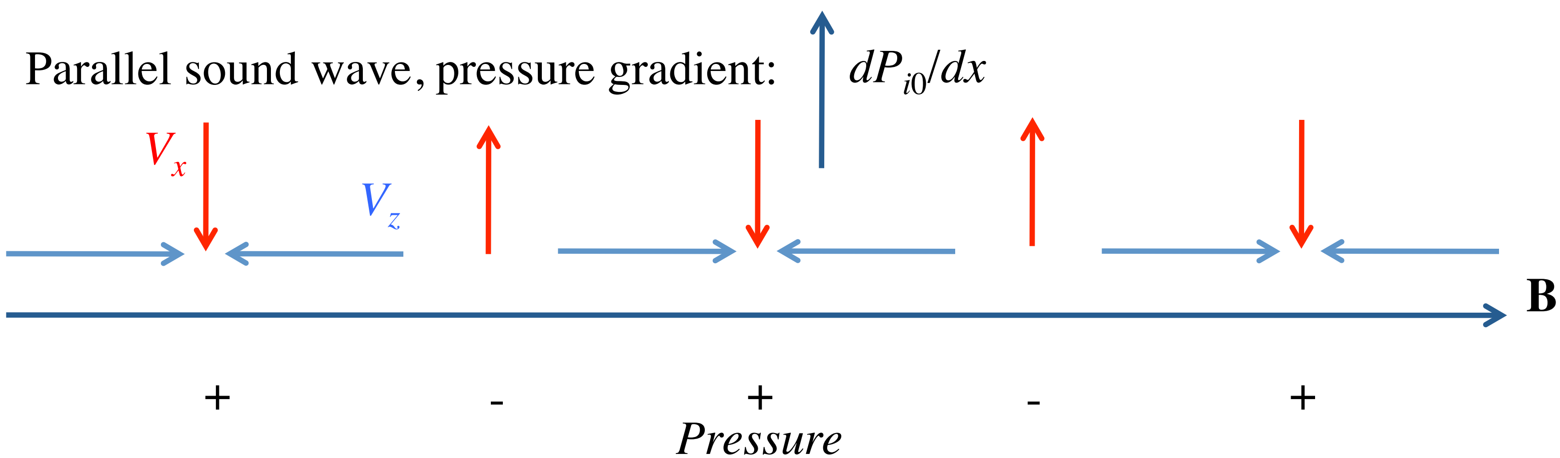

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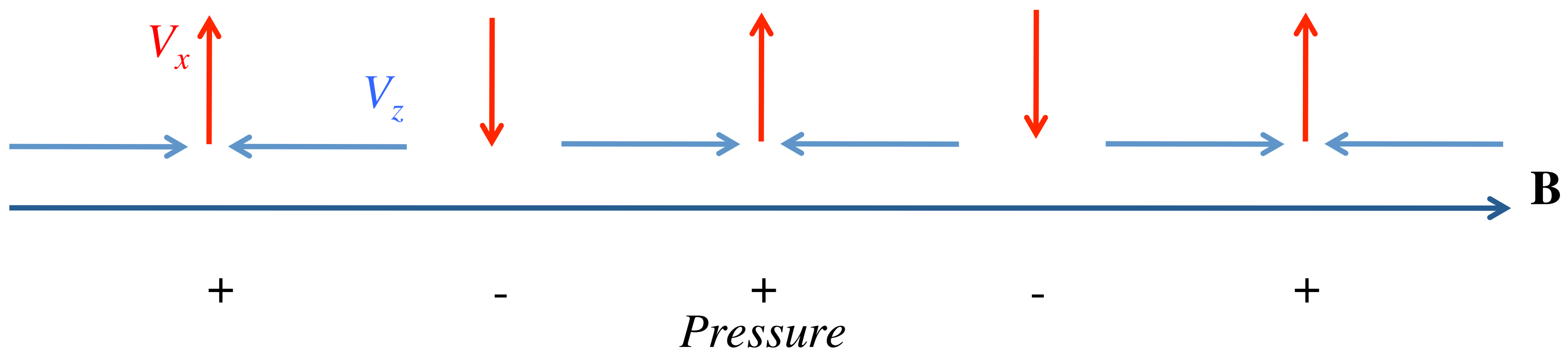
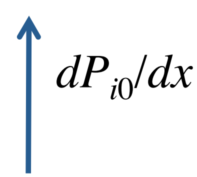
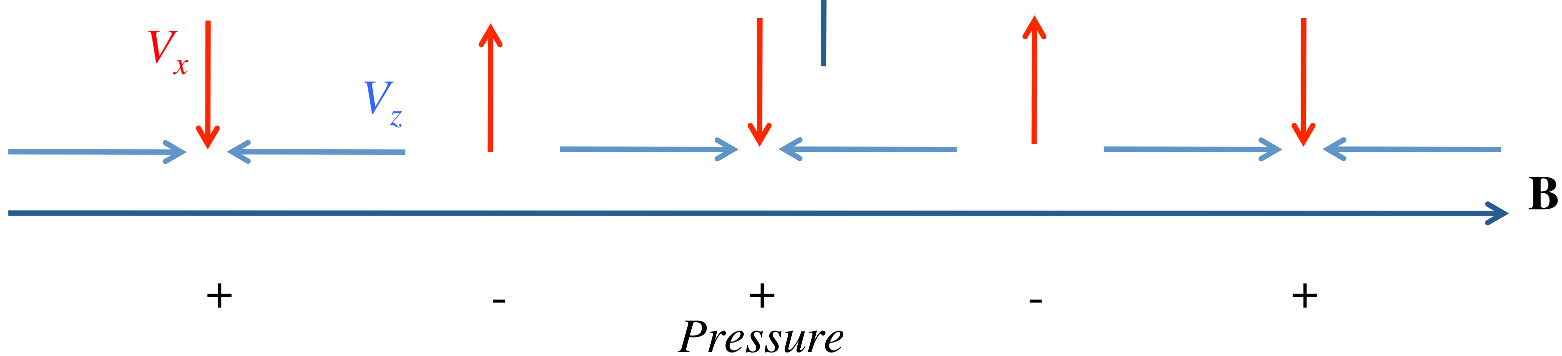


Parallel sound wave, pressure gradient:





π phase shift in V_{xi}: $\delta p = V_{xi}\, dP_{i0}/dx$ reinforces pressure perturbation;
FLR cancels diamagnetic advective contributions

Electrostatic Marginal Stability

- Cubic of form: $w^3 - 3Aw + B = 0$

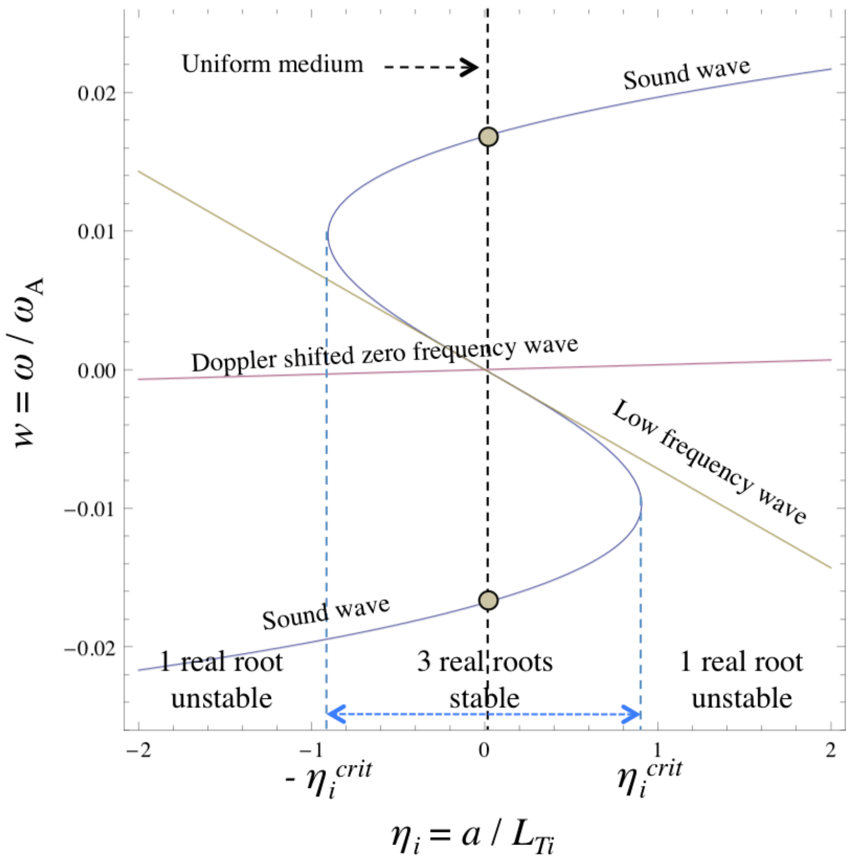
$$w = \sqrt{A}\left(z + \frac{1}{z}\right) \quad \rightarrow \quad \xi^2 + \frac{B}{A^{3/2}}\xi + 1 = 0 \quad , \quad \xi = z^3$$

$$z^3 = -\frac{B}{2A^{3/2}} + \sqrt{\frac{B^2}{4A^3} - 1} \quad \rightarrow \quad z = \left(-\frac{B}{2A^{3/2}} + \sqrt{\frac{B^2}{4A^3} - 1}\right)^{1/3} e^{2\pi i l/3} \quad , \; l = 0,1,2$$

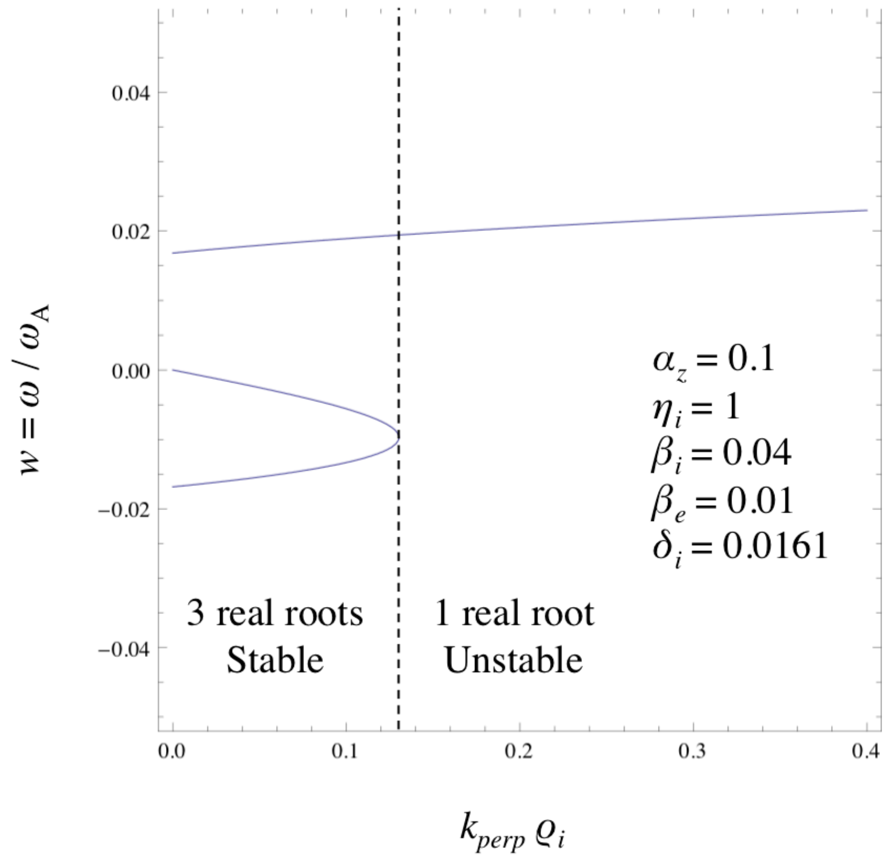
- Unstable if $\dfrac{B^2}{4A^3} > 1$
- Approximate instability condition:

$$\eta_i \rho_i \equiv \frac{\rho_i}{L_{Ti}} > \eta_i^{crit} \rho_i \approx \frac{k_\parallel}{k_\perp}$$

Behavior of Roots, $f_3(w;\eta)=0$

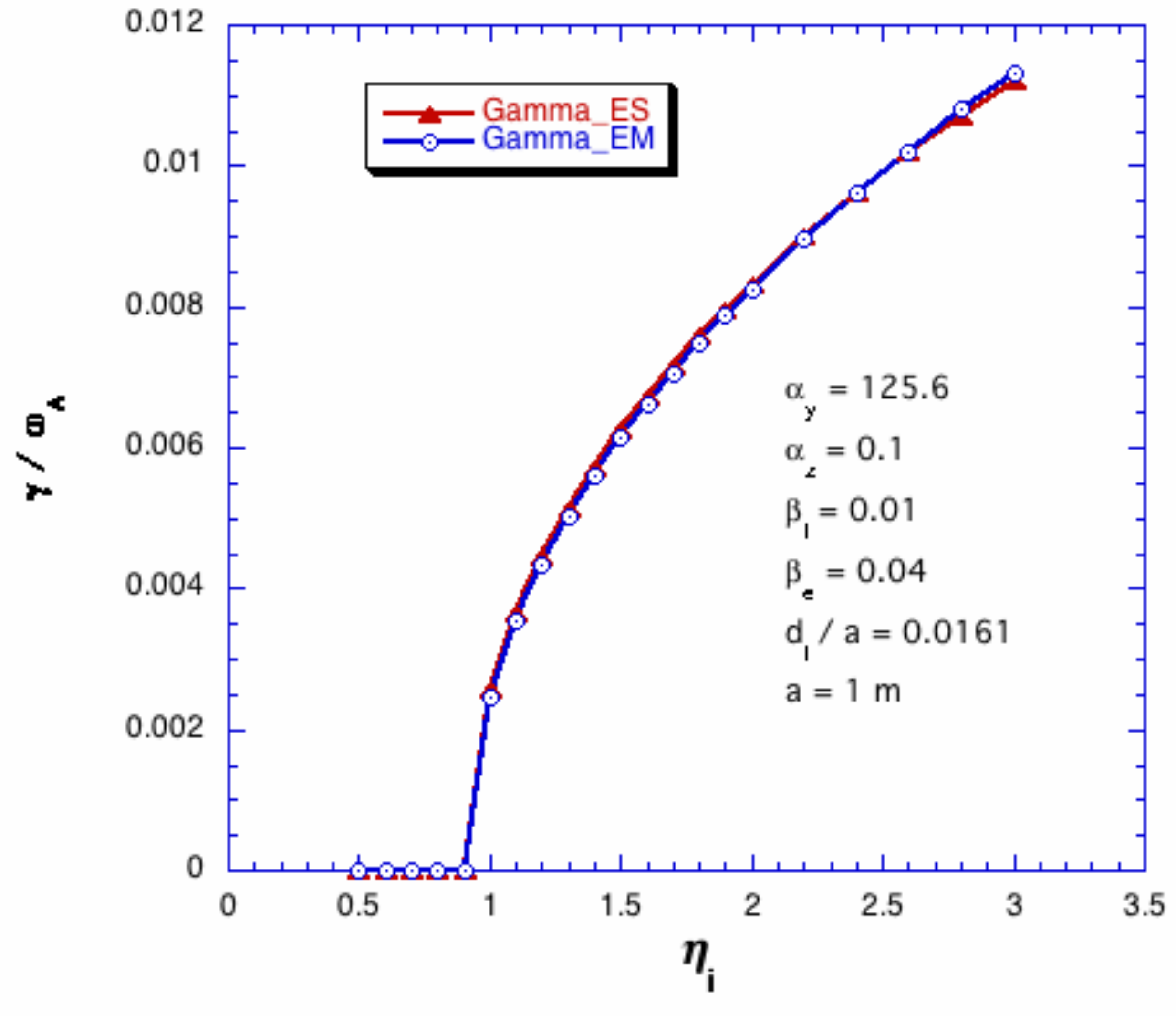


Threshold in η_i

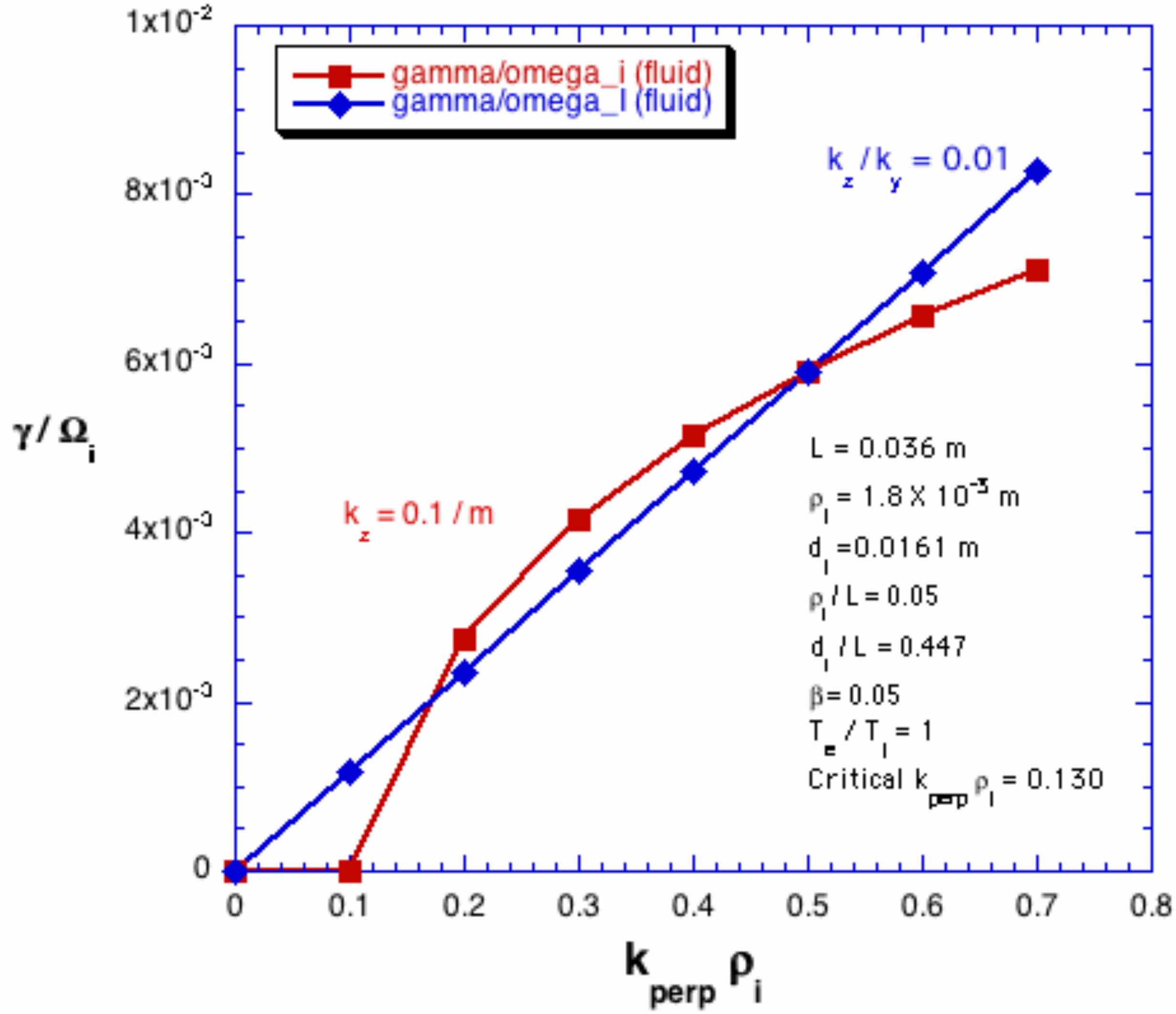


Threshold in $k_{perp}\,\varrho_i$

ITG is Electrostatic

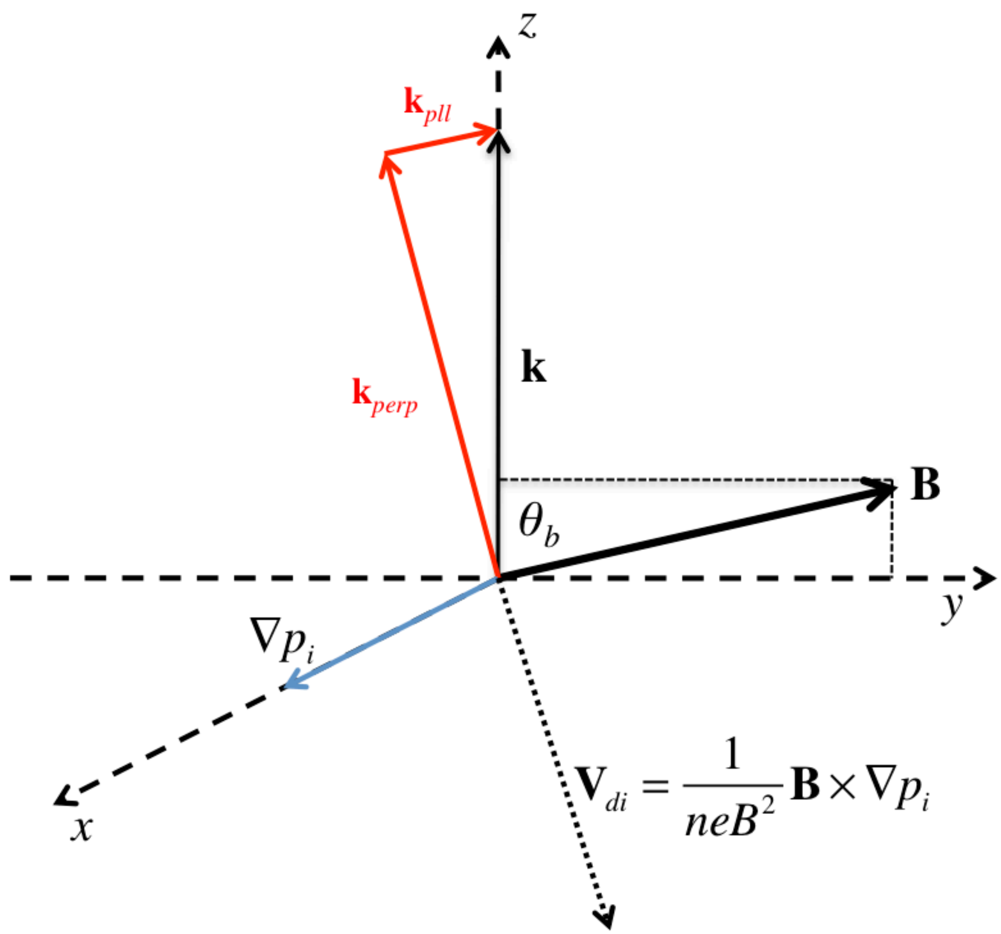


Growth Rate Scaling Depends on How k_z/k_y Varies



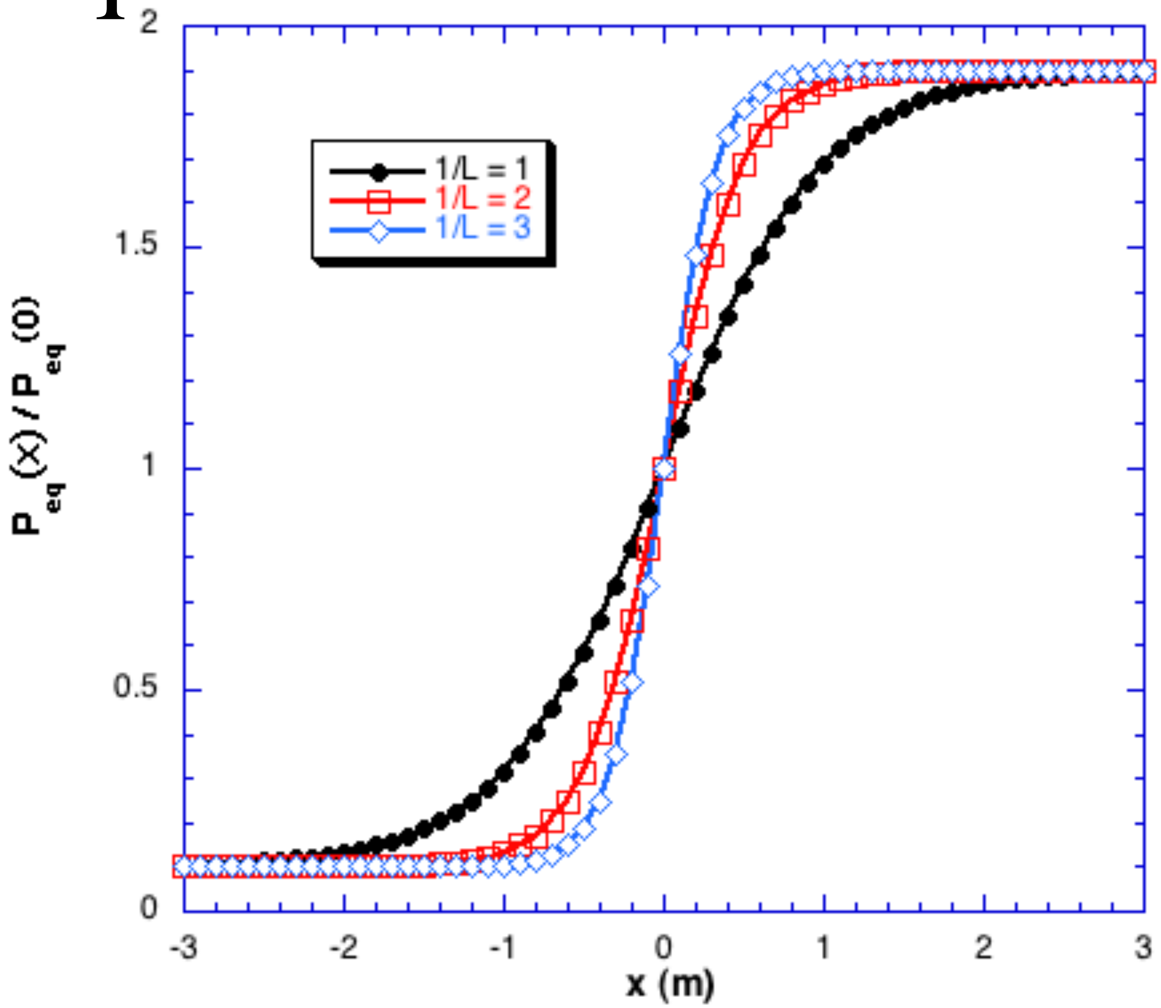
NIMROD Results

Computational Geometry



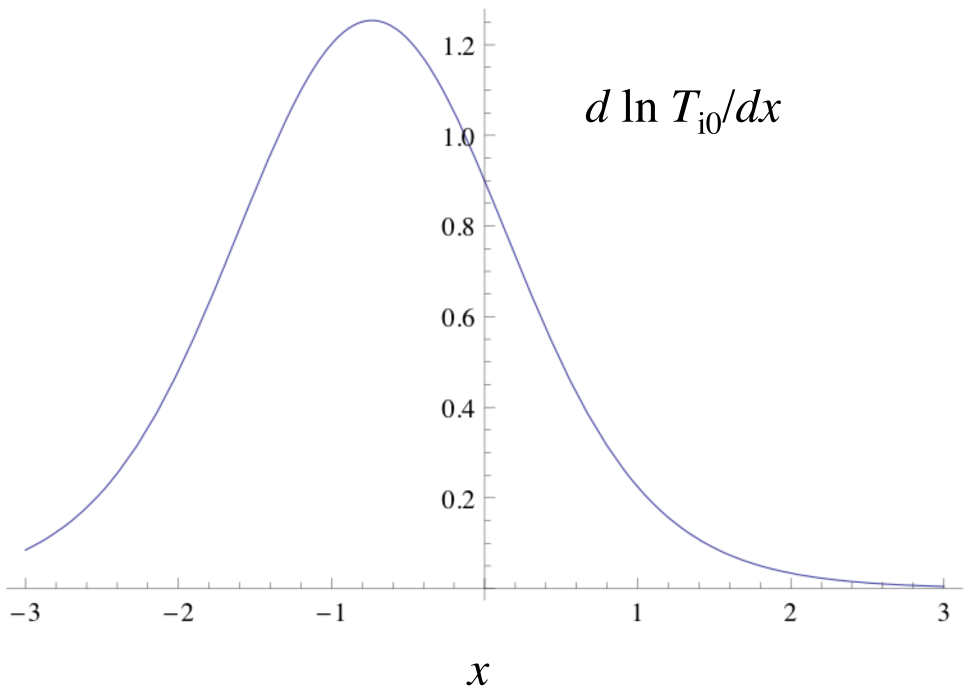
Computational problem is 2D in (x,z) plane,
1-D in x and k_z

Equilibrium Pressure Profile



$$p_{i0}(x) = p_{i0}(0)\left[1 + 0.9\tanh\frac{x}{L_{Ti0}}\right]$$

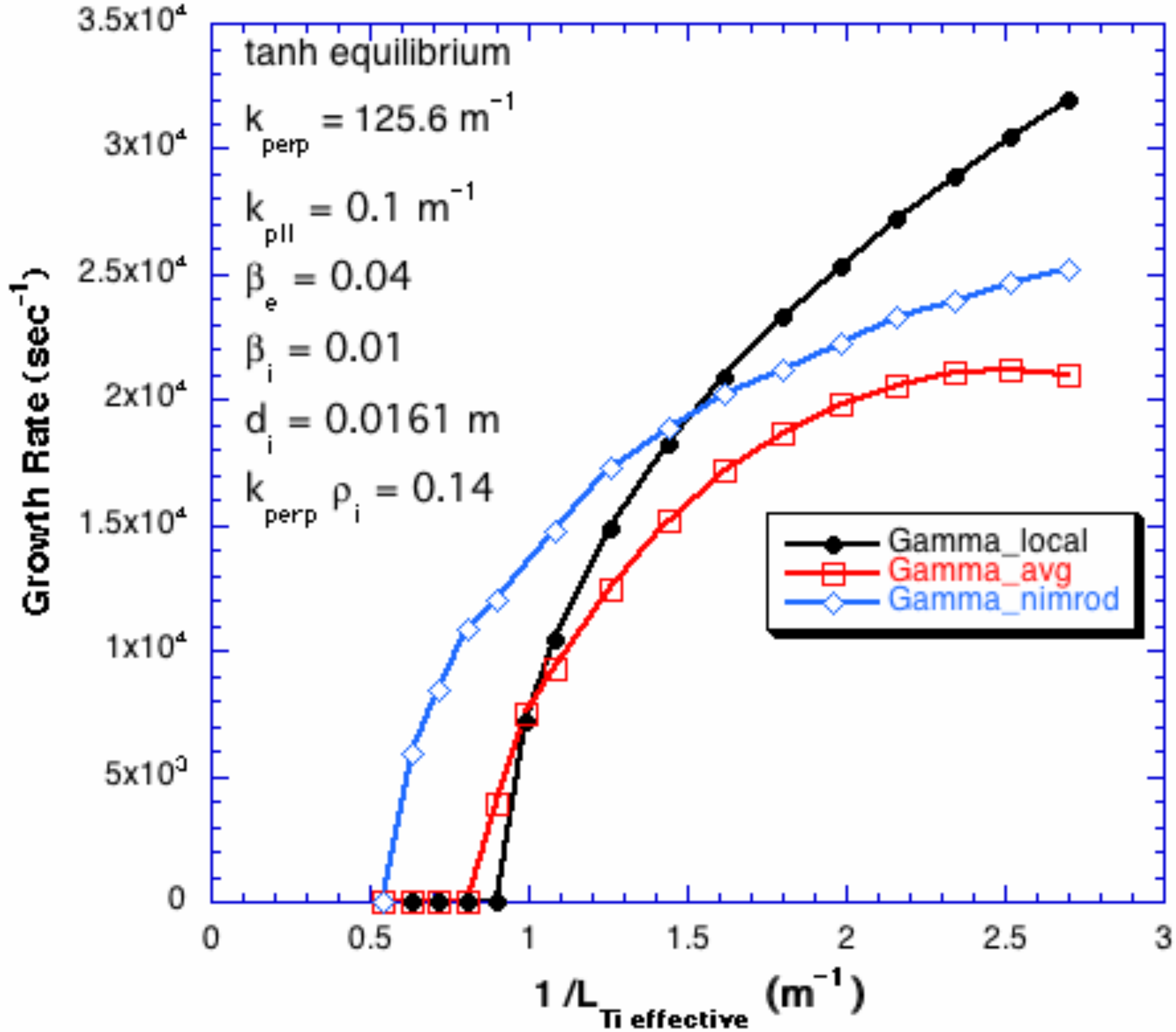
Instability Drive in Tanh Model



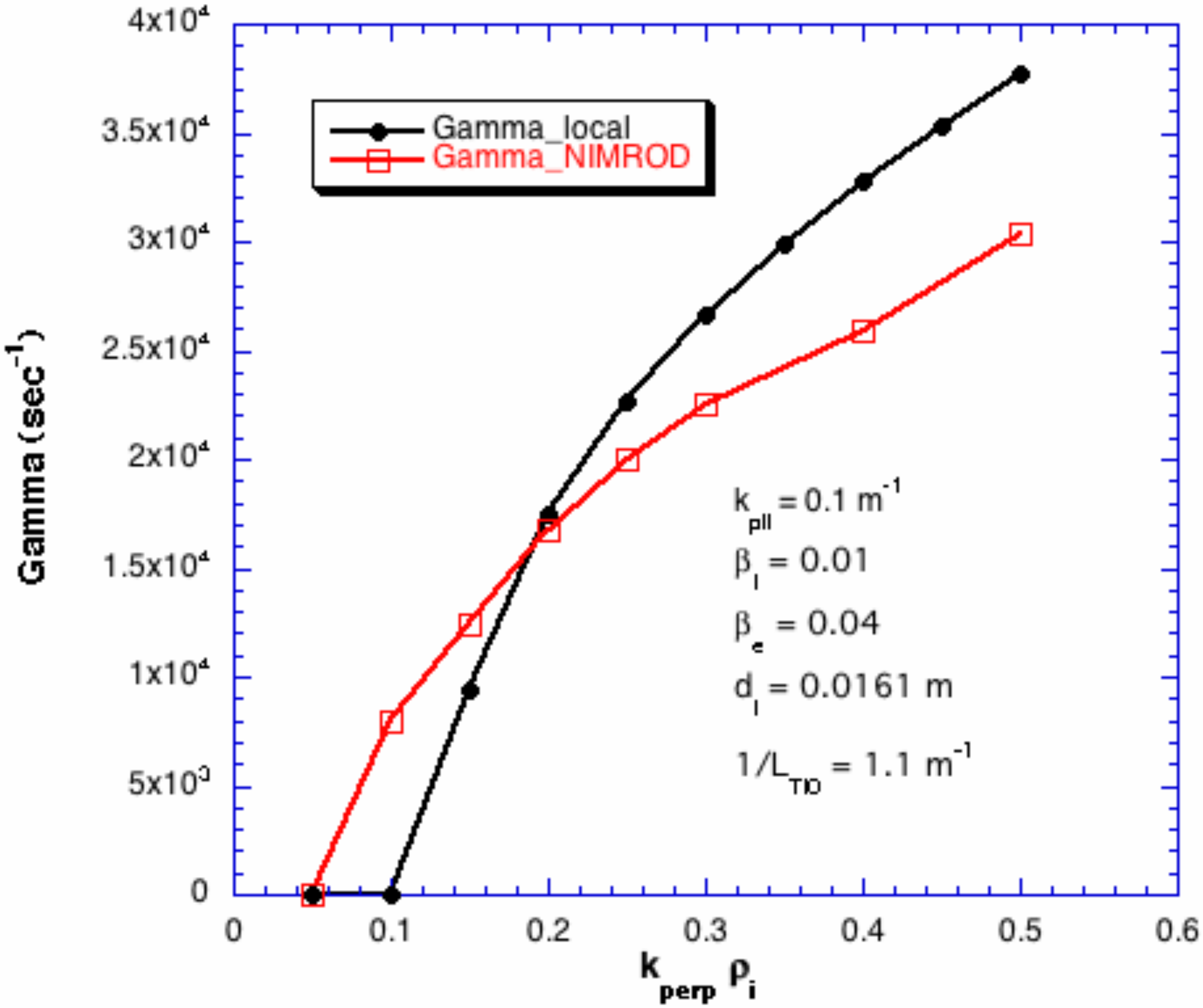
Biased towards $x < 0$

Theory/Computation Comparison:

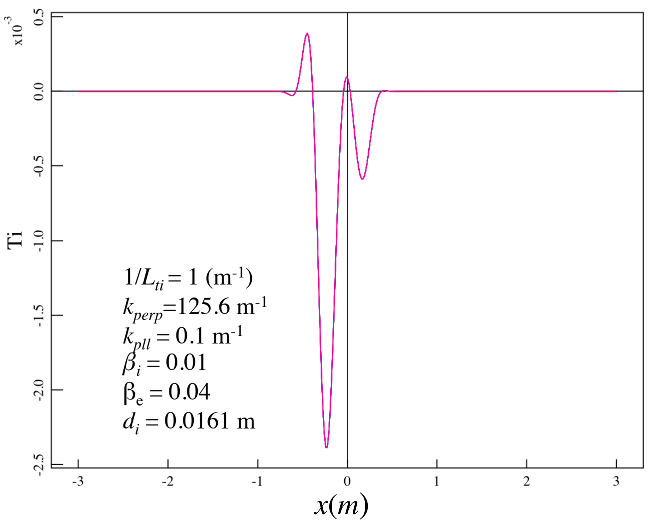
$\gamma_{LOCAL}, \gamma_{AVG}, \gamma_{NIMROD}$ vs. η_i

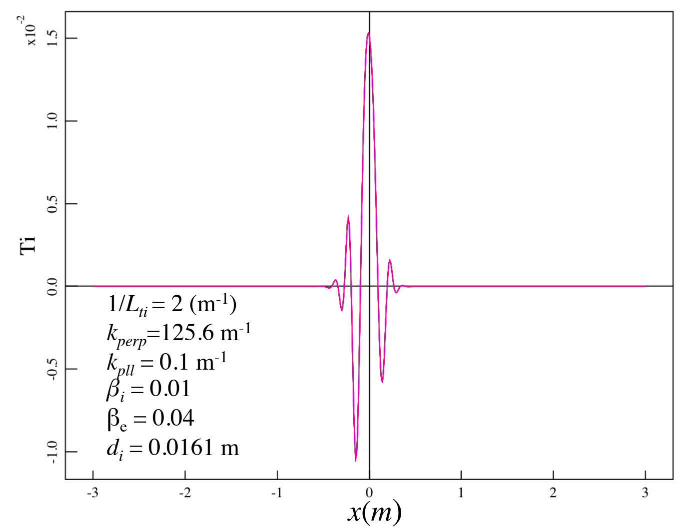


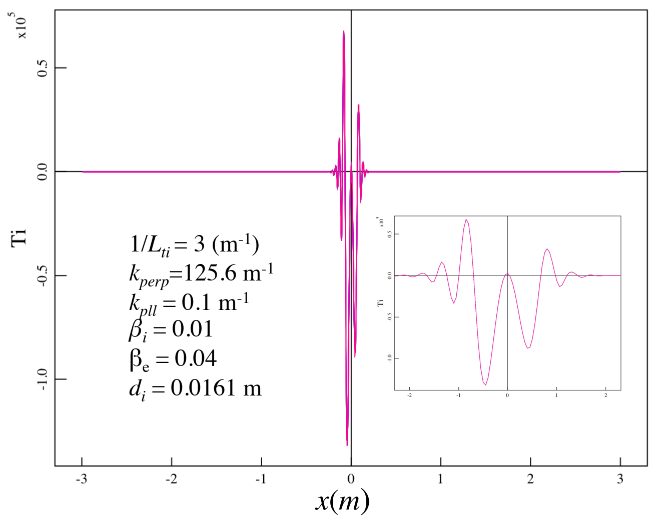
Theory/Computation Comparison: $\gamma_{LOCAL}, \gamma_{NIMROD}$ vs. $k_{perp}\,\varrho_i$



Computational Eigenfunction Structure

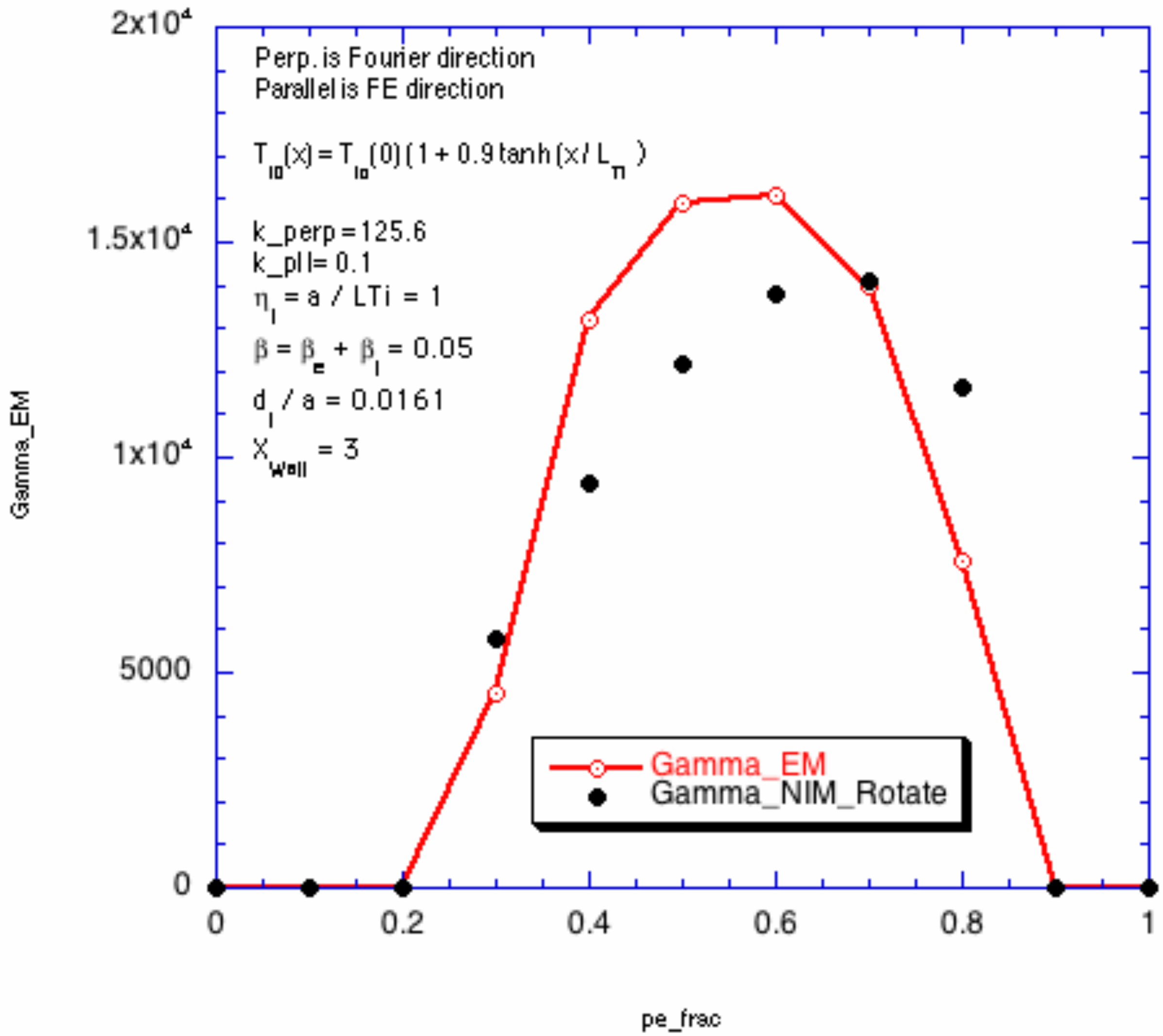






- Perturbed ion temperature
- More structure as $1/L$ increases
- Resolved
- Local theory gives no eigenfunction structure
- Slightly biased toward $x < 0$.

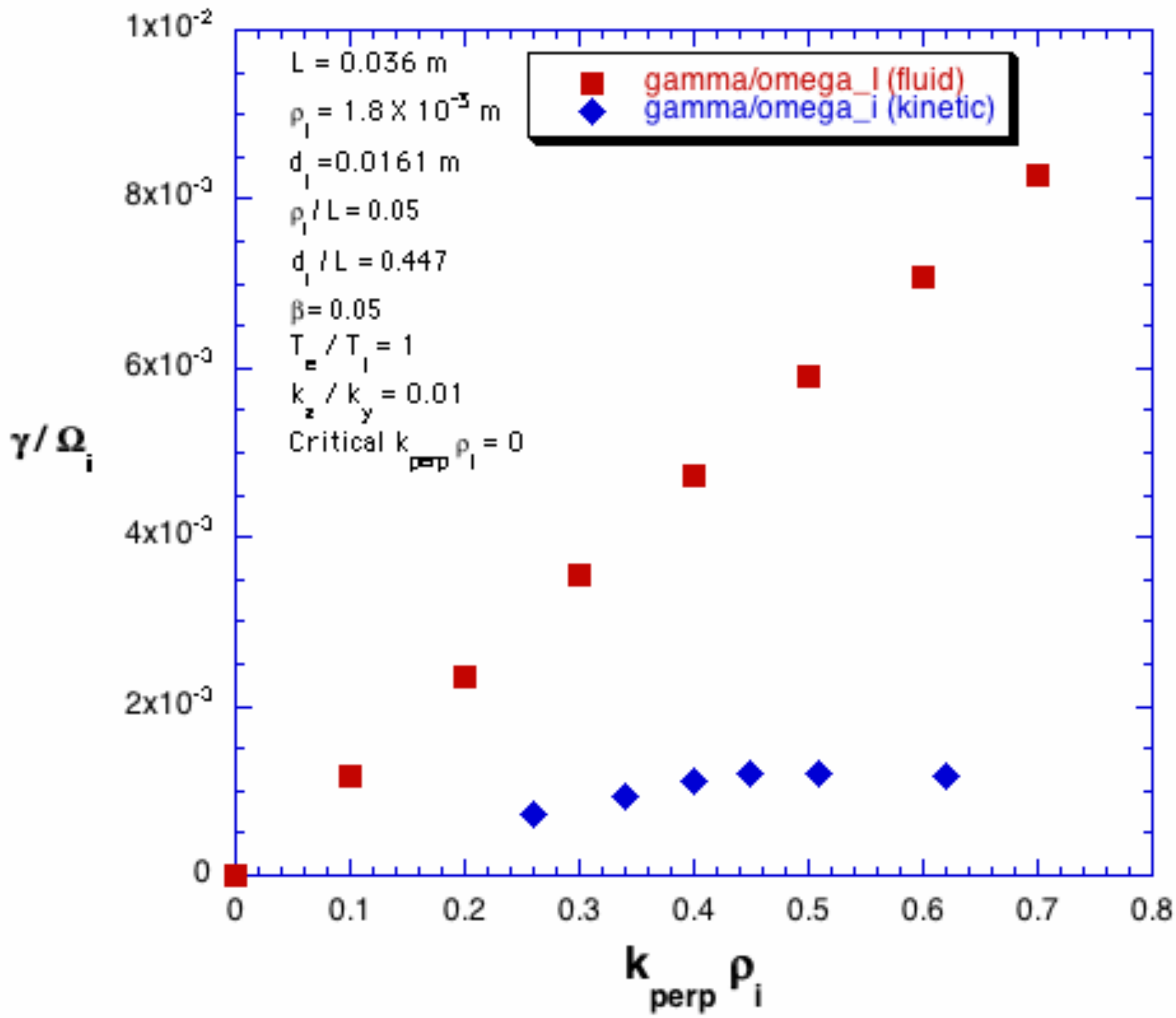
Computation/Theory Comparison: $\gamma_{LOCAL}, \gamma_{NIMROD}$ vs. $f_e = \beta_e/\beta$



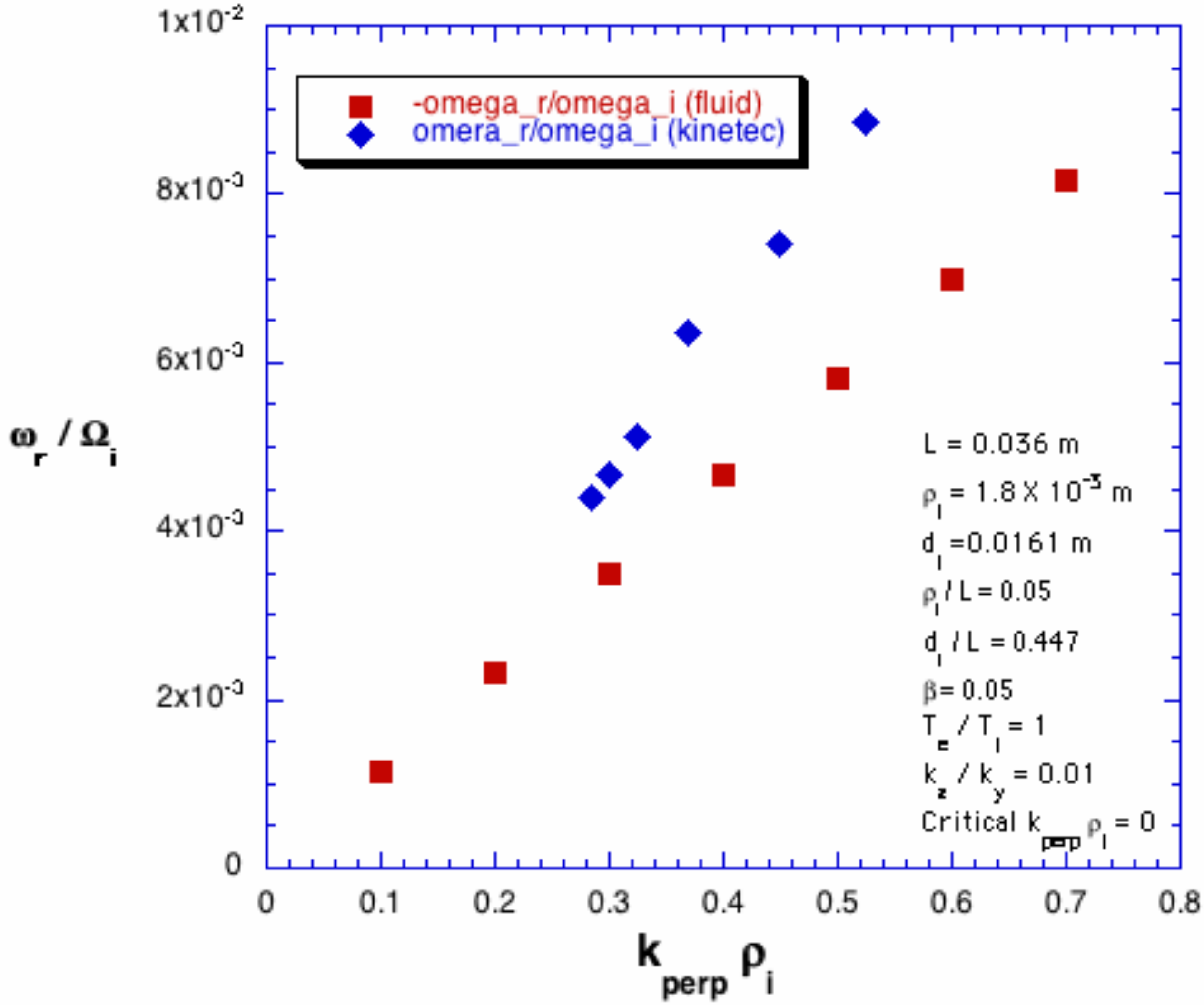
Comparison with Kinetic Theory

- Preliminary!
- Comparison between local analytic fluid and kinetic models (Cheng, Parker)
- No computational comparisons yet…..
- Still a lot of work to be done!

Growth Rate, $k_z/k_y = 0.01$



Real Frequency, $k_z/k_y = 0.01$

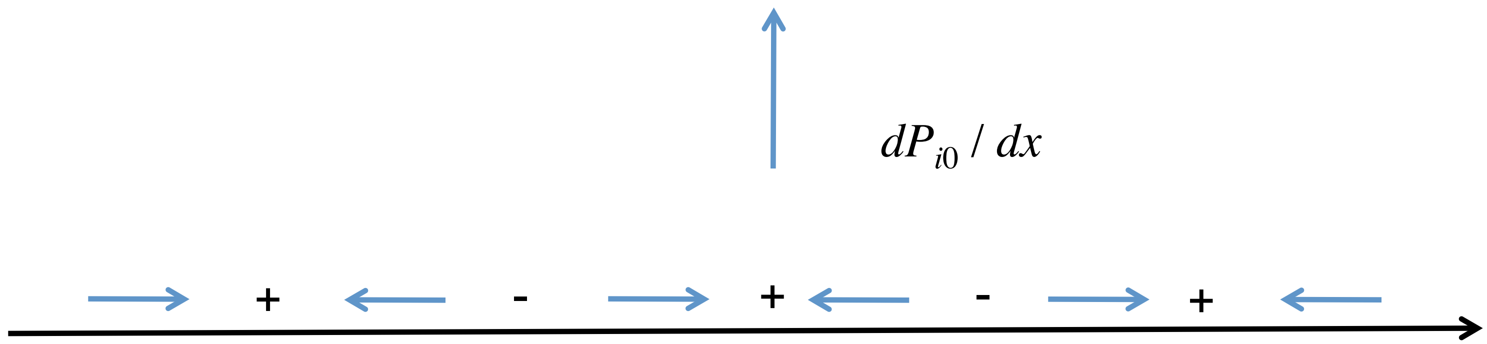


Discussion

- Fluid theory requires both:
  - Ion gyro-viscous stress
  - Ion diamagnetic heat flux
  - Only details depend on form of "gyro-viscous cancellation"
- Instability threshold in both $1/L_{Ti0}$ and $k_{perp}\, \varrho_i$ (at fixed k_z)
- Reasonable agreement between NIMROD and local theory on growth rate behavior
- Comparison not possible on eigenmode structure
- *NIMROD is verified where theory and computation can be compared reasonably*
- Fluid theory has no natural stabilizing mechanism at high k
  - Implications for non-linear extended MHD computations
- Preliminary comparison of local fluid and kinetic analytic models
- Await direct comparison between NIMROD and kinetic codes

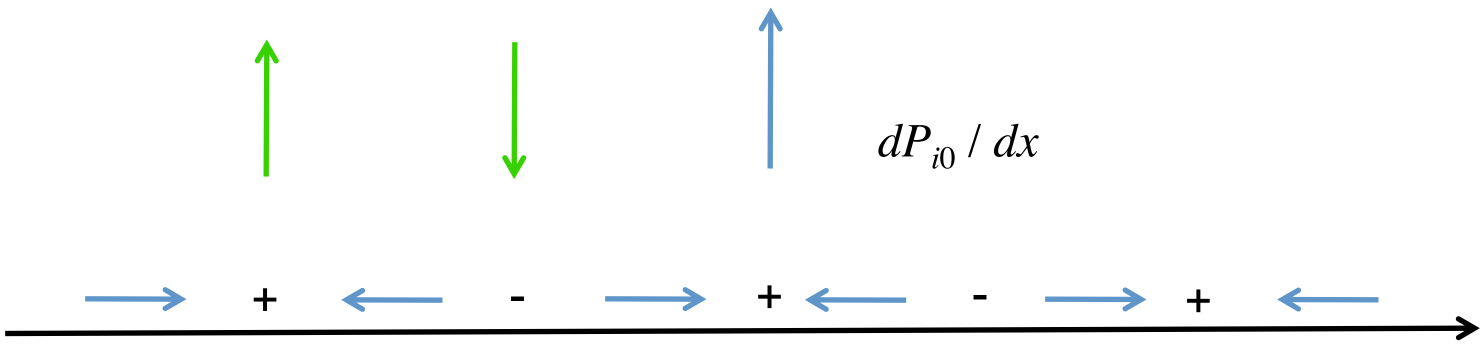
The End

Physical Behavior(?)



Parallel sound wave: Pressure and V_z perturbation parallel to **B**

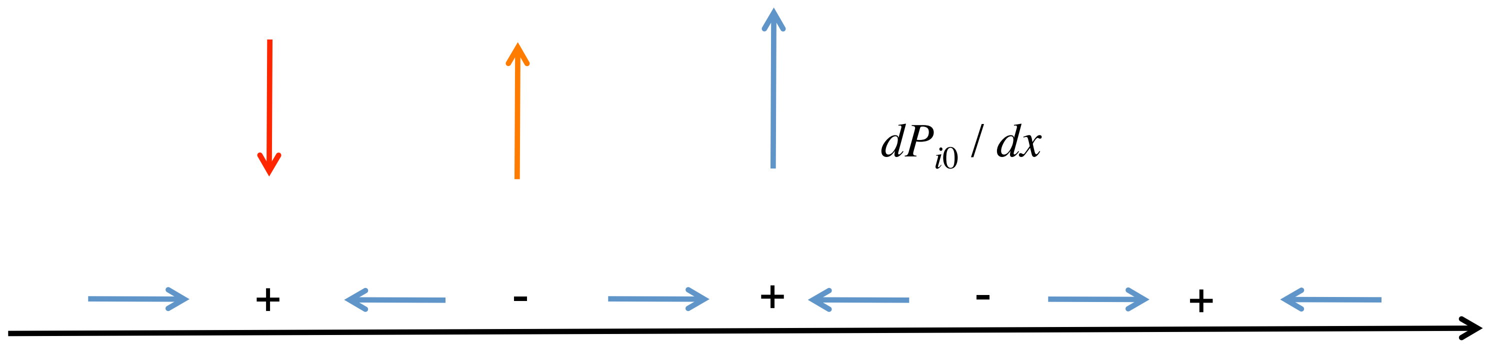
Physical Behavior(?)



Parallel sound wave: Pressure and V_z perturbation parallel to **B**

V_x when $dP_{i0}/ dx = 0$

Physical Behavior(?)



Parallel sound wave: Pressure and V_z perturbation parallel to **B**

V_x when $dP_{i0}/ dx = 0$

V_x when $dP_{i0}/ dx \neq 0$

- dP_{i0} / dx induces phase shift in V_x
- $\delta p = V_x \, dP_{i0} / dx$ re-inforces pressure perturbation
- Instability