STUDIES OF NONLINEAR RESISTIVE AND EXTENDED MHD IN ADVANCED TOKAMAKS USING THE NIMROD CODE

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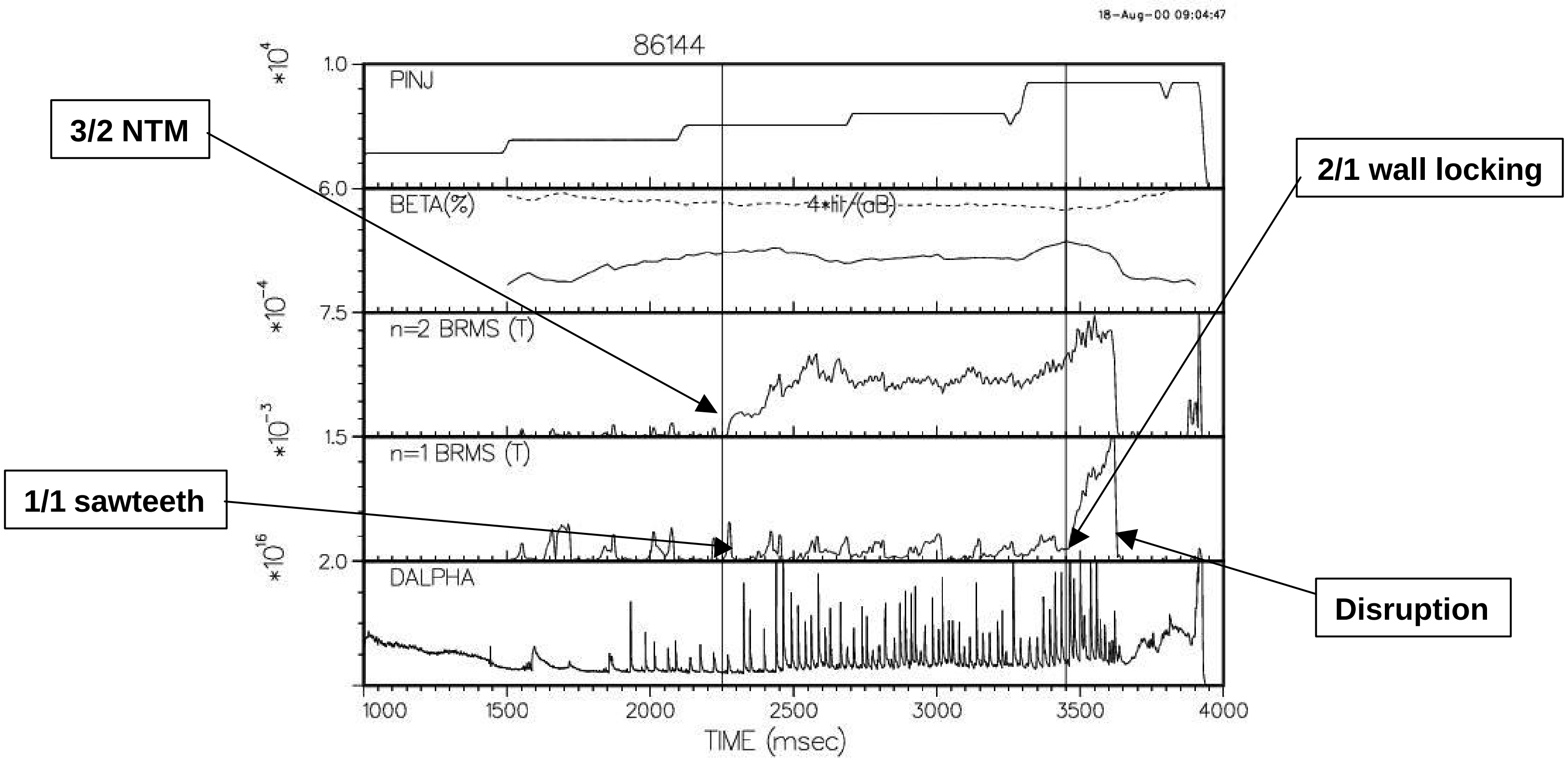
MODERN TOKAMAKS ARE RICH IN MHD ACTIVITY

Example: DIII-D shot 86144



- **Sawtoothing discharge**
- **3/2 NTM triggered at 2250 msec**
- **2/1 locks to the wall**

IMPORTANCE OF REALISTIC MODELING

- **Cost of next generation of fusion experiments estimated to be at least several billion $$**
- **Cost proportional to volume: *$ ~ V***
- **Power density proportional to square of max. pressure: $P/V \sim p^2_{max}$**
  - **=> $\$ \sim 1/p^2_{max}$ for fixed *P* and *B* (engineering constraints)**
- **Physics uncertainties limit max. pressure to ~ 2/3 theoretical p_{max}**

Uncertainties in nonlinear physics account for ~ 1/2 the cost of advanced fusion experiment!

- **Predictive fluid modeling with realistic parameters has high leverage to remove this uncertainty**

MODELING REQUIREMENTS

- **Slow evolution**

  Nonlinear fluid model required

- **Plasma shaping**

  Realistic geometry required

- **High temperature**

  Realistic *S* required

- **Low collisionality**

  Extensions to resistive MHD required

- **Strong magnetic field**

  Highly anisotropic transport required

- **Resistive wall**

  Non-ideal boundary conditions required

2-FLUID MODEL

- **Maxwell (no displacement current):**

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad , \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad ,$$

- **Momentum, energy, and continuity for each species (α = e, *i*):**

$$m_\alpha n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t} + \mathbf{v}_\alpha \cdot \nabla \mathbf{v}_\alpha \right) = -\nabla \cdot \mathbf{P}_\alpha + q_\alpha n_\alpha \left(\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B} \right) + \sum_\beta \mathbf{R}_{\alpha\beta} + \mathbf{S}_\alpha^m$$

$$\frac{\partial p_\alpha}{\partial t} + \mathbf{v}_\alpha \cdot \nabla p_\alpha = -\frac{3}{2} p_\alpha \nabla \cdot \mathbf{v}_\alpha - \mathbf{P}_\alpha : \nabla \mathbf{v}_\alpha - \nabla \cdot \mathbf{q}_\alpha + \mathbf{Q}_\alpha$$

$$\frac{\partial n_\alpha}{\partial t} = -\nabla \cdot \left(n_\alpha \mathbf{v}_\alpha \right) + S_\alpha^n$$

- **Current and quasi-neutrality:**

$$\mathbf{J}_\alpha = n_\alpha q_\alpha \mathbf{v}_\alpha \, , \qquad n = n_e = Z n_i$$

SINGLE FLUID FORM

- **Add electron and ion momentum equations:**

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla\cdot\mathbf{P}'+\mathbf{J}\times\mathbf{B}$$

- **Subtract electron and ion momentum equations (Ohm's law):**

$$\mathbf{E}=-\underbrace{\mathbf{v}\times\mathbf{B}}_{\textit{Ideal MHD}}+\underbrace{\eta\mathbf{J}}_{\textit{Resistive MHD}}+\underbrace{\frac{1}{ne}\frac{1-\nu}{1+\nu}\mathbf{J}\times\mathbf{B}}_{\textit{Hall Effect}}$$

$$-\underbrace{\frac{1}{ne(1+\nu)}\nabla\cdot\left(\mathbf{P}_e'-\nu\mathbf{P}_i'\right)}_{\substack{\textit{Pressure Effects}\\ \textit{and Closures}}}+\underbrace{\frac{1}{\varepsilon_0\omega_{pe}^2(1+\nu)}\left[\frac{\partial\mathbf{J}}{\partial t}+\nabla\cdot(\mathbf{vJ}+\mathbf{Jv})\right]}_{\textit{Electron Inertia}}$$

MODELING GOALS

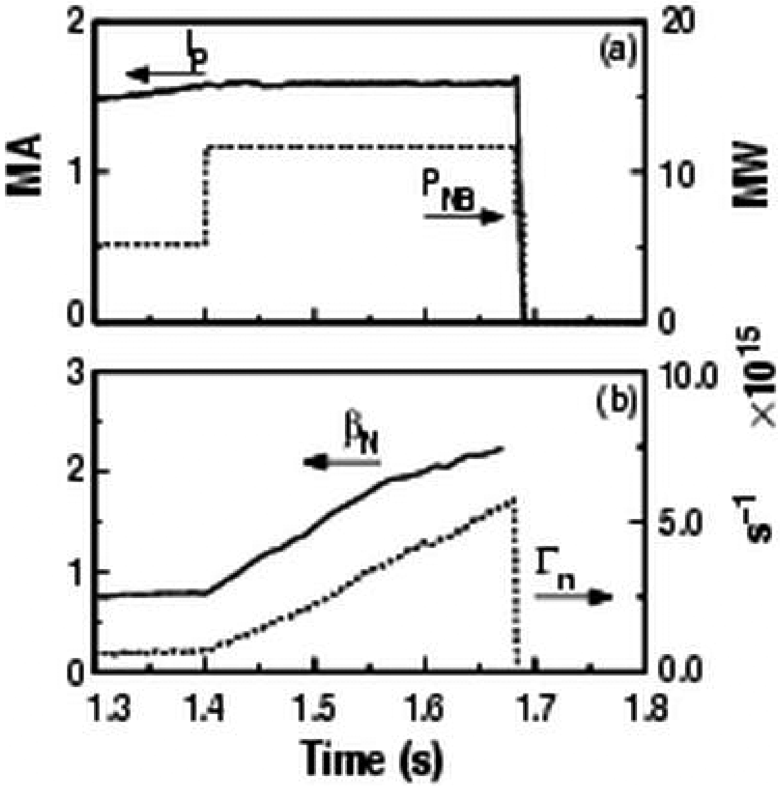
- *Analysis and interpretation of experimental data*
  - – **NIMROD can interface with common design and analysis codes**
    - **EFIT**
    - **TOQ**
    - **CHEASE**
  - – **DIII-D has extensive data base**
  - – **Study nonlinear phenomena**
  - – **Tests both resistive and extended MHD models**
- *Analysis and evaluation of designs for advanced experiments*
  - – **Example: VDE in ITER**

DIII-D EXPERIMENTAL DISCHARGES

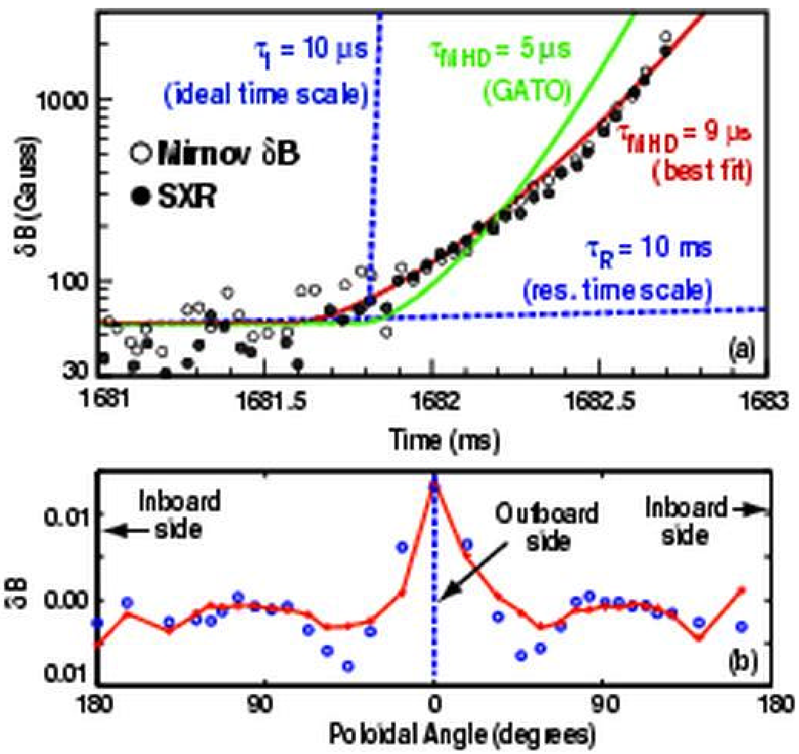
- ***Shot 87009***
  - Highly shaped plasma
  - Disruption when heated through b limit
    - Why is growth faster than simple exponential?
    - What causes disruption?
  - Nonlinear resistive MHD
- ***Shot 86144***
  - ITER-like discharge
  - Sawteeth
    - Nonlinear generation of secondary islands
    - Destabilization of NTM?
  - Tests both resistive MHD and closure models for Extended MHD

DIII-D SHOT #87009

- **High-b disruption when heated slowly through critical b_N**



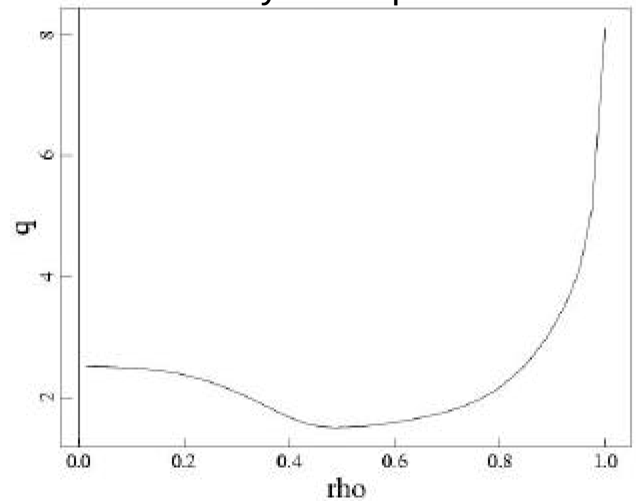
- **Growth is faster than simple exponential**



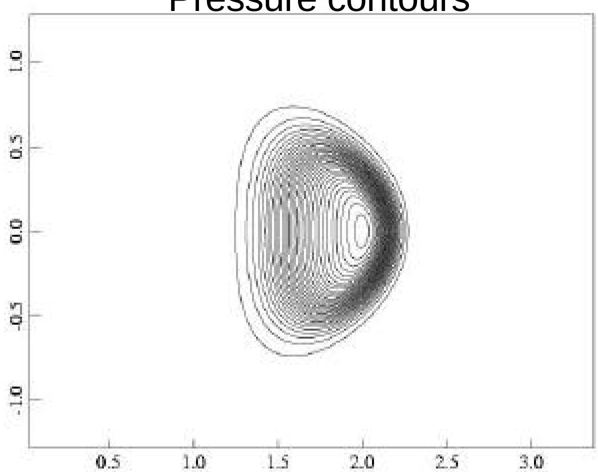
EQUILIBRIUM AT t = 1681.7 msec

- Equilibrium reconstruction from experimental data
- Negative central shear
- Gridding based on equilibrium flux surfaces
  - Packed at rational surfaces
  - Bi-cubic finite elements

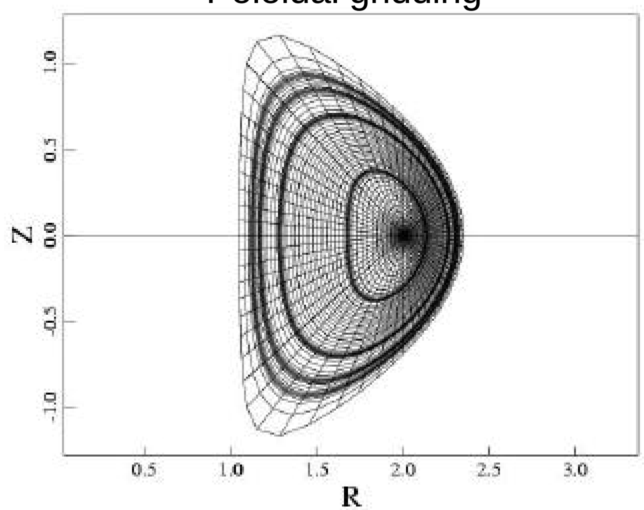
Safety factor profile



Pressure contours

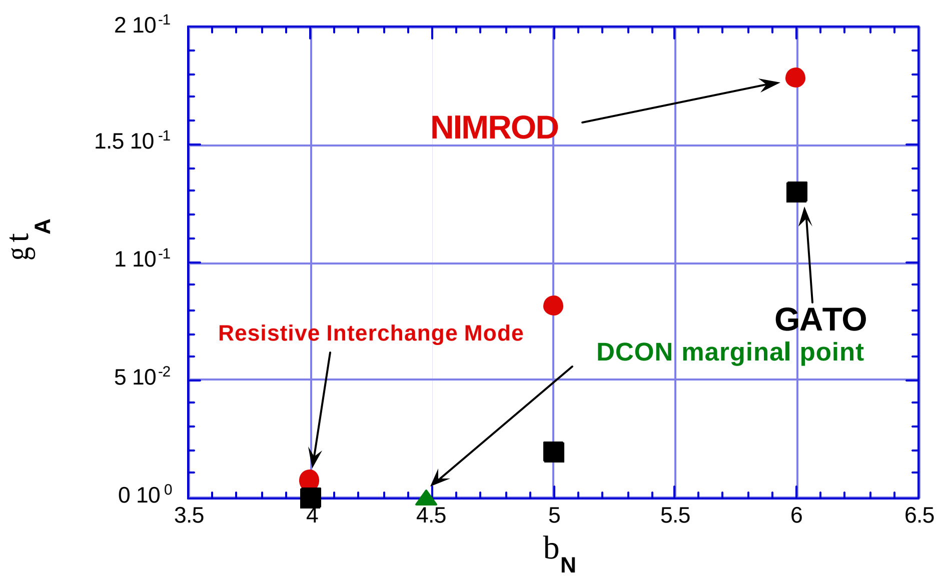


Poloidal gridding



LINEAR STABILITY

- **Present version of NIMROD requires conducting wall at outer flux surface**
  - **Critical b_N for ideal instability larger than in experiment**
- **NIMROD gives slightly larger ideal growth rate than GATO**
- **NIMROD finds resistive interchange mode below ideal stability boundary**



THEORY OF SUPER-EXPONENTIAL GROWTH

- **In experiment mode grows faster than exponential**
- **Theory of ideal growth in response to slow heating (*Callen, Hegna, Rice, Strait, and Turnbull, Phys. Plasmas 6, 2963 (1999)*):**

Heat slowly through critical b: $b = b_c(1 + g_h t)$

Ideal MHD: $w^2 = -\hat{g}^2_{MHD}(b/b_c - 1)$ ➔ $g(t) = \hat{g}_{MHD}\sqrt{g_h t}$

Perturbation growth:

$$\frac{dx}{dt} = g(t)x \quad ➔ \quad x = x_0 \exp[(t/t)^{3/2}], \qquad t = (3/2)^{2/3}\hat{g}_{MHD}^{-2/3} g_h^{-1/3}$$

- **Good agreement with experimental data**

NONLINEAR SIMULATION WITH NIMROD

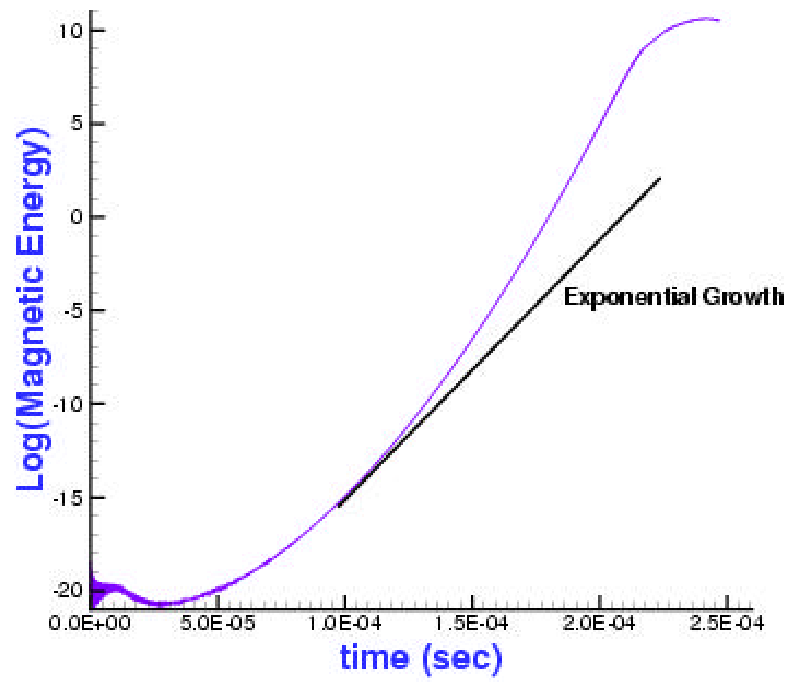
- **Initial condition: equilibrium below ideal marginal b_N**
- **Use resistive MHD**
- **Impose heating source proportional to equilibrium pressure profile**

$$\frac{\P P}{\P t} = \quad ..... \quad + g_H P_{eq}$$

$$\Rightarrow \quad b_N = b_{Nc}\left(1 + g_H t\right)$$

- **Follow nonlinear evolution through heating, destabilization, and saturation**

Log of magnetic energy in *n* = 1 mode vs. time
$S = 10^6$ Pr = 200 $g_H = 10^3$ sec^{-1}



SCALING WITH HEATING RATE

- **NIMROD simulations also display super-exponential growth**
- **Simulation results with different heating rates are well fit by $x \sim \exp[(t-t_0)/t]^{3/2}$**
- **Time constant scales as**

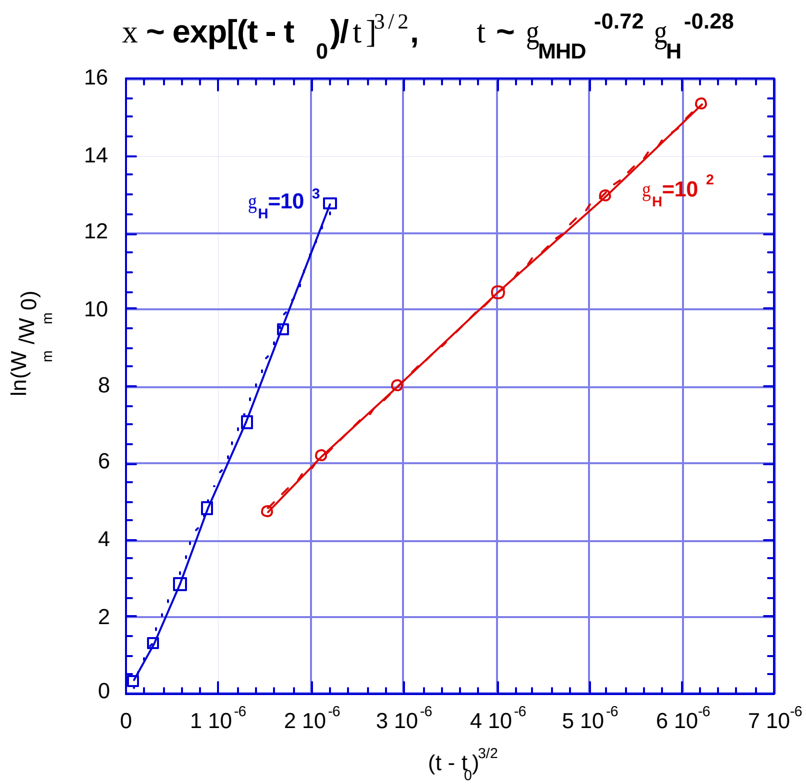
$$t \sim g_{MHD}^{-0.72} g_H^{-0.28}$$

- **Compare with theory:**

$$t = (3/2)^{2/3} \hat{g}_{MHD}^{-2/3} g_h^{-1/3}$$

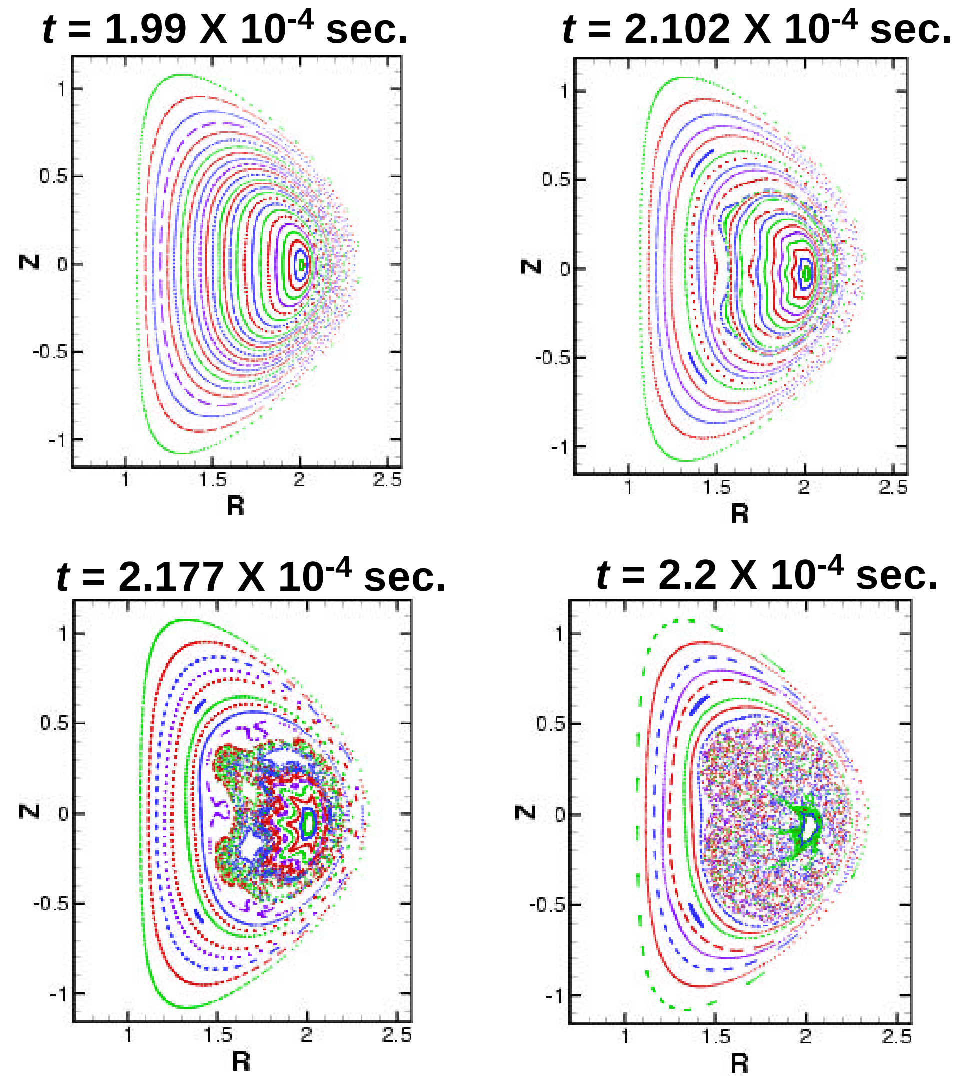
- **Discrepancy possibly due to non-ideal effects**

Log of magnetic energy vs. $(t - t_0)^{3/2}$ for 2 different heating rates

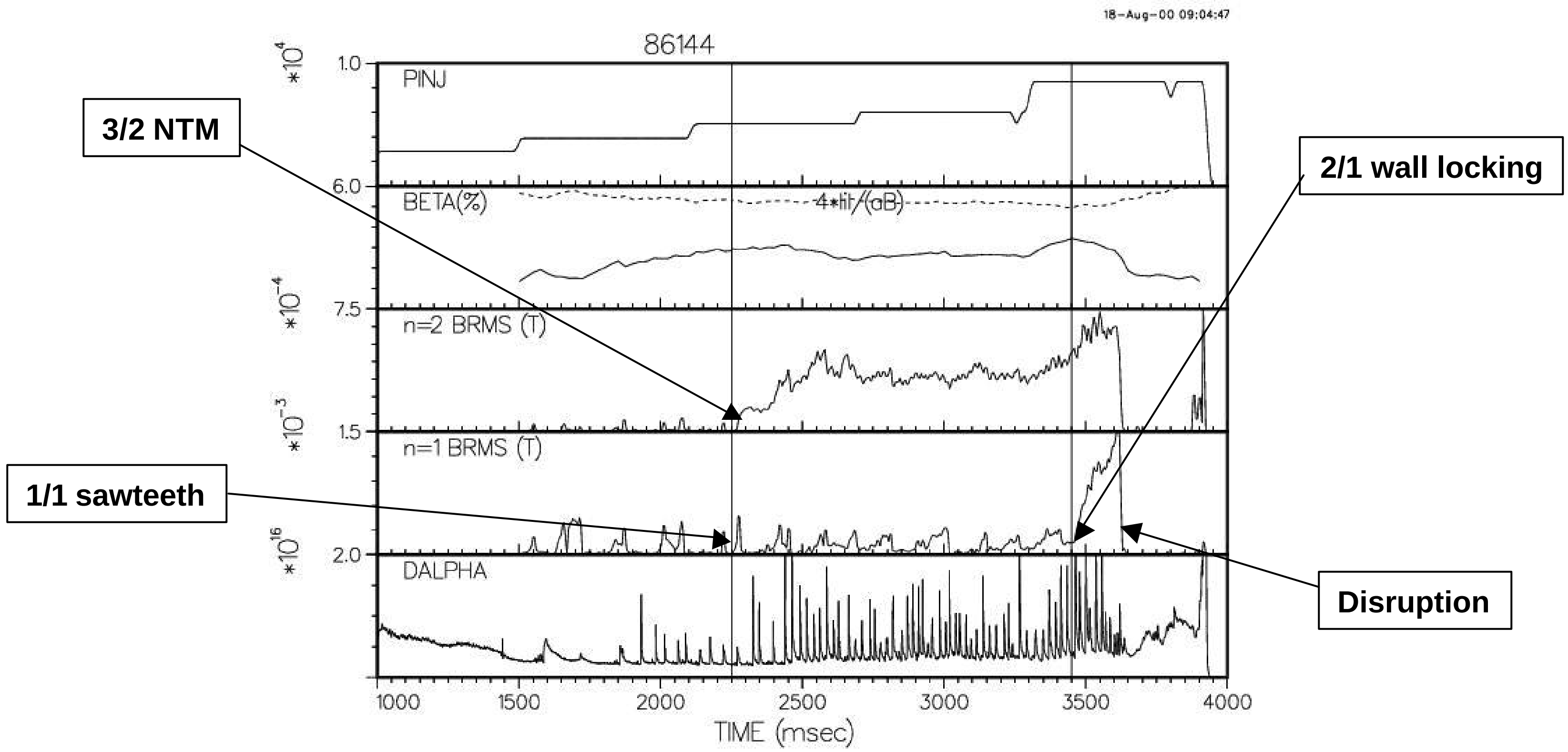


EVOLUTION OF MAGNETIC FIELD LINES

- **Simulation with small but finite resistivity**
- **Ideal mode yields stochastic field lines in late nonlinear stage**
- **Implications for degraded confinement**
- **Disruption?**



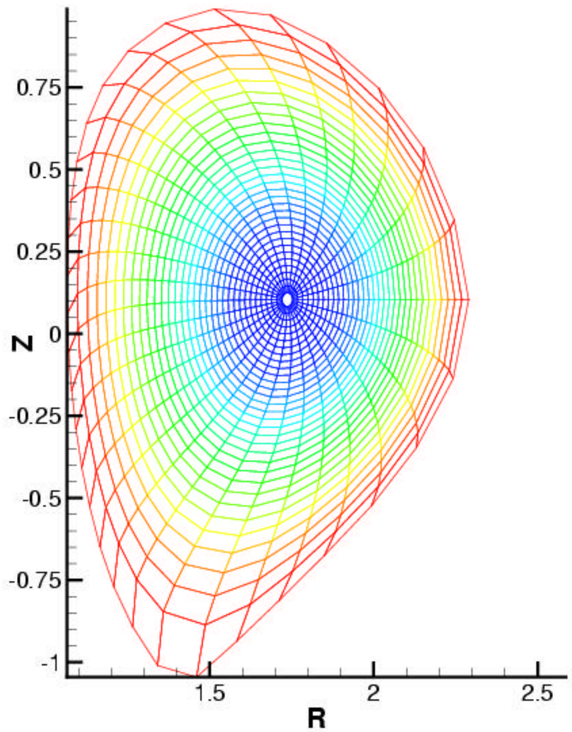
DIII-D SHOT #86144



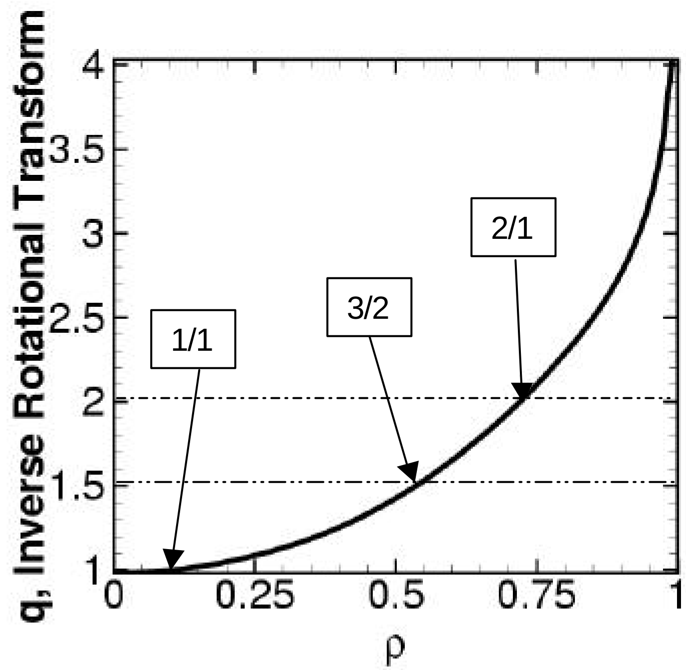
- •Sawtoothing discharge
- •3/2 NTM triggered at 2250 msec
- •2/1 locks to the wall

EQUILIBRIUM AT *t* = 2250 msec

Grid (Flux Surfaces)

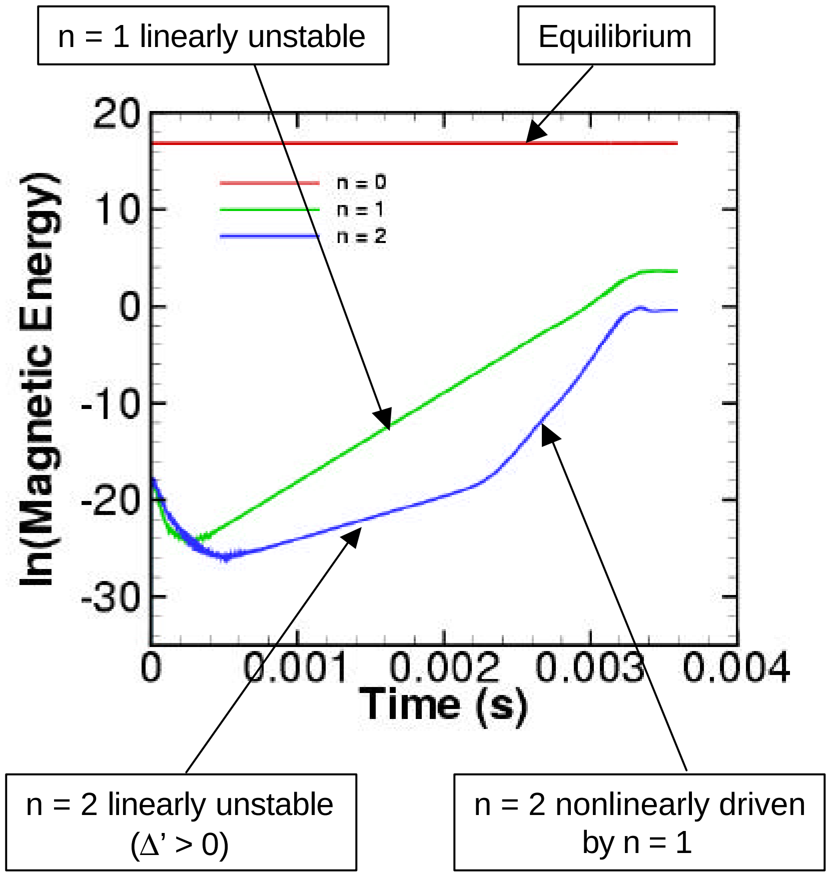


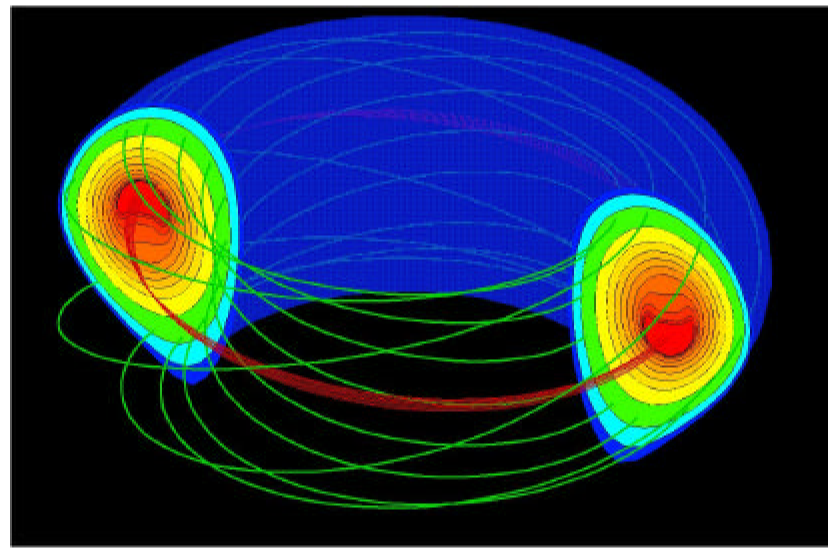
q - profile



- ITER-like discharge
- *q*(0) slightly below 1

DISCHARGE IS UNSTABLE TO RESISTIVE MHD



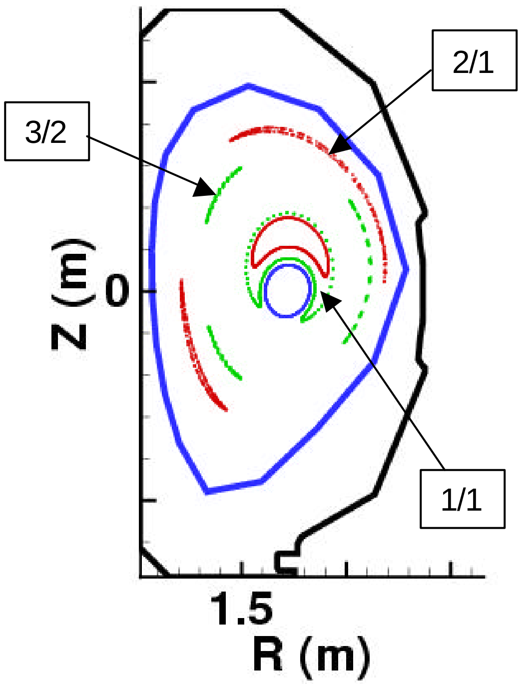


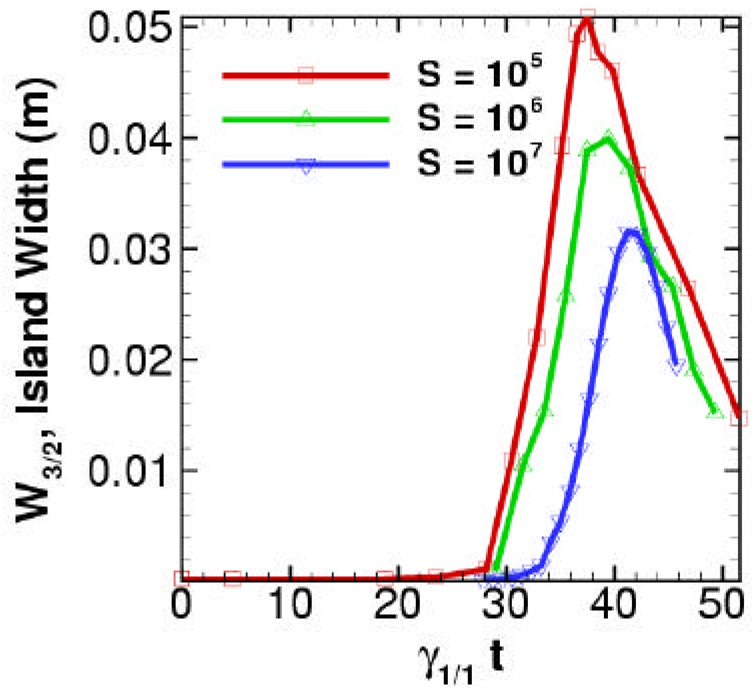
Pressure and field lines

$\boldsymbol{S = 10^7 \quad \mathrm{Pr} = 10^3}$

$\boldsymbol{g = 4.58 \times 10^3\ /\ \mathrm{sec} \quad g_{exp} \sim 1.68 \times 10^4\ /\ \mathrm{sec}}$

SECONDARY ISLANDS IN RESISTIVE MHD





- **Secondary islands are small in resistive MHD**
  - **—W_{exp} ~ 0.1 m**
- **3/2 island width decreases with increasing *S***
- **Need extended MHD to match experiment**

NUMERICALLY TRACTABLE CLOSURES

- Resistive MHD is insufficient to explain DIII-D shot 86144
    - 3/2 magnetic island is too small
- Parallel variation of *B* leads to trapped particle effects
- Particle trapping causes neo-classical effects
    - Poloidal flow damping
    - Enhancement of polarization current
    - Bootstrap current
- Simplified model captures most neo-classical effects

  (T. A. Gianakon, S. E. Kruger, C. C. Hegna, Phys. Plasmas (to appear) (2002))

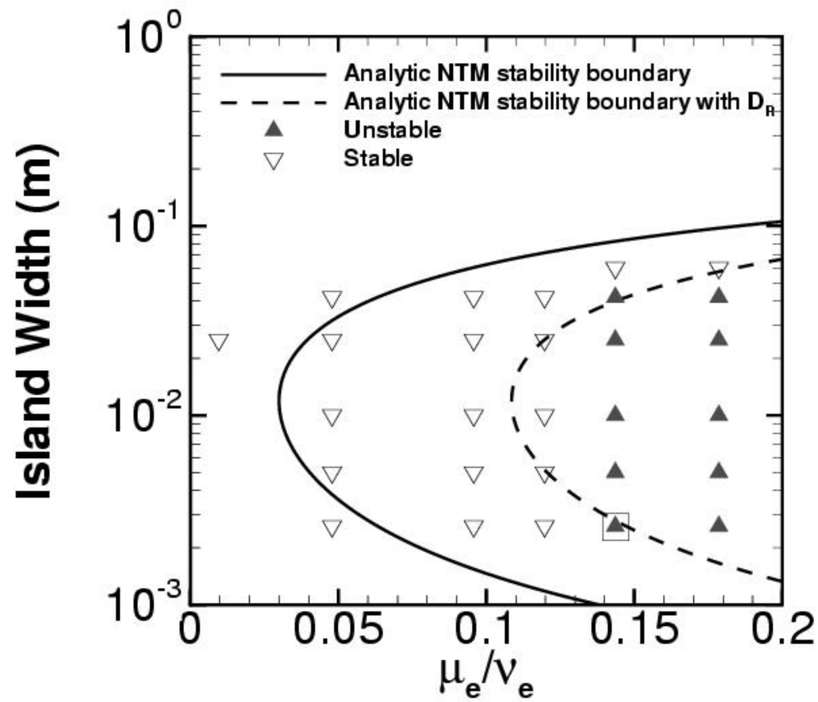
$$\nabla \cdot \Pi_a = m_a n_a m_a \langle B_0 \rangle^2 \frac{\mathbf{V}_a \cdot \nabla q}{\left(\mathbf{B}_0 \cdot \nabla q\right)^2} \nabla q$$

- For electrons, ideal MHD equilibrium yields bootstrap current

$$\nabla \cdot \Pi_e = -\frac{r_e m_e}{ne} \frac{\langle B \rangle^2}{B^2} \frac{\mathbf{B}_0 \times \nabla p \cdot \nabla q}{\left(\mathbf{B}_0 \cdot \nabla q\right)^2} \nabla q$$

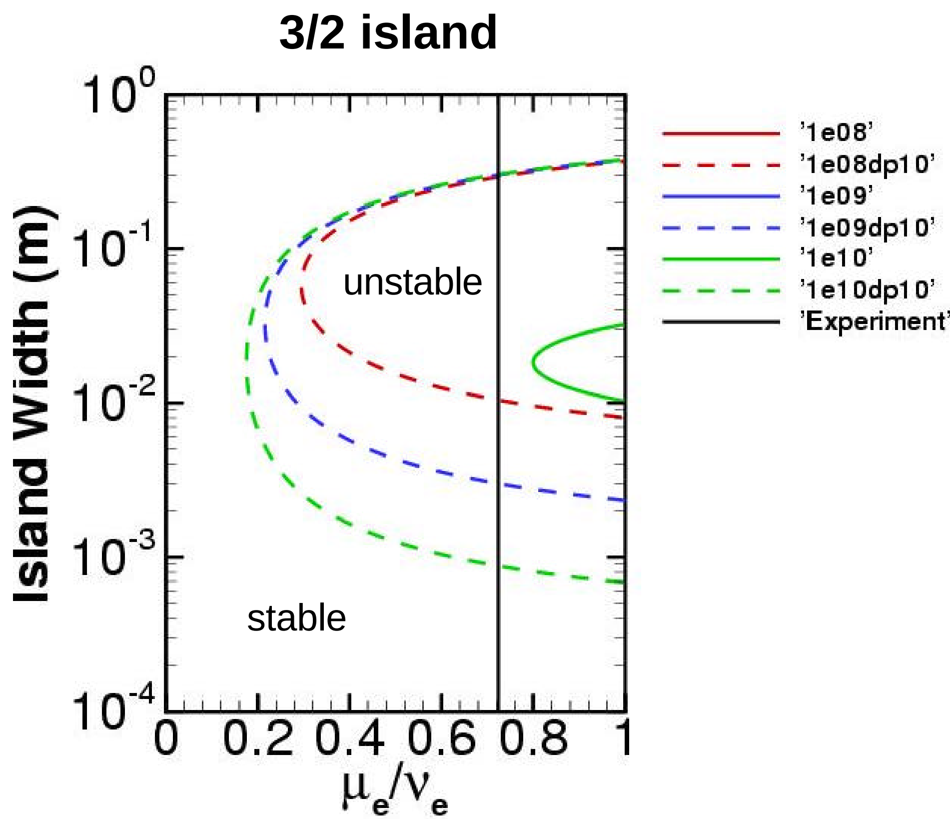
CLOSURES REPRODUCE NTM INSTABILITY

- TFTR-like equilibrium
- Comparison with modified Rutherford equation
- Initialize NIMROD with various seed island sizes
- Look for growth or damping
- Seek self-consistent seed and growth



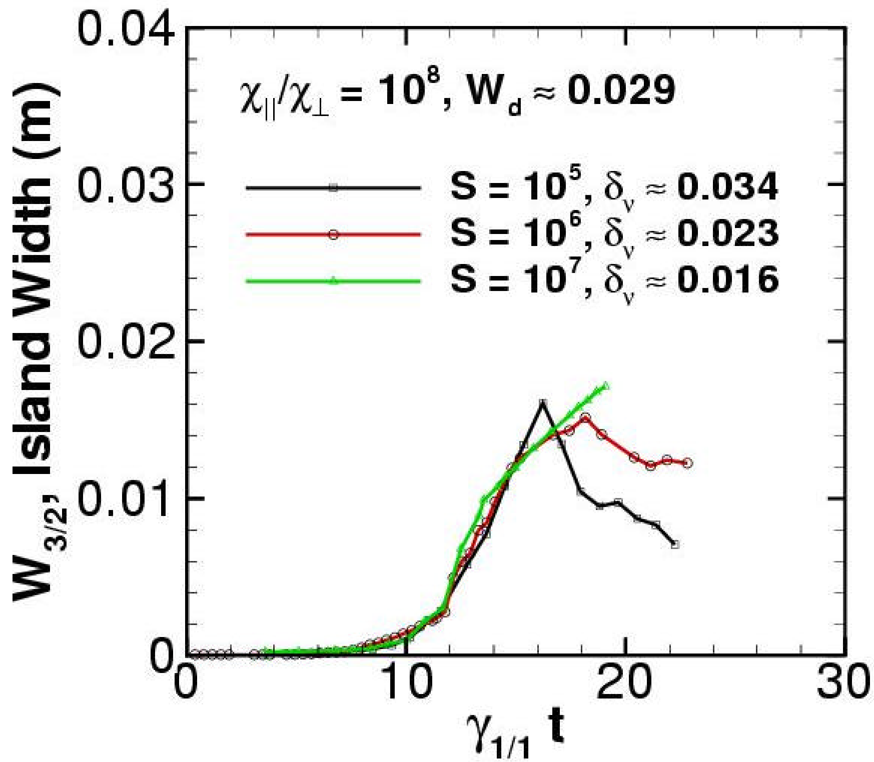
86144.2250 NTM STABILITY BOUNDARIES

- **Use modified Rutherford equation**
- **3 values of anisotropic heat flux**
- **2 values of D¢**
  - **Vacuum**
  - **Reduced by factor of 10**
- **Experimental island width ~ 0.1 m**



SELF-CONSISTENT NTM MAY REQUIRE HIGHER *S*

- **$W_{3/2}$ versus time with NTM closure**
- **W_{exp} ~ 0.1 m**
- **Island width exceeds analytic threshold**
- **No NTM observed**
- **Island width less than visco-resistive layer width at low *S***
- **Calculations at larger *S* are underway**



Nonlinear NTM calculations are extremely challenging!

SUMMARY

- **Nonlinear modeling of experimental discharges is possible, but extremely challenging**
- ***DIII-D shot #87009***
  - **Heating through b limit**
  - **Super-exponential growth, in agreement with experiment and theory**
  - **Nonlinear state leads to stochastic fields**
  - **Calculations with anisotropic thermal transport underway**
- ***DIII-D shot #86144***
  - **Secondary islands driven by sawtooth crash**
  - **Resistive MHD insufficient, requires neo-classical closures**
  - **Must go to large *S* (~10^7) to get proper length scales**
    - **Calculations are underway**