WHAT ARE CLOSURES?

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Theses:

- **Historically, closures have meant kinetically-derived expressions for higher order moments for closing fluid moment equations at finite order.**
- **Currently "closures" mean many things to many people — collisional, $\perp$ or $\|$ kinetic effects, gyrofluid, inclusion of turbulence effects, etc.**
- **Extended MHD equations developed from two-fluid equations with general closures provide reasonable basis for describing macroscopic plasmas.**
- **Closures must be developed for classes of problems — because general closures are only possible for collision-dominated regime (i.e., Braginskii).**
- **"Fast" ($\omega > \nu$) and "slow" ($\omega < \nu$) MHD have different closure needs.**

Outline

- **Fluid moment equations and closures**
- **Collisional (Braginskii) analysis, closures**

  Two-fluid analysis, equations $\Longrightarrow$ extended MHD

  Chapman-Enskog (C-E) procedure

  Anisotropic nature of closures

  Approximations used in deriving collisional closures
- **Various types of closures:**

  General moment approach

  Neoclassical-based closures

  Parallel kinetics — drift-kinetic C-E equation; PIC-based and "continuum" solutions

  Perpendicular closures via fluid moments — diamagnetic/gyroviscosity, perpendicular

  Drift-kinetic and gyrokinetic-based approaches

  Energetic plasma component effects

  Turbulence closures? — neutral fluid, drift-wave, multi-scale
- **Generic fast and slow MHD closure issues**
- **Summary — develop closures for classes of problems**

Fluid Moment Equations — The Classic Closure Problem

- The rigorous Plasma Kinetic Equation (PKE) to begin from is

$$\boxed{\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f + \frac{q}{m}(\vec{E} + \vec{v}\times\vec{B})\cdot\vec{\nabla}_v f = \mathcal{C}\{f\}.}$$

- Exact fluid moment equations for each plasma species result from velocity-space moments ($\int d^3v\,\vec{v}^{\,n},\ n = 0, 1, 2$) of this fundamental kinetic equation:

$n = 0, |\vec{v}|^0$, density $\quad \dfrac{\partial n}{\partial t} + \vec{\nabla}\cdot n\vec{V} = 0, \qquad \{\vec{\nabla}\vec{V}\} \equiv \dfrac{1}{2}\left[\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^T\right] - \dfrac{1}{3}\overleftrightarrow{\mathbf{I}}\,(\vec{\nabla}\cdot\vec{V}),$

$n = 1, \vec{v}$, momentum $\quad mn\dfrac{d\vec{V}}{dt} = nq(\vec{E} + \vec{V}\times\vec{B}) - \vec{\nabla}p - \vec{\nabla}\cdot\pi + \vec{R}, \qquad \dfrac{d}{dt} \equiv \dfrac{\partial}{\partial t} + \vec{V}\cdot\vec{\nabla},$

$n = 2, v^2$, energy $\quad \dfrac{3}{2}n\dfrac{dT}{dt} + nT\,\vec{\nabla}\cdot\vec{V} = -\vec{\nabla}\cdot\vec{q} - \overleftrightarrow{\pi} : \{\vec{\nabla}\vec{V}\} + Q, \qquad p \equiv nT,$

or entropy $\quad \dfrac{\partial (ns)}{\partial t} + \vec{\nabla}\cdot\left(ns\vec{V} + \dfrac{\vec{q}}{T}\right) = \dfrac{1}{T}(-\vec{q}\cdot\vec{\nabla}\ln T - \overleftrightarrow{\pi} : \{\vec{\nabla}\vec{V}\} + Q), \quad s \equiv \ln(T^{3/2}/n).$

- These moment equations need closure moments for $\vec{q}$ and $\overleftrightarrow{\pi}$ ($\vec{v}_r \equiv \vec{v} - \vec{V}$):

heat flux $\vec{q} \equiv \displaystyle\int d^3v\,\vec{v}_r\left(\frac{mv_r^2}{2} - \frac{5}{2}\right)f, \qquad$ stress tensor $\overleftrightarrow{\pi} \equiv \displaystyle\int d^3v\,m\left(\vec{v}_r\vec{v}_r - \frac{v_r^2}{3}\overleftrightarrow{\mathbf{I}}\right)f.$

Definitions Of Relevant Velocity-Space Integrals

- **Fluid moments in terms of velocity-space integrals of distribution function:**

density: $n(\vec{x},t) \equiv \int d^3v\, f,$

flow velocity: $\vec{V}(\vec{x},t) \equiv \dfrac{1}{n}\displaystyle\int d^3v\, \vec{v}\, f,$

temperature: $T(\vec{x},t) \equiv \dfrac{1}{n}\displaystyle\int d^3v\, \dfrac{m|\vec{v}_r|^2}{3}\, f, \qquad \vec{v}_r \equiv \vec{v} - \vec{V}(\vec{x},t).$

- **Velocity-space moments of the Coulomb collision operator:**

density: $0 \equiv \int d^3v\, \mathcal{C}\{f\},$

momentum: $\vec{R}_e \equiv \displaystyle\int d^3v\, m_e \vec{v}\, \mathcal{C}\{f\} = -m_e n_e \nu_e[(\vec{V}_e - \vec{V}_i) - \alpha \vec{q}_e / n_e T_e + \cdots] \sim n_e e \eta \vec{J},$

energy: $Q_e \equiv \displaystyle\int d^3v\, \dfrac{m_e|\vec{v}_r|^2}{2}\, \mathcal{C}\{f\} = -\dfrac{3}{2} n_e \nu_e \dfrac{m_e}{m_i}(T_e - T_i) \underbrace{-\vec{R}_e\cdot(\vec{V}_e - \vec{V}_i)}_{\sim +\eta J^2}.$

- **Note that closure moment for $\vec{q}_e$ is needed to specify frictional force $\vec{R}_e$ and energy transfer Q_e. Need parallel electron heat flow for Spitzer resistivity. (Braginskii high collisionality closure writes $\vec{q}_e$ in terms of $E_\|$ and $\nabla_\| T_e$.)**

Collisional Analysis Uses Chapman-Enskog Procedure

- **In a neutral fluid, assuming the lowest order kinetic equation is dominated by the collision operator (i.e., $\mathcal{C}\{f_0\} \simeq 0$), the lowest order solution $f_0 = f_M$ is a "dynamic" Maxwellian with "parameters" $n(\vec{x},t)$, $T(\vec{x},t)$, $\vec{V}(\vec{x},t)$:**

$$\boxed{f_M(\vec{x},\vec{v},t) = n(\vec{x},t)\left(\frac{m}{2\pi T(\vec{x},t)}\right)^{3/2} e^{-m|\vec{v}_r|^2/2T(\vec{x},t)},} \qquad \vec{v}_r \equiv \vec{v} - \vec{V}(\vec{x},t).$$

- **Chapman-Enskog procedure: Next order equation is obtained by substituting $f = f_M + \delta f$ into kinetic equation, making use of density, momentum and energy conservation equations to remove dependences on $\partial n/\partial t$, $\partial\vec{V}/\partial t$ and $\partial T/\partial t$, and neglecting higher order corrections ($\sim 1/\nu$):**

$$\mathcal{C}\{\delta f\} \simeq \left[\left(\frac{m|\vec{v}_r|^2}{2T} - \frac{5}{2}\right)\vec{v}_r\cdot\vec{\nabla}\ln T + \frac{m}{T}\{\vec{\nabla}\vec{V}\} : \left(\vec{v}_r\vec{v}_r - \frac{|\vec{v}_r|^2}{3}\overleftrightarrow{\mathrm{I}}\right)\right] f_M \sim -\nu\,\delta f.$$

- **Inverting collision operator yields δf whose velocity-space moments provide needed closure relations (collision length $\lambda \equiv v_T/\nu$, $v_T \equiv \sqrt{2T/m}$):**

$$\vec{q} = -\kappa^m\vec{\nabla}T = -n\chi^m\vec{\nabla}T, \text{ with "molecular" heat diffusivity } \chi^m \sim v_T^2/\nu = \nu\lambda^2,$$

$$\overleftrightarrow{\pi} = -2\nu^m\{\vec{\nabla}\vec{V}\} = -nm\mu^m\{\vec{\nabla}\vec{V}\}, \text{ "molecular" viscosity } \mu^m \sim \nu\lambda^2,\ \vec{\nabla}\cdot\overleftrightarrow{\pi} \simeq -\nu^m\nabla^2\vec{V}.$$

Magnetized Plasmas Are Very Anisotropic ($\perp$, $\|$ to $\vec{B}$)

- Braginskii [1] used a Chapman-Enskog procedure and an ordering scheme for magnetized ($\omega_c \equiv qB/m >> \nu$), collisional ($\nu >> \omega, k_\| v_T$) plasmas:

  $\perp$ to $\vec{B}$: small gyroradius, $\varrho \equiv v_T/\omega_c \implies \epsilon_\perp \sim |\varrho \vec{\nabla}_\perp| << 1.$

  $\|$ to $\vec{B}$: short collision length, $\lambda \equiv v_t/\nu \implies \epsilon_\| \sim |\lambda \nabla_\| | << 1.$

- Conductive heat flux closure moment is found to have parallel ($\|$), cross ($\wedge$, in flux surface) and perpendicular ($\perp$, across flux surfaces) components:

$$\vec{q} = -\,n\,\chi_\| \nabla_\| T - n\,\chi_\wedge (\vec{B}/B)\times\vec{\nabla}T - n\,\chi_\perp \vec{\nabla}_\perp T, \quad \text{in which } \vec{\nabla}_\perp \equiv -\,(1/B^2)\vec{B}\times(\vec{B}\times\vec{\nabla}),$$

  parallel heat conduction: $\chi_\| \sim \nu\lambda^2 \sim \epsilon_\|^2 \epsilon_\perp^0 \Longrightarrow$ fast ($t \sim 1/\nu$), $\|$ T_e equilibration,

  cross (diamagnetic heat flow): $\chi_\wedge \sim v_T \varrho \sim \epsilon_\perp \Longrightarrow$ slower, diamag. flows in surface,

  perpendicular heat conduction: $\chi_\perp \sim \nu\varrho^2 \sim \epsilon_\perp^2 \Longrightarrow$ slowest, radial heat transport.

- Stress tensor has similar form: $\overleftrightarrow{\pi} = \overleftrightarrow{\pi}_\| + \overleftrightarrow{\pi}_\wedge + \overleftrightarrow{\pi}_\perp$ with similar scalings $\overleftrightarrow{\pi}_\| \sim \epsilon_\|^2 \epsilon_\perp^0$ (parallel stress), $\overleftrightarrow{\pi}_\wedge \sim \epsilon_\perp$ (gyroviscosity) and $\overleftrightarrow{\pi}_\perp \sim \epsilon_\perp^2$ ($\perp$ visc.).

[1] S.I. Braginskii, in *Reviews of Plasma Physics* (Consultants Bureau, NY, 1965), Vol I, p 205.

Comments On Collisional Magnetized Plasma Equations

- The collisional fluid equations use these anisotropic closures and the resultant "two-fluid" equations are known as the Braginskii equations.

- Braginskii closures and equations are derived using the following major approximations, which determine their range of validity:

  short collision length, $\epsilon_\parallel \sim \lambda \nabla_\parallel << 1$ — not valid for most tokamak plasma regimes,

  small gyroradius, $\epsilon_\perp \sim \varrho \vec{\nabla}_\perp << 1$ — equil. ok, but need $k_\perp \varrho << 1$ for perturbations,

  slow processes, $\partial/\partial t << \nu$ — equil. ok, but need $\omega/\nu << 1$ for perturbations

  negligible anomalous transport — add transport coefficients from microturbulence?

- Critiques of the Braginskii equations:

  They neglect effects due to collisions with neutrals or energetic (e.g., fast ion) particles — but these transport-time-scale (slow) effects can mostly just be added as "sources."

  They do not include direct loss processes (e.g., near separatrix, on open field lines).

  The stress tensor $\overleftrightarrow{\pi}$ is driven not just by $\{\vec{\nabla}\vec{V}\}$ but also by a comparable $\{\vec{\nabla}\vec{q}\}$ [2].

[2] A.B. Mikhailovskii, *Theory of Plasma Instabilities* (Atomdat, Moscow, 1977), Vol 2, p 307-325 (in Russian); A.B. Mikhailovskii and V.S. Tsypin, Plasma Physics 13, 785 (1971); *ibid.*, Beitr. Plasmaphys. 24, 335 (1984).

Extended MHD Model Derived From Two-Fluid Equations

- Assume for the moment that anisotropic closures for $\vec{q}$ and $\overleftrightarrow{\pi}$ can be obtained for both electrons and ions for relevant situations.

- Then, adding, subtracting electron and ion density and momentum equations one obtains general "extended MHD" equations:

density $$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}\cdot\rho_m \vec{V} = 0, \qquad \rho_m \equiv \frac{\sum_s n_s m_s}{\sum_s m_s} \simeq n_i, \;\; \vec{V} \equiv \frac{\sum_s n_s m_s \vec{V}_s}{\sum_s n_s m_s} \simeq \vec{V}_i,$$

charge density $$\vec{\nabla}\cdot\vec{J} = 0, \qquad \vec{J} \equiv e(n_i Z_i \vec{V}_i - n_e \vec{V}_e),$$

momentum $$\rho_m \frac{d\vec{V}}{dt} = \vec{J}\times\vec{B} - \vec{\nabla}P - \vec{\nabla}\cdot\overleftrightarrow{\Pi}, \qquad P \equiv p_e + p_i, \;\; \overleftrightarrow{\Pi} \simeq \overleftrightarrow{\pi}_i + \overleftrightarrow{\pi}_e \simeq \overleftrightarrow{\pi}_i,$$

Ohm's law $$\vec{E} + \vec{V}\times\vec{B} = \underbrace{\frac{\vec{R}_e}{n_e e}}_{\sim \eta \vec{J}} + \underbrace{\frac{\vec{J}\times\vec{B} - \vec{\nabla}p_e - \vec{\nabla}\cdot\overleftrightarrow{\pi}_e}{n_e e}}_{\text{Hall terms}} + \underbrace{\frac{m_e}{e^2}\frac{d}{dt}\left(\frac{\vec{J}}{n_e}\right)}_{\text{electron inertia}}.$$

- Main effects of closures come in parallel Ohm's law and equation of state for the total plasma pressure P obtained from plasma entropy evolution:

$$\frac{d}{dt}\left(\ln\frac{P}{\rho_m^\Gamma}\right) = \frac{\Gamma-1}{P}\left(p_e\frac{ds_e}{dt} + p_i\frac{ds_i}{dt}\right) \simeq \frac{\Gamma-1}{P}(-\underbrace{\vec{\nabla}\cdot\vec{q}_e}_{\sim\nu_e\epsilon_\parallel^2} - \underbrace{\{\vec{\nabla}\vec{V}_i\}:\overleftrightarrow{\pi}_i}_{\sim\nu_i\epsilon_\perp^2} + \underbrace{\eta J^2}_{\sim 1/\tau_E}), \quad \Gamma \equiv \frac{5}{3}.$$

Comments On Closures For These MHD-Type Equations

- **The only general, analytic closures are the collisional (Braginskii) ones.**

- **The main limitation in using Braginskii closures is the high collisionality requirement for the parallel kinetics: $\epsilon_\parallel \sim (v_T/\nu)\nabla_\parallel = \lambda\nabla_\parallel << 1$.**

- **The closures should be determined from a Chapman-Enskog-type procedure so the kinetics used to obtain them does not simultaneously produce "extra" δn, $\delta\vec{V}$, and/or δT contributions to the equations:**

  The usual drift-kinetic and gyro-kinetic equations are not developed using a Chapman-Enskog-like procedure and hence usually produce δn, $\delta\vec{V}$, and/or δT terms.

  Formal Chapman-Enskog-type procedures and resultant drift-kinetic equations have been developed for arbitrary $\parallel$ collisionality [3-5], but they are rather complicated.

- **The anisotropic components of the closures can be handled differently:**

  parallel: in general a kinetic analysis must be used, including collisional effects,

  cross: fluid-type analysis, gyroviscosity for these diamagnetic flow type effects,

  perpendicular: fluidlike radial transport due to collisional effects on diamagnetic flows.

[3] K.C. Shaing and D.A. Spong, Phys. Fluids B 2, 1190 (1990) — first Chapman-Enskog-like formalism.
[4] J.P. Wang and J.D. Callen, Phys. Fluids B 4, 1139 (1992) — axisymmetric $\vec{B}$, neoclassical formalism.
[5] Z. Chang and J.D. Callen, Phys. Fluids B 4 1167 (1992) — sheared slab model, with Landau damping.

Comments On Closures For MHD (continued)

- Friction forces $\vec{R}$ and stress tensors $\overleftrightarrow{\pi}$ are most fundamentally, generally written in terms of $\vec{V}$ and $\vec{q}$ — rather than $\vec{V}$ and $\vec{\nabla} T$ Braginskii uses.

  Are the usual two-fluid analysis and equations using the best variables (i.e., n, $\vec{V}$, T)?

- The parallel Ohm's law is governed experimentally by the neoclassical Ohm's law and apparently not affected [6,7] by microturbulence — because $k_\| << k_\perp$ and hence their parallel momentum transfer is small

- Temporal regimes — it seems there are two MHD regimes of interest:

  "fast MHD" ($\omega >> \nu$) — little entropy production, closures not very important?

  "slow MHD" ($\omega << \nu$) — collision-dominated closures and dissipation critical.

- Spatial regimes — very anisotropic and different physics each direction:

  parallel: need more general kinetic-based formalism, closures for $k_\| \lambda \sim 1$
  cross (in flux surface): need separation from drift-wave-driven microturbulence
  $\Longrightarrow k_\theta \varrho_S < 0.3? \Longrightarrow$ poloidal mode numbers $m \lesssim 0.3\, r/\varrho_S \sim$ 30–100(ITER)?
  perpendicular (across flux surface): avoid FLR effects on resistive layer widths
  $\Longrightarrow k_x \varrho_i < 1$ with $\delta_\eta \sim r/(mS)^{1/3} \Longrightarrow mS \lesssim (r/\varrho_i)^3 \sim 10^6$–$3 \times 10^7$(ITER)?

[6] K.C. Shaing, Phys. Fluids 31, 8 (1988) — for electrostatic microturbulence.
[7] F.L. Hinton, R.E. Waltz, and J. Candy, Phys. Plasmas 11, 2433 (2004) — including $\delta \vec{B}_\perp$ effects.

General Fluid Moment Expansion Approach

- Grad [8] showed that complete, orthogonal basis set for Boltzmann collision operator (and thus also its small momentum transfer limit $\Longrightarrow$ Coulomb collision Fokker-Planck operator) can be written in terms of Hermite $\Longrightarrow$ spherical harmonics and Laguerre (Sonine) polynomials:

  $f(\vec{x},\vec{v},t) = f_M(\vec{x},\vec{v},t)\,\sum_{lmn}\Phi_{lmn}(\vec{x},t)\,Y_{lm}(\vartheta,\varphi)\,L_n^{(l+1/2)}(v).$

- Herdan and Liley [9] emphasized physical fluid moments in this general expansion by writing it as [$L_0^{(l+1/2)}(v) = 1,\ L_1^{(l+1/2)}(v) = l + 3/2 - v^2/v_T^2$]:

$$
\begin{aligned}
f(\vec{x},\vec{v},t) = f_M\ \Big(\ 1\ &+ \left[\frac{\delta n}{n}L_0^{(1/2)} + \frac{\delta T}{T}L_1^{(1/2)} + \cdots\right] && P_0\left(\frac{\vec{v}}{v_T}\right) \quad \text{moments}\\
&+ \frac{2}{v_T^2}\,\vec{v}\cdot\left[\delta\vec{V}\,L_0^{(3/2)} + \vec{V}_q\,L_1^{(3/2)} + \cdots\right] && P_1\left(\frac{\vec{v}}{v_T}\right) \quad \text{moments}\\
&+ \frac{\vec{v}\vec{v} - (v^2/3)\overleftrightarrow{\mathrm{I}}}{2mnv_T^4} : \left[\overleftrightarrow{\pi}\,L_0^{(5/2)} + \overleftrightarrow{\pi}_q\,L_1^{(5/2)} + \cdots\right] && P_2\left(\frac{\vec{v}}{v_T}\right) \quad \text{moments}\\
&+ \cdots\Big), \qquad \vec{V}_q \equiv -\frac{2\vec{q}}{5nT}\ \text{heat flow},\ \overleftrightarrow{\pi}_q \equiv -\frac{2\overleftrightarrow{\Theta}}{7T}\ \text{heat stress} && \vdots\ .
\end{aligned}
$$

[8] H. Grad, "Princ. of the Kin. Th. of Gases" in *Handbuch der Physik*, Vol. 12 (Springer-Verlag, Berlin, 1957).
[9] R. Herdan and B.S. Liley, Rev. Mod. Phys. 32, 731 (1960) — dynamics, transport for a thermal plasma.

Various Levels Of Moment Approaches Have Been Proposed

- **Chapman-Enskog formalism is a "5 moment" approach — it uses $n, T, \vec{V}$ as dynamic variables and sets $\delta n = 0$, $\delta T = 0$ and $\delta \vec{V} = \vec{0}$.**

- **Grad 13 moment approach uses $n, T, \vec{V}, \vec{q}, \overleftrightarrow{\pi}$ as dynamic variables.**

- **A more "balanced" approach motivated by Herdan and Liley is the 18 moment approach — (n, T), $(\vec{V}, \vec{q}\,)$, $(\overleftrightarrow{\pi}, \overleftrightarrow{\pi}_q)$ as dynamic variables.**

- **One could carry on to even higher levels of moment approaches — but if high order moments are important, shouldn't one use a kinetic theory?**

- **However moment approach is a useful framework to use in kinetic theory when determining closure moments with simple density, flow, stress drives — at least in collisional (Braginskii) regimes:**

electric field $\vec{E}$ drives electron flow, current $\vec{J}$	$\Longrightarrow$	Spitzer problem, resistivity,
temperature gradient $\vec{\nabla} T$ drives heat flow	$\Longrightarrow$	heat flux $\vec{q}$,
velocity gradient $\{\vec{\nabla}\vec{V}\}$ drives stress	$\Longrightarrow$	viscous stress tensor $\overleftrightarrow{\pi}$.

Moment Expansion Solution Of "Kinetic" Spitzer Problem

- **Electron flow, current induced by electric field is called Spitzer problem:**

$$\frac{q_e}{m_e}\vec{E}\cdot\frac{\partial f_M}{\partial \vec{v}} = \mathcal{C}\{\delta f\} \quad \Longrightarrow \quad \delta f = -\,\mathcal{C}^{-1}\left\{\frac{q_e\vec{v}\cdot\vec{E}}{T}f_M\right\} \quad \Longrightarrow \quad \vec{J} = q_e\int d^3v\,\vec{v}\,\delta f \equiv \sigma_{\rm Sp}\vec{E}.$$

- **In moment approach one takes $\int d^3v\,\vec{v}\,L_i^{(3/2)}$ moments of kinetic equation and obtains a matrix equation to be solved for $\vec{V}$, $\vec{q}$, etc. induced by $\vec{E}$:**

$$n_e e\begin{pmatrix}\vec{E}\\ 0\\ \vdots\end{pmatrix} = -\frac{m_e n_e \nu_e}{Z}\underbrace{\begin{pmatrix}\ell_{00} & \ell_{01} & \cdots\\ \ell_{10} & \ell_{11} & \cdots\\ \vdots & \vdots & \ddots\end{pmatrix}}_{L_{ij}}\begin{pmatrix}\vec{V}_e-\vec{V}_i\\ \vec{V}_q\\ \vdots\end{pmatrix} \quad \Longrightarrow \quad \left\{\begin{aligned}\vec{J} &\equiv -n_e e(\vec{V}_e-\vec{V}_i)\\ &= \frac{n_e e^2}{m_e\nu_e}Z[L_{ij}^{-1}]_{00}\vec{E}.\end{aligned}\right.$$

- **One can show [10] that inverting friction matrix L_{ij} yields variational solution of Spitzer problem and hence plasma electrical conductivity σ:**

1×1 matrix inversion yields $\sigma_0 \equiv \dfrac{n_e e^2}{m_e\nu_e}$, which is reference ($\perp$) conductivity,

2×2 matrix inversion yields $\sigma_{\rm Sp} = \dfrac{1}{\alpha_e}\sigma_0$, with $\sim 5\%$ accuracy ($< 1/\ln\Lambda \sim 0.07$) in α_e,

3×3 matrix inversion yields $\sigma_{\rm Sp} = \dfrac{1}{\alpha_e}\sigma_0$, with $\sim 1\%$ accuracy in α_e ($\simeq 0.51$ for $Z=1$).

[10] S.P. Hirshman, 21, 1295 (1978) — variational solution of Spitzer problem via moments.

Comments On Moment Approach Solutions Of Kinetics

- Moment approach matrix solution of Spitzer problem also produces electron heat flux induced by the electric field, $\vec{V}_q = (Ze/m_e\nu_e)[L_{ij}^{-1}]_{01}\vec{E}$.

  This is a key contribution to Spitzer conductivity (with $\geq 2\times 2$ matrix inversion) since it converts friction force $\vec{R}_e$ from reference ($\perp$) to $\parallel$ Spitzer electrical conductivity:

  $$\vec{R}_e = -m_e n_e \nu_e[(\vec{V}_e - \vec{V}_i) + \tfrac{3}{2}\vec{V}_q] = \frac{n_e e}{\sigma_0}\left(\vec{J} - \frac{9Z/4}{\sqrt{2}+Z}\sigma_0\vec{E}\right) \Longrightarrow \vec{J} = \sigma_0\left(\frac{\sqrt{2}+13Z/4}{\sqrt{2}+Z}\right)\vec{E} = \sigma_{\rm Sp}\vec{E}.$$

- In general there are additional "thermodynamic" drives (beyond electric field $\vec{E}$) due to $\vec{\nabla}\ln p$ and $\vec{\nabla}\ln T \Longrightarrow$ transport fluxes $\vec{\Gamma}$, $\vec{q}$.

- Braginskii collisional closures were obtained using moment approach:

  Effects of all "forces" ($\vec{E}$, $\vec{\nabla}\ln p$, $\vec{\nabla}\ln T$, $\{\vec{\nabla}\vec{V}\}$) were determined simultaneously $\Longrightarrow$ Onsager symmetry, thermal force effect ($0.71\nabla_\parallel T_e$), Ettinghausen effect, etc.;

  4×4 matrix inversion was used for accurate numerical coefficients;

  However, really only need 2×2 approach for order $1/\ln\Lambda \sim 5\%$ accuracy in resistivity but factor of 2 accuracy in thermal diffusivity χ — need 3×3 for similarly accurate χ.

- Moment approach shows that, at least in the collisional regime, the relevant fluid moment variables are (n, T), $(\vec{V}, \vec{q})$, $(\overleftrightarrow{\pi}, \overleftrightarrow{\pi}_q)$, with closures for $\vec{q}, \overleftrightarrow{\pi}, \overleftrightarrow{\pi}_q$ determined kinetically in collisional equilibrium ($\partial/\partial t << \nu$).

Neoclassical Closures

- **There are two basic approaches to neoclassical transport theory for axisymmetric toroidal plasmas:**

  kinetic [11] — collisions of particles drifting off flux surfaces cause radial transport,

  fluid [12] — viscous drag on untrapped particles due to collisions with "immobile" trapped particles causes parallel/poloidal force that leads to radial plasma transport.

- **Only fluid moment approach is relevant for extended MHD equations.**

- **Key assumptions in deriving usual neoclassical closures for transport:**

  axisymmetric magnetic field geometry — no ripples, $\delta\vec{B}$, or magnetic islands,

  collisional equilibrium ($\partial/\partial t < \nu$) between trapped and (flowing) untrapped particles,

  flux-surface-average is appropriate because on collisional time scale the particles circumnavigate poloidal cross-section of torus many times — $\lambda \equiv v_T/\nu >> 2\pi R_0 q$.

- **The relevant neoclassical closures are the flux-surface-average of the parallel viscous forces induced by the poloidal flow U_θ and heat flow Q_θ:**

$$\begin{pmatrix} \langle \vec{B}\cdot\vec{\nabla}\cdot\overleftrightarrow{\pi}_\parallel \rangle \\ \langle \vec{B}\cdot\vec{\nabla}\cdot\overleftrightarrow{\Theta}_\parallel \rangle \end{pmatrix} = mn\langle B^2\rangle \begin{pmatrix} \mu_{00} & \mu_{01} \\ \mu_{10} & \mu_{11} \end{pmatrix} \begin{pmatrix} U_\theta \\ Q_\theta \end{pmatrix}, \quad \mu \sim \sqrt{\epsilon}\,\nu, \quad U_\theta \equiv \frac{\vec{V}\cdot\vec{\nabla}\theta}{\vec{B}\cdot\vec{\nabla}\theta} = \frac{V_\parallel}{B} + \frac{\vec{V}_\perp\cdot\vec{\nabla}\theta}{\vec{B}\cdot\vec{\nabla}\theta}.$$

[11] F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. 48, 239 (1976) — kinetic neoclassical transport.
[12] S.P. Hirshman and D.J. Sigmar, Nuclear Fusion 21, 1079 (1981) — fluid moment neoclassical transport.

Neoclassical MHD Is One Set Of Extended MHD Equations

- **Neoclassical MHD equations [13] use collisionally equilibrated parallel viscous forces and approximate gyroviscous forces to yield (for $t > 1/\nu$):**

  neoclassical parallel Ohm's law, including trapped particle effects on electrical conductivity [$\sigma \simeq \sigma_{\rm Sp}/(1 + \mu_e/\nu_e)$] and bootstrap current [$J_{\rm bs} \sim (\mu_e/\nu_e) dP/d\psi$],
  poloidal flow damped to zero [or small $V_\theta \sim (1.17/q_i) dT_i/d\psi$] $\Longrightarrow$ only toroidal flow,
  enhanced $\perp$ dielectric and inertia $\Longrightarrow 1 + c^2/c_A^2 \to 1 + c^2/c_{A\theta}^2$, larger by $B^2/B_\theta^2 \sim 10^2$,
  neoclassical tearing modes (NTMs) driven by island perturbation of bootstrap current.

- **Hegna suggested a heuristic local (i.e., not flux-surface-averaged) neoclassical closure to facilitate NIMROD simulations of NTMs [14]:**

$$\vec{\nabla}\cdot\overleftrightarrow{\pi}_\parallel = mn\mu\langle B^2\rangle \frac{\vec{V}\cdot\vec{e}_\theta}{(\vec{B}\cdot\vec{e}_\theta)^2}\,\vec{e}_\theta, \quad \vec{e}_\theta \equiv \sqrt{g}\,\vec{\nabla}\zeta\times\vec{\nabla}\psi = \frac{\vec{\nabla}\zeta\times\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\theta}.$$

- **To proceed further with neoclassical MHD-type simulations we need:**

  time-dependent parallel viscous force — to study NTM threshold behavior,
  local parallel viscous force (or pressure-ansisotropy) — to facilitate local analysis,
  inclusion of nonaxisymmetric effects of islands, $\delta\vec{B}$ — for toroidal flow damping.

[13] J.D. Callen, W.X. Qu, K.D. Siebert, B.A. Carreras, K.C. Shaing and D.A. Spong in *Plasma Physics and Controlled Nuclear Fusion Research 1986* (IAEA, Vienna, 1987), Vol. 2, p 157.
[14] T.A. Gianakon, S.E. Kruger and C.C. Hegna, Phys. Plasmas 9, 536 (2002) — NIMROD sim. of NTMs.

Recent Progress On Neoclassical Closures

- **Temporal behavior of the parallel viscous force has been calculated [15], with most useful formulas obtained in a small $\epsilon \equiv \Delta B/2B$ expansion:**

$$\langle \vec{B}\cdot\vec{\nabla}\cdot\overleftrightarrow{\pi}\rangle \simeq mn\langle B^2\rangle\mu \left[U_\theta(t) + \frac{1}{\bar{\nu}}\frac{\partial U_\theta}{\partial t} + \sum_n c_n \int_0^t d\tau\, e^{-\bar{\nu}\kappa_n(t-\tau)}\frac{\partial U_\theta}{\partial t}\right],$$

which implies a "time-history" integral equation for the parallel flow evolution,

$U_\theta(t) = h_\theta(t) + \int_0^t d\tau\, K_\theta(t;\tau)\, U_\theta(t)$ **in which h_θ represents initial conditions.**

- **Local pressure anisotropy $\pi_\parallel \equiv p_\parallel - p_\perp$ also determined recently [16]:**

$\pi_\parallel \sim mn\mu U_\theta \langle B^2\rangle \times$**(incomplete elliptic functions) — continuous function of θ;**

however, $\vec{B}\cdot\vec{\nabla}\cdot\overleftrightarrow{\pi}_\parallel = (2/3)\vec{B}\cdot\vec{\nabla}\pi_\parallel + \pi_\parallel(\vec{B}\cdot\vec{\nabla}\ln B)$ is divergent at $B = B_{\rm max}$.

- **Nonaxisymmetry effects introduced by magnetic islands or $\delta\vec{B}$ cause neoclassical radial particle fluxes, toroidal viscous flow damping [17,18]:**

Magnetic islands change radial location of $\psi \Longrightarrow \partial(\vec{V}\cdot\vec{\nabla}\zeta)/\partial t \sim (w/a)^2 \sim (\delta B_{x_{mn}}/B)$;

Ideal MHD $\delta\vec{B}$ produces helical change in equilibrium $\Longrightarrow \partial(\vec{V}\cdot\vec{\nabla}\zeta)/\partial t \sim |\delta\vec{B}/B|^2$.

[15] A.L. Garcia-Perciante, J.D. Callen, K.C. Shaing, and C.C. Hegna, Phys. Plasmas 12, 052516 (2005).
[16] A.L. Garcia-Perciante, J.D. Callen, K.C. Shaing, and C.C. Hegna, UW-CPTC 05-4, sub. Phys. Plasmas.
[17] K.C. Shaing, Phys. Rev. Lett. 87, 245003 (2001) — particle flux due to magnetic islands.
[18] K.C. Shaing, Phys. Plasmas 10, 1443 (2003) — toroidal momentum damping due to MHD modes.

Extended MHD Equations Require Various Closures

- Closures for $\vec{q}$ and $\overleftrightarrow{\pi}$ need to have their parallel ($\|$), cross ($\wedge$) and perpendicular ($\perp$) components specified:

$$\vec{q} = \vec{q}_{\|} + \vec{q}_{\wedge} + \vec{q}_{\perp}, \qquad \text{and} \qquad \overleftrightarrow{\pi} = \overleftrightarrow{\pi}_{\|} + \overleftrightarrow{\pi}_{\wedge} + \overleftrightarrow{\pi}_{\perp}.$$

- Parallel heat flow $\vec{q}_{\|} = q_{\|}\vec{b}$, $\vec{b} \equiv \vec{B}/B$, $q_{\|} \equiv -\int d^3v\, v_{\|} L_1^{3/2} F$ determined using F obtained solving a Chapman-Enskog-type drift kinetic equation [4,5]:

$$\frac{\partial F}{\partial t} + v_{\|}\,\vec{b}\cdot\vec{\nabla}F = \mathcal{C}_R\{F\} + v_{\|} L_1^{(3/2)} f_M\,\vec{b}\cdot\vec{\nabla}T - \frac{m}{T}\left(v_{\|}^2 - \frac{v_{\perp}^2}{2}\right) f_M\,(\vec{b}\cdot\{\vec{\nabla}\vec{V}\}\cdot\vec{b}) + \cdots .$$

- Various approaches used to obtain $q_{\|}$ from this parallel kinetic equation:

  Collisional regime (Braginskii) — neglect $\partial F/\partial t$, $v_{\|}\,\vec{b}\cdot\vec{\nabla}F$; invert collision operator;
  Collisionless — linearize and obtain Hammett-Perkins [19] Landau-type closures [5,20];
  PIC-type δf code (Barnes) — but higher order moments are "noisier?"
  "Continuum" type solutions [21] — expand F in pitch-angle eigenfunctions of $\mathcal{C}_R\{F\}$.

- Stress $\overleftrightarrow{\pi}_{\|} \equiv \pi_{\|}(\vec{b}\vec{b} - \overleftrightarrow{\mathrm{I}}/3)$, $\vec{b} \equiv \vec{B}/B$, $\pi_{\|} \equiv p_{\|} - p_{\perp} = \int d^3v\, m(v_{\|}^2 - v_{\perp}^2/2)F$ is also determined from the solution of the parallel kinetic equation [22]
$\Longrightarrow$ neoclassical closures for $\pi_{\|}$ and $\langle \vec{B}\cdot\vec{\nabla}\cdot\overleftrightarrow{\pi}_{\|}\rangle$ for $k_{\|}v_T << \nu$.

[19] G.W. Hammett and F.W. Perkins, Phys. Rev.Lett. 64, 3019 (1990) — simplest Landau closure for $q_{\|}$.
[20] A.I. Smolyakov, M. Yagi, J.D. Callen, Fields Inst. Comm. 46, 243 (2005) — neutral fluid nonlocal clos.
[21] E.D. Held *et al.*, Phys. Plasmas 11, 2419 (2004) — and references cited therein.
[22] E.D. Held, "Unified form for parallel ion viscous stress in magnetized plasmas," PoP 10, 4708 (2003).

Some Complications In Obtaining Parallel Closures

- The "usual" Chapman-Enskog-like drift-kinetic equation (DKE) [4,5] has many ($\gtrsim$ 5) "drives" on its right side in terms of the form

  $v_\parallel f_M \vec{B} \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi} \Longrightarrow$ causes $\sim \sqrt{\epsilon}$ correction to parallel viscous forces,

  $v_\parallel R_{e\parallel} \Longrightarrow$ additional corrections to parallel flow?, part of Spitzer problem?

  dissipative $L_1^{(1/2)}$ terms due to $\overleftrightarrow{\pi} : \{\vec{\nabla}\vec{V}\}$, $\vec{\nabla}\cdot\vec{q}$ and $Q \Longrightarrow$ temperature change δT?

- Also, $\epsilon_\perp^2$ additions to DKE — Catto & Simakov [23], paleoclassical [24].

- Unfortunately, to obtain parallel closures correct to $\mathcal{O}(\epsilon_\perp^2)$ one needs to keep many (most?) of the $\mathcal{O}(\epsilon_\perp)$ terms, particularly for 3D geometry.

- Shaing and Spong [3] have exhibited some of the 3D complications that arise in long collision length plasmas by obtaining a "local" closure relation for $\pi_\parallel \equiv p_\parallel - p_\perp$ in the plateau collisionality regime.

- Also, Shaing emphasizes that in general there are not enough free parameters in kinetic analysis to satisfy all Chapman-Enskog constraints $\Longrightarrow$ residual "extra" (but usually higher order) δn, δT, $\delta\vec{V}$ terms.

[23] A.N. Simakov, P.J. Catto, Phys. Plasmas 12, 012105 (2005) — additional $\mathcal{O}(\epsilon_\perp^2)$ terms in DKE (for $\overleftrightarrow{\pi}_\wedge$).
[24] J.D. Callen, Phys. Rev. Lett. 94, 055002 (2005); to be published in Nuclear Fusion and Phys. Plasmas.

Extended MHD Equations Require Various Closures (cont'd)

- Components of $\vec{q}$ perpendicular to $\vec{B}$ can be obtained from the fluid moment equation for $\partial\vec{q}/\partial t$ [4] (here $\vec{R}_q \sim mn\nu[l_{10}(\vec{V}_e - \vec{V}_i) + l_{11}\vec{V}_q]$):

$$\frac{d\vec{q}}{dt} = \frac{\omega_c}{B}\vec{q}\times\vec{B} - \frac{5}{2}\frac{nT}{m}\vec{\nabla}T + \frac{T}{m}(\vec{\nabla}\cdot\overleftrightarrow{\Theta} + \vec{R}_q) - \cdots,\ \text{which upon taking } \vec{B}\times \text{ yields}$$

$$\vec{q}_\wedge = \underbrace{\frac{5}{2}\frac{\vec{B}\times\vec{\nabla}T}{\omega_c}}_{\text{diamagnetic}} \sim \epsilon_\perp, \quad \text{and} \quad \vec{q}_\perp = \frac{1}{\omega_c B}\vec{B}\times\left[\frac{T}{m}\underbrace{(\vec{R}_q + \vec{\nabla}\cdot\overleftrightarrow{\Theta})}_{\text{classical + neo}} - \frac{d\vec{q}}{dt} + \cdots\right] \sim \epsilon_\perp^2.$$

- A similar analysis of the $d\overleftrightarrow{\pi}/dt$ equation can be performed to yield

$$\overleftrightarrow{\pi}_\wedge = \underbrace{\frac{2p}{\omega_c}\overleftrightarrow{\mathcal{K}}^{-1}\{\{\vec{\nabla}\vec{V}\} + \frac{4}{5nT}\{\vec{\nabla}\vec{q}\}\}}_{\text{gyroviscous stress}} \sim \epsilon_\perp, \quad \text{and} \quad \overleftrightarrow{\pi}_\perp = \underbrace{\frac{\nu_{\text{eff}}}{\omega_c}\overleftrightarrow{\mathcal{K}}^{-1}\{\overleftrightarrow{\pi}_\wedge\} + \cdots}_{\text{perpendicular stress}} \sim \epsilon_\perp^2,$$

in which the inverse tensor operator [25] $\overleftrightarrow{\mathcal{K}}^{-1}\{\overleftrightarrow{\mathrm{S}}\} = \frac{1}{4}\Big([\vec{b}\times\overleftrightarrow{\mathrm{S}}\cdot(\overleftrightarrow{\mathrm{I}} + 3\vec{b}\vec{b})] + \text{transpose}\Big)$.

- Recently, Ramos [26] used fluid moments to obtain a compact form for the gyroviscous stress tensor $\overleftrightarrow{\pi}_\wedge$ for arbitrary magnetic geometry.

[25] C.T. Hsu, R.D. Hazeltine and P.J. Morrison, Phys. Fluids 29, 1480 (1986) — see Appendix A.
[26] J.J. Ramos, "Fluid formalism for collisionless magnetized plasmas," Phys. Plasmas 12, 052102 (2005).

Drift-Kinetics And Gyrokinetics Take A Different Approach

- **Their basic philosophy is to not use any fluid moment equations, but instead derive everything from a kinetic description:**

  Drift-kinetic equation (DKE) uses small gyroradius expansion, now keeps $\mathcal{O}(\epsilon_\perp^2)$ terms.

  Gyro-kinetic equation (GKE) considers arbitrary $\epsilon_\perp \sim |\varrho\vec{\nabla}_\perp| \sim 1 \Longrightarrow$ Bessel functions.

  Fluid moment equations are obtained from velocity-space moments of these equations.

  Gyrofluid approach [27] obtained moment equations for n,$V_\parallel$, $q_\parallel$, $q_\perp$, $p_\parallel$, $p_\perp$; languishes.

- **The δf codes and turbulence simulations usually assume:**

  Lowest order distribution is a Maxwellian on static flux surfaces, i.e. $f_M(\psi)$.

  "Drives" on right of equations are due to $\delta\vec{E}$ and $\delta\vec{B}$, i.e., Chapman-Enskog-type analysis is not used so the drives are not due to gradients of n, T, $\vec{V}$.

  Collisional effects are small, not dominant — parallel Ohm's law is usually collisionless one (no Spitzer restivity, no bootstrap current, etc.), collisional transport neglected.

- **My conclusion: kinetic distribution from present δf codes cannot be used to obtain closure moments for extended MHD — magnetic islands, stochasticity not allowed; produces significant δn, δT, $\delta\vec{V}$ components; and entropy production is not predominantly due to collisional effects.**

[27] W. Dorland and G.W. Hammett, Phys. Fluids B 5, 812 (1993); P.B. Snyder and G.W. Hammett, Phys. Plasmas 8, 3199 (2001) — see also the many references cited in this last paper.

Closure Issues For Other Physical Effects

- **Energetic components (for ions induced by NBI or RF heating, for electrons due to LH or EC heating) introduce additional issues:**

  Lowest order distribution is not Maxwellian, e.g., $f_{0i} \sim \dot{n}_f \tau_S / [4\pi(v^3 + v_c^3)]$ — ? on relevant energy "polynomials" (replace Laguerre), but pitch-angle eigenfunctions ok.

  Present 2D fast particle simulations neglect $\vec{E}$ field, non-axisymmetric geometry, etc.

  Fundamentally, how does one obtain self-consistency on transport time scale between base plasma and density, momentum, and energy inputs from energetic component? — at present these "source" effects are just added to plasma fluid moment equations.

  Adding current-drive effects from energetic particle components that induce significant modifications of the parallel Ohm's law can cause numerical instability [28] — handle via only calculating difference in Ohm's law as $\vec{E}$ departs from equilibrium.

- **Can effects of turbulence be included in extended MHD simulations?**

  Neutral fluid turbulence closure models seek Reynolds stress closures that represent non-dissipative transfer of energy to higher $\vec{k}$ values in inertial range, then dissipation

  Drift-wave-type turbulence has non-inertial unstable $\vec{k}$-space region ($k_\perp \varrho_i \sim 0.1$–$1$) but some mode coupling to other $\vec{k}$ space regions (e.g., to $k_\theta = k_\zeta = 0$ zonal flows) — can these reactive and dissipative effects be approximated by the transport they induce?

  "Multi-scale" models seek to include phenomena on shorter time, length scales via "projective" or "multiple timescale" integrations, "sub-grid-scales" — avoid closures?

[28] S.C. Jardin and D.W. Ignat — Phys. Fluids 1995?

There Are Two Generic Types Of Extended MHD Problems

- **"Fast MHD" ($\omega > \nu_i \sim 10^3$ s^{-1}) phenomena occur on Alfvenic timescale:**

  Examples: sawtooth crashes, disruption precursors (DIII-D #87009), ELMs.

  Physically, need ideal MHD plus diamagnetic flow & gyroviscosity (two-fluid) effects — ω_{*i} stabilization for 1/1 sawtooth crashes, plus ω_{*e} for stabilizing high mode numbers.

  Dissipative closures operate on longer time scales ($t > 1/\nu$) and hence are negligible — except for parallel T_e equilibration in irregular magnetic fields, destabilizing resistivity effects and possibly stabilizing diffusion coefficient effects on high mode numbers [29].

- **"Slow MHD" ($\omega > \nu_i \sim 10^3$ s^{-1}) phenomena occur on the resistive time scale and involve many physical processes:**

  Examples: $\Delta' > 0$ tearing modes, NTMs, RWMs.

  Neoclassical MHD effects important — poloidal flow damping $\Longrightarrow$ only toroidal flow, enhanced inertia (by $B^2/B_\theta^2 >> 1$), neoclassical parallel restivity, bootstrap current.

  Since nonlinear evolution (tearing modes $\Longrightarrow$ magnetic islands, RWMs $\Longrightarrow$ kink in plasma growing on wall time scale $\sim$ 10 ms) is on transport time scale, all transport effects are important — need complete ($\|$, $\wedge$, $\perp$) dissipative closures for $\vec{q}$, $\overleftrightarrow{\pi}$.

  Diamagnetic flow (ω_*) effects are ultimately not so critical — vanish on separatrix where $\vec{\nabla} P \to 0$, or just lead to slight changes in toroidal flow velocity.

  The second order (in gyroradius) effects are needed for perturbations, but they may not be needed for equilibrium since they represent negligible classical diffusion effects.

[29] B.A. Carreras, L. Garcia and P.H. Diamond, Phys. Fl. 30, 1388 (1987) — $\chi_\perp$, $\mu_\perp$ effects on res-g modes.

Discussion: Develop Closures For Classes Of Problems?

- **Closures can only be systematically derived for collisional regime $\Longrightarrow$ Braginskii equations — but toroidal plasmas violate $\boldsymbol{\lambda \nabla_\parallel} < 1$ condition.**

- **No general closures can be derived for long collision length $\boldsymbol{\lambda}$ regimes — because $\|$ kinetics depends on geometry over the collision length.**

- **Also, needed (for $\wedge$, $\perp$ closures) first and second order terms in finite gyroradius expansion are complicated and depend on gradients of $\vec{B}$.**

- **Thus, one is led to consider key closures needed for classes of problems:**

  <u>Fast MHD</u> ($\omega > \nu_i$)

  mainly just diamagnetic flows & gyroviscosity, but maybe with some diffusivities to stabilize high mode numbers.

  <u>Slow MHD</u> ($\omega < \nu_i$)

  tearing modes, NTMs — mainly just neo $\|$ viscous force (but local with dynamics),

  RWMs — mainly just equilibrium neoclassical parallel viscous force, plus toroidal flow damping induced as mode kinks the plasma and magnetic field.

- **But for ultimate extended MHD simulations of toroidal plasmas one will need to develop procedures for determining and numerically implementing $\|$, $\wedge$ and $\perp$ closures (& extra terms) to sufficient accuracy.**

Summary

- **Extended MHD equations developed from two-fluid equations provide a reasonable basis for simulating macroscopic plasma behavior — if suitable closures for $\vec{q}$ and $\overleftrightarrow{\pi}$ are available and/or numerically implementable.**

- **Rigorous analysis in collisional regime (Braginskii) shows that in a magnetized plasma closures are very anisotropic — $\vec{q}_\parallel \sim \epsilon_\perp^0 \epsilon_\parallel^2$ (parallel), $\vec{q}_\wedge \sim \epsilon_\perp$ (diamagnetic), $\vec{q}_\perp \sim \epsilon_\perp^2$(perpendicular) and similarly for $\overleftrightarrow{\pi}$.**

- **Determinations of closure components depend on direction:**

  $\parallel$ (kinetic, $\epsilon_\parallel \sim |\lambda \nabla_\parallel| \gtrsim 1$) — parallel Chapman-Enskog-type drift-kinetic equation,

  $\wedge$ (diamagnetic, $\epsilon_\perp \sim |\varrho \vec{\nabla}_\perp| << 1$) — can use $\vec{B}\times$ fluid moment equations for $\vec{q}$, $\overleftrightarrow{\pi}$,

  $\perp$ (perp, $\epsilon_\perp^2 << 1$) — gyroradius smaller parts of $\vec{B}\times$ fluid moment equations.

- **General closures are not possible — should we focus on developing closures for classes of problems of interest? which ones?**

- **Closures needed for "fast MHD" ($\omega > \nu_i \sim 10^3\ \mathrm{s}^{-1}$) and "slow MHD" ($\omega < \nu_i$) are quite different — dominant diamagnetic effects versus neoclassical MHD effects.**