Sawteeth, Snakes, and FTLEs

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Topics

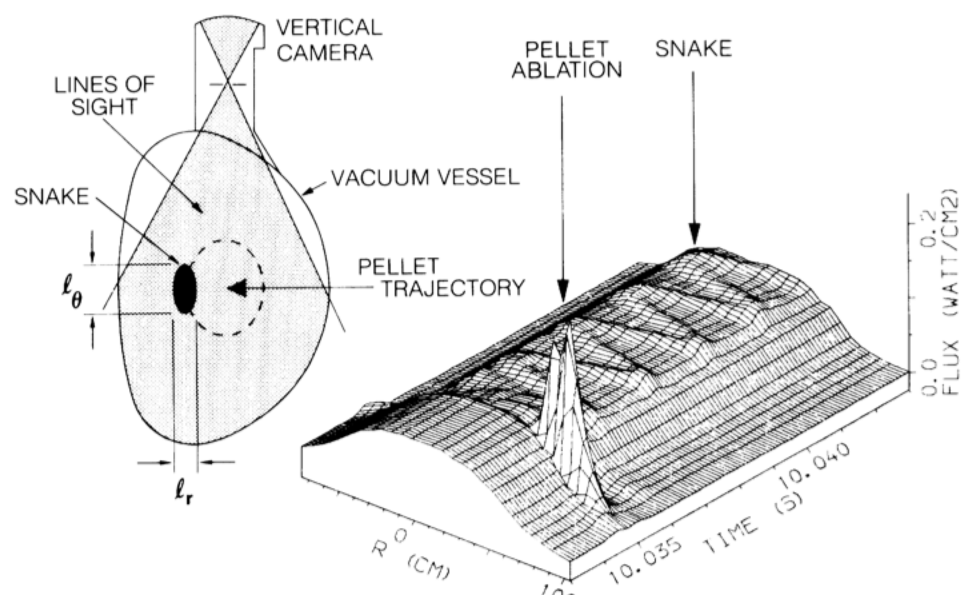
- m/n=1/1 modes with MHD density evolution
  - Fast sawtooth crash with MHD at low resistivity
  - Snakes: a new type of 1/1 internal kink
- Finite time Lyapunov exponents for plasmas
  - First results

1/1 modes: Sawteeth and snakes

- Density evolution in MHD with low resistivity can give
  - Fast sawtooth crash similar to experiment
    - Temperature and magnetic structure lost before density
    - Not Kadomtsev model (plasma not tied to magnetic flux)
    - Later, fast high-m ballooning burst removes density outside $q>1$
  - Snake-like behavior consistent with snakes+sawteeth
    - Small $q<1$ region: New type of 1/1 internal kink, driven by an applied helical density concentration near $q=1$. Resembles Alcator C-Mod heavy-impurity-ion snakes
    - Larger, unstable $q<1$ region: Applied helical density triggers a sawtooth crash
      - Crash redistributes density asymmetrically inside $q<1$ with 1/1 helical form (simulation incomplete)
      - Resembles pellet-induced snakes that start with crash

Snakes are common in fusion plasmas

- Two main types: pellet-induced and heavy-impurity ion
  - Heavy-impurity ion snakes
    - PLT (S.A. Cohen, et al., APS-DPP 1983)
    - "O-mode" in early DIII (G.L. Jahns, et al, 1986)
  - Pellet-induced snakes in JET (D_2 pellets in D_2 plasma) (A. Weller et al., PRL 1987; J.A. Wesson et al., IAEA 1986; R. Gill, et al. NF 1992)
  - Seen in many experiments;
    – still a subject of research
- Ion density concentration, observed in Soft X-Ray (SXR) emission, has snake-like trace in radius versus time (plasma toroidal rotation)
- Somewhat random trigger, behavior



Pellet-induced
(A. Weller, PRL 1987)

Snakes

- Fundamental problem with snake models so far:

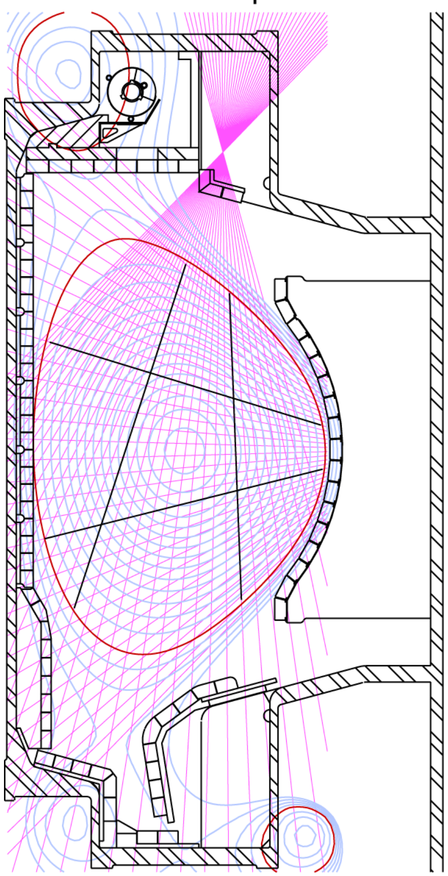
  Snake is a long-lasting, helical ion density accumulation. Assumed to have a related 1/1 helical magnetic structure.

  Snakes coexist with periodic sawtooth oscillations. Snake lifetime is determined by slow (diffusive?) ion loss, not sawtooth.

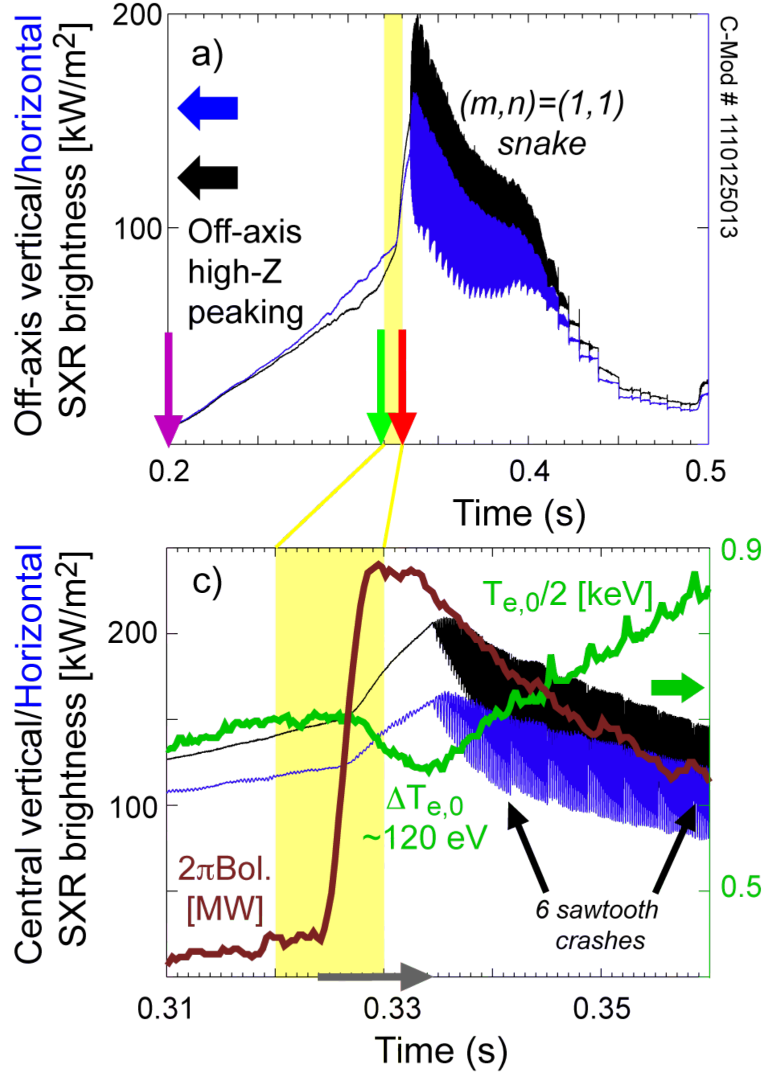
- MHD models have used pressure only: snake high density is a high pressure region in the MHD force balance
  - Inconsistent with presence of sawteeth: more heating will drive internal kink of high pressure region, crash will remove snake density also. (Or, heating of low pressure region will destroy the force balance supporting the snake).
  - Models require unusual q,p profiles or non-MHD effects
- NEW: including density evolution in MHD gives consistent results
  - Supported by new observations from Alcator C-Mod

C-Mod: Combination of high resolution diagnostics for snake and sawteeth measure n_{Mo}

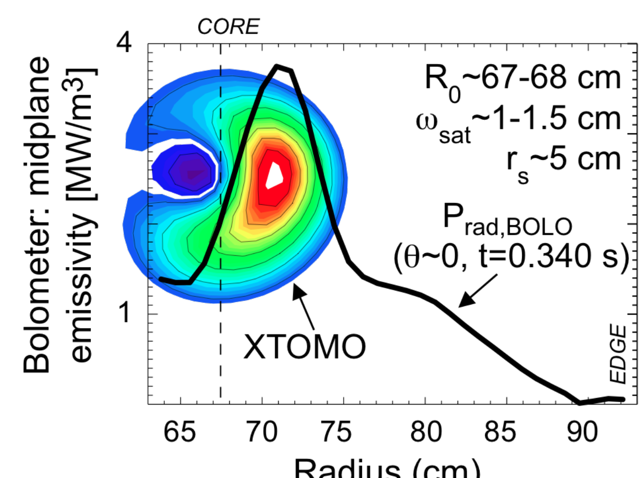
C-Mod # 1110125013
t=360 ms, I_p~0.93 MA



XTOMO arrays



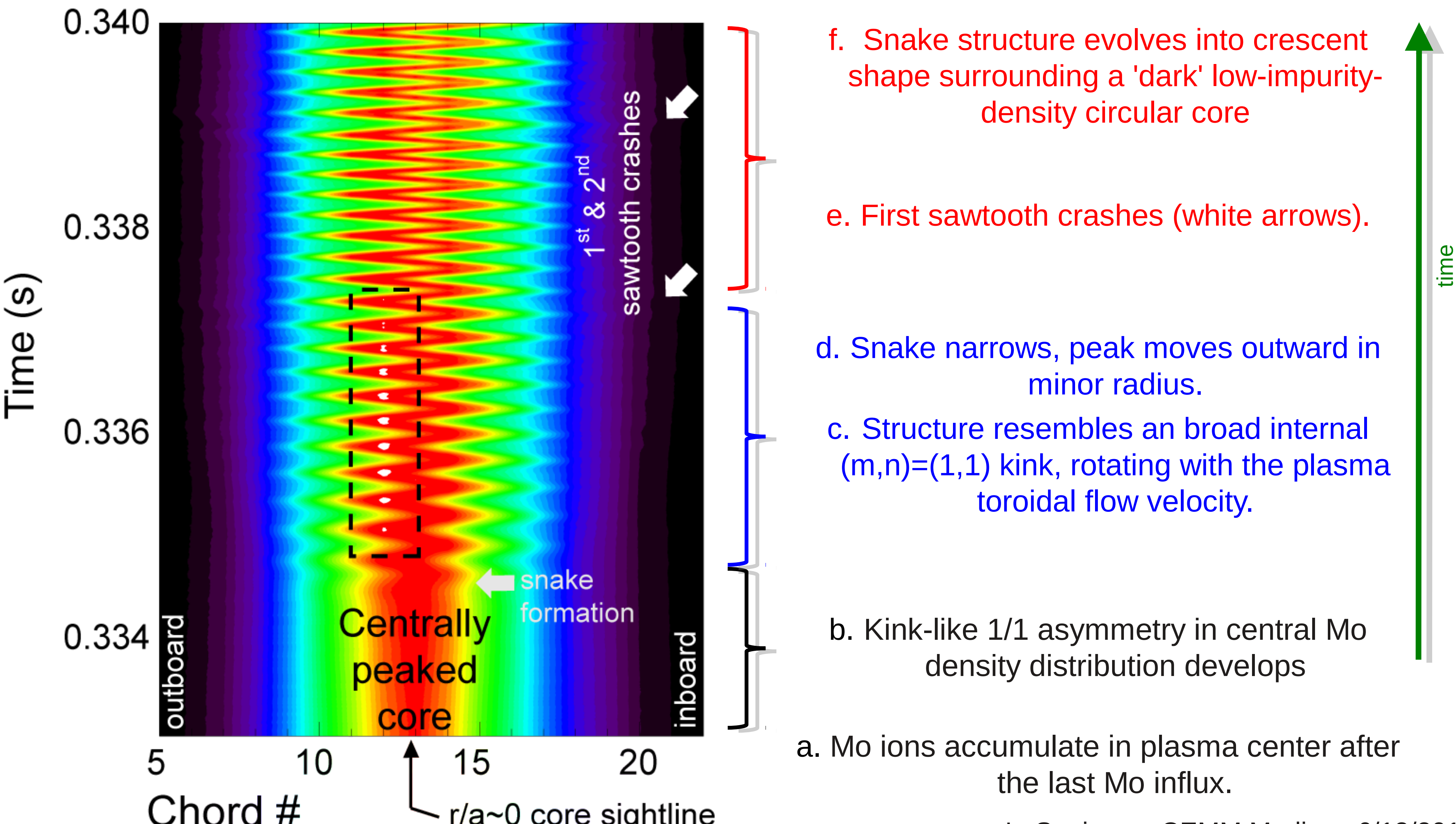
- X-Ray tomographic arrays (XTOMO) horizontal and vertical views
- High resolution X-Ray imaging spectrometer (HIREX)
- AXUV bolometric reconstructions



Modified from: L. Delgado-Aparicio, APS-DPP 2011

C-Mod Mo snake formation in early ohmic plasma: central impurity accumulation→ second impurity injection → kinked circular core→ crescent shape inside q=1

L. Delgado-Aparicio, APS-DPP 2011



f. Snake structure evolves into crescent shape surrounding a 'dark' low-impurity-density circular core

e. First sawtooth crashes (white arrows).

d. Snake narrows, peak moves outward in minor radius.

c. Structure resembles an broad internal (m,n)=(1,1) kink, rotating with the plasma toroidal flow velocity.

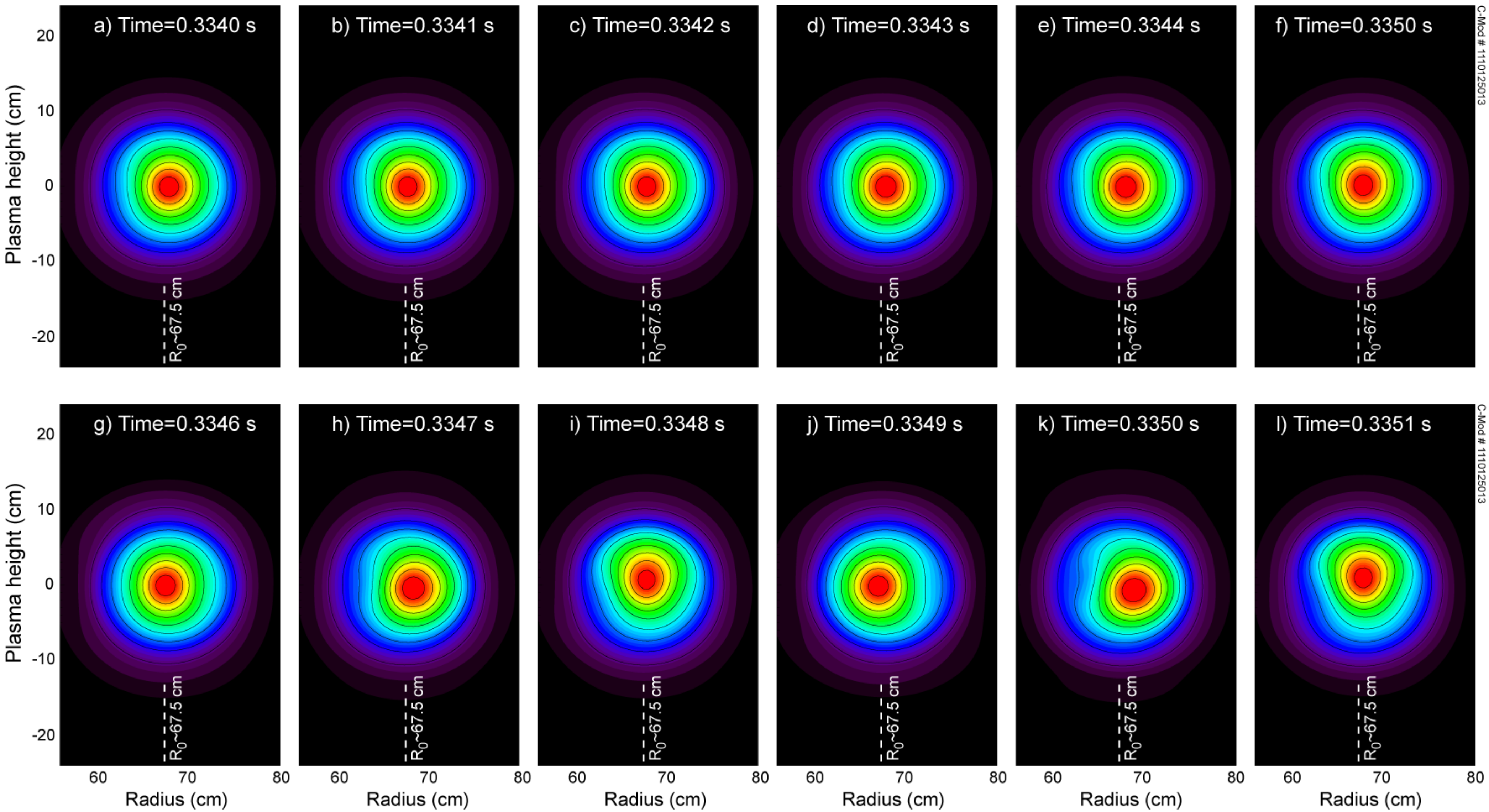
b. Kink-like 1/1 asymmetry in central Mo density distribution develops

a. Mo ions accumulate in plasma center after the last Mo influx.

Snake forms initially as a small 1/1 kink-like impurity density perturbation (δn_{Mo})

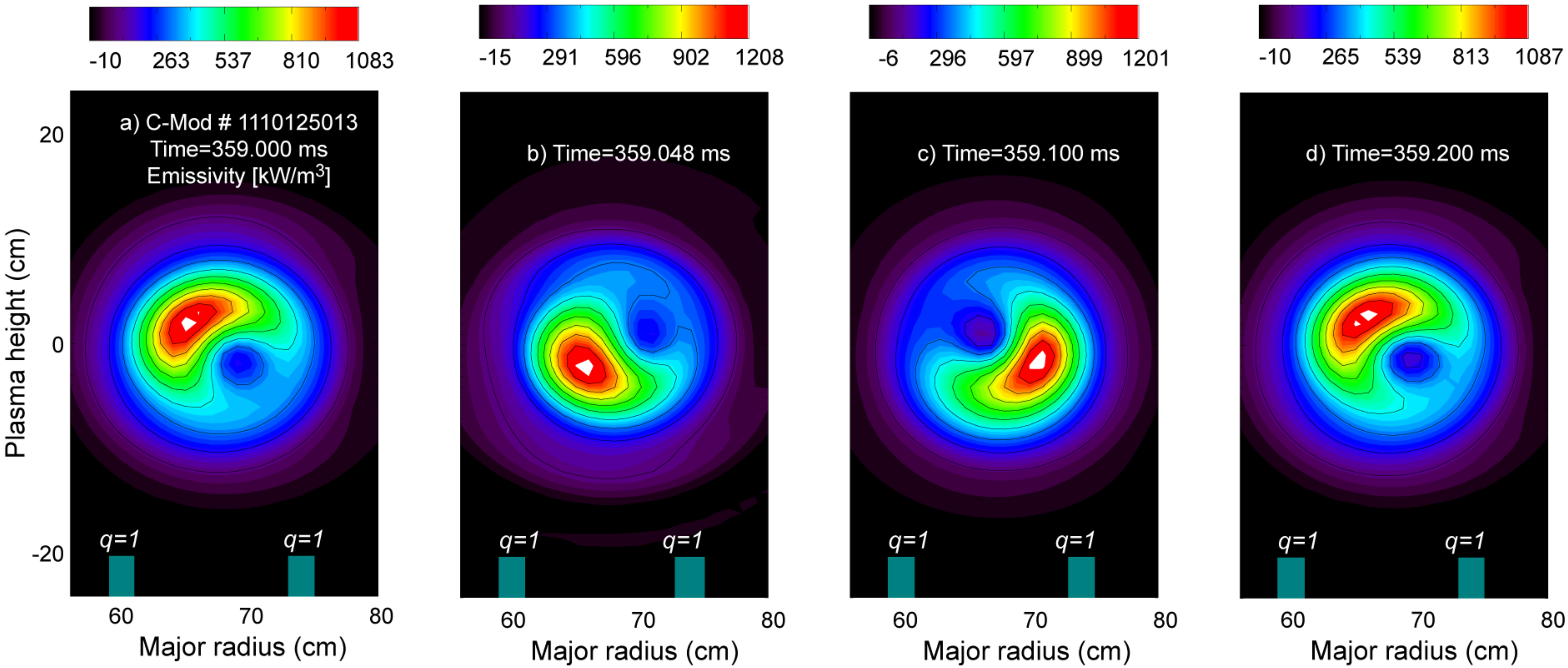
L. Delgado-Aparicio APS-DPP11

SXR tomography during snake formation



Later snake: SXR tomographic reconstruction shows 1/1 crescent shape and sawtooth crash

L. Delgado-Aparicio, APS-DPP 2011



a. Crescent dimensions: peak $r_s \sim 5$ cm, full-width $\omega_{sat} \sim 6$ cm and $\pi < \sigma_\theta < 5\pi/4$. Perturbation rotates in the electron diamagnetic drift direction.

b. Sawtooth crash: circular 'dark' SXR core moves outward to edge, releases heat pulse. Then returns more slowly to center.

c. New: 'dark SXR' core is higher T_e

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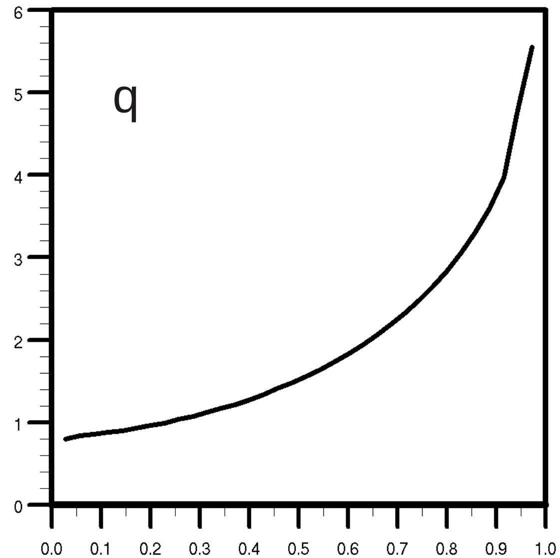
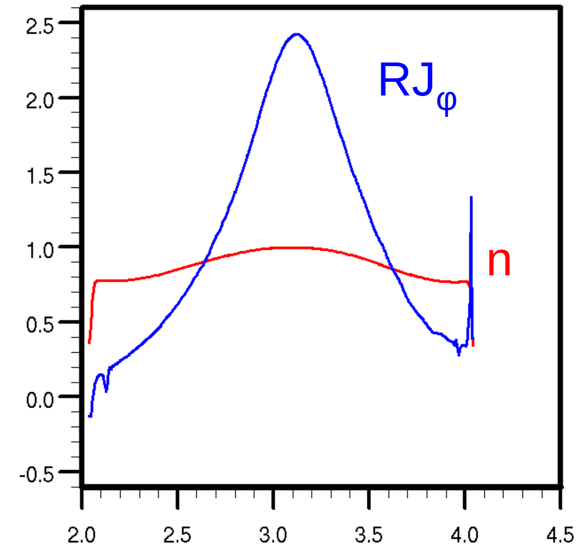
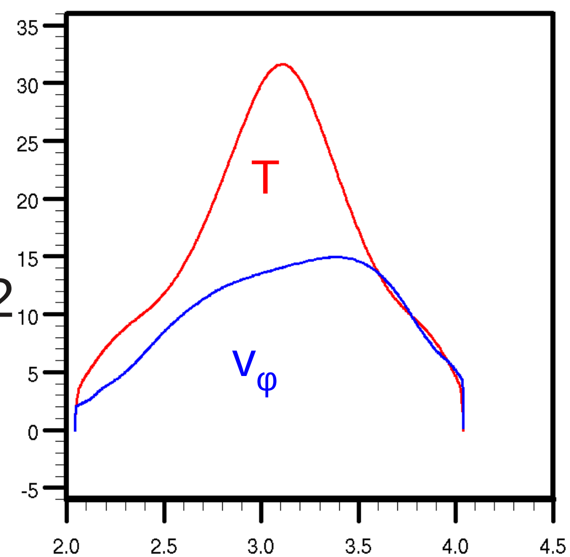
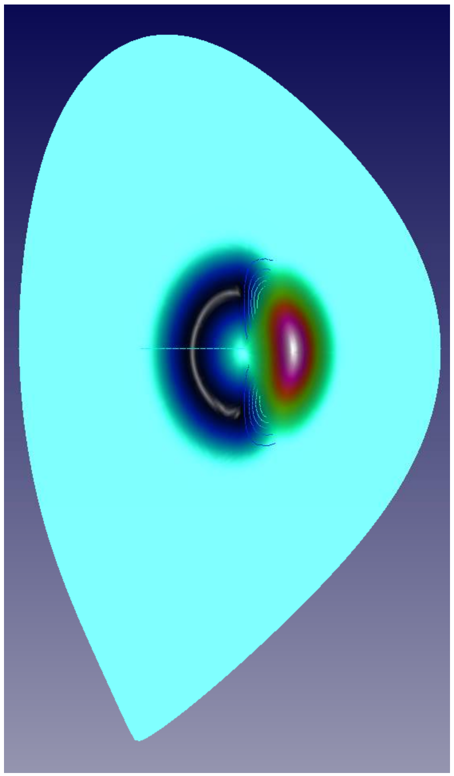
Simulation

- Equilibrium from C-Mod EFIT of EDA H-mode (!)
  - Chosen for q-profile; reasonable central profiles p, n ,etc
  - Similar size, shape
  - Small rotation (ohmic)
- Full cross-section, boundary is full magnetic separatrix with X-point
  - 30k grid points/plane, 72 φ- planes (n≤23 modes)
- Apply positive helical density centered around q=1, ~H(cos(θ±φ))









Non-axisymmetric n

Small amplitude 1/1 mode over q<1

$$\frac{\partial\tilde{\psi}}{\partial t} = -\tilde{\mathbf{v}}\cdot\nabla\psi_o - \mathbf{v}_o\cdot\nabla\tilde{\psi} + \frac{\partial\tilde{U}}{\partial\phi} + \eta\tilde{J}$$
$$\frac{\partial\tilde{p}}{\partial t} = -\tilde{\mathbf{v}}\cdot\nabla p_o - \mathbf{v}_o\cdot\nabla\tilde{p}.$$

Writing $\tilde{U} = \tilde{U}(r)e^{i(m\theta-n\phi)}e^{\gamma t}$**, to lowest order**

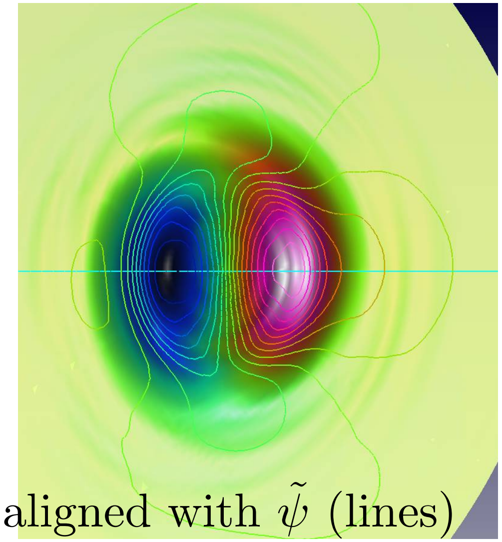
$$\frac{\partial\tilde{\psi}}{\partial t} = -\nabla_{\|}\tilde{U} - \mathbf{v}_o\cdot\nabla\tilde{\psi}$$
$$\frac{\partial\tilde{p}}{\partial t} = \hat{\boldsymbol{\phi}}\cdot\nabla\tilde{U}\times\nabla p - \mathbf{v}_o\cdot\nabla\tilde{p},$$

Defining $k_\perp = m/r$ **and** $k_{\|} = ((m/r)B_\theta - (n/R)B_\phi)/B$

$$\dot{\tilde{\psi}} = -ik_{\|}(RB)\tilde{U}$$
$$\dot{\tilde{p}} = ik_\perp p'(r)\tilde{U},$$

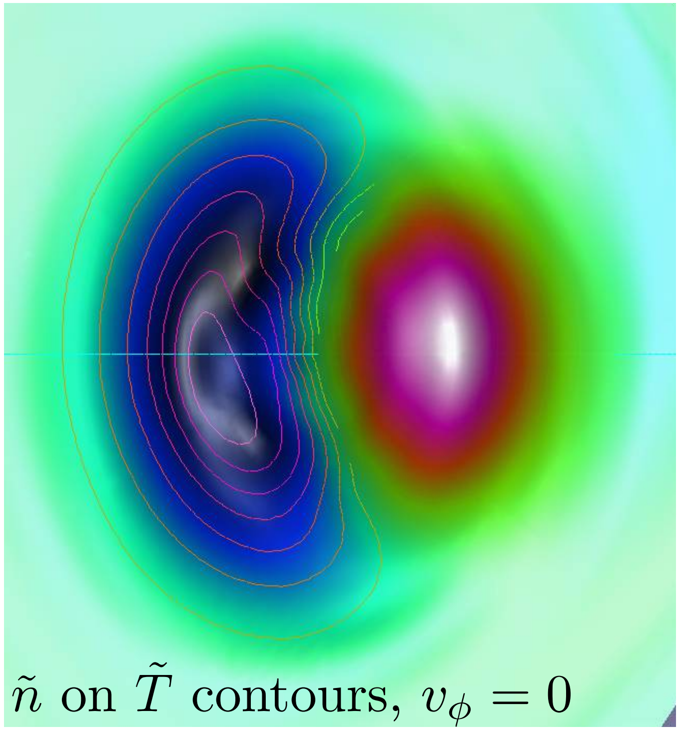
where $\dot{f} \equiv \partial f/\partial t$**. Then**

$$\tilde{\psi}/\tilde{p} = -(R_o a/R^2)(p_o/p_o'').$$

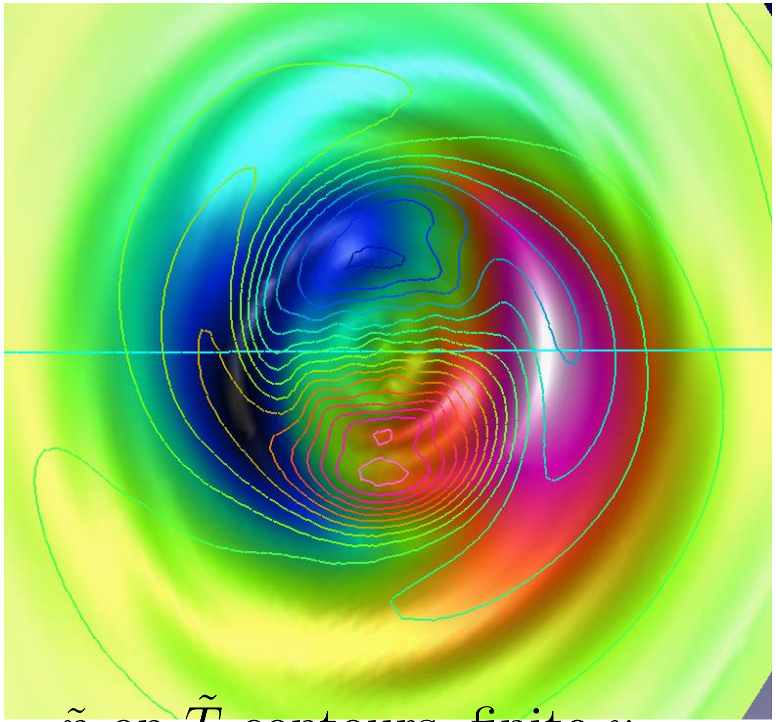


$\tilde{p}$ aligned with $\tilde{\psi}$ (lines)

Result: New form of 1/1 internal kink (nonlinear)



$\tilde{n}$ on $\tilde{T}$ contours, $v_\phi = 0$

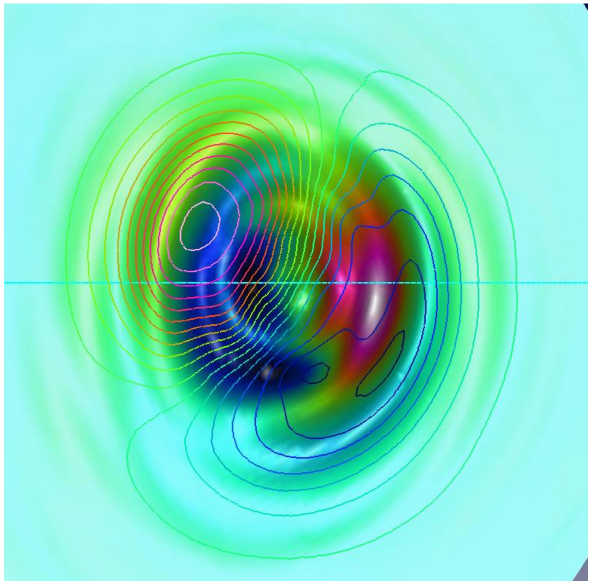


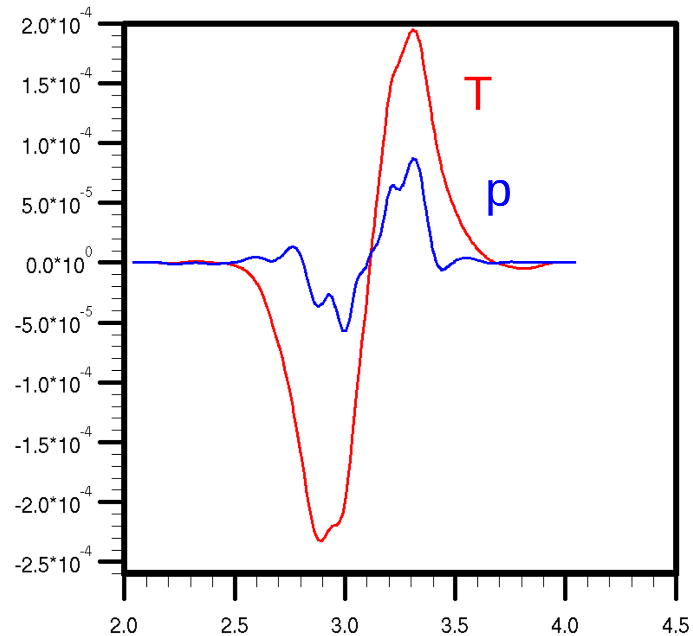
$\tilde{n}$ on $\tilde{T}$ contours, finite v_ϕ

- Applied helical density perturbation (few %) applied around q=1 drives a 1/1 kink-like mode with very low growth rate (few x 10^{-4})
- Inside q<1, 1/1 temperature tends to have opposite sign to 1/1 density, reduces 1/1 ∇p. Outside q=1, helical density is long-lived.
- Density resembles early C-Mod snake (broad kink over q>1)
- Difference in behavior with background toroidal (and intrinsic MHD poloidal) rotation and with initial density helicity (ie, cos(θ±φ)).

1/1 pressure and temperature

$\tilde{p}$ on $\tilde{n}$ contours, $v_\phi = 0$

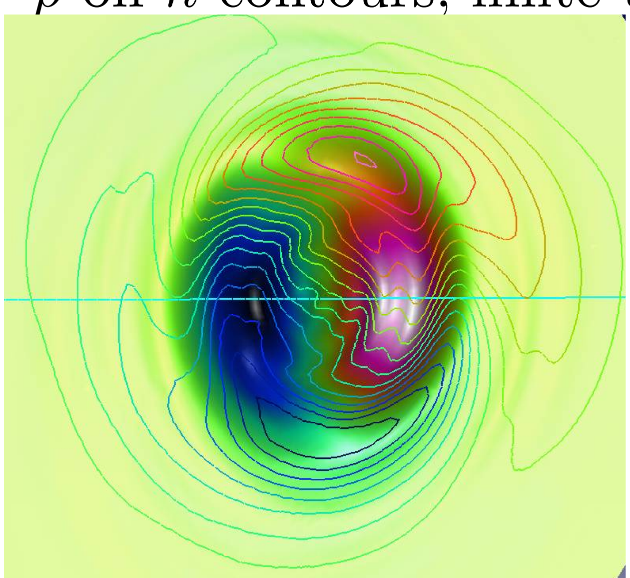


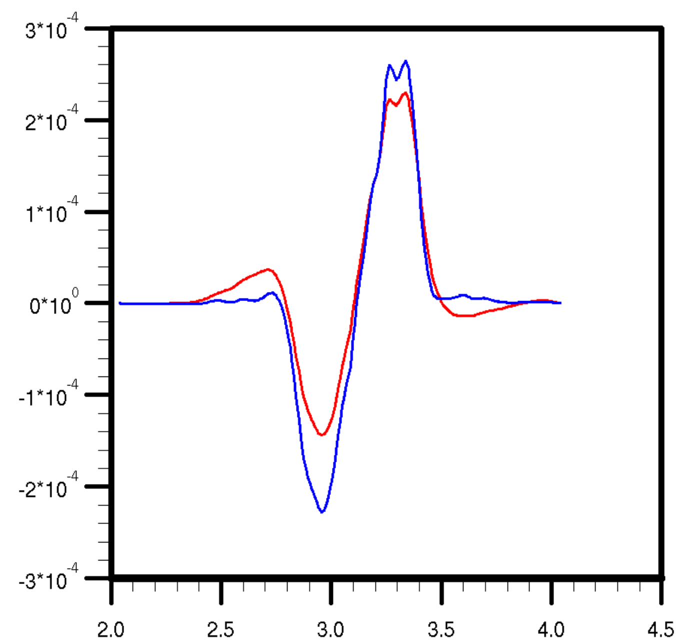


T and p profiles at the angle φ where the maximum p lies on the outboard midplane.

$$\tilde{p} \simeq n\tilde{T} + \tilde{n}T$$
$$n \simeq 1,\ T \simeq 0.03$$

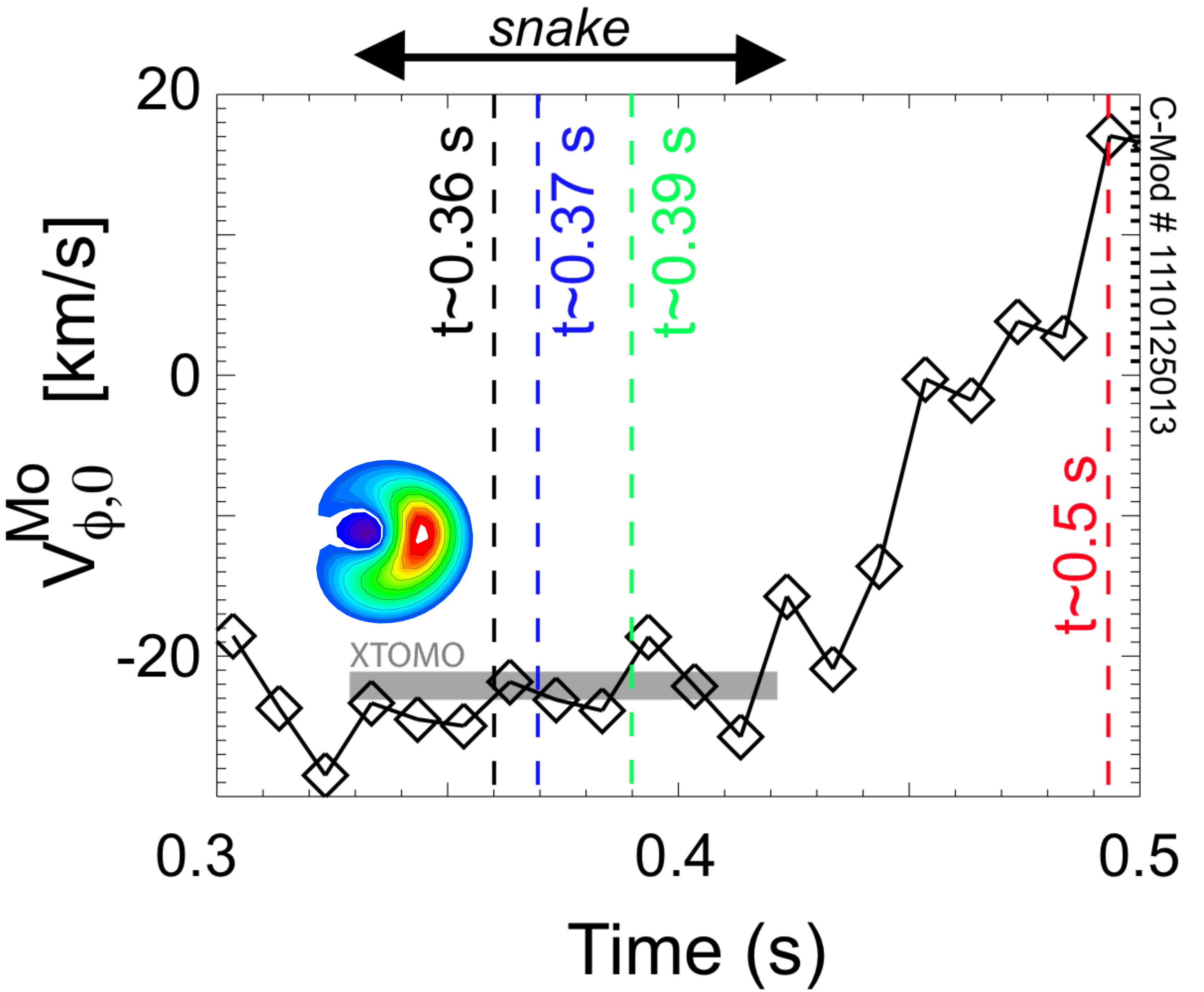
$\tilde{p}$ on $\tilde{n}$ contours, finite v_ϕ





(p and ψ are always closely aligned)

Question: plasma and snake rotation
Which is which?



a. The "plasma" toroidal rotation speed (Mo ions, spectroscopic) is similar to snake rotation (SXR).

b. Both rotations are roughly constant over snake lifetime

c. Opposite direction to usual ohmic rotation

(Snakes starting later in discharge rotate ohmically)

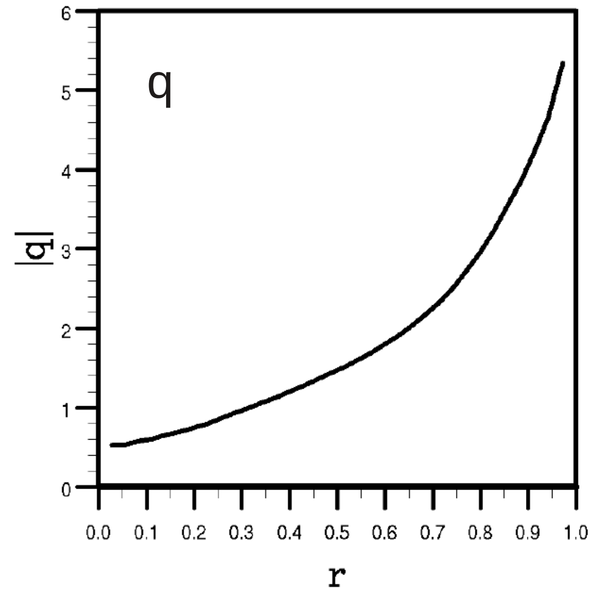
Helical density perturbation for large q<1 region

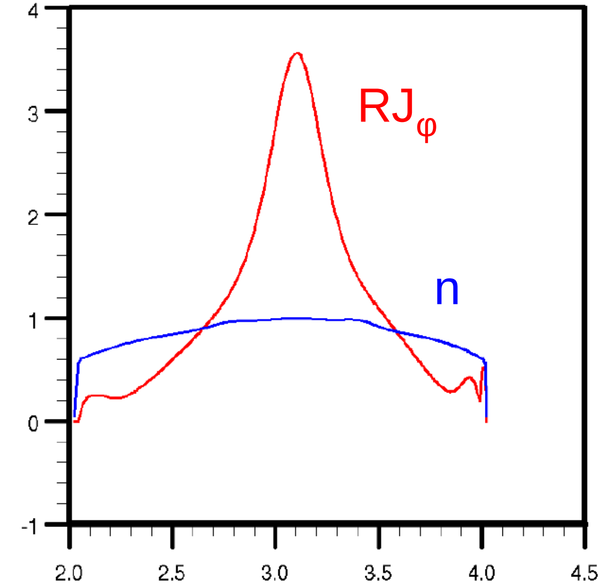
- Similar helical perturbation was applied to a large q<1 region with low $q_0 \approx 0.5$
  - Profiles similar, but some differences
  - No background toroidal rotation
- Unstable to resistive (and ideal?) internal kink for low $\eta = 10^{-6} - 10^{-8}$ Crash onset time scales inversely with resistivity.
- Density perturbation of few % triggers a rapid crash
  - Crash itself behaves similarly to regular crash
- Regular crash: For the kink at smaller amplitude, the kink n, p, T are aligned over q<1, with the same sign (unlike the small q<1 mode)

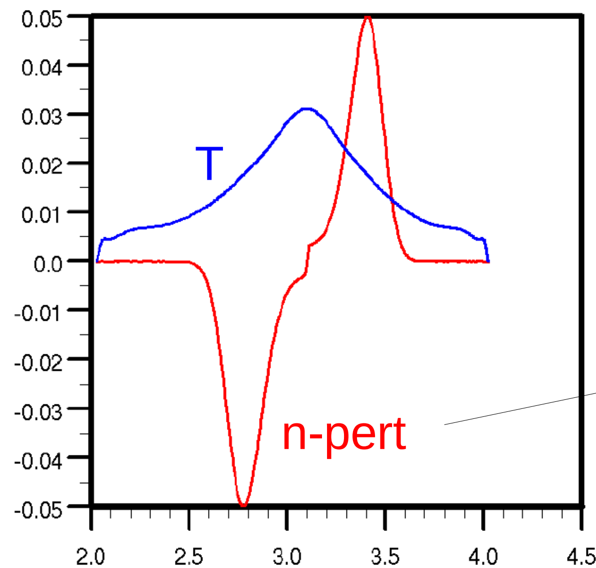
- Initial profiles and non-axisymmetric part of the initial helical density perturbation

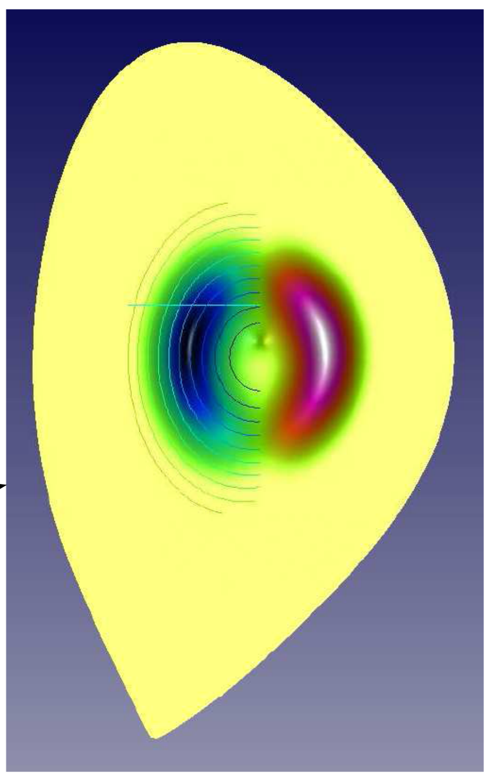
- Low $q_o \approx 0.5^+$ is unstable to resistive internal kink a low η

- Simulations have relatively low resolution (30k points/plane, n≤23 toroidal modes), but larger scale computation should use better cases.







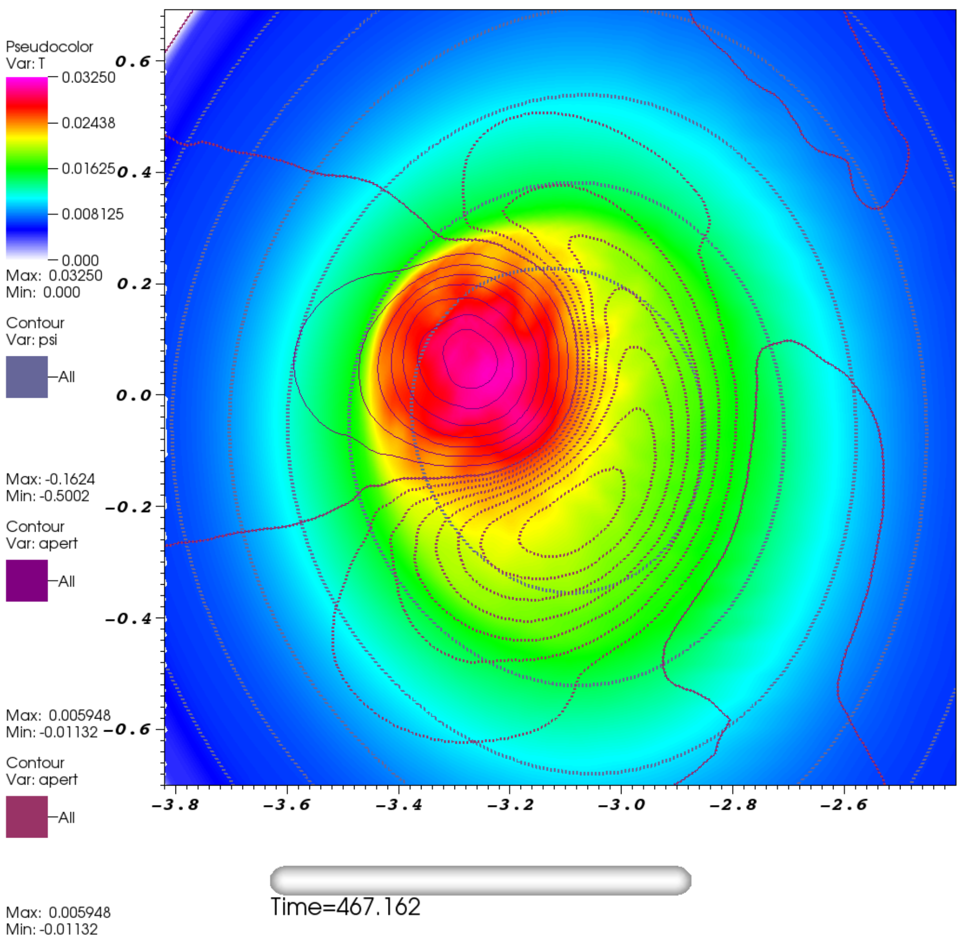


Strong sawtooth crash is 3D, not 2D

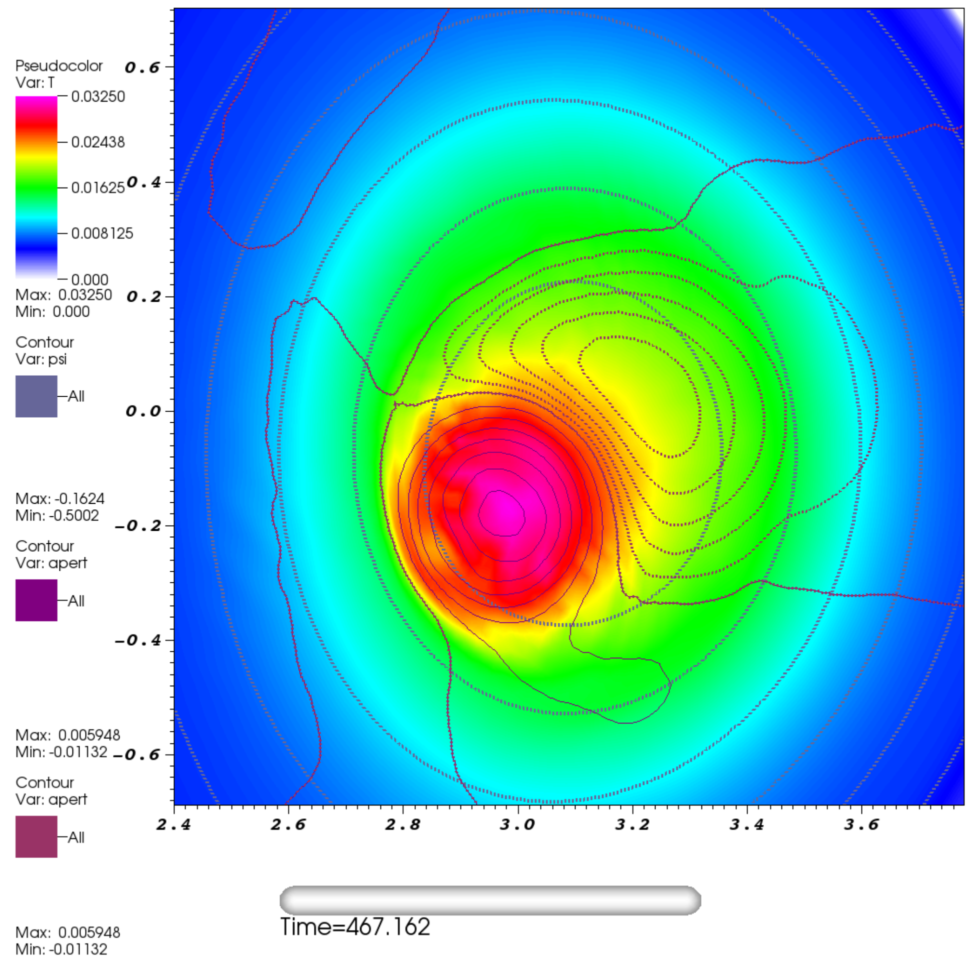
- Crash is fully 3D with strong poloidal (θ) variation, unlike RMHD or cylindrical plasma, 2D helical symmetry models with modes $m(\theta\text{-}\varphi)$
  - High-field side and low-field side reconnection
  - **V** and $\mathbf{B}_{\perp\varphi}$ not completely described by poloidal stream functions
- $\partial n/\partial t$ and $\partial T/\partial t$ evolve very differently
  - Temperature rapidly moves out into a thin layer near the X-point, then re-peaks into the plasma center from there; transient 'cold bubble', not related to a 1/1 magnetic bubble
  - Density moves through X-point more slowly, eventually ends up on poloidally opposite side from X-point, strongly localized in (θ,φ); the part outside q=1 is lost in a second phase of high-m ballooning driven by the density concentration
- Fast crash (ballpark, 100μs DIII-D, 20μs C-Mod) to complete loss of central temperature after peak T begins to move off-axis
  - "Stochastic" magnetic layer surrounds the hot core at large displacement

Temperature: displaced core

φ=π (LFS)



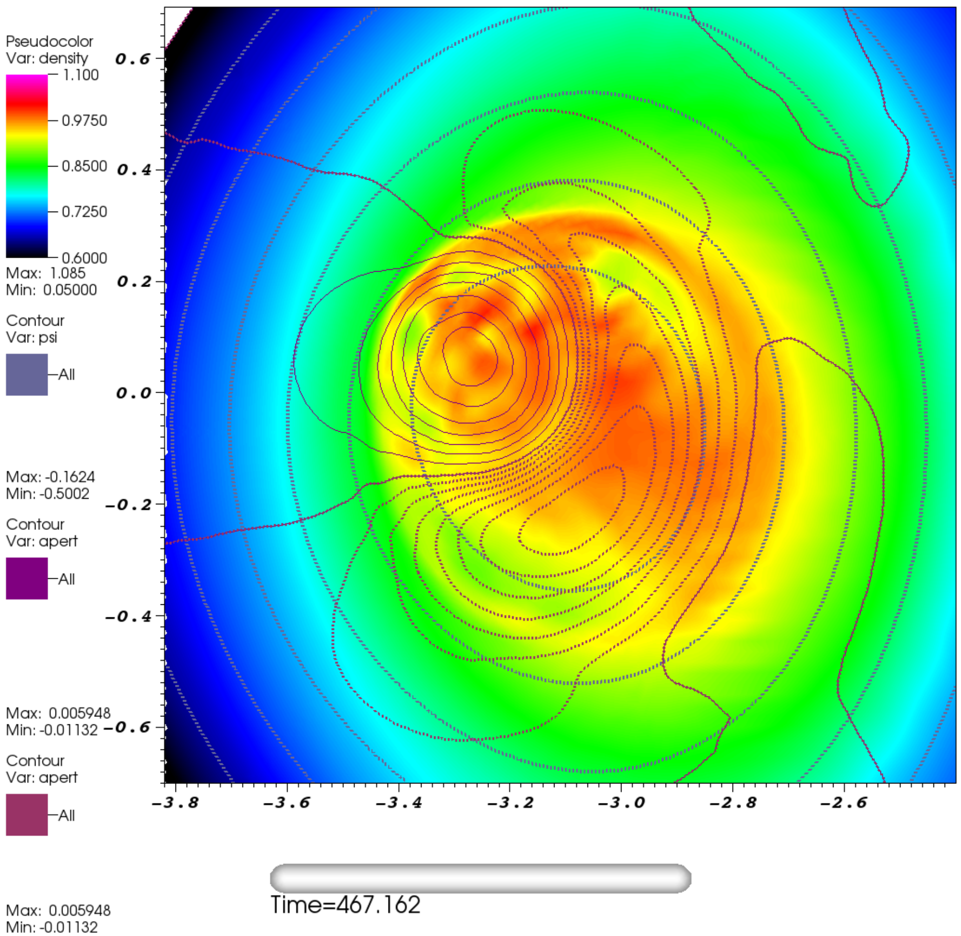
φ=0 (HFS)



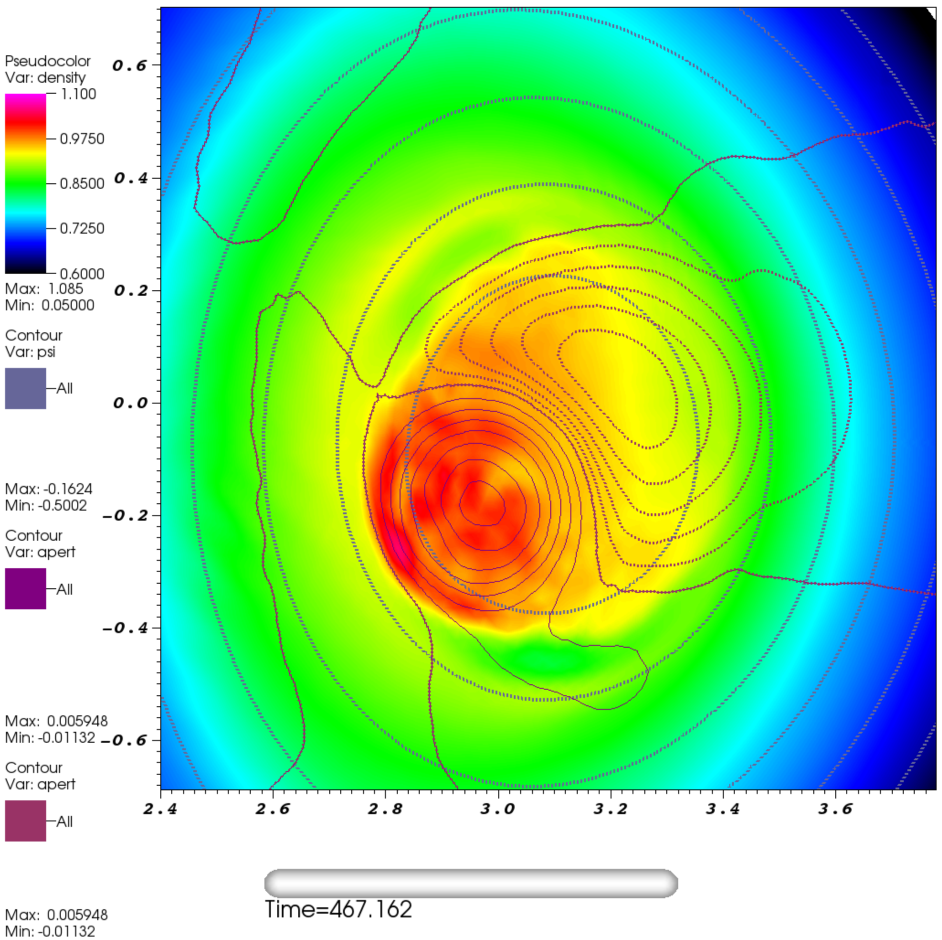
t=467.2

Density: displaced core

φ=π (LFS)



φ=0 (HFS)



t=467.2

Crash has strong poloidal differences: High vs Low Field side

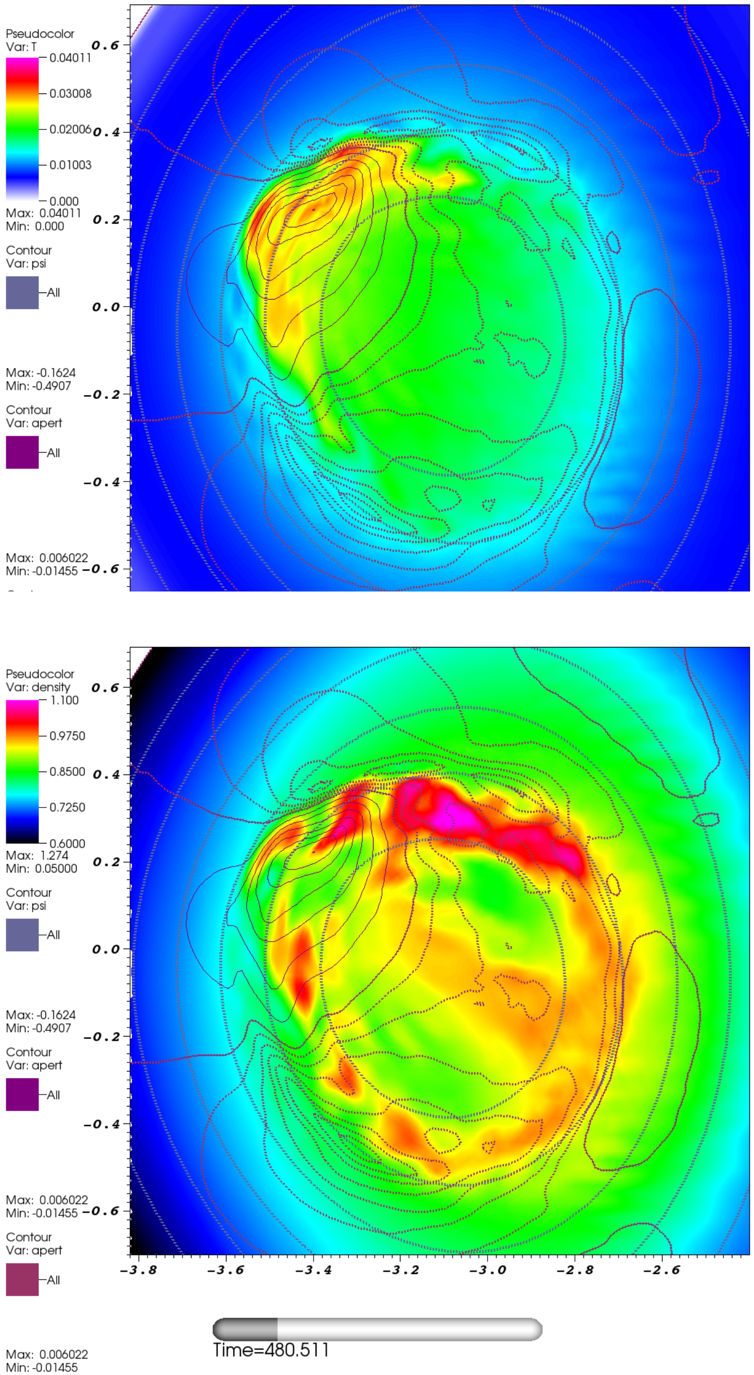
=480.5

φ=π
LFS

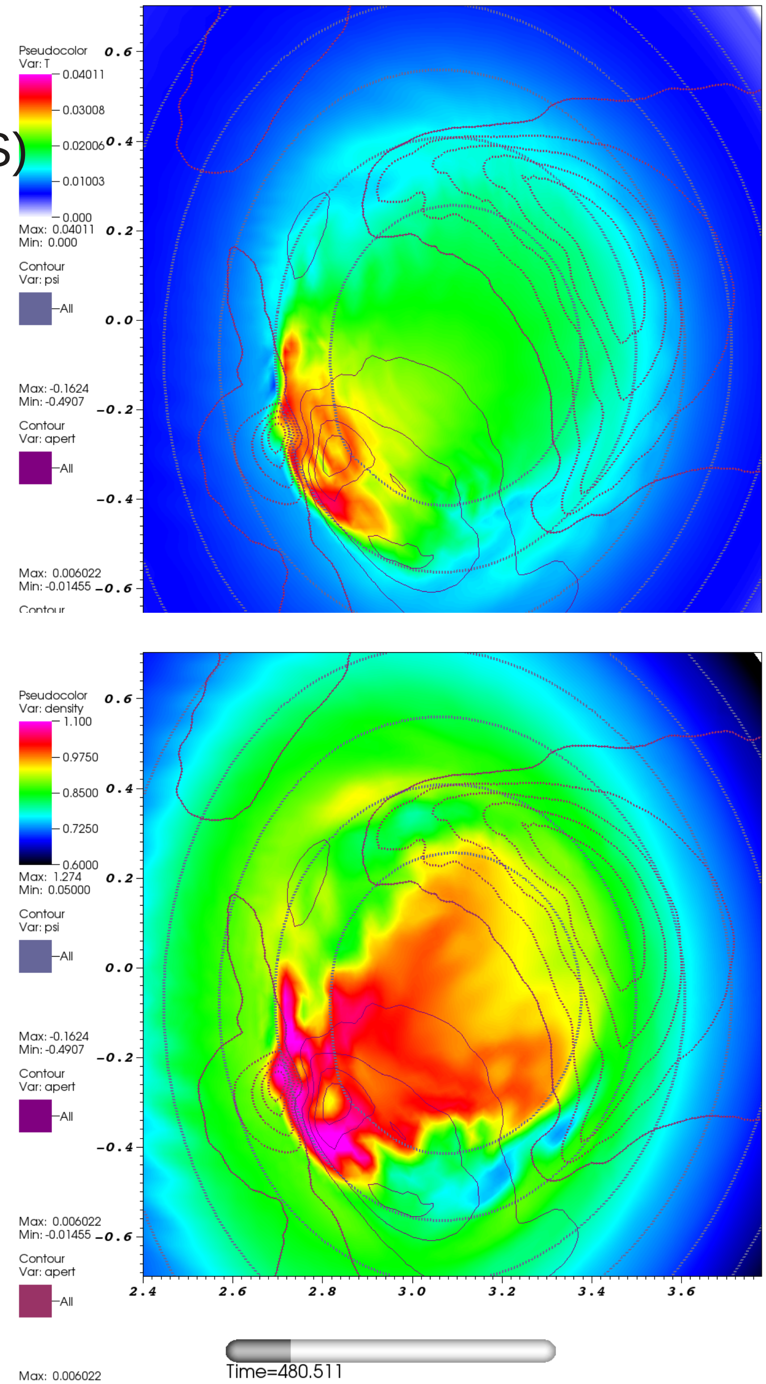
T

n

T relative,
n absolute
scales



φ=0
(HFS)



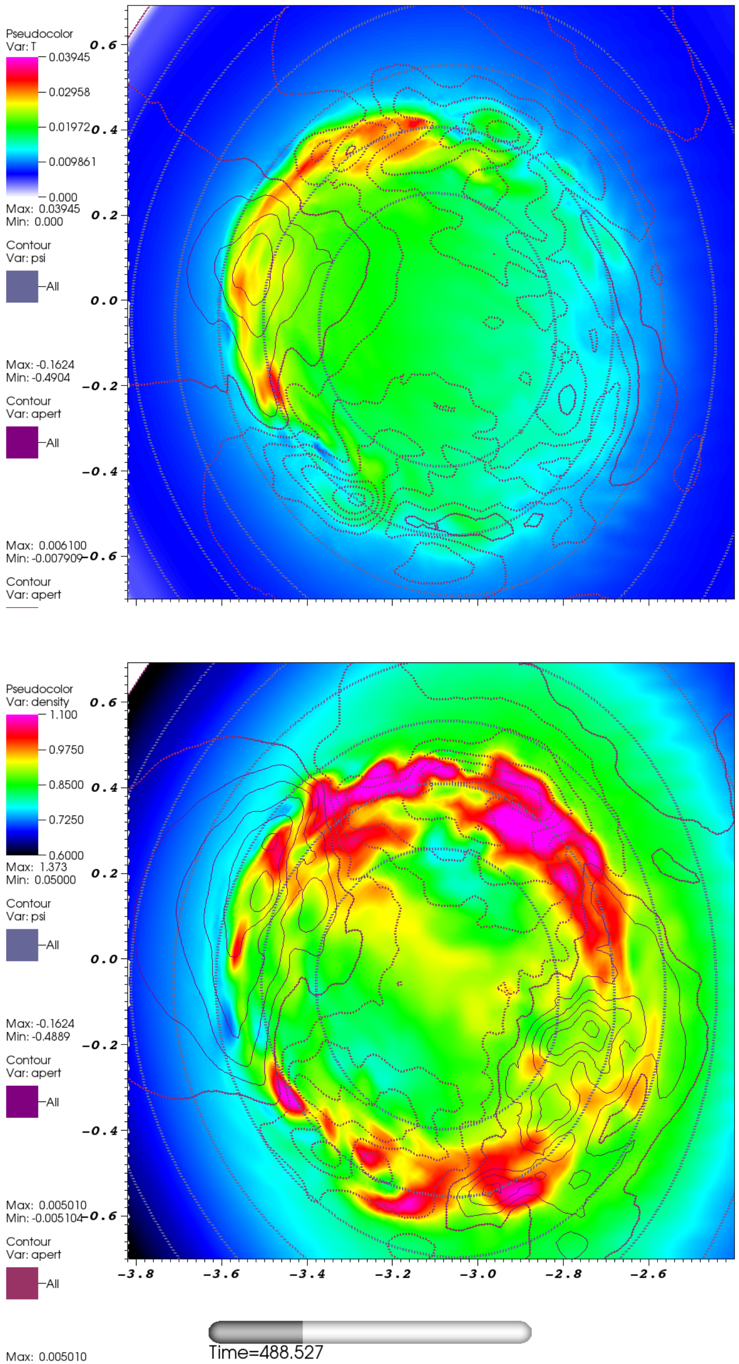
Crash initially leaves temperature in a layer near X-point

t=488.5

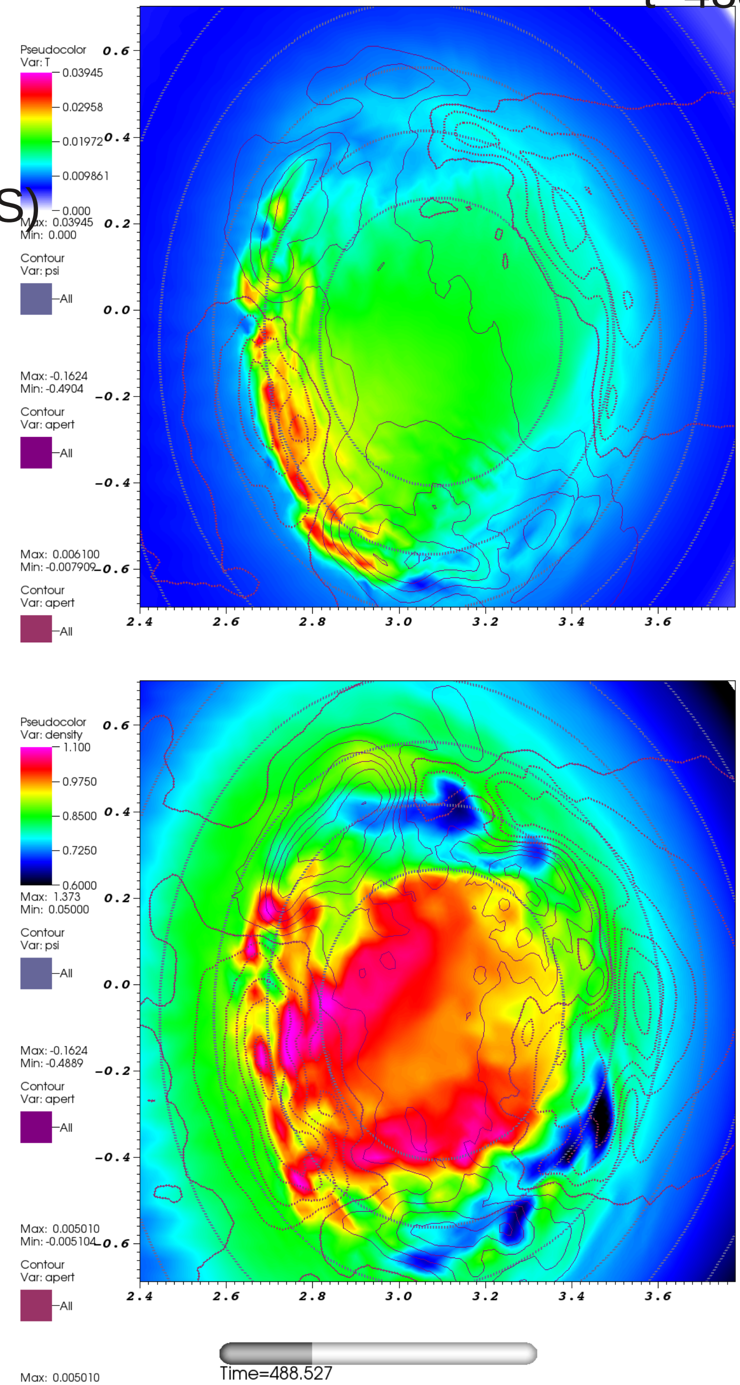
φ=π
(LFS)

T

n



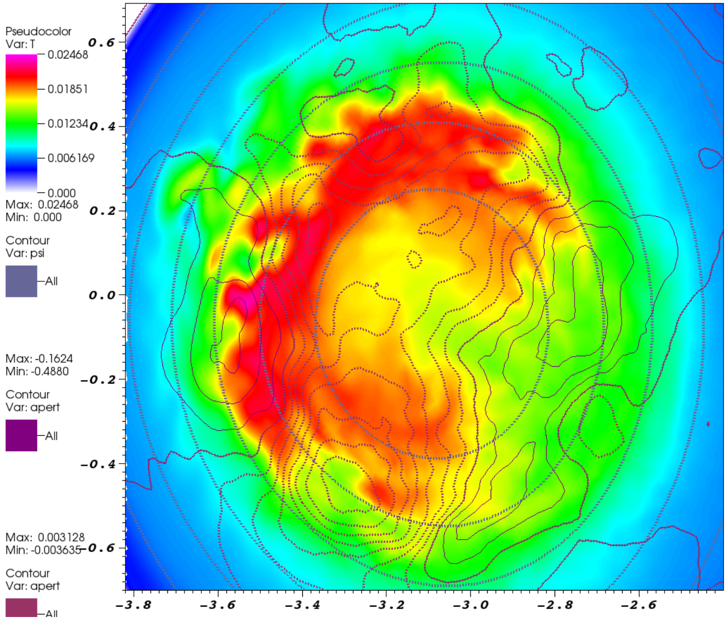
φ=0
(HFS)



Temperature rebuilds from X-point side

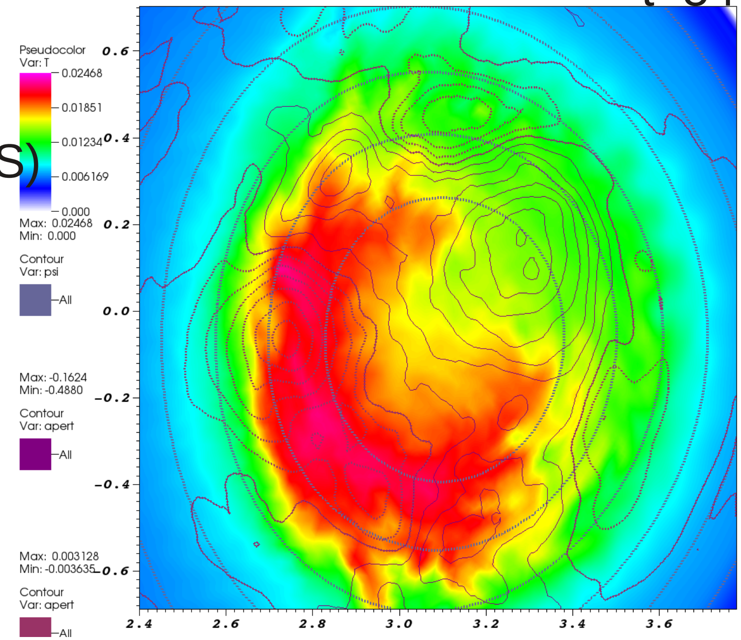
t=547

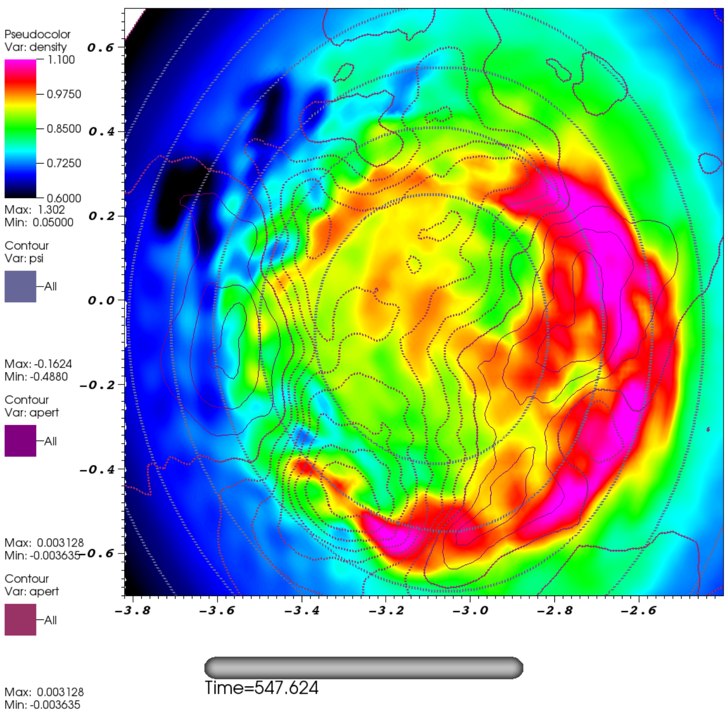
φ=π (LFS)



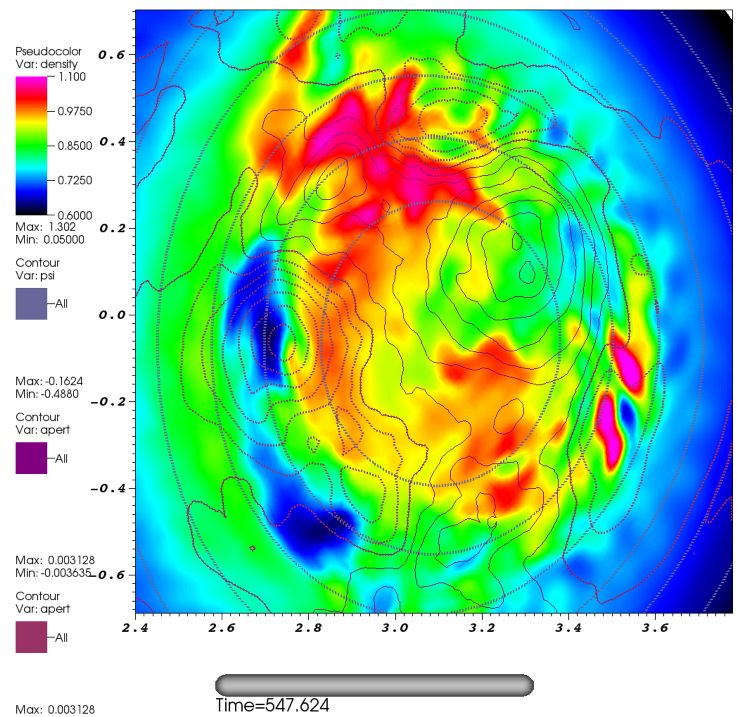
φ=0 (HFS)

T





n



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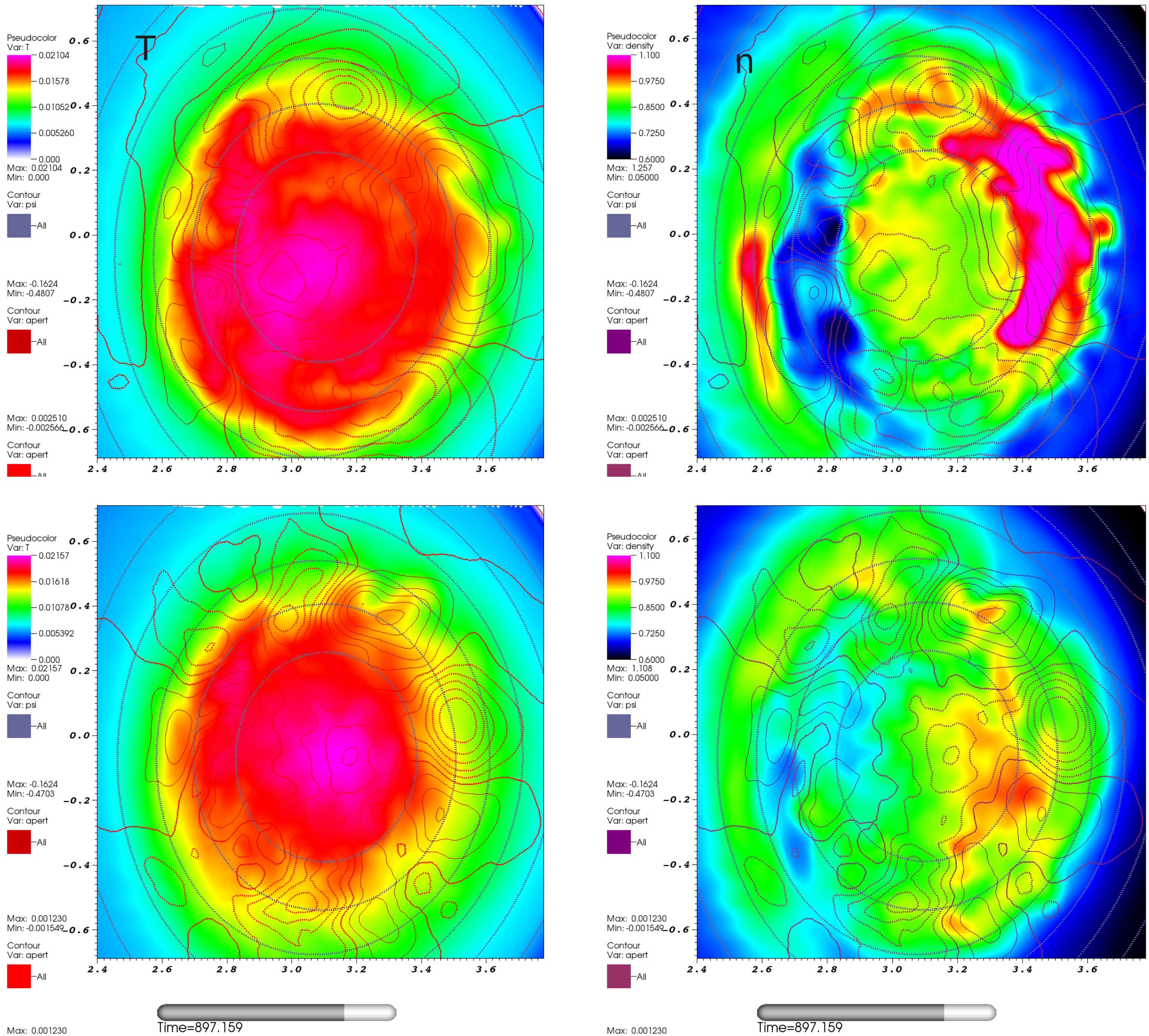
Temperature repeaks in center; density moves opposite X-point, outer part lost

φ=0
(HFS)

t=667

t=897

$T=2/3T_o$



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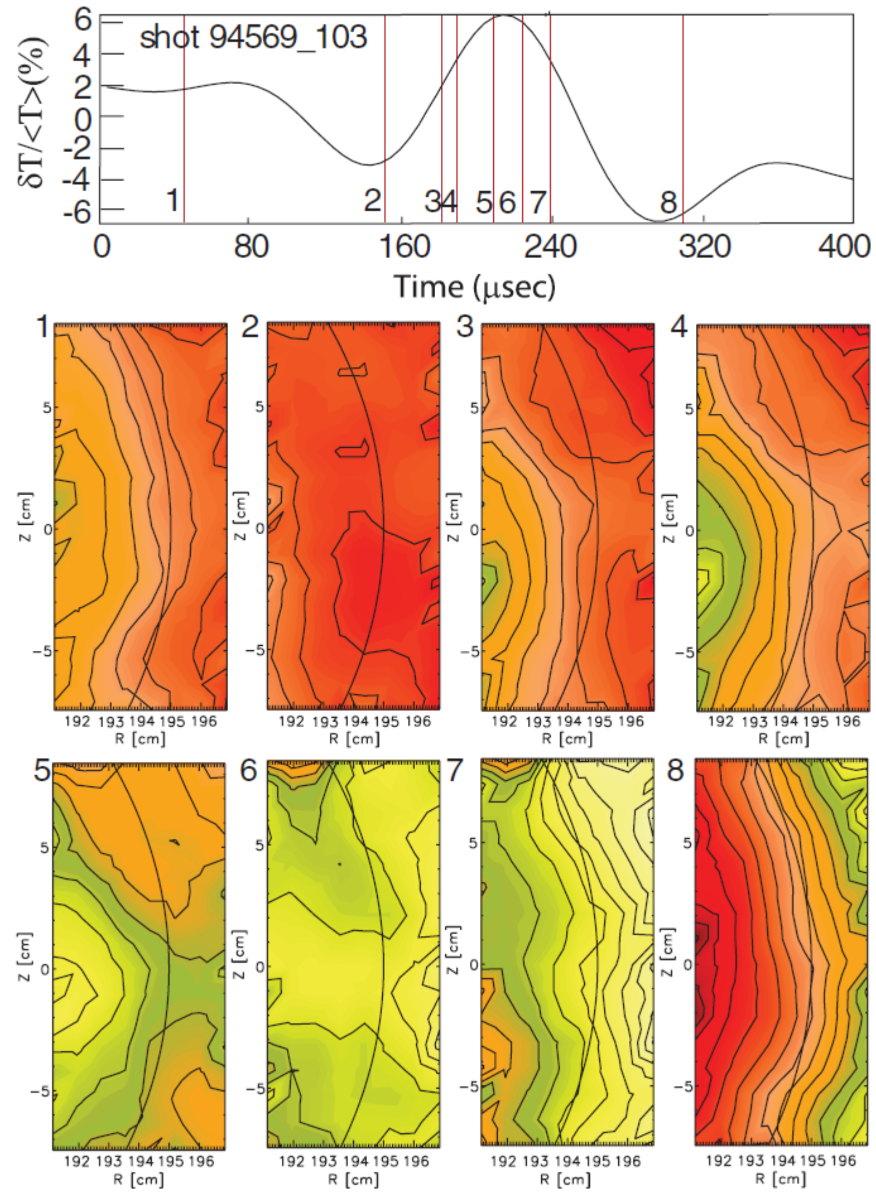
Temperature loss is similar to experiment

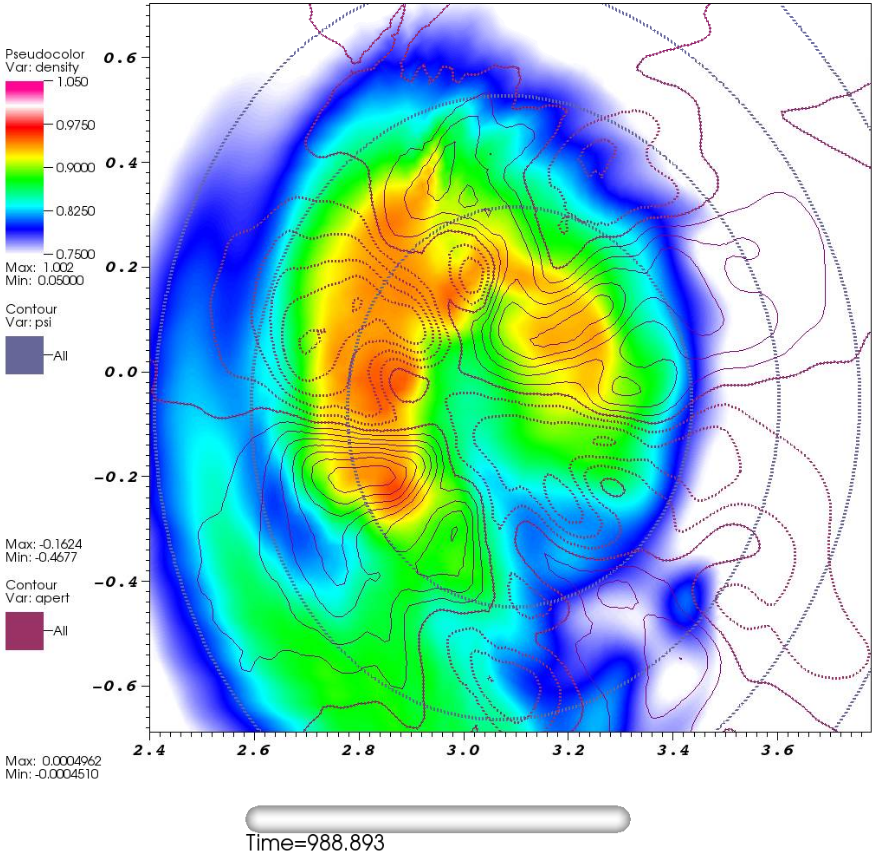
- Fast, localized loss through X-point region
- Formation of hot layer on surface just outside q=1
- Lower temperature interior
- Fast ECE measurements by H. Park, et al, on a number of experiments

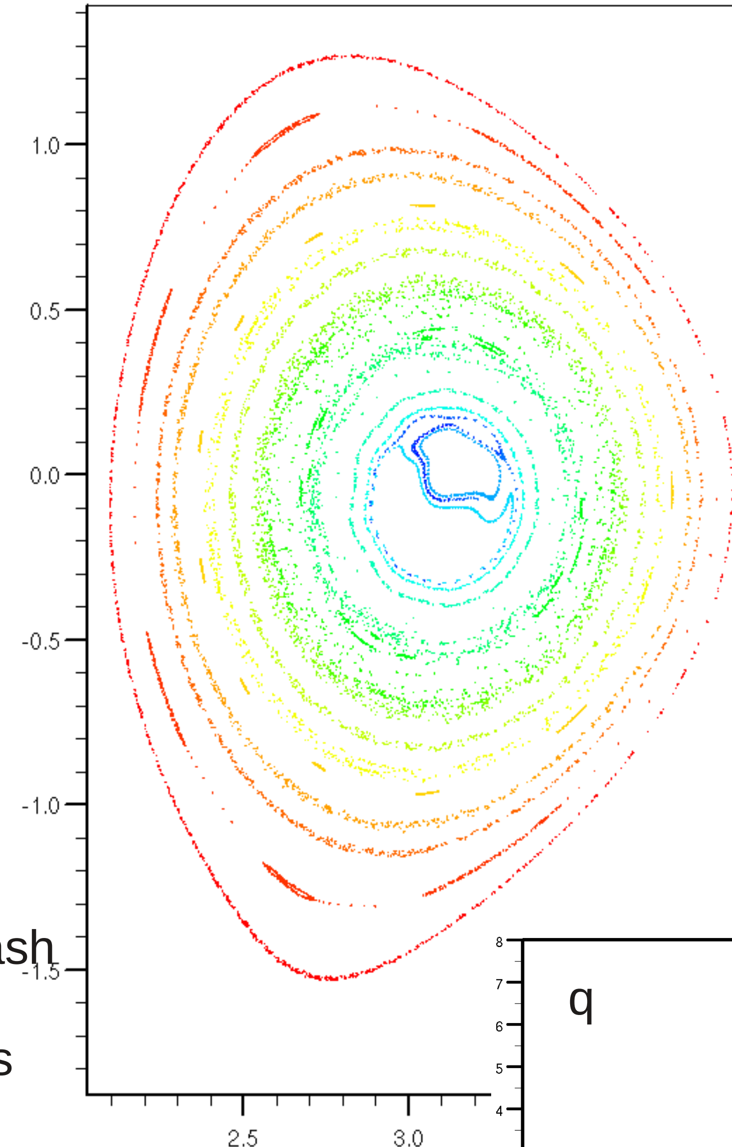
  (Box on outboard side of q=1 region)

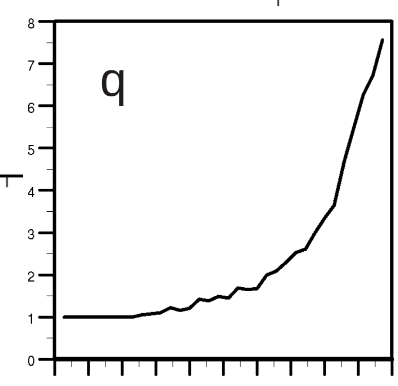
  (H. Park, et al, PRL 2006 in TEXTOR)



Sawtooth crash results in helical density inside q<1



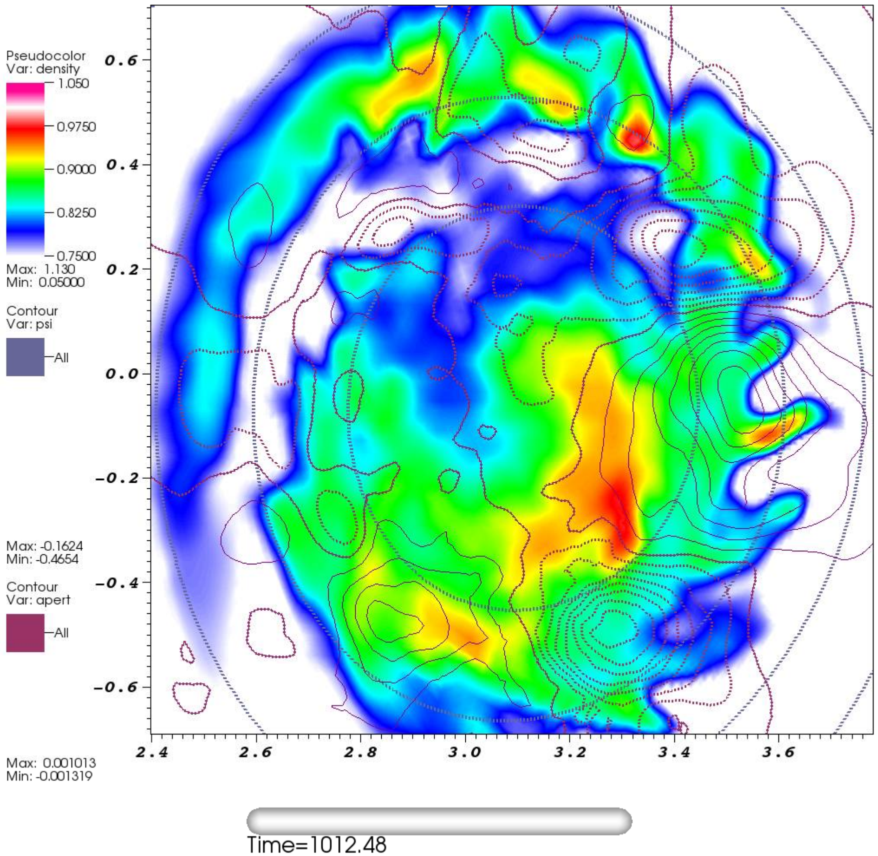




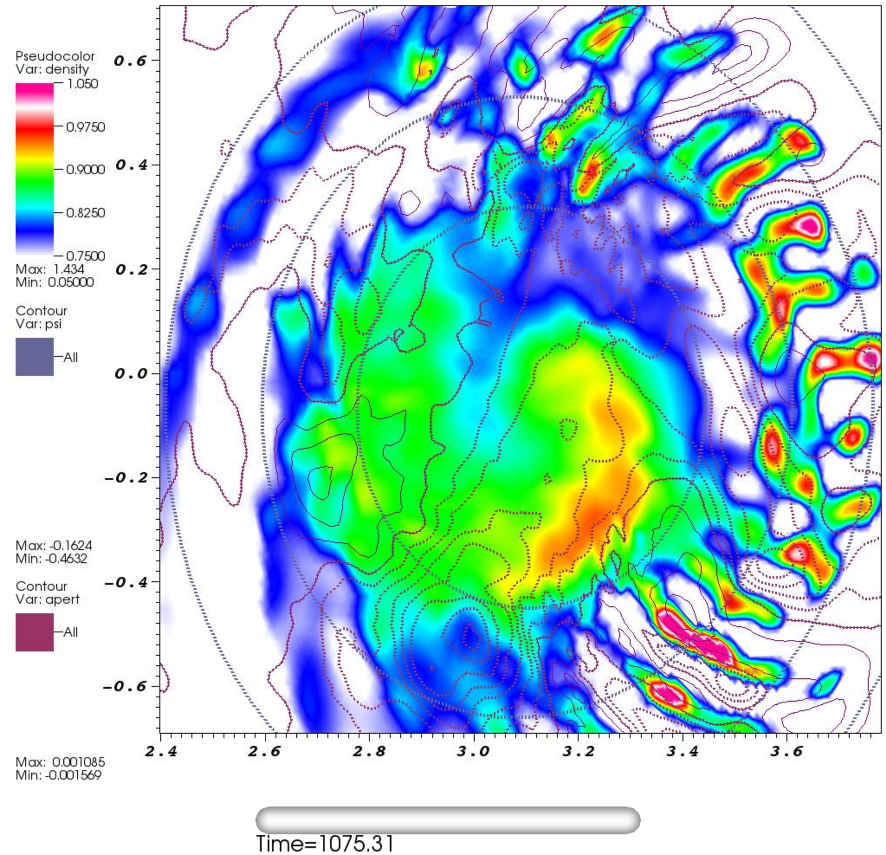
- Final density distribution for triggered ($\delta n/n=0.1$) crash at $\eta=10^{-6}$.
- Crescent-shaped density over q<1 and q>1 (q=1 falls outside right blob, but inside edge of left blob.
- Puncture plot at $\varphi=0.22\pi$ (density at $\varphi=0$) shows two m=1 islands; T peak falls in upper island. Density rotates CW with φ; R blob falls in bottom island.

Sawtooth crash at low η is followed by ballooning outburst

t=1012

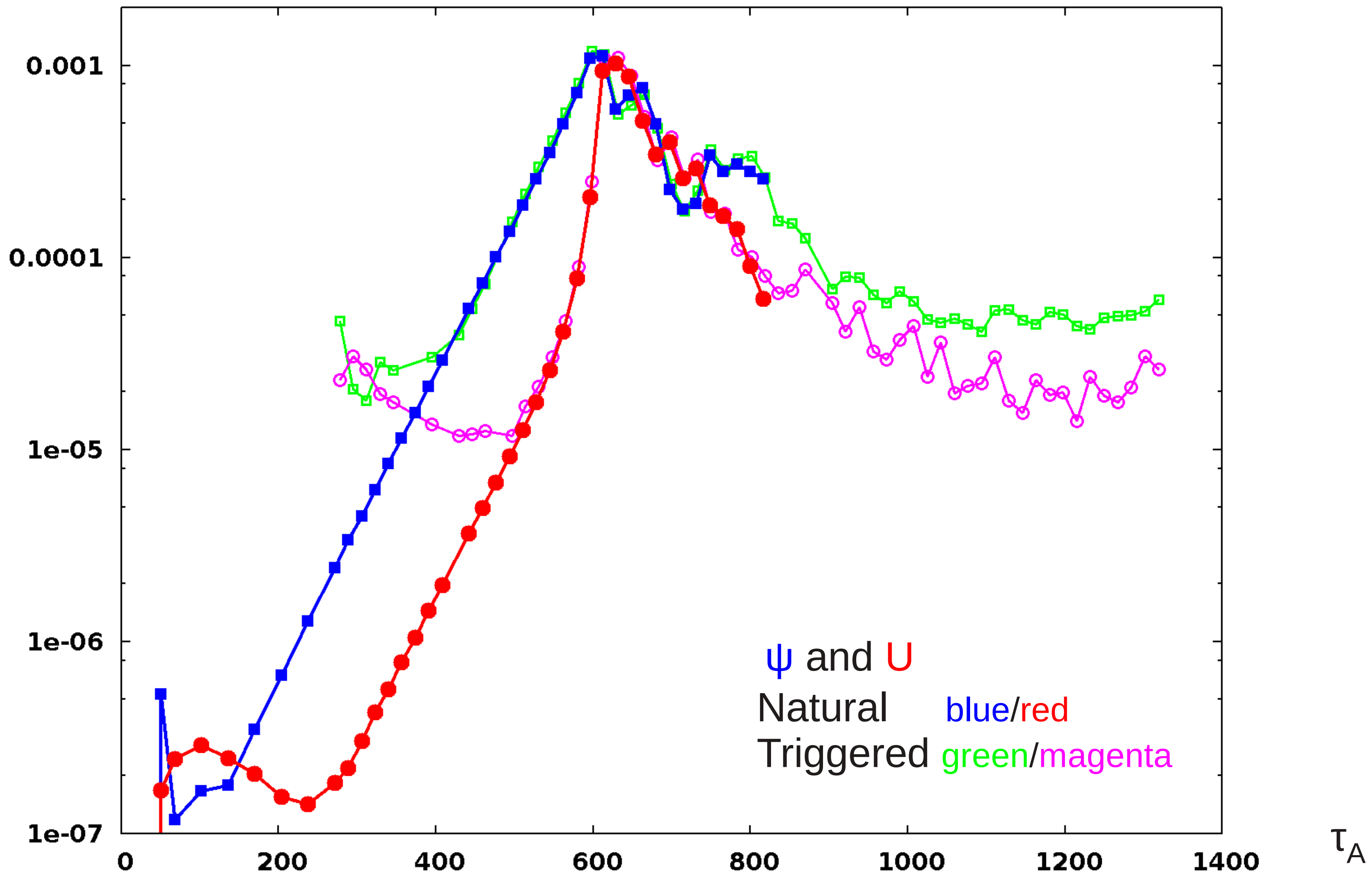


t=1075



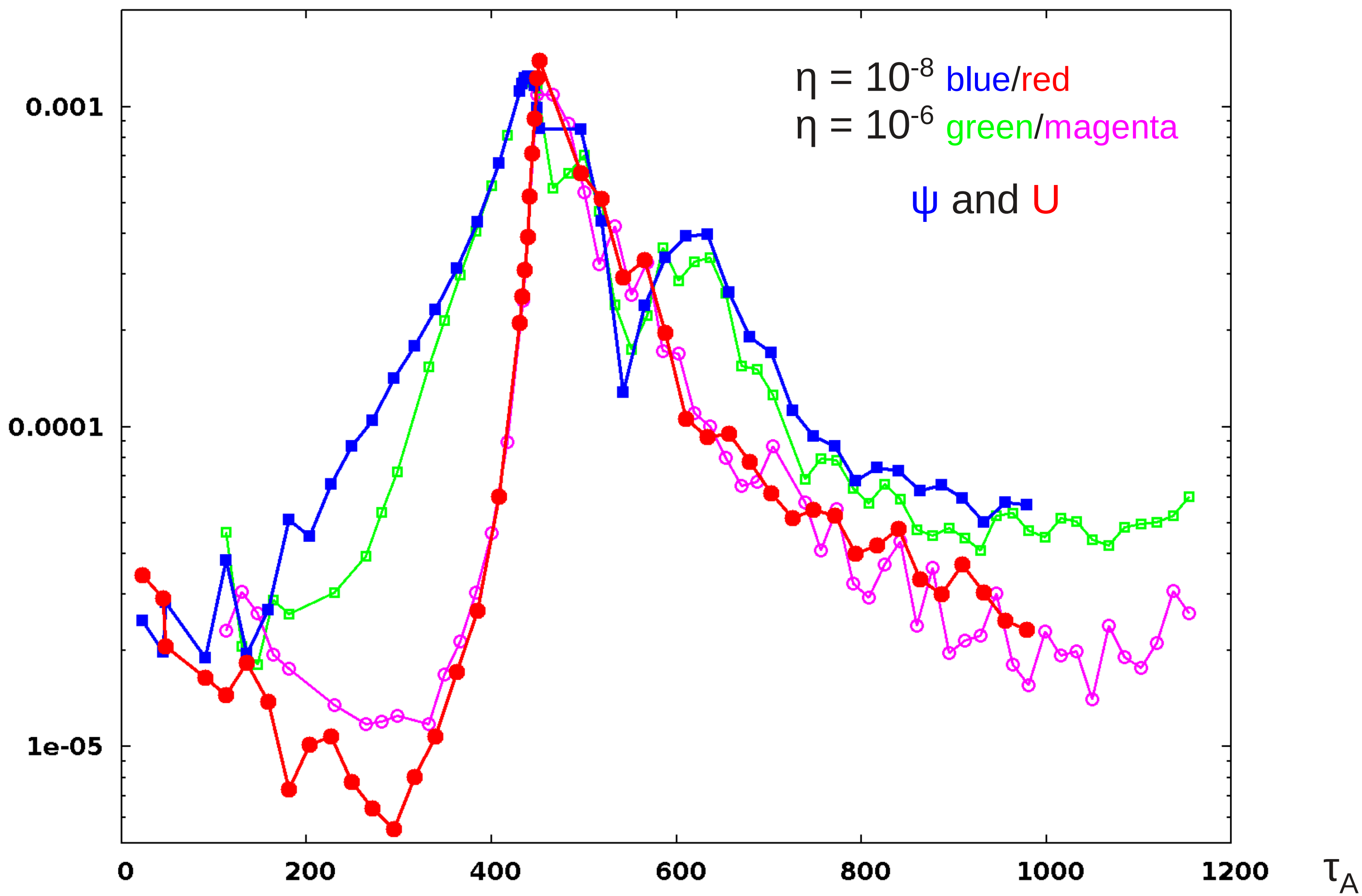
- Density distribution for triggered crash at $\eta=10^{-8}$, showing rapid development of ballooning burst outside q>1 (q~1.5)
  - Central structure rotates CCW (ω_{*e}) in plane at $\varphi=0$.
  - Helical density inside q<1 is preserved.
  - Ballooning at $\eta=10^{-7}$, not at 10^{-6}
- Simulation was stopped here....

Triggered crash behaves very similarly to natural crash



$\eta = 10^{-6}$: Triggered crash curves shifted in time to match, no scaling.
Amplitude determines fast crash onset?

Triggered crash has only weak scaling with resistivity
Time history of n=1 harmonics



Triggered crash at higher resistivity grows/decays faster in ψ, but more similarly in U.

Poloidal stream functions shown: red/magenta is ψ, blue/green U, velocity)

Finite Time Lyapunov Exponents

- The Finite Time Lyapunov Exponent (FTLE) measures the local divergence or convergence of adjacent field lines of a vector field over a *finite* distance or time interval
  - Pick out boundaries in flow, including vortices in turbulent fluids
- Large FTLE values correspond to Lagrangian Coherent Structures (LCS) in a fluid velocity field, the time-evolving 'skeleton' of the flow
- Recent developments, starting with G. Haller (2001,2002) have made it a practical tool for the study of fluid motion
  - Mathematical foundation in *n*D by Lekien, Shadden, et al (2007)
- First application to plasmas: **B** and **V** fields, at a single time.
  - H. Krishnan and H. Childs, LBNL, have done FTLE calculation for M3D, using VisIt visualization package (ongoing collaboration).
  - Methods were developed, tested over past year on ocean circulation problem. (Still working on plasma case!)

Why FTLEs and LCS for plasma?

- Finite Time Lyapunov Exponent (FTLE) pick out boundaries in the 3D vector field flow, including turbulent flow $\rightarrow$ Structure of turbulence
- Lagrangian Coherent Structures in a fluid velocity field form the 'skeleton' (manifolds) of the turbulent flow, that evolve in time
- Plasmas have two vector fields, **V** and **B**
  - Different properties, but both can become stochastic/turbulent
  - Related: the Elsässer variables **V**$\pm$**B**, are exactly equivalent to **V** and **B** for an incompressible plasma (with some restrictions on density). Can measure shear Alfvén wave vs flow effects on steady states, compressible vs incompressible effects, ...
- Plasma FTLE as *quantitative* diagnostic for stochastic/turbulent plasma
  - Calculation methods involve particle/streamline/surface tracing. Parallelize well. Not just for visualization; new simulation methods?
- Lagrangian Coherent Structures in plasmas – what are they and what do they signify?

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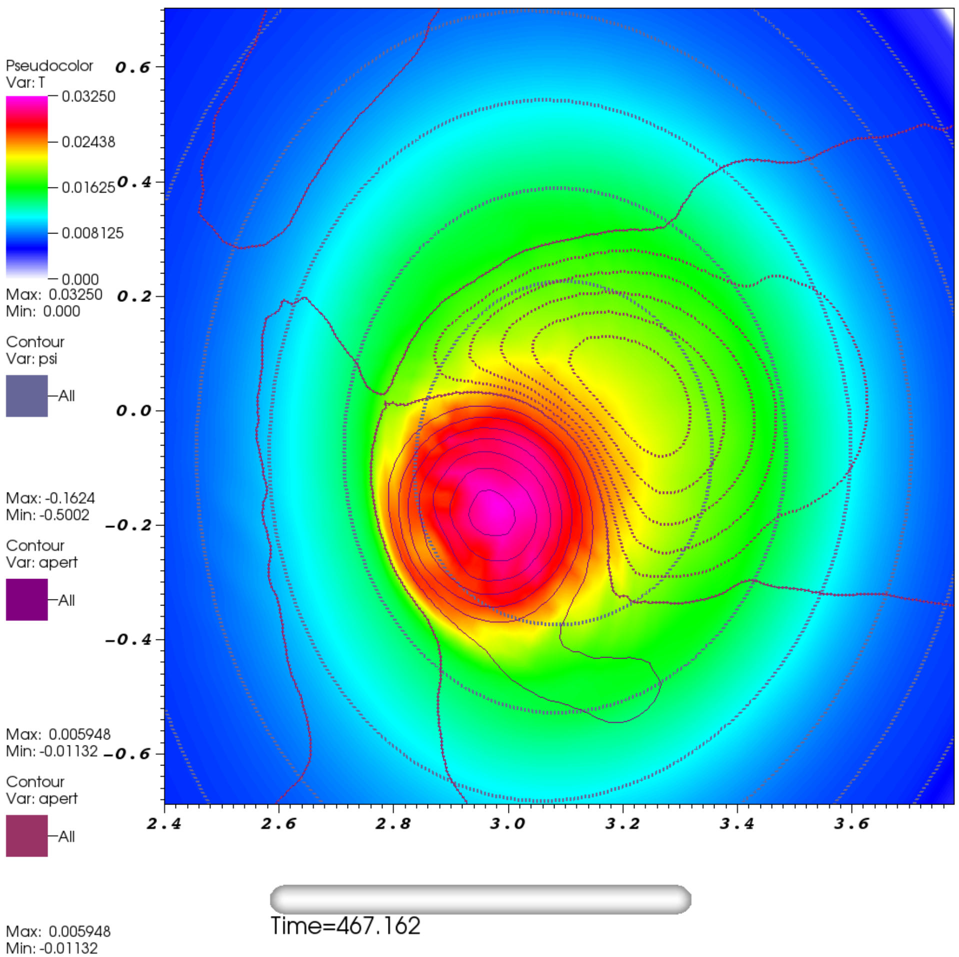
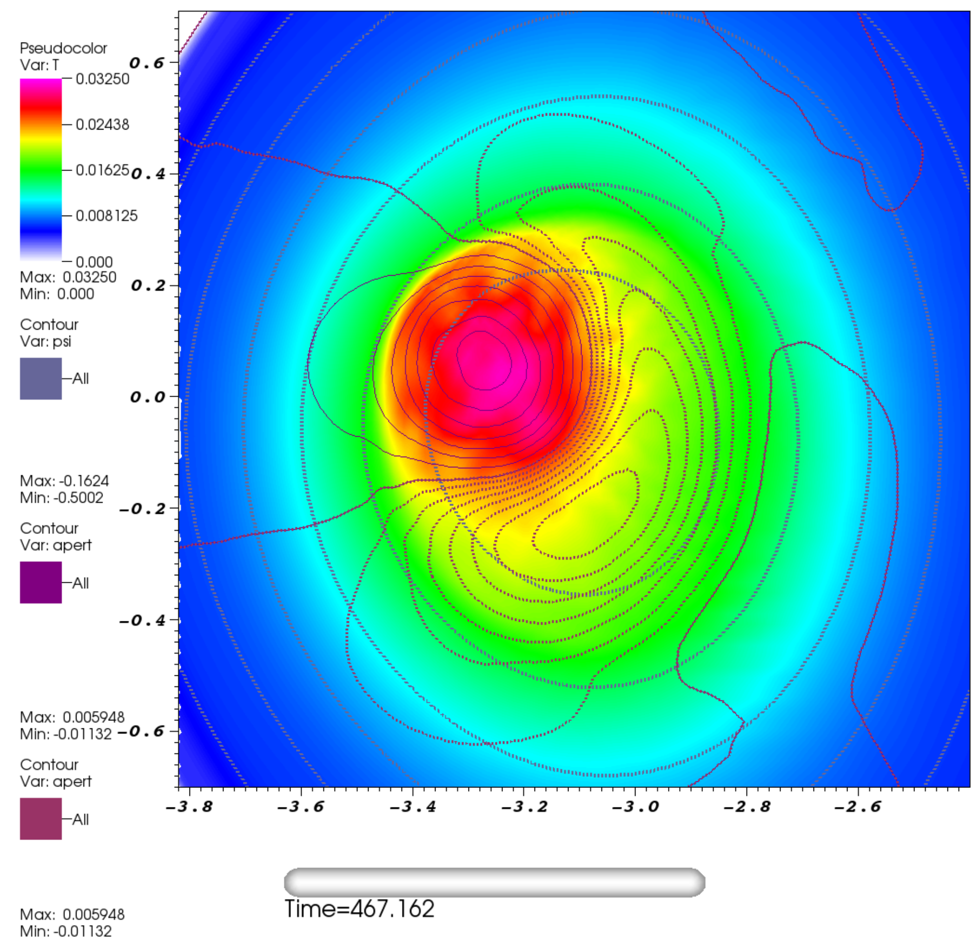
Temperature: displaced core

φ=0 (HFS)



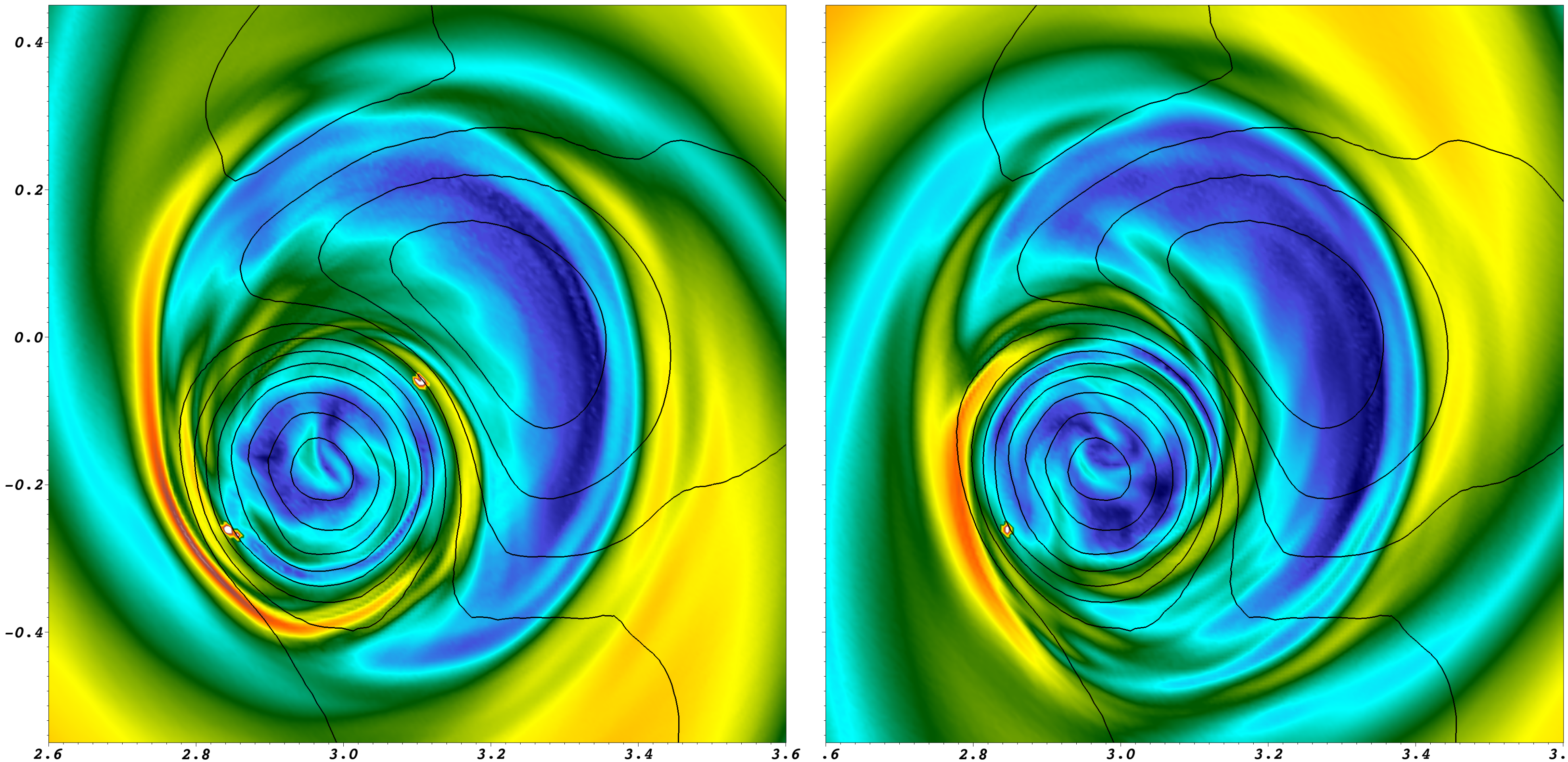
φ=π (LFS)



t=467.2

Finite Time Lyapunov Exponents: Magnetic Field

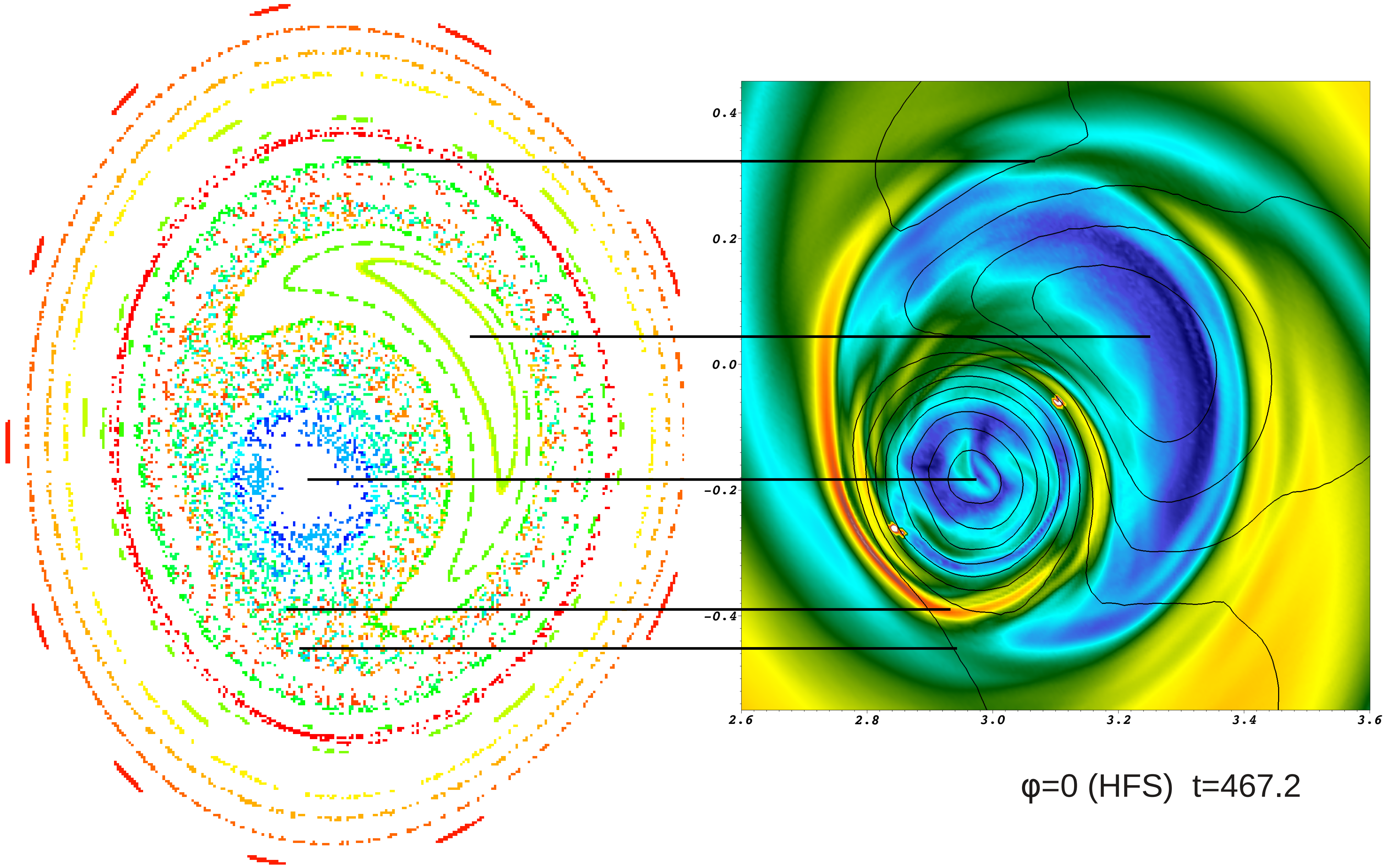
FTLE(+**B**) with perturbed poloidal magnetic flux FTLE(-**B**)



- Magnetic core in lower half of central q<1 region, island in upper half
- Singular FTLE point at magnetic 'X-point'
- Poloidal magnetic flux ψ does not completely describe **B**-field

φ=0 (HFS), t=467.2

FTLE(**B**) and magnetic field



φ=0 (HFS) t=467.2

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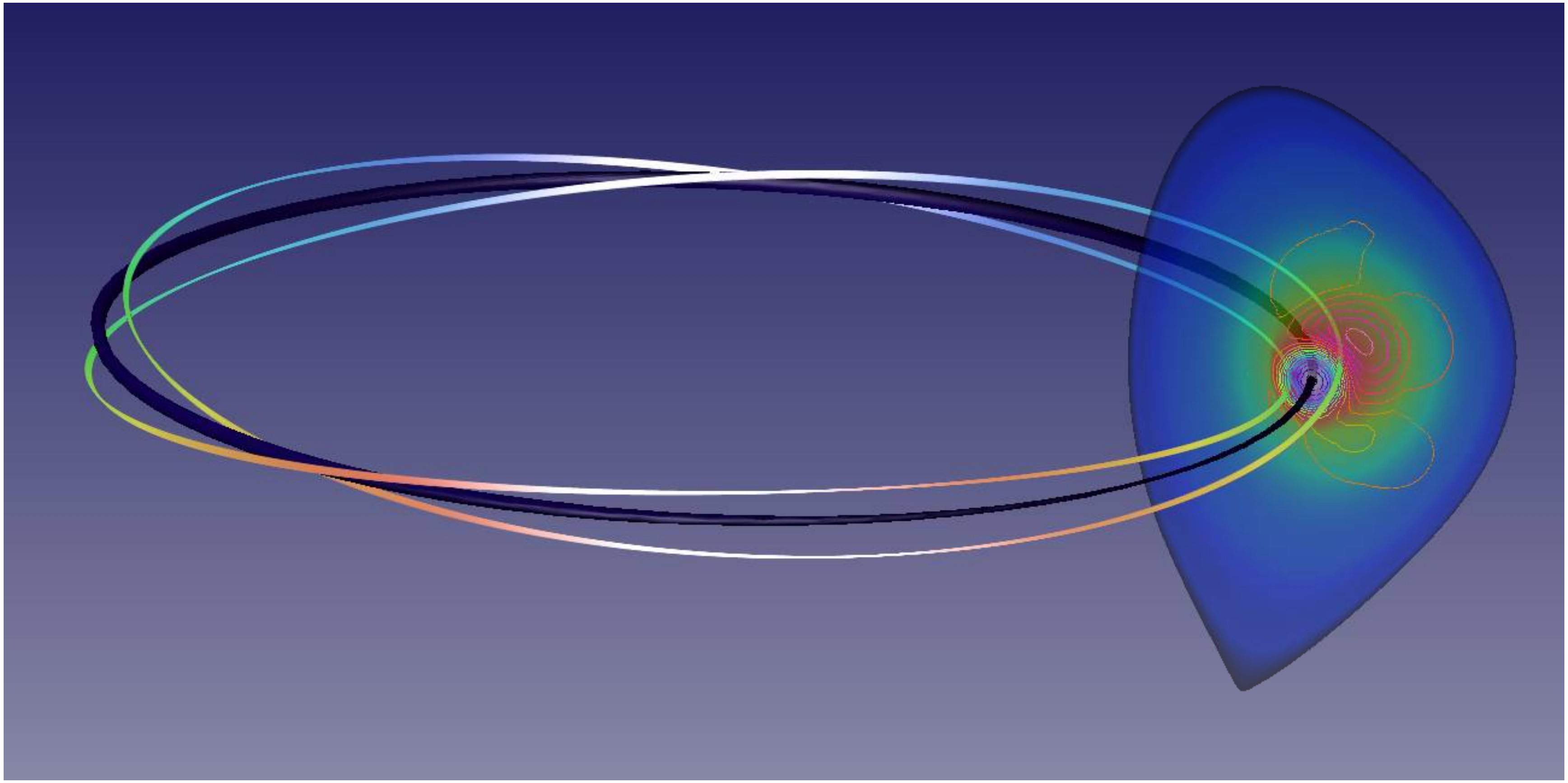
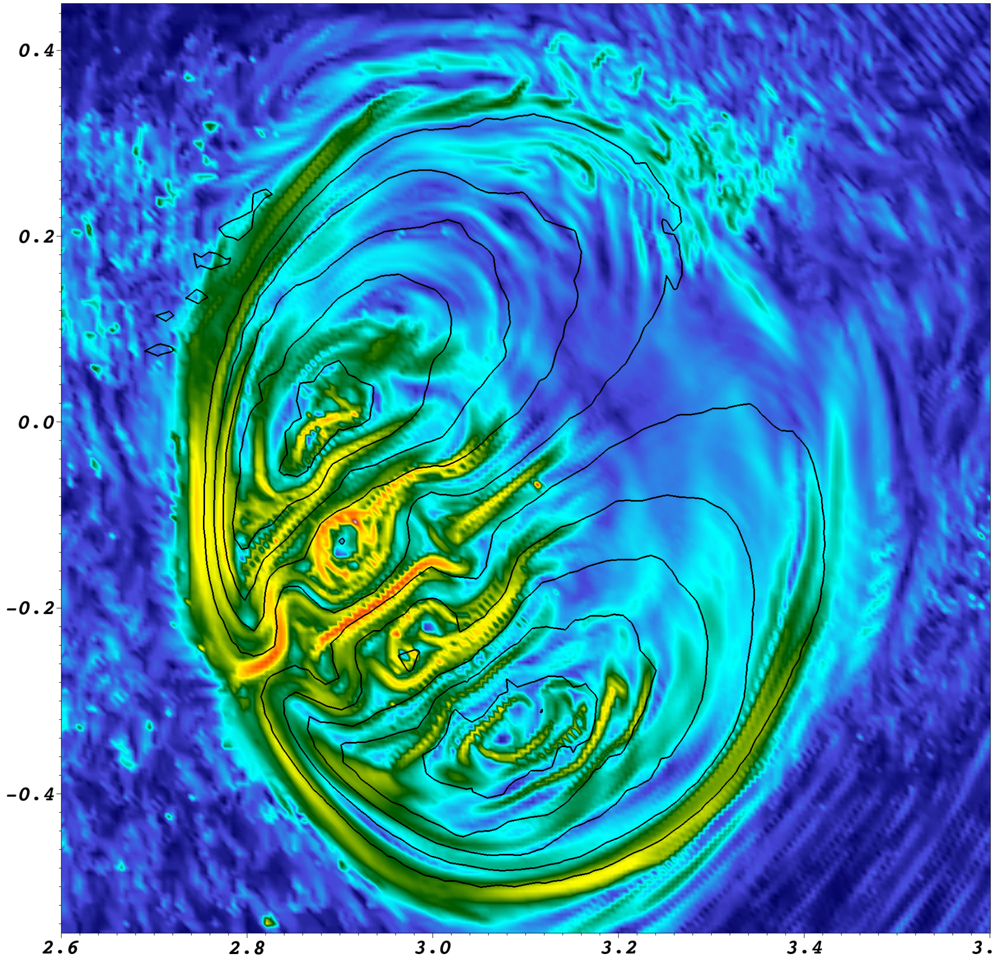
FTLE picks out magnetic structure



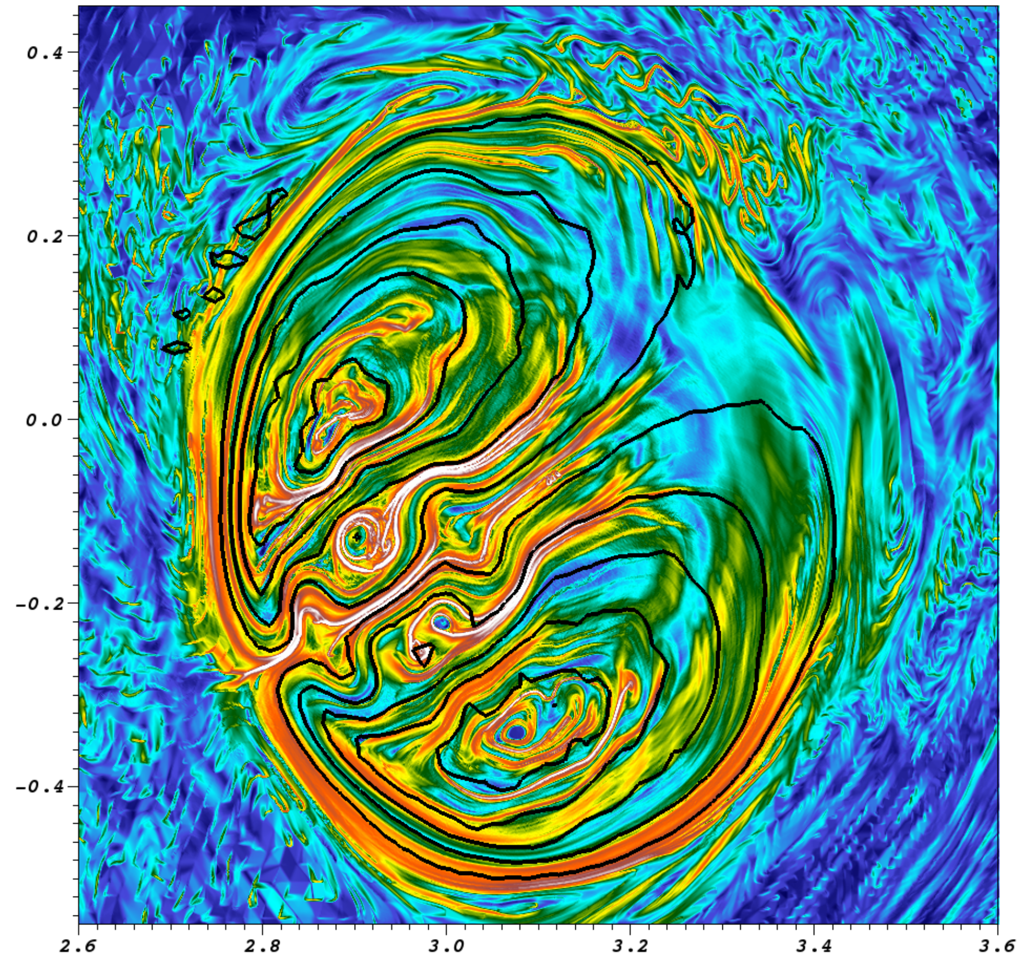
- High FTLE(±**B**) value "spots" correspond to a 2/1 X-line around the core – surrounding field lines show it is a X-point.
  - Shows accuracy of FTLE calculation (developed in past year; tests on ocean model)
  - Existence verified by field line tracing, not easily seen in puncture plot.
- Corresponding spots in **v**±**B**, interpretation still uncertain.

Finite Time Lyapunov Exponent: **V**

FTLE(**V**) reference

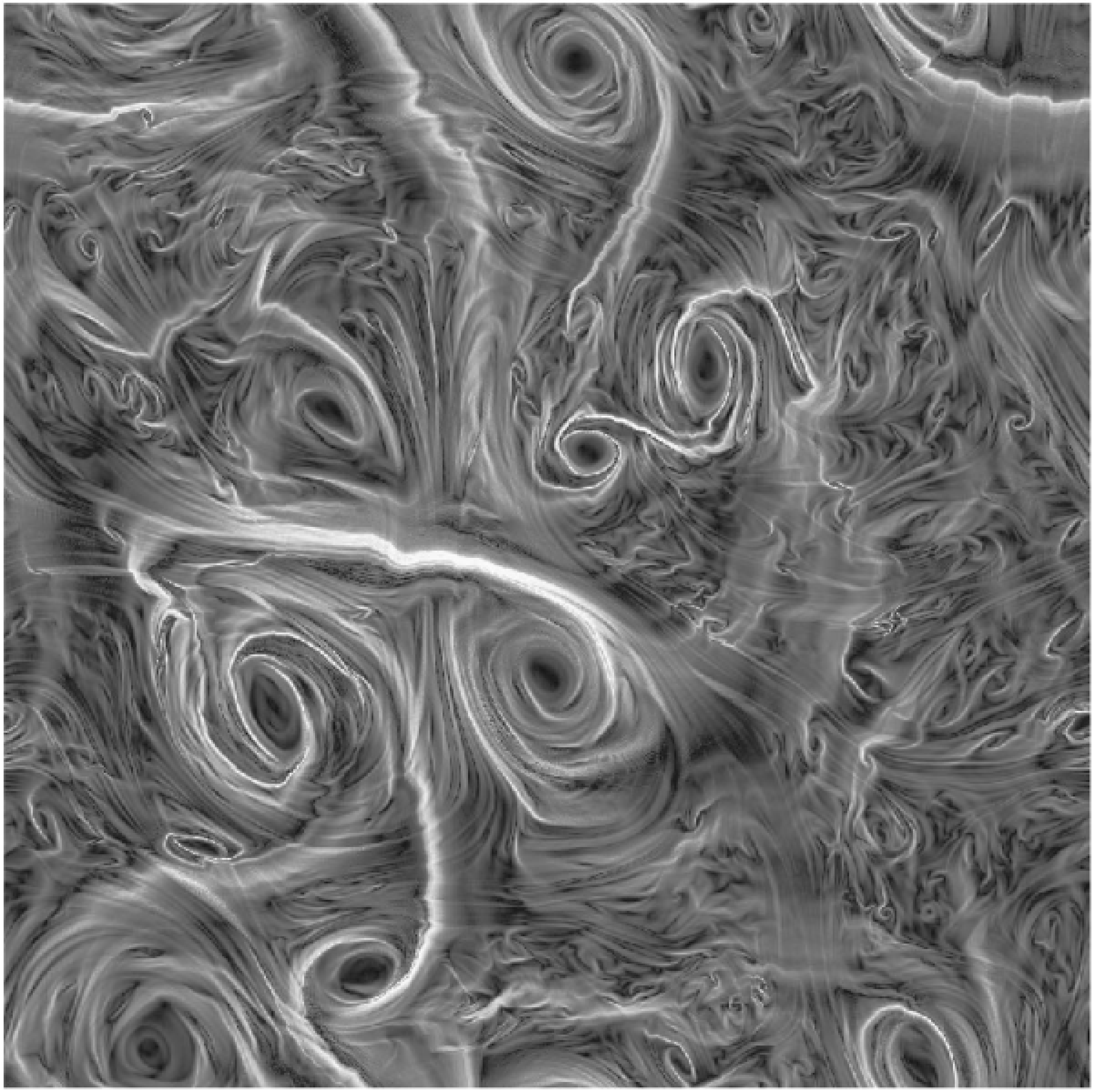


FTLE(**V**) traced 2x longer



- Following **V** for longer brings out more small scale structure (as for regular fluid **V**).
- |FTLE| increases, so colors change; still close to poloidal velocity stream function U

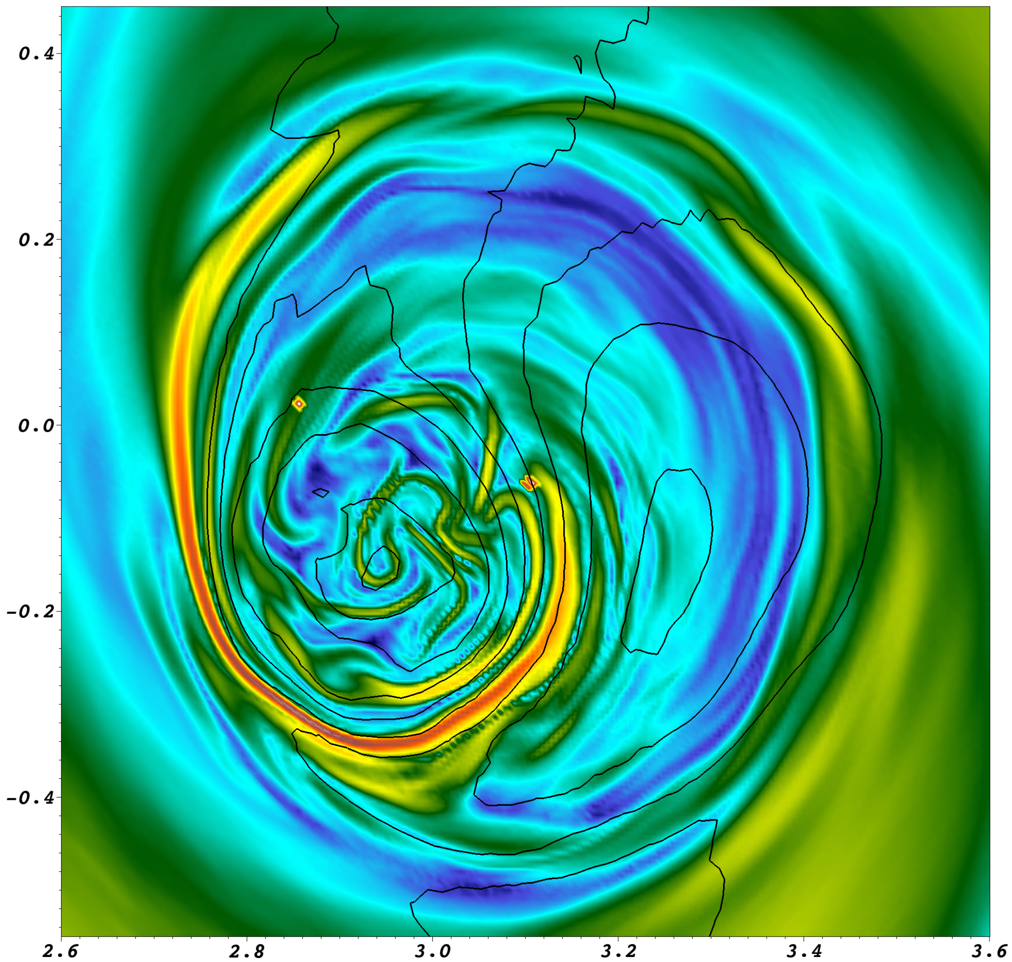
φ=0 (HFS), t=467.2



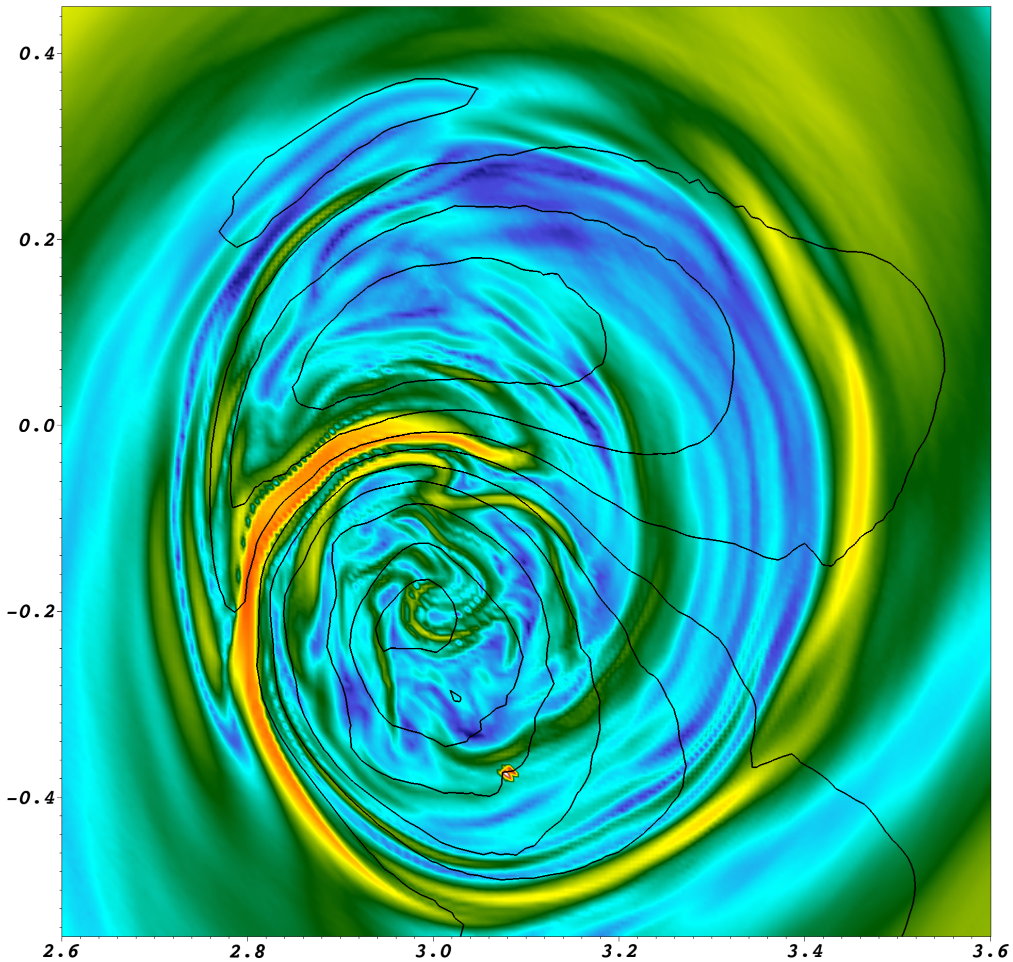
Plasma FTLE($\pm\mathbf{V}$) resembles FTLE of decaying 2D fluid turbulence, with multiple vortices and fine scale structure (e.g., G. Lapeyre, Chaos, 2002)

Finite Time Lyapunov Exponents: **V** ± **B**

FTLE(**V**+**B**) with $U+\tilde{\psi}/R_o$



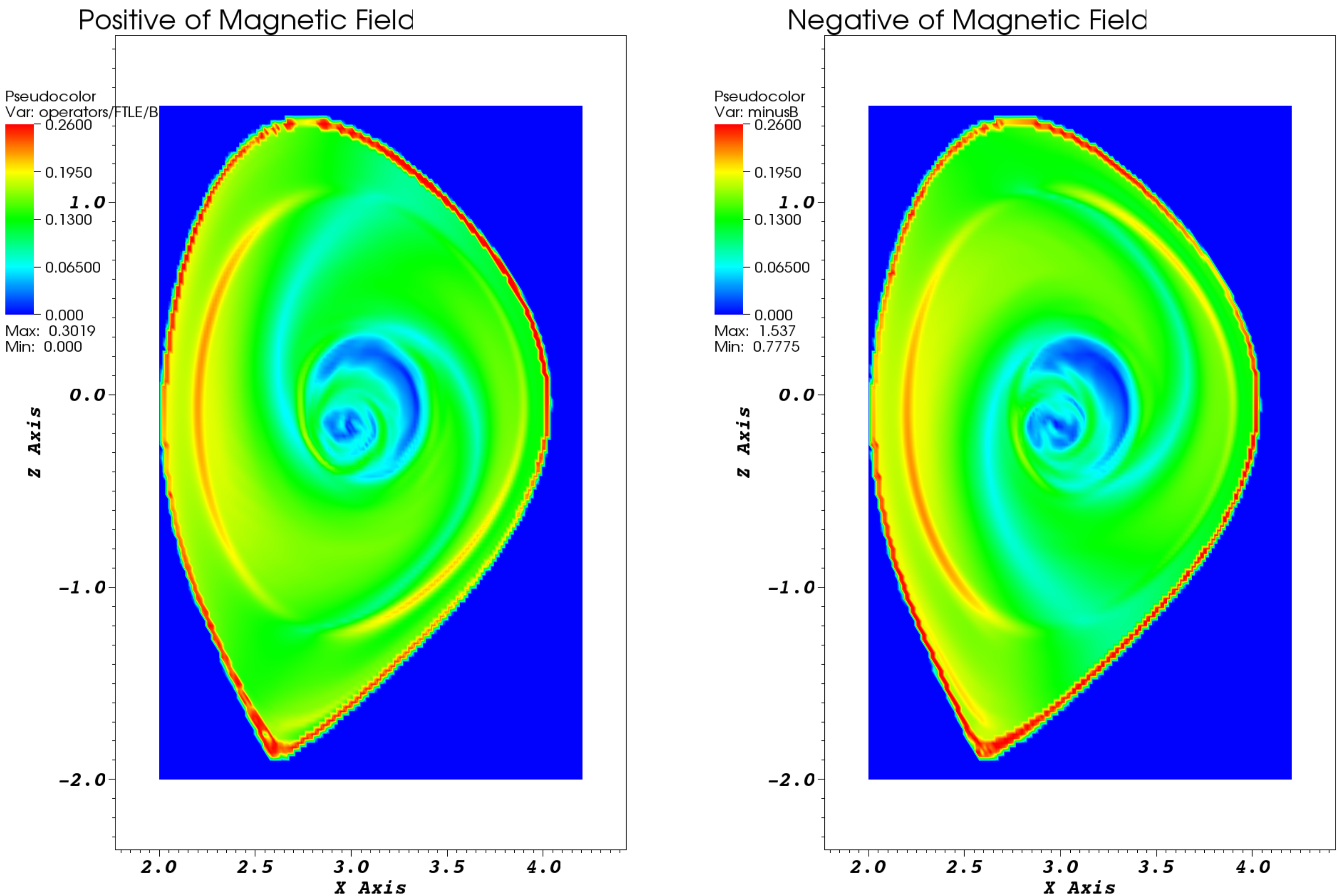
FTLE(**V**-**B**) with $U-\tilde{\psi}/R_o$



- Kink also has a **V**±**B** structure! Elsässer variables => incompressible MHD. but not RMHD!
- Approximated by simplest combination of poloidal stream functions $U-\psi/R_o$
- **V**±**B** structure has X-points, like **B**, but different locations

φ=0 (HFS), t=467.2

FTLE(±**B**), full cross section

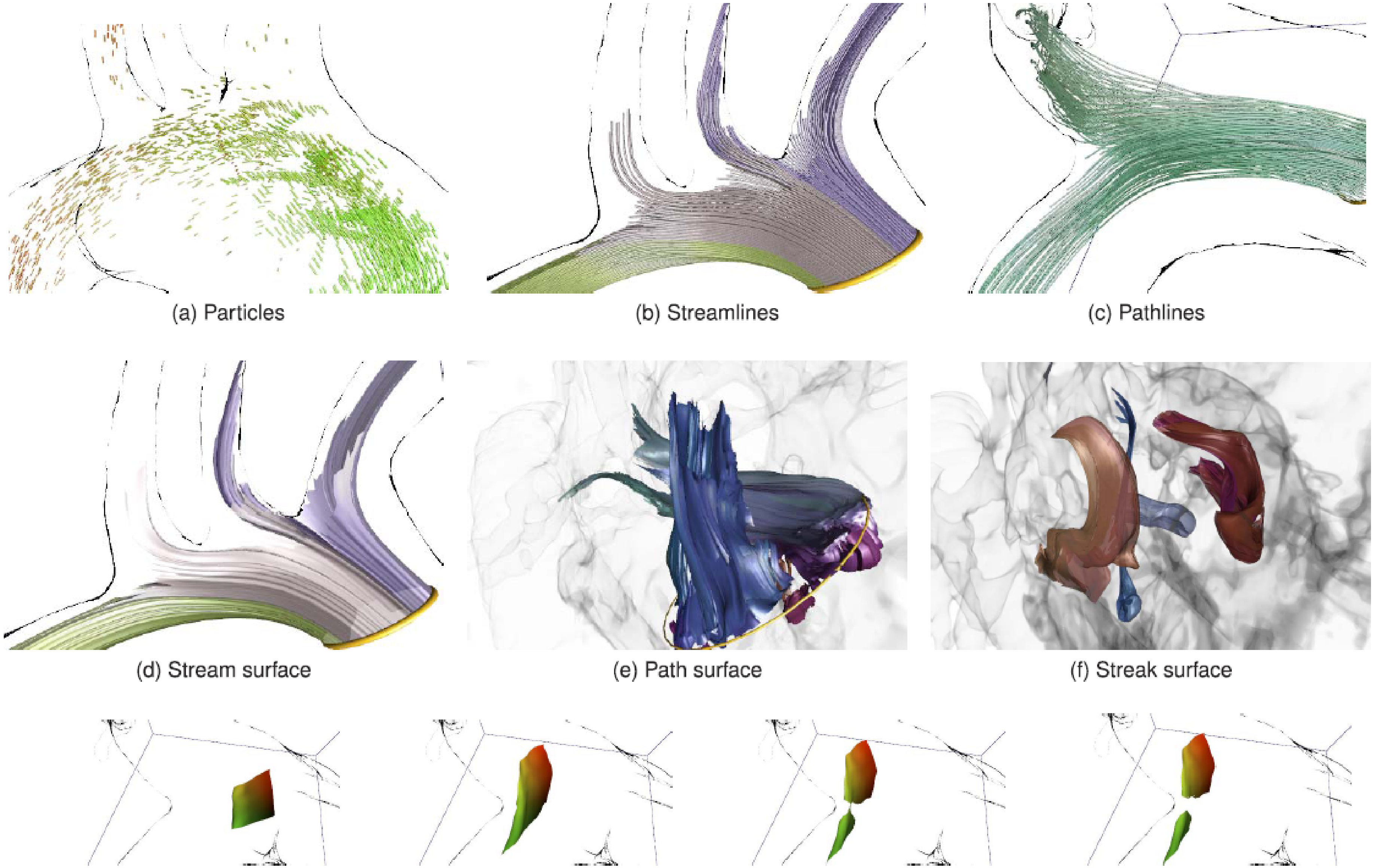


Small 3/1 island at q=3; high edge shear with X-point on boundary

user: hari
Thu Feb 9 15:53:01 2012

user: hari
Thu Feb 9 15:36:40 2012

FTLE and LCS visualization has many similarities to plasma simulations - cross fertilization



(a) Particles (b) Streamlines (c) Pathlines

(d) Stream surface (e) Path surface (f) Streak surface

(g) Time surface

Fig. 1. Our prototype visualization system provides a variety of integration-based methods coupled with interactive seeding to allow blood flow visualization. Top row of images illustrates particle and line primitives. The middle row depicts stream surface, path surface, and streak surfaces. The bottom row consists of four images from an animation showing a time surface splitting into two as it encounters a bifurcation point.

(H Krishnan, et al, IEEE Trans Vis Comp Graphics 2012)

Summary

- MHD with density evolution: 1/1 kink-type mode
  - Snake (helical ion density concentration) – new mode
  - Sawtooth fast crash at low η
  - Not so simple: rotation effects, ...
- Finite Time Lyapunov Exponent for vector fields
  - First results in plasmas show great promise
  - To be extended to Lagrangian Coherent Structures in plasmas, for time-dependent problems
  - FTLE methods to be added to publicly available VisIt (needs parallelization?)