Nonlinear Simulation of NSTX including Sheared Flows

W. Park, J. Breslau, J. Chen, G.Y. Fu, S.C. Jardin, S. Klasky,
J. Menard, A. Pletzer, D. Stutman (PPPL)
H.R. Strauss (NYU)
L.E. Sugiyama (MIT)

Outline

- M3D code
- NSTX
- 2D steady state with toroidal flow
- Linear eigenmodes with and without flow
- 3D nonlinear evolution with and without flow
- IRE

M3D project

Multilevel 3D Project
for
Plasma Simulation
studies

Simple
:
:
Realistic

All levels are needed to understand the physics.

Physics	Geometry	State
MHD 2 Fluids Gyrokin. Hot P. /MHD Gyrokin. Ion /Fluid Elect. Full kin. Ion /Fluid Elect.	MPP Serial Unstructured FE Structured FD	Equilibrium Linear Nonlinear

MHD model

- Solves MHD equations.

$$\left\{\begin{aligned}
&\rho\partial \mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v}\\
&\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}), \quad \mathbf{J} = \nabla\times\mathbf{B}\\
&\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0\\
&\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\,\nabla\,(p/\rho)
\end{aligned}\right.$$

The fast parallel equilibration of T is modeled using wave equations;

$$\left\{\begin{aligned}
&\partial T/\partial t = s\,\mathbf{B}/\rho\cdot\nabla u\\
&\partial u/\partial t = s\,\mathbf{B}\cdot\nabla\mathbf{T} + \upsilon\nabla^2 u
\end{aligned}\right.$$

s = wave speed/ v_A

Two-fluid MH3D-T

- Solves the two fluid equations with gyro-viscosity and neoclassical parallel viscousity terms in a torus.

- **Equations**

$$\left\{\begin{aligned}
&\mathbf{v} \equiv \mathbf{v}_i - \mathbf{v}_i^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}_{\parallel}/en,\\
&\mathbf{v}_e^* \equiv -\mathbf{B}\mathrm{x}\nabla p_e\ /(enB^2), \quad \mathbf{v}_i^* \equiv \mathbf{v}_e^* + \mathbf{J}_{\perp}/en,
\end{aligned}\right.$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} + \rho(\mathbf{v}_i^*\cdot\nabla)\mathbf{v}_{\perp} = -\nabla p + \mathbf{J}\times\mathbf{B} - \mathbf{b}\cdot\nabla\cdot\Pi i,$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}) - \nabla_{\parallel} p_e/en - \mathbf{b}\cdot\nabla\cdot\Pi e,$$
$$\mathbf{J} = \nabla\times\mathbf{B},$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}_i) = 0,$$

$$\begin{aligned}
\partial p/\partial t + \mathbf{v}\cdot\nabla p = &-\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p/\rho)\\
&- \mathbf{v}_i^*\cdot\nabla p + (1/en)\mathbf{J}\cdot\nabla p_e\\
&- \gamma p\nabla\cdot\mathbf{v}_i^* + \gamma p_e\mathbf{J}\cdot\nabla(1/en)
\end{aligned}$$

$$\begin{aligned}
\partial p_e/\partial t + \mathbf{v}\cdot\nabla p_e = &-\gamma p_e\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p_e/\rho)\\
&+ (1/en)\mathbf{J}_{\parallel}\cdot\nabla p_e - \gamma p_e\nabla\cdot(\mathbf{v}_e^* - \mathbf{J}_{\parallel}/en)
\end{aligned}$$

GK Hot Particle /MHD Hybrid MH3D-K

- **Fluid equations**

$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p - (\nabla\cdot\mathbf{P}_h)_\perp + \mathbf{J}\times\mathbf{B}$ (Pressure coupling)

or

$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + (\nabla\times\mathbf{B} - \mathbf{J}_h)\times\mathbf{B} + q_h\mathbf{V}\times\mathbf{B}$

(Current coupling)

$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$, $\mathbf{E} = \mathbf{v}\times\mathbf{B} - \eta(\mathbf{J} - \mathbf{J}_h)$, $\mathbf{J} = \nabla\times\mathbf{B}$

$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$

$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\cdot\nabla(p/\rho)$

- **Gyrokinetic equations for energetic particles**

$d\mathbf{R}/dt = u[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,] + (1/\Omega)\mathbf{b}\times(\mu\nabla\mathbf{B} - q\mathbf{E}/m)$,

$du/dt = -[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,]\cdot(\mu\nabla\mathbf{B} - q\mathbf{E}/m)$.

GK Particle Ion / Fluid Electron Hybrid

- **Pressure coupling**

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla\cdot\mathbf{P}_i - \nabla P_e + \mathbf{J}\times\mathbf{B}$$

$$= -\nabla\cdot\mathbf{P}_i^{CGL} - \nabla\cdot\Pi_i - \nabla P_e + \mathbf{J}\times\mathbf{B}$$

$\nabla\cdot P_i^{CGL}$: from particles following GK eqns.

$\nabla\cdot\Pi_i$: fluid picture as 2 fluid eqns,

or from particles.

- **Fluid electrons**

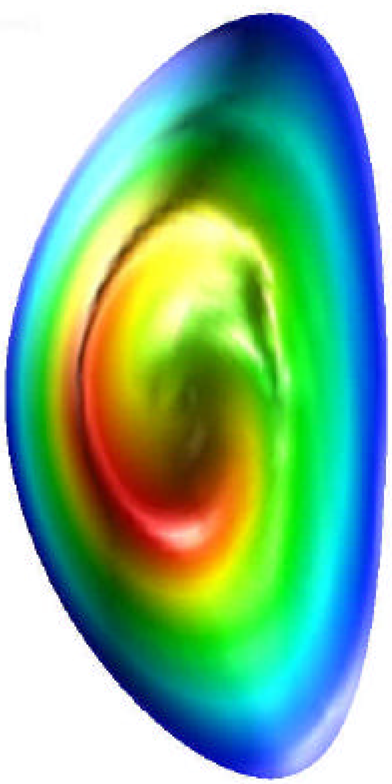
$$\mathbf{E} = -\mathbf{V}_e\times\mathbf{B} + \eta\mathbf{J} + \nabla\cdot\mathbf{P}_e/ne$$

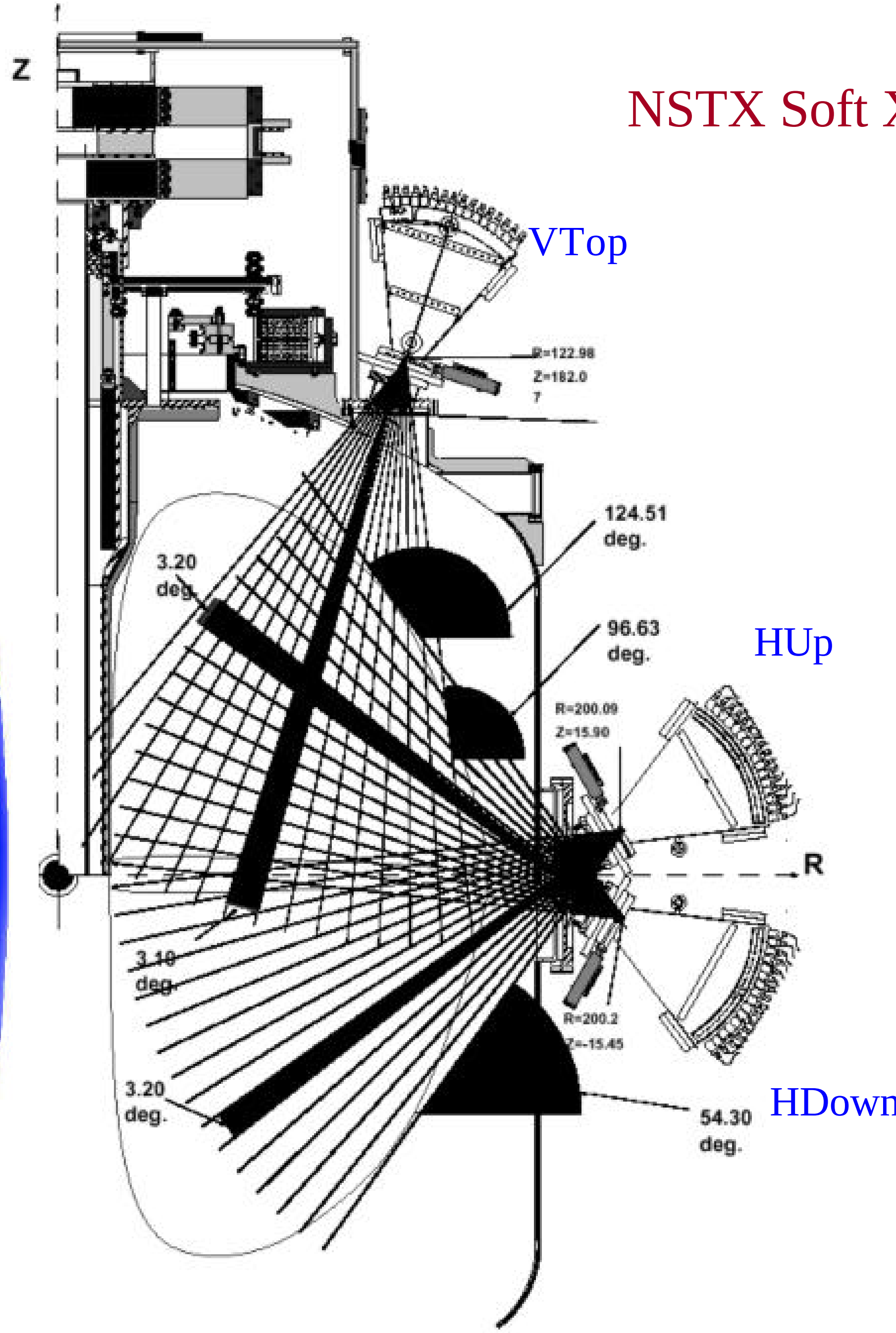
$$= -\mathbf{V}_e\times\mathbf{B} + \eta\mathbf{J} + \nabla P_e/ne + \mathbf{bb}\cdot\nabla\cdot\Pi_e/ne$$

$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$, $\mathbf{J} = \nabla\times\mathbf{B}$

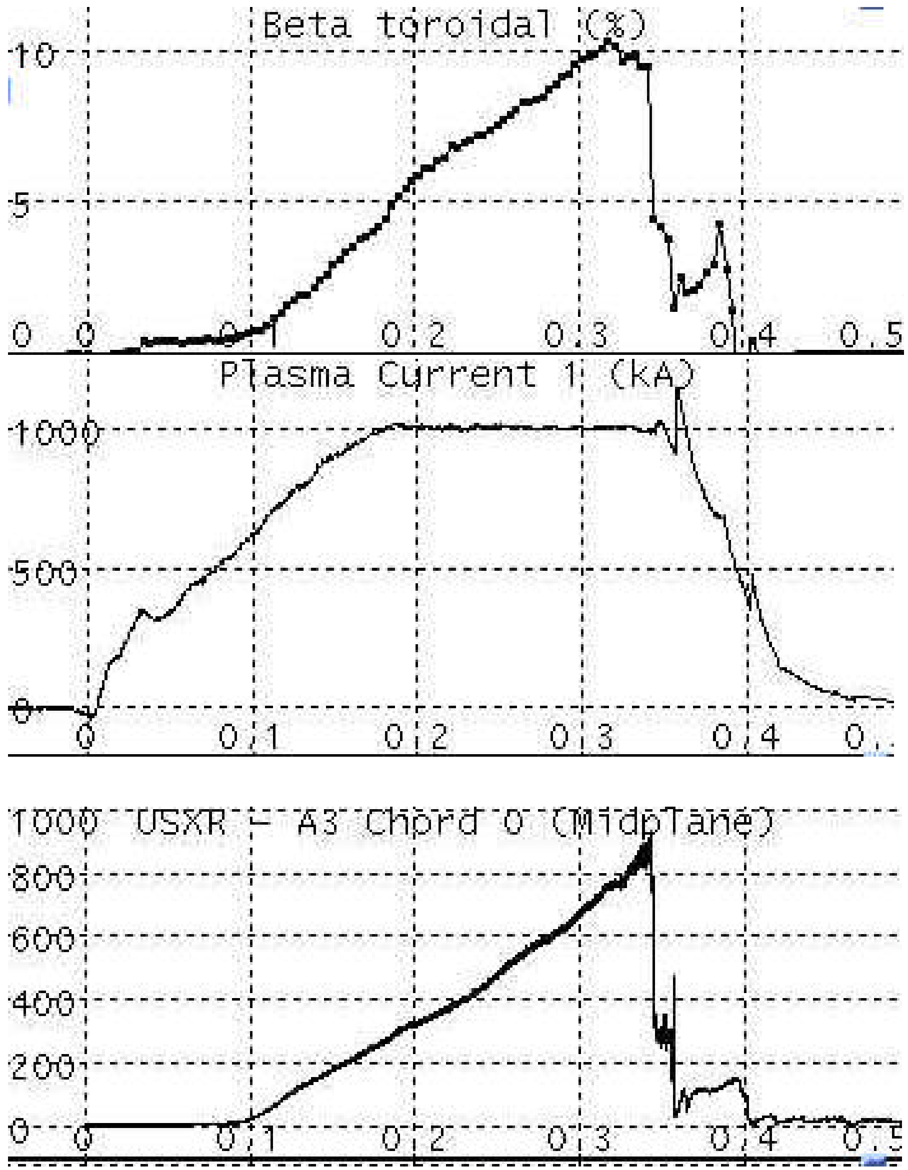
P_e eqn currently, but $P_\parallel$ and $P_\perp$ eqns are planned.

NSTX Soft X-ray arrays

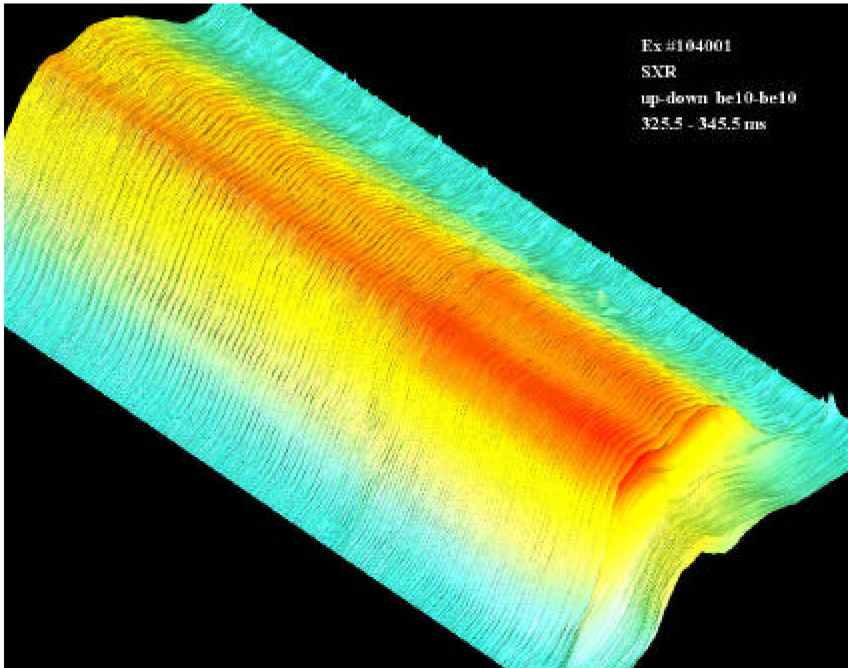


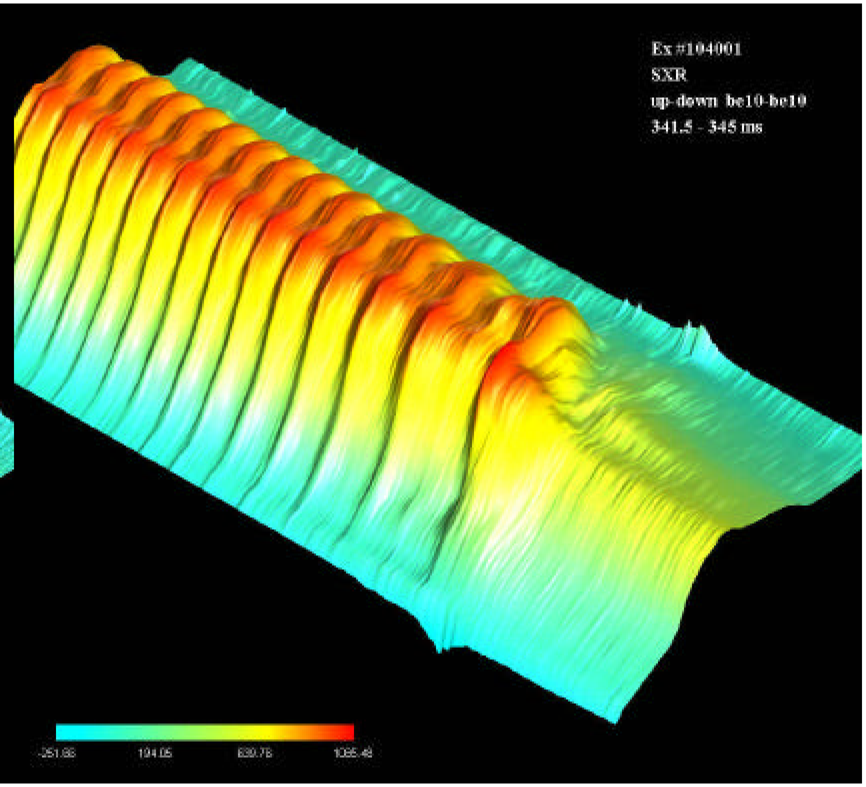


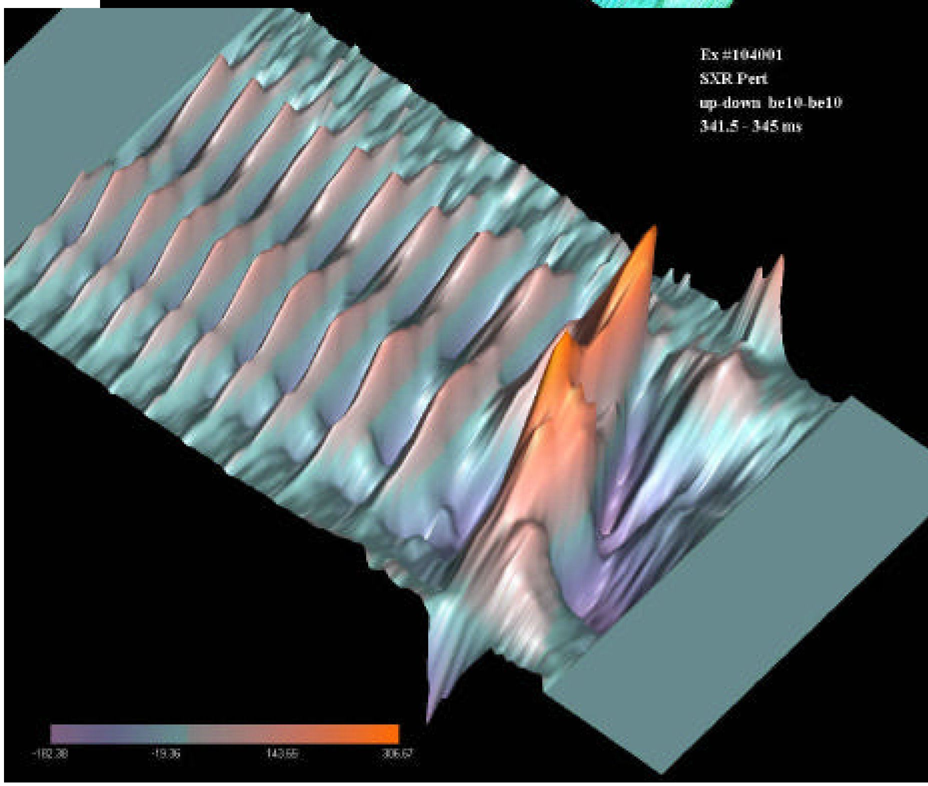
EX #104001 data



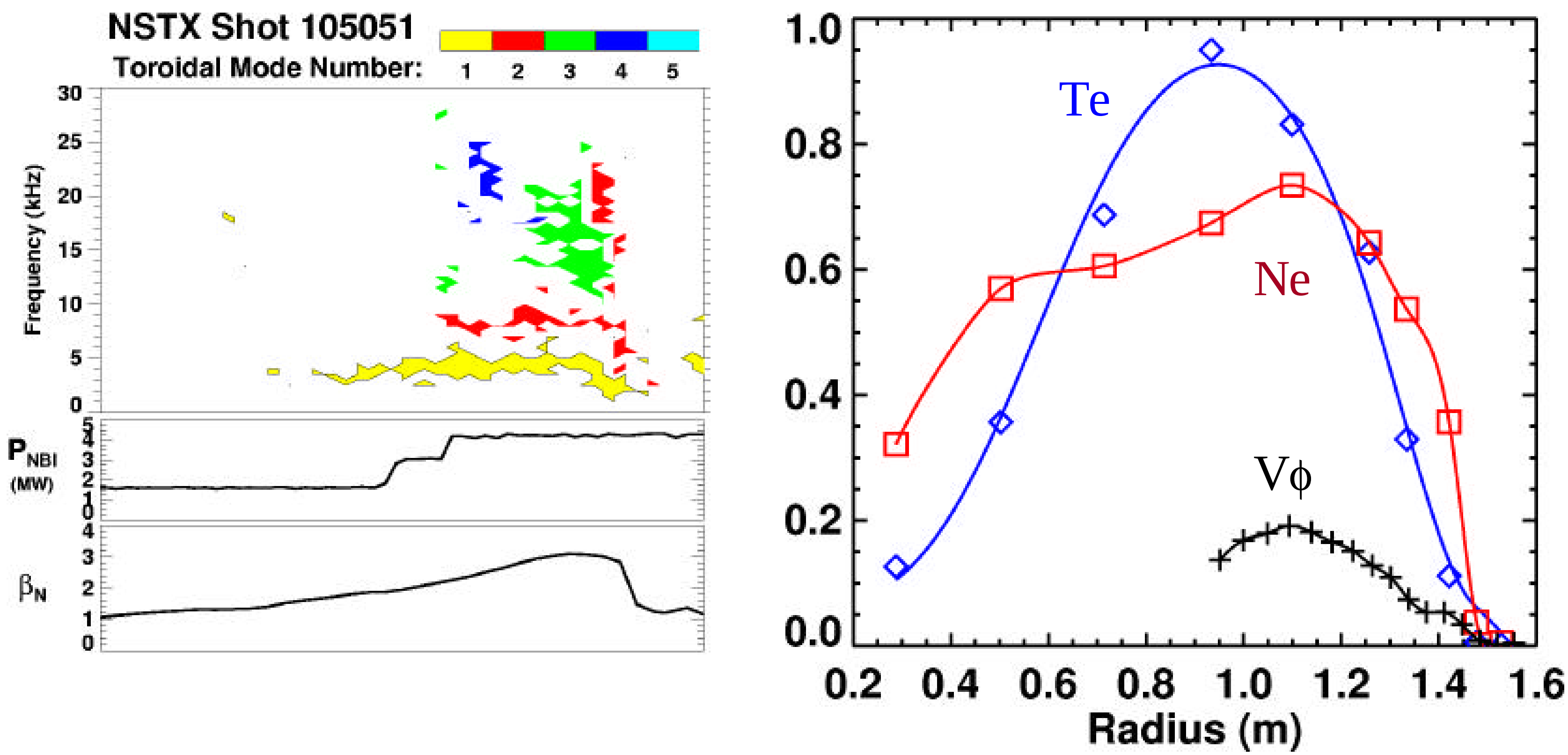
IRE







Soft-Xray data

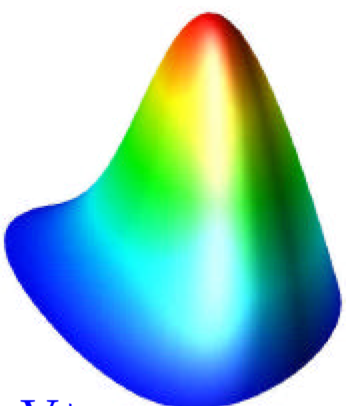


NSTX Shot 105051
Toroidal Mode Number:
1
2
3
4
5
Frequency (kHz)
30
25
20
15
10
5
0
PNBI (MW)
βN
Te
Ne
Vϕ
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Radius (m)

2D steady state with toroidal sheared flow

Quasi neutrality: $$r\,\mathbf{V}\bullet\nabla\mathbf{V} + \nabla\bullet\overleftrightarrow{\mathbf{P}} - \mathbf{J}\times\mathbf{B} = 0$$

MHD: $$\nabla\bullet\overleftrightarrow{\mathbf{P}} = \nabla P$$



Vϕ

At the magnetic axis: $$\mathbf{J}\times\mathbf{B} = 0$$

$$-\frac{r\,\mathrm{V}_f^{\,2}}{R} + \frac{T\partial r}{\partial R} = 0$$

Relative shift of $r \equiv$ $$\frac{R\partial r}{r\partial R} = \frac{\mathrm{V}_f^{\,2}}{T} = \frac{2M_A^{\,2}}{b}$$

On a flux surface: $$\Delta \ln r = \frac{\Omega^2 \Delta R^2}{2T}$$

2D steady state with toroidal sheared flow

Quasi neutrality: $\rho\,\mathbf{V}\bullet\nabla\mathbf{V} + \nabla\bullet\overleftrightarrow{\mathbf{P}} - \mathbf{J}\times\mathbf{B} = 0$

$$\nabla\bullet\overleftrightarrow{\mathbf{P}} = \nabla\bullet\overleftrightarrow{\mathbf{P}}^{CGL} + \nabla\bullet\overleftrightarrow{\Pi}_g$$

$$= \mathrm{p}\overleftrightarrow{\mathrm{I}} \qquad + (P_{\parallel} - P_{\perp})\overleftrightarrow{\Pi}_{ii} \qquad + \overleftrightarrow{\Pi}_g$$

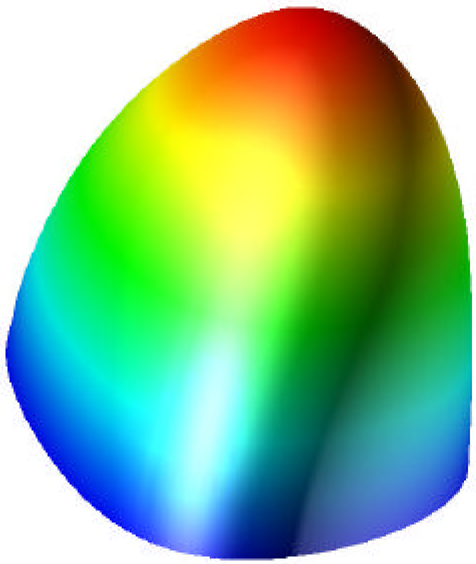
MHD   Hot Particle/MHD   2-Fluids

2-Fluids with Neocl.clos.

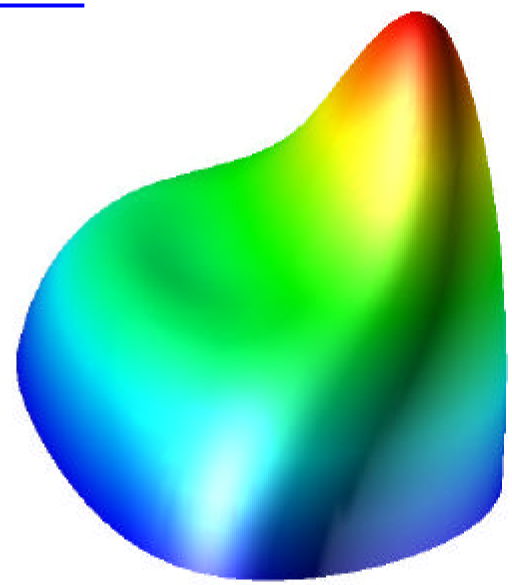
Ion Particle/Electron Fuid

Particles or Phase-space fluids

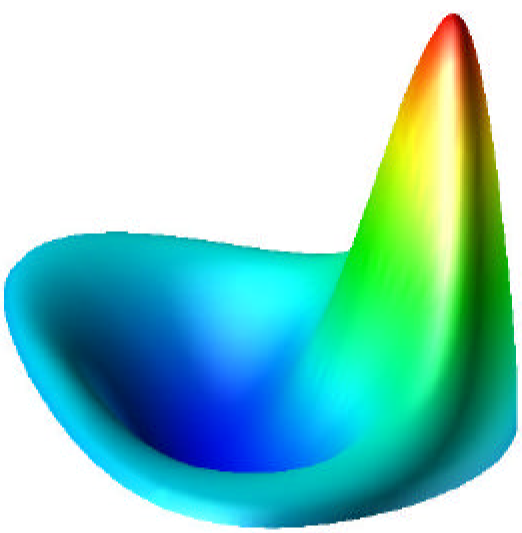
Density profile



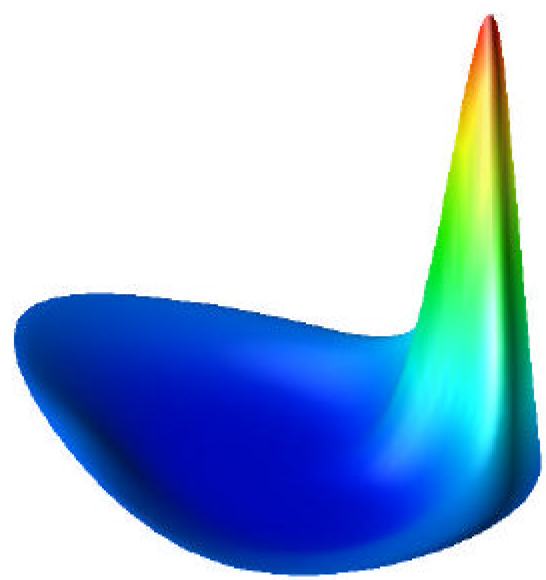
$M_A=0$
Sh=0
$\rho_{max}=1$
$\rho_{min}=0.5$



$M_A=0.2$
Sh=0.3
$\rho_{max}=1.1$
$\rho_{min}=0.5$



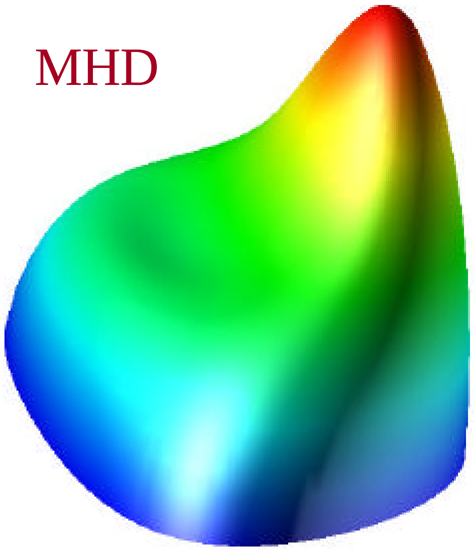
$M_A=0.5$
Sh=0.4-0.07=0.33
$\rho_{max}=1.9$
$\rho_{min}=0.2$



$M_A=0.8$
Sh=0.5-0.15=0.35
$\rho_{max}=5.2$
$\rho_{min}=0.005$

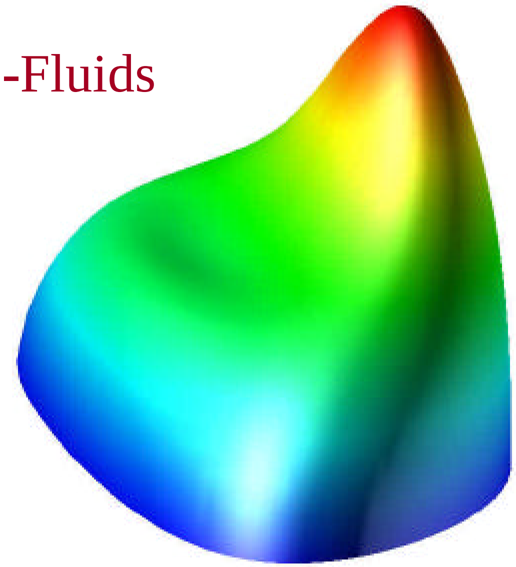
Density profile

MHD



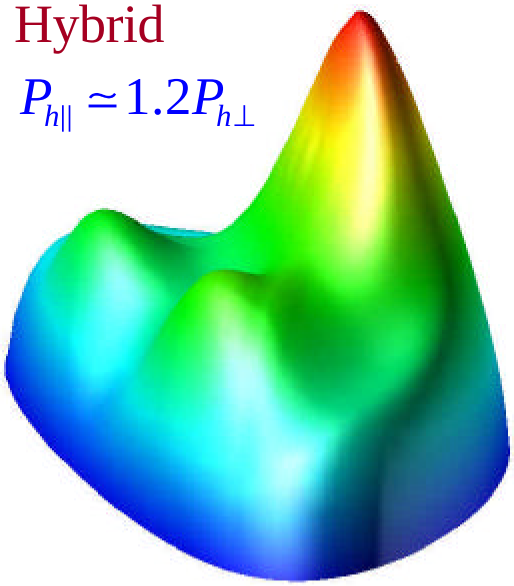
$M_A=0.2$
Sh=0.3
$\rho_{max}=1.1$
$\rho_{min}=0.5$
RelSh=1

Two-Fluids



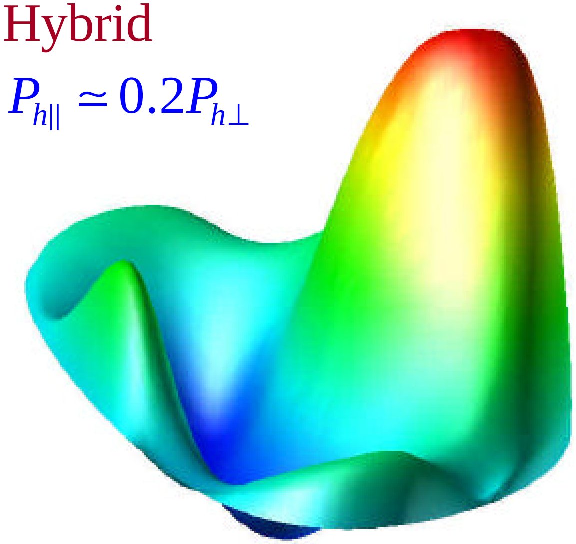
$M_A=0.2$
Sh=0.3
$\rho_{max}=1.1$
$\rho_{min}=0.5$
RelSh=1

Hybrid
$P_{h\|} \simeq 1.2 P_{h\perp}$

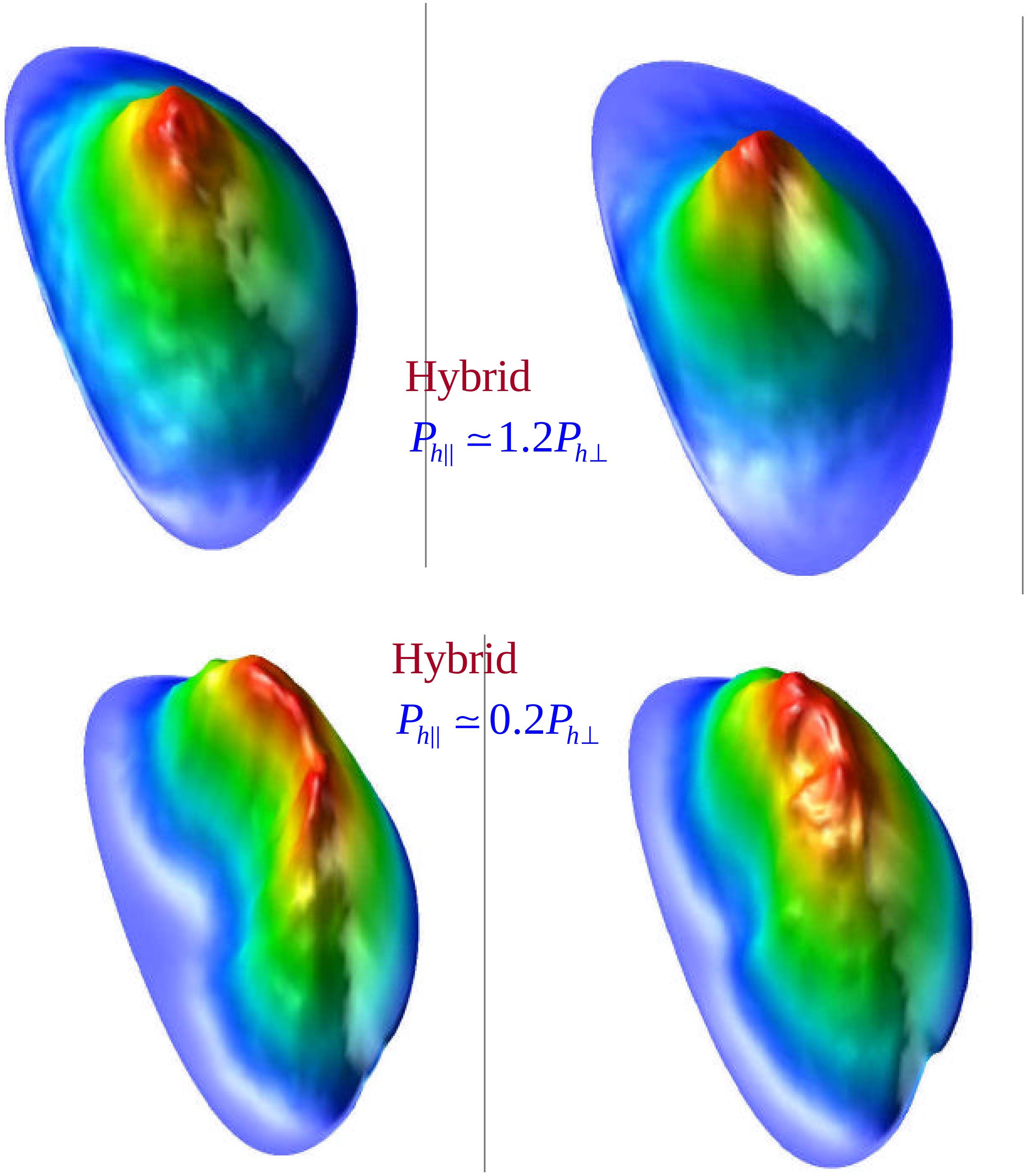


$M_A=0.2$
Sh=0.3
$\rho_{max}=1.2$
$\rho_{min}=0.5$
RelSh=0.8

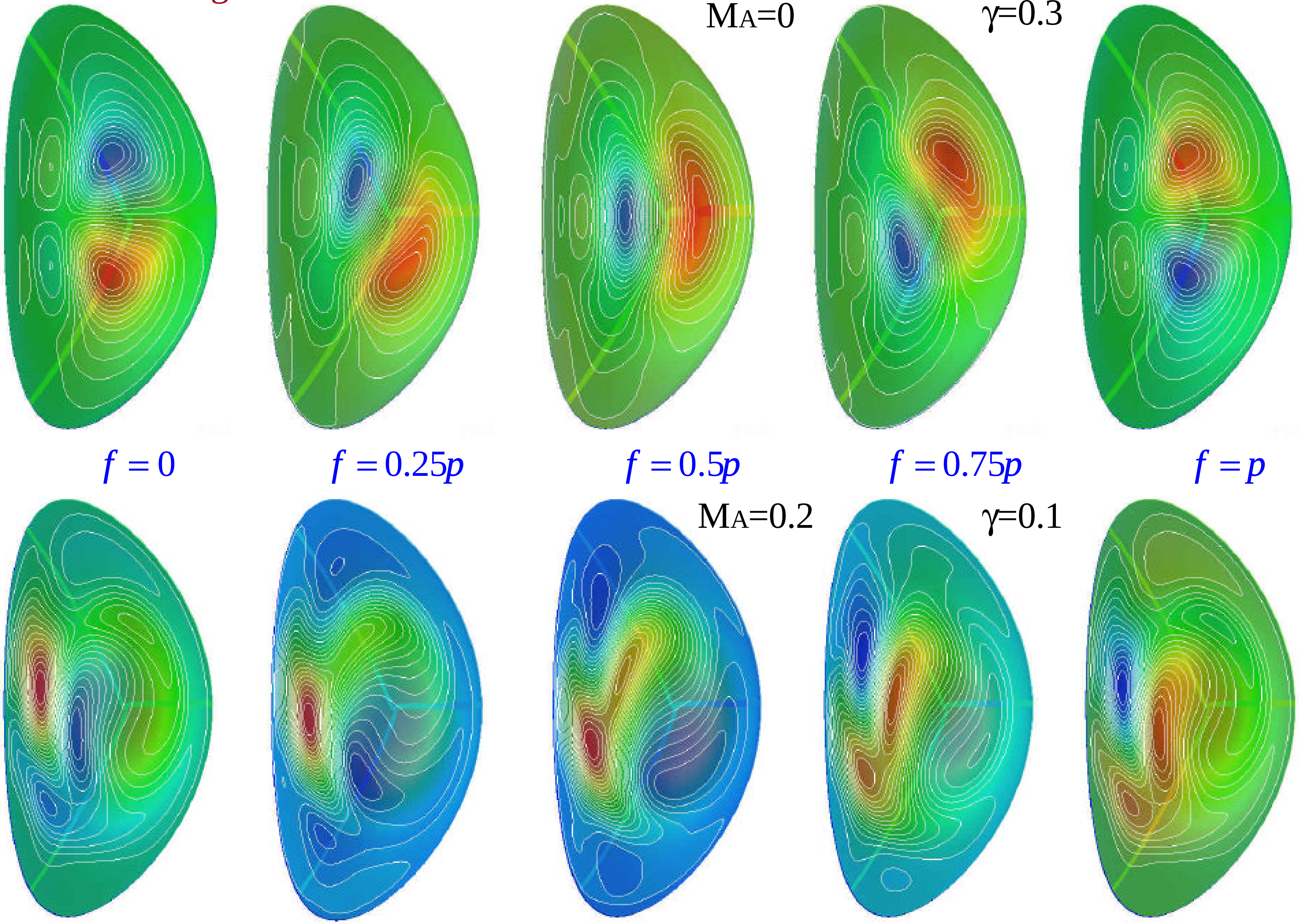
Hybrid
$P_{h\|} \simeq 0.2 P_{h\perp}$



$M_A=0.2$
Sh=0.3
$\rho_{max}=1.8$
$\rho_{min}=0.15$
RelSh=1.9


Hybrid
$P_{h\|} \simeq 1.2 P_{h\perp}$
Hybrid
$P_{h\|} \simeq 0.2 P_{h\perp}$

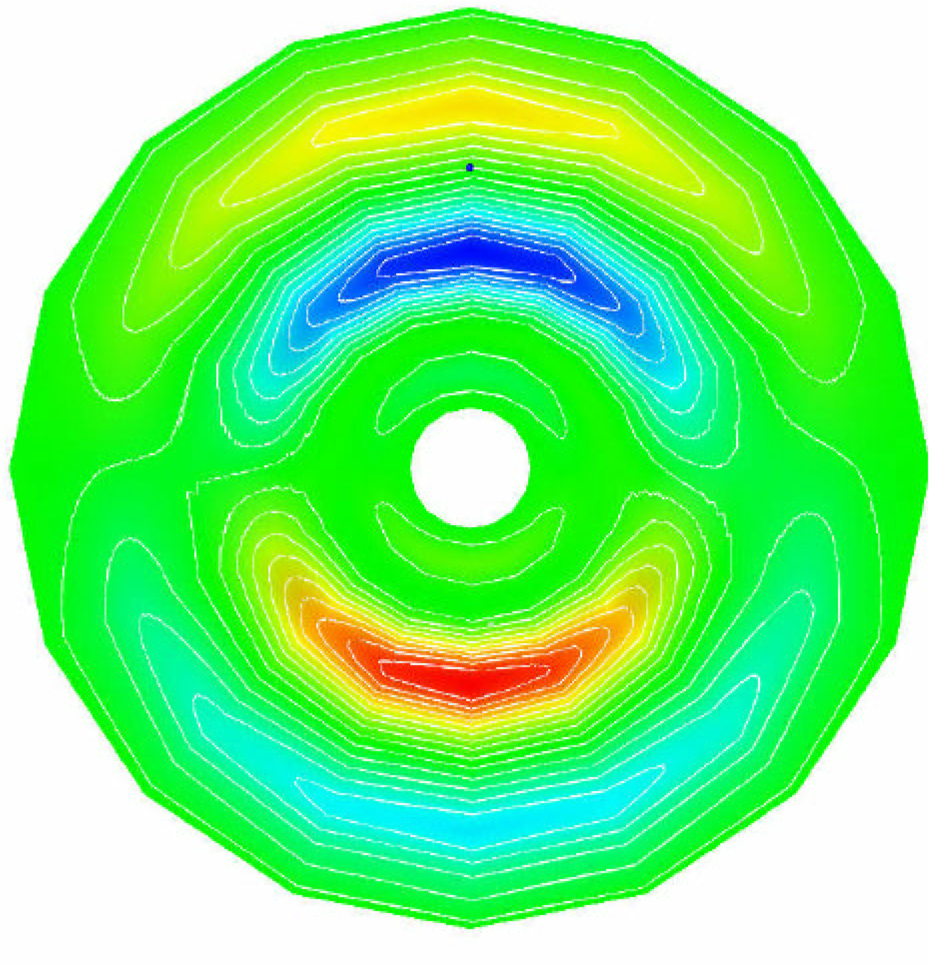
Linear Eigenmodes

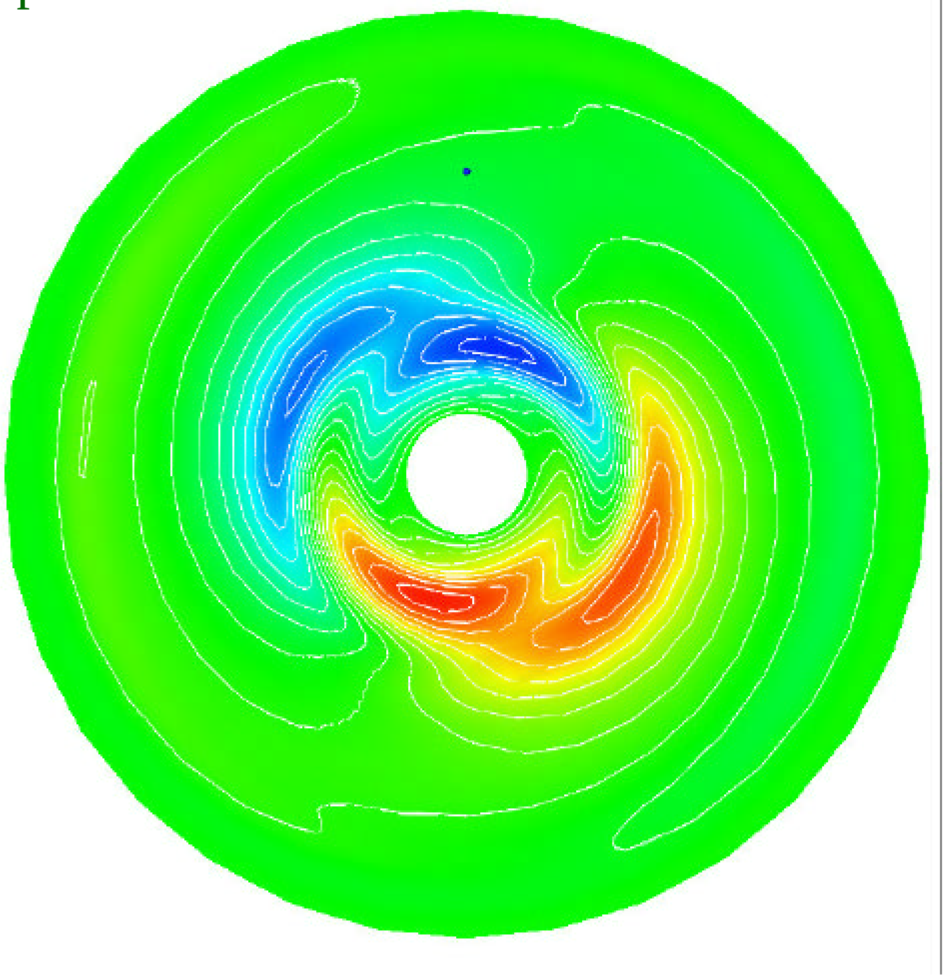


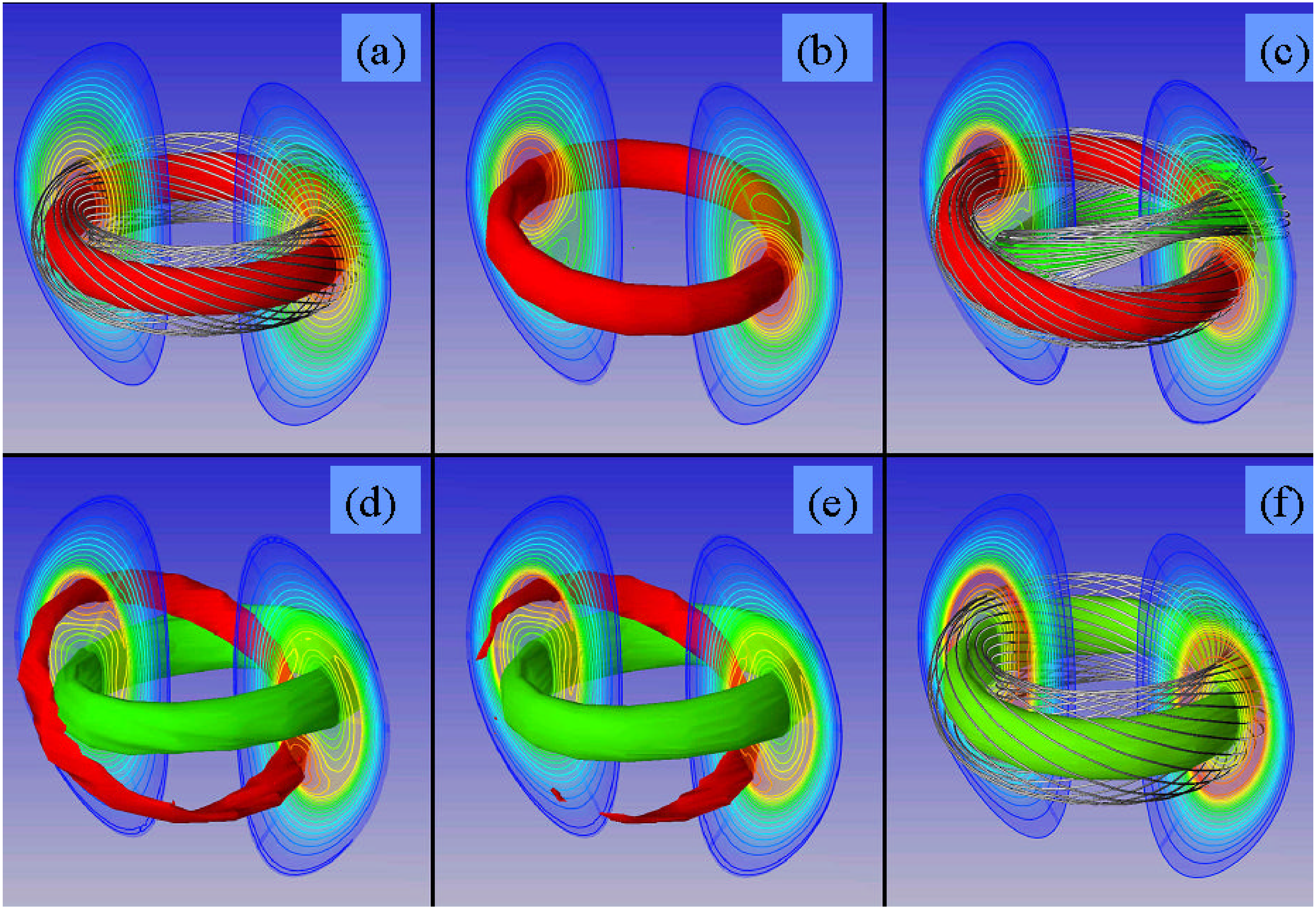
$M_A=0$ $\Omega m=0$

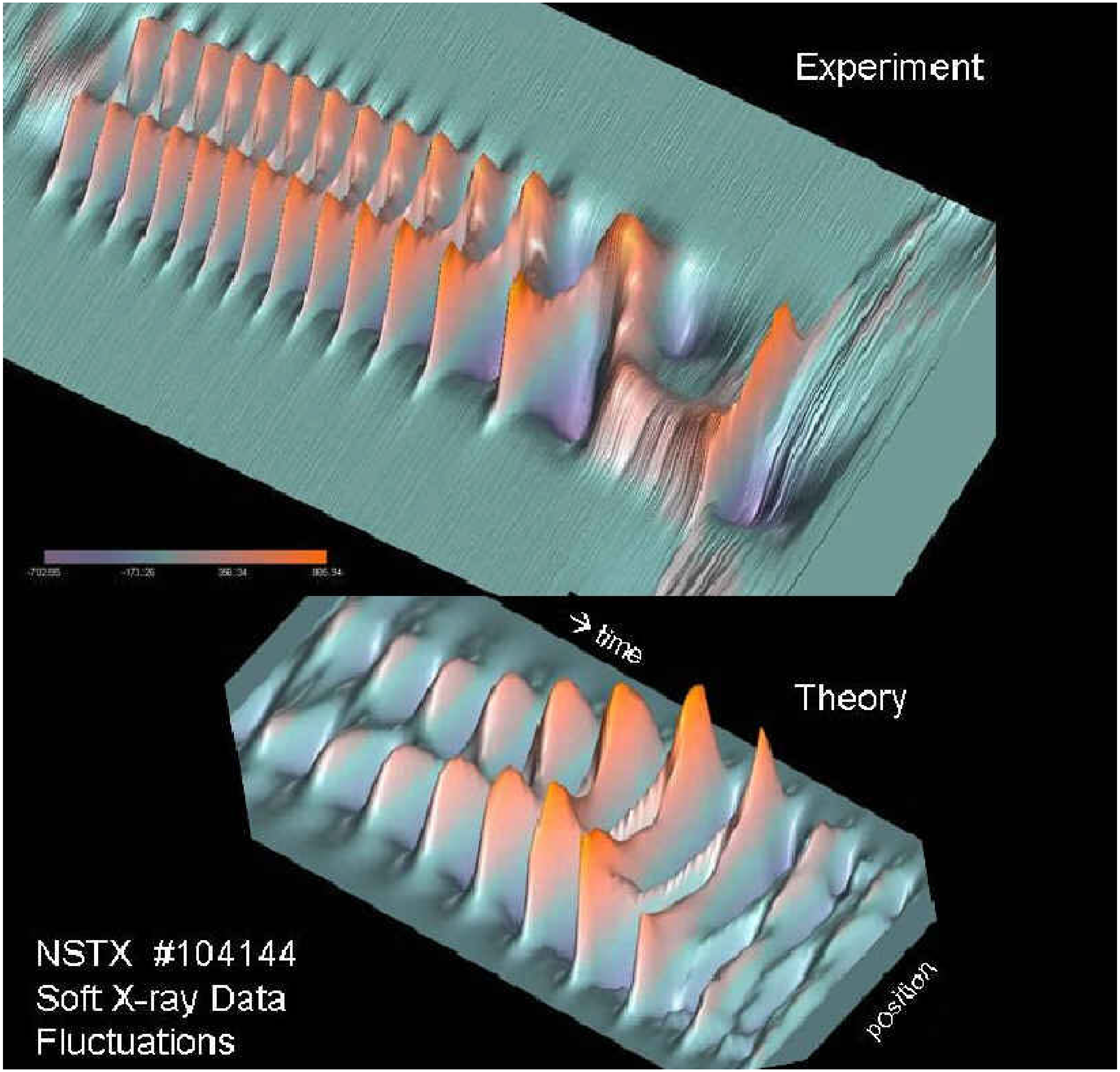
$M_A=0.2$ $\Omega m=0.13$

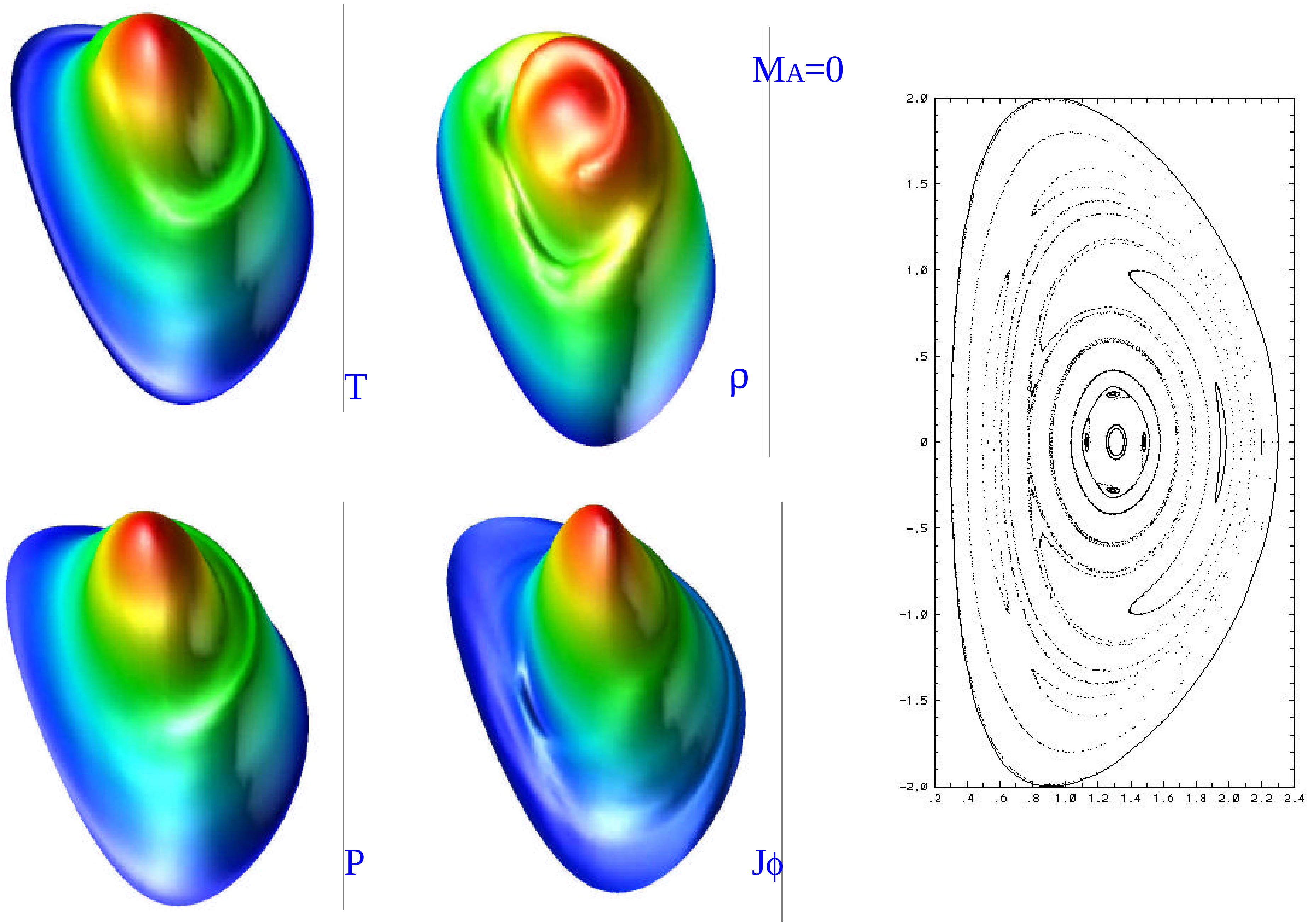
On the mid-plane



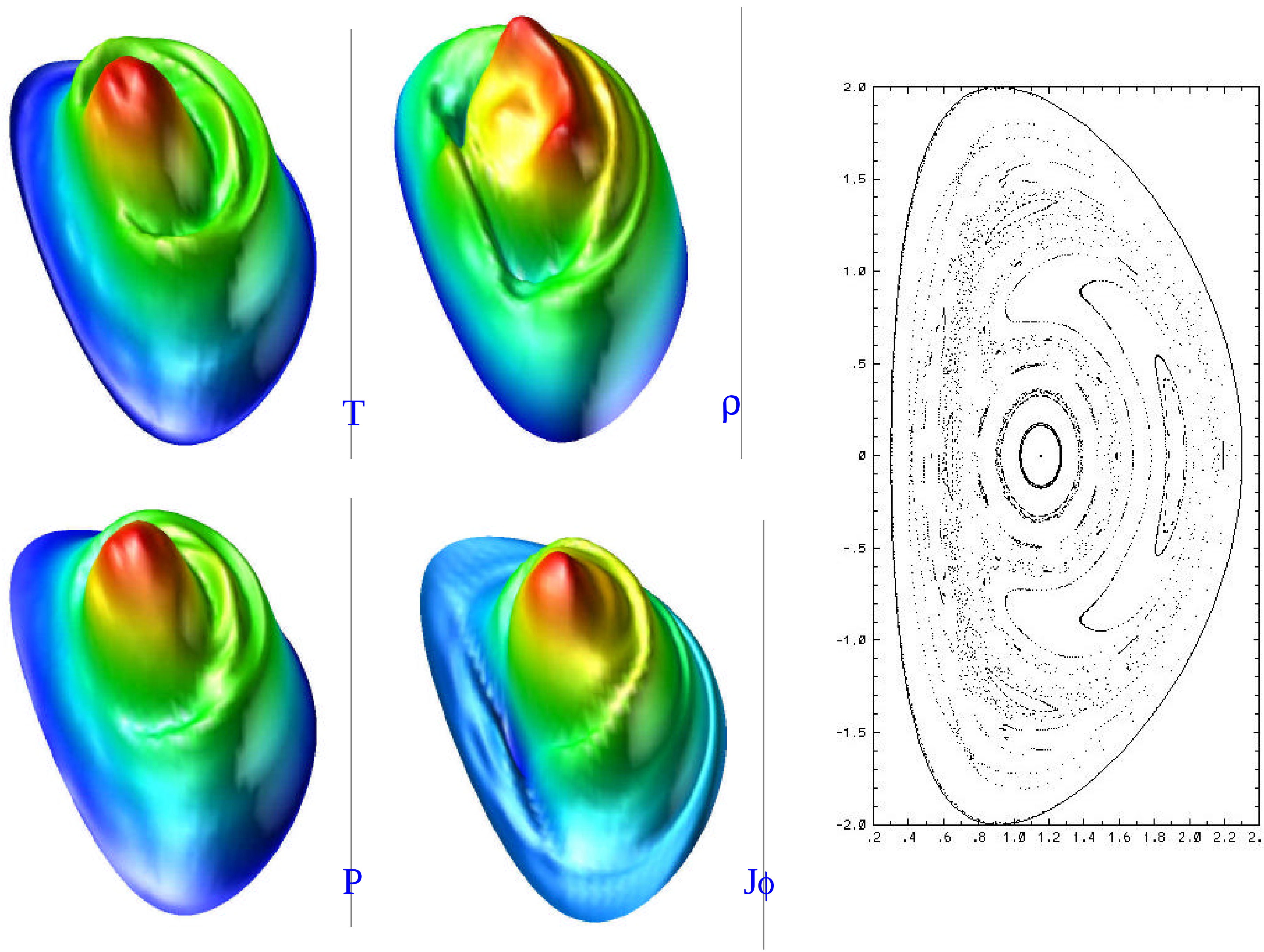


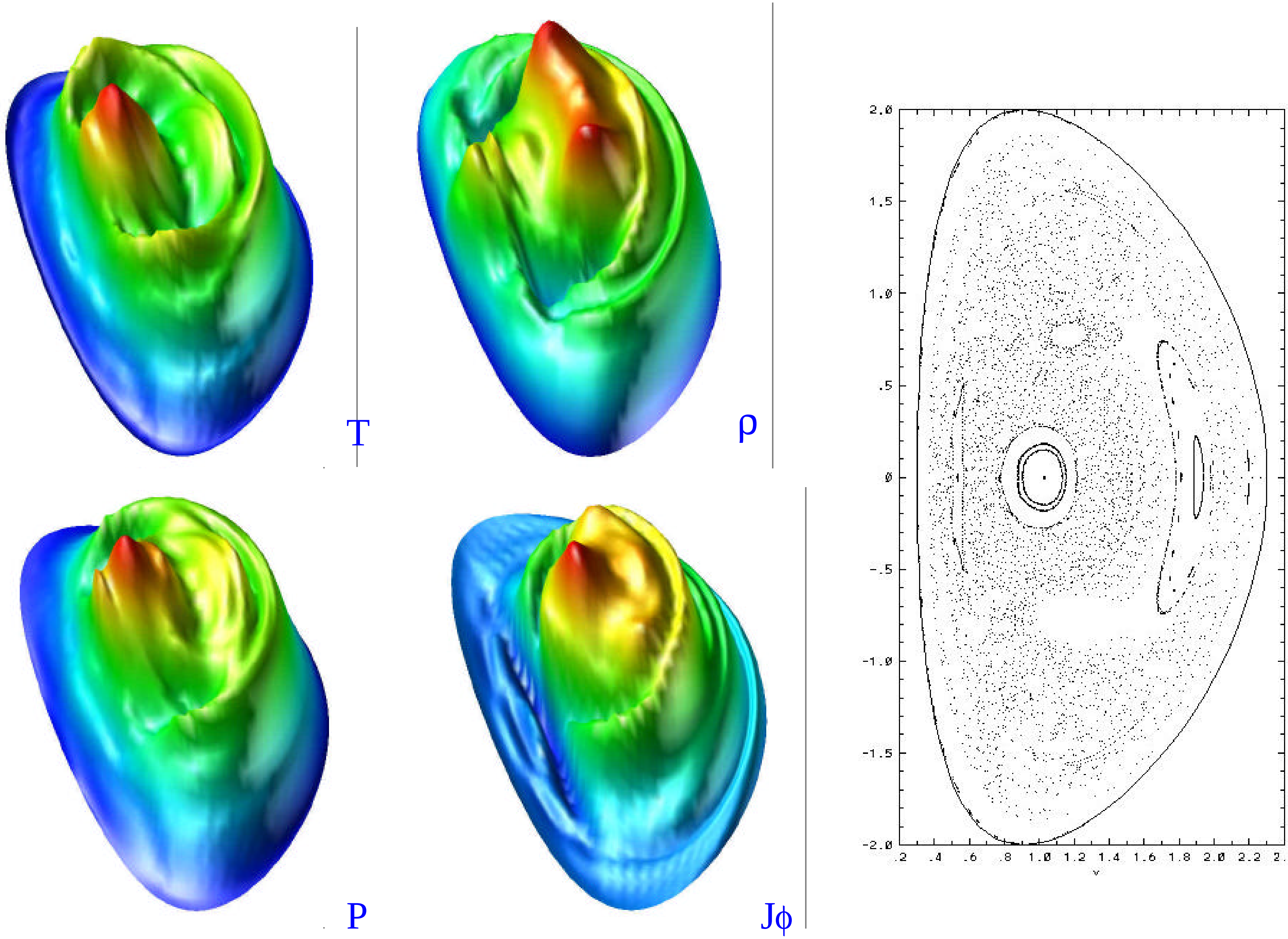

(a)
(b)
(c)
(d)
(e)
(f)

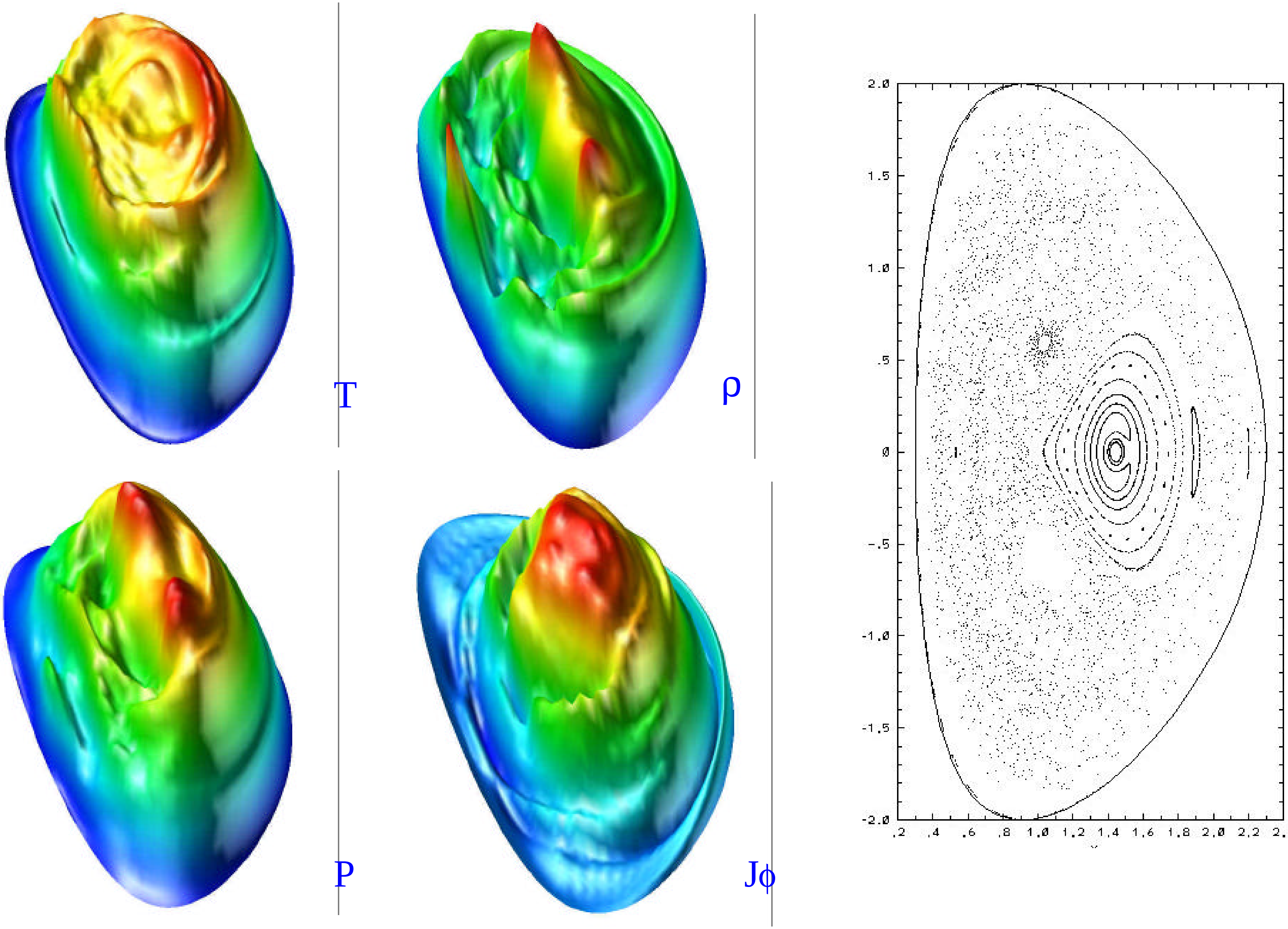

Experiment
→ time
Theory
position
NSTX #104144
Soft X-ray Data
Fluctuations

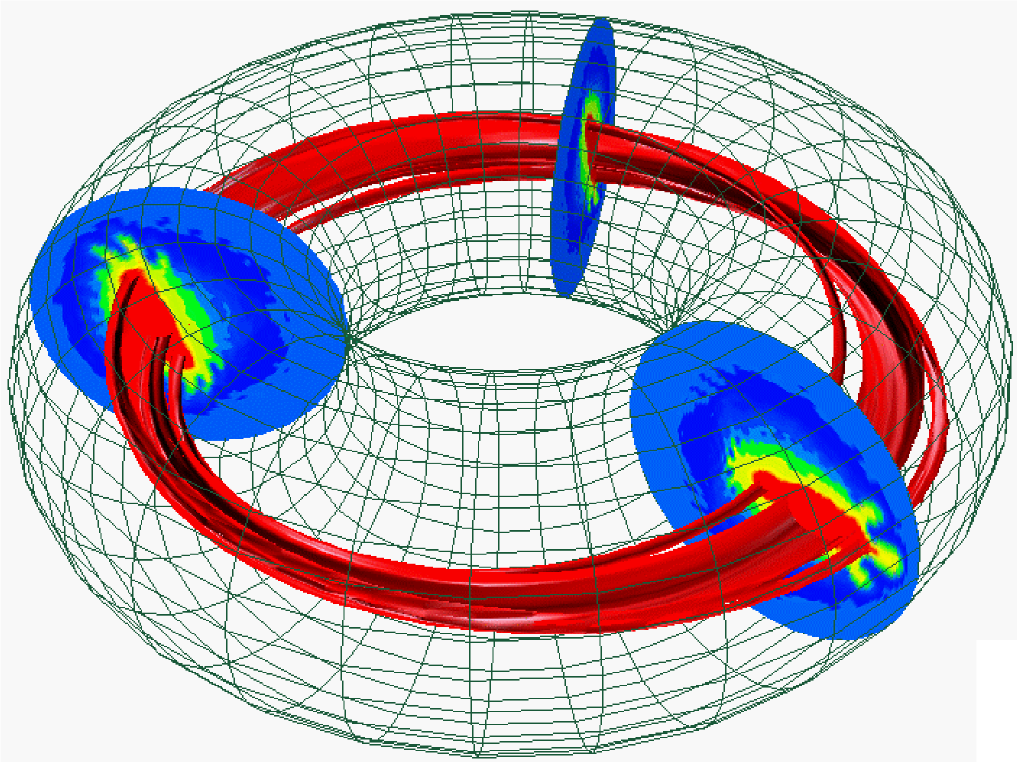


T
ρ
MA=0
P
Jϕ
2.0
1.5
1.0
.5
0
-.5
-1.0
-1.5
-2.0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4


T
ρ
P
Jϕ
2.0
1.5
1.0
.5
0
-.5
-1.0
-1.5
-2.0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

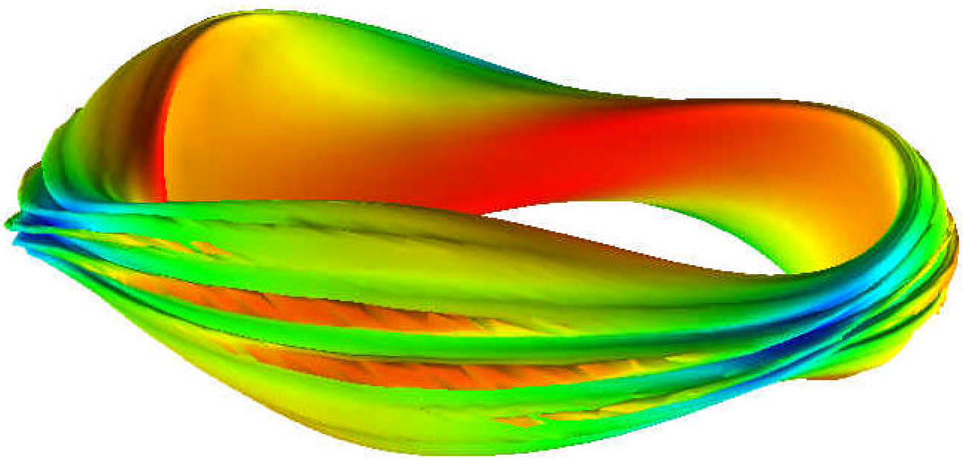

T
ρ
P
Jϕ
2.0
1.5
1.0
.5
0
-.5
-1.0
-1.5
-2.0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

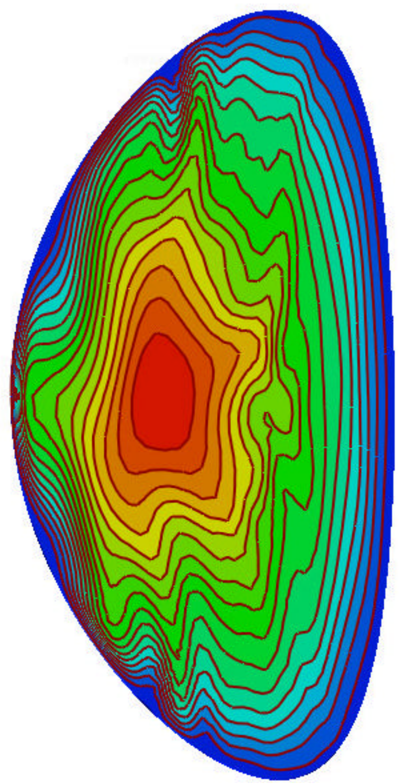

T
ρ
P
Jϕ
2.0
1.5
1.0
.5
0
-.5
-1.0
-1.5
-2.0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

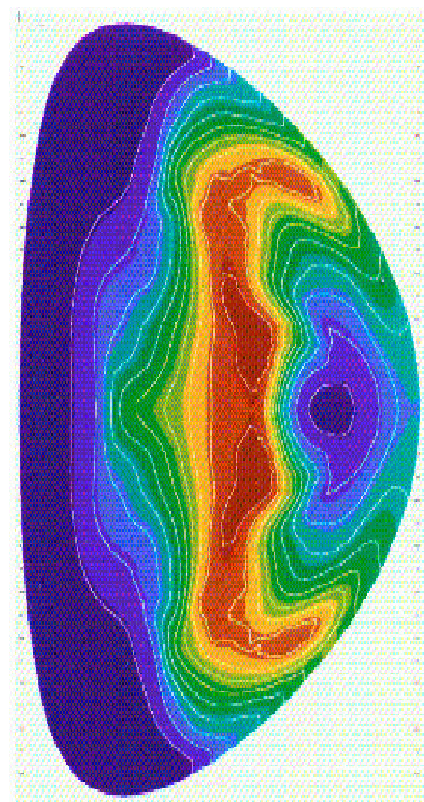


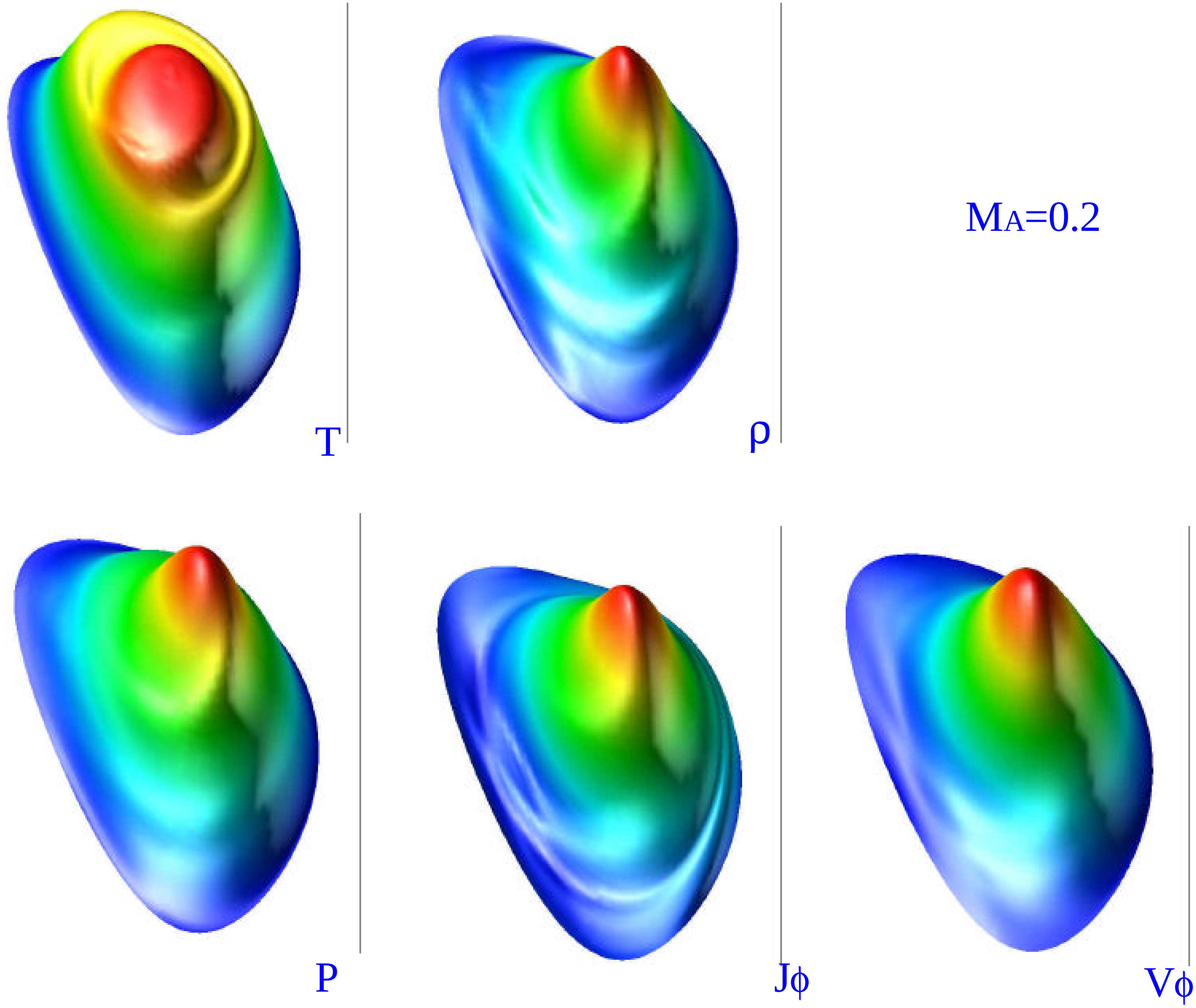
IRE : Disruption

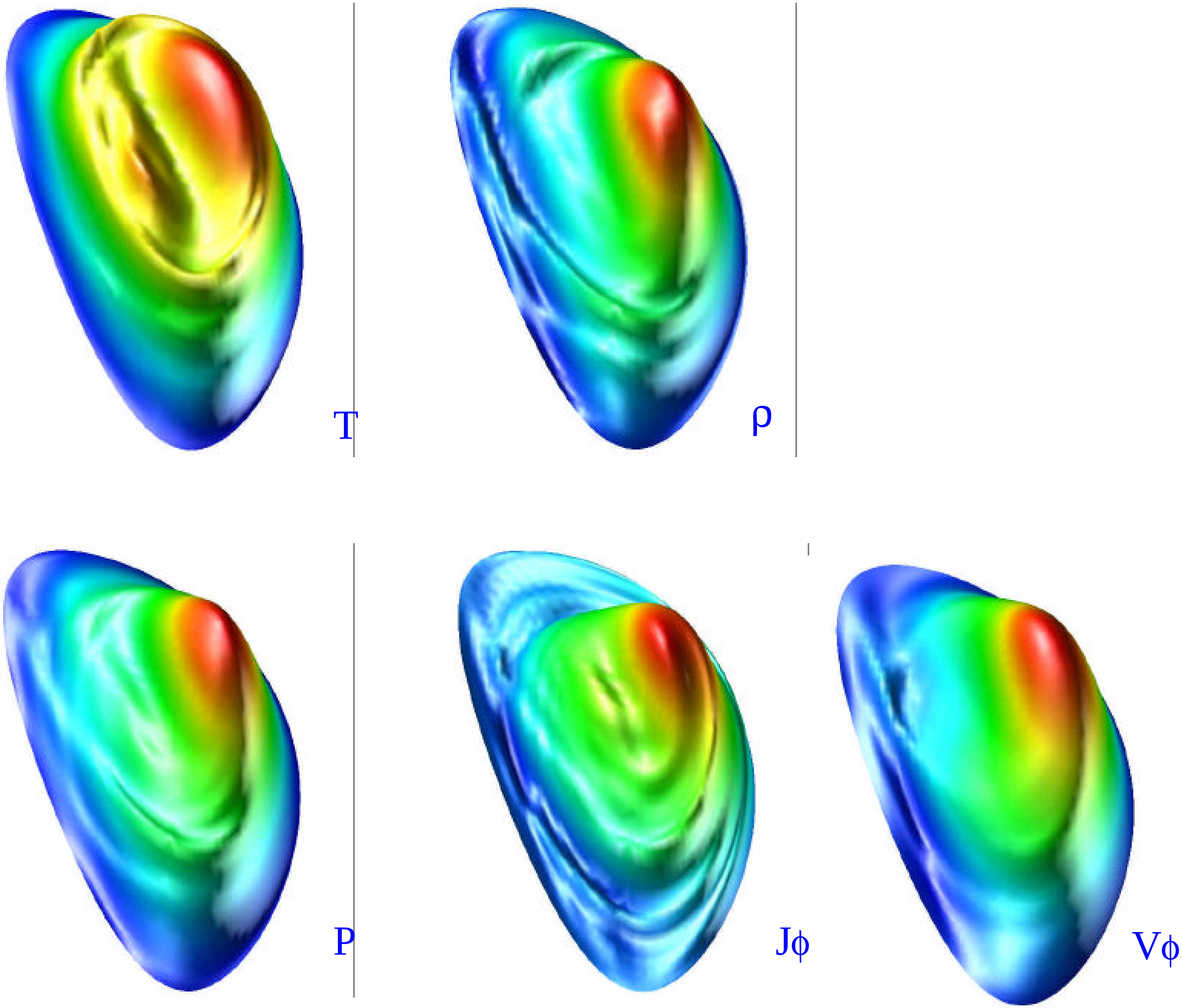
- Stochasticity as shown before.
- Localized steepening of pressure driven modes as shown here.

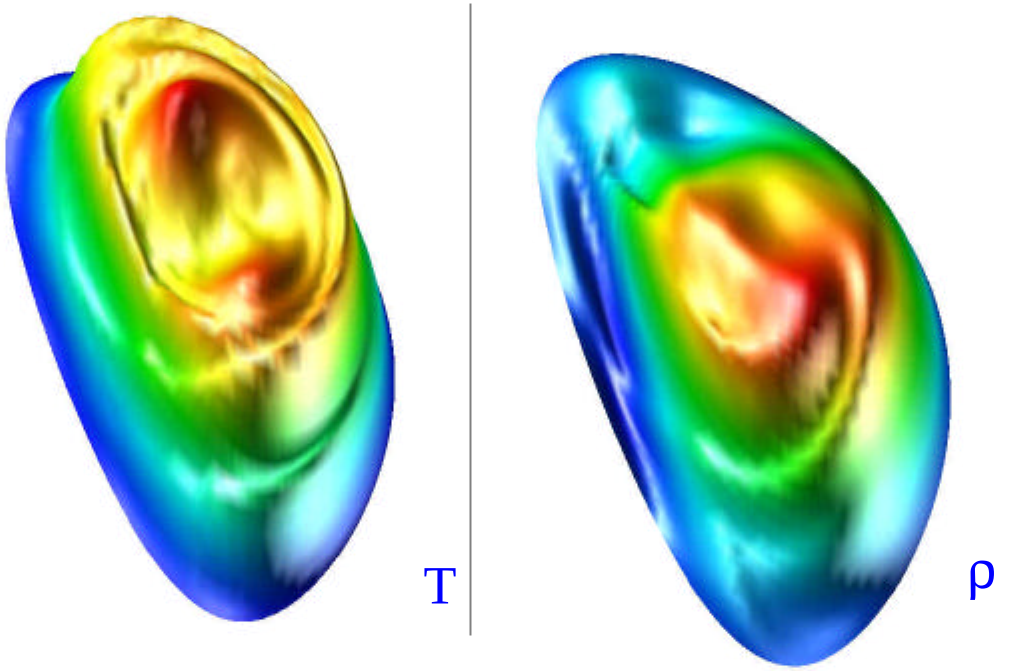




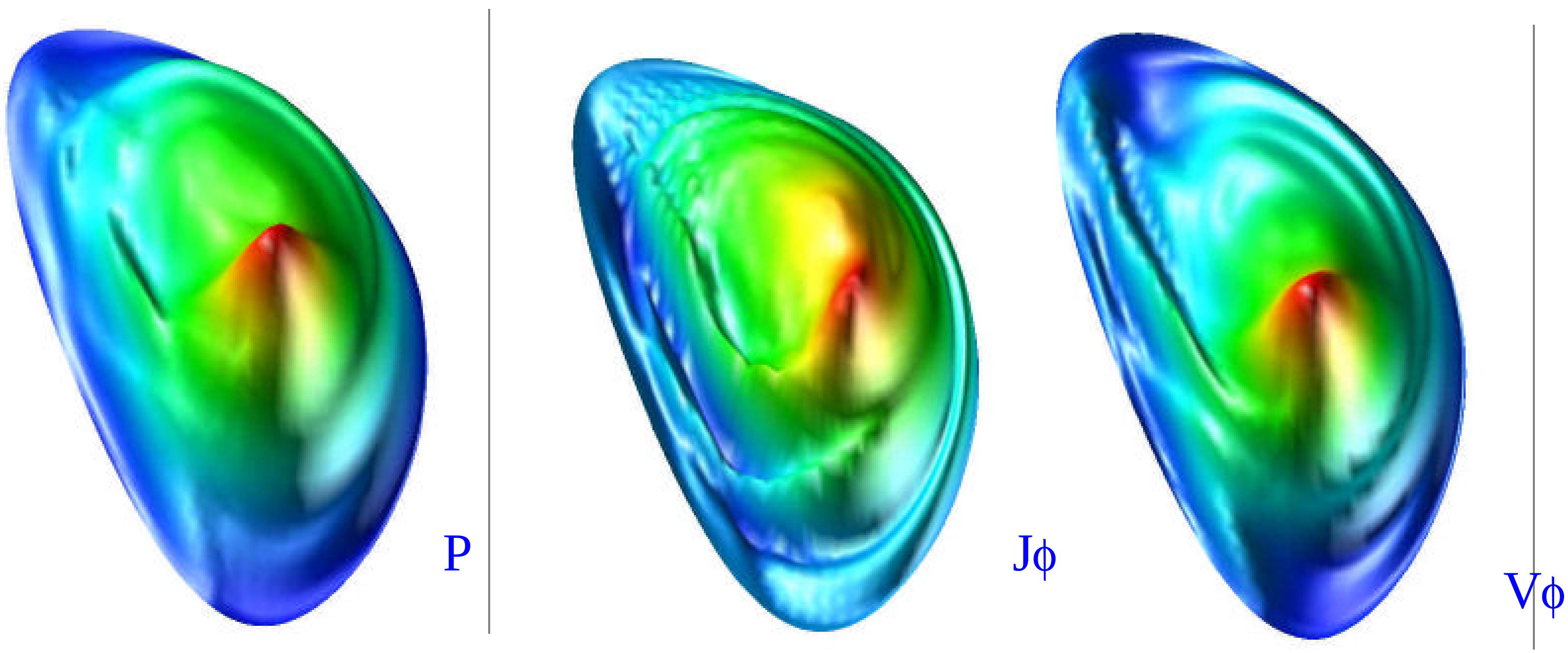


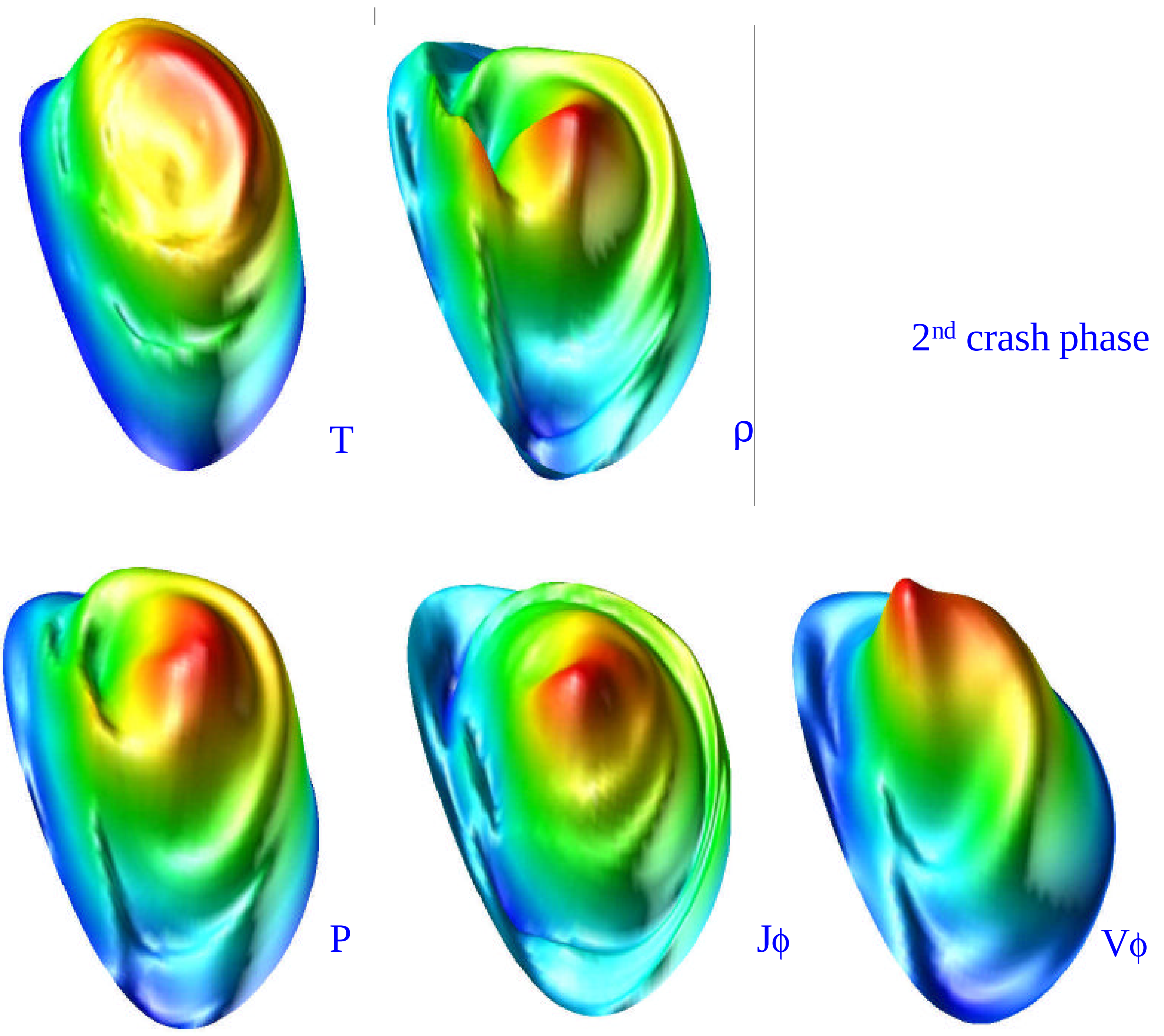

T
ρ
M_A=0.2
P
Jϕ
Vϕ

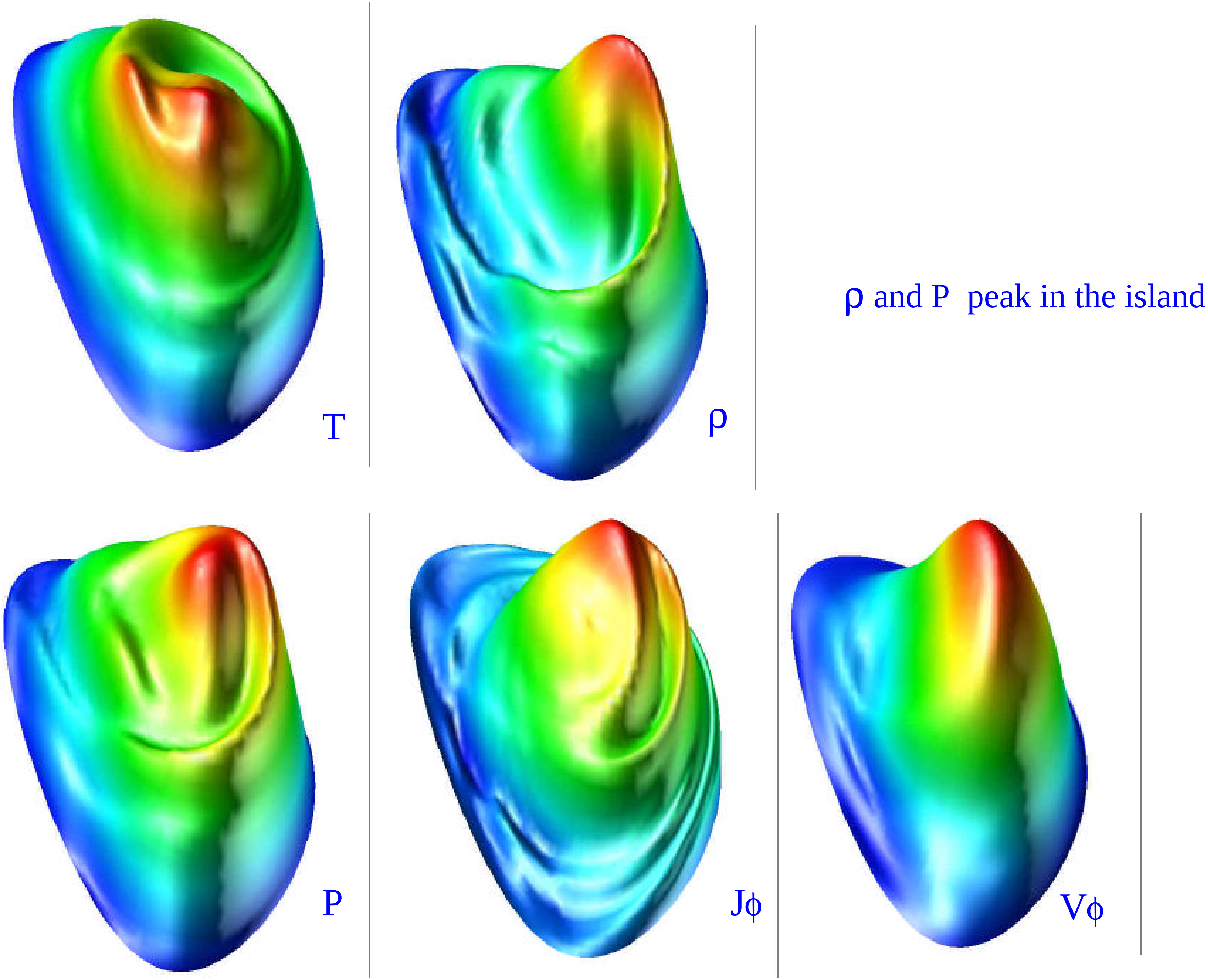

T
ρ
P
Jϕ
Vϕ

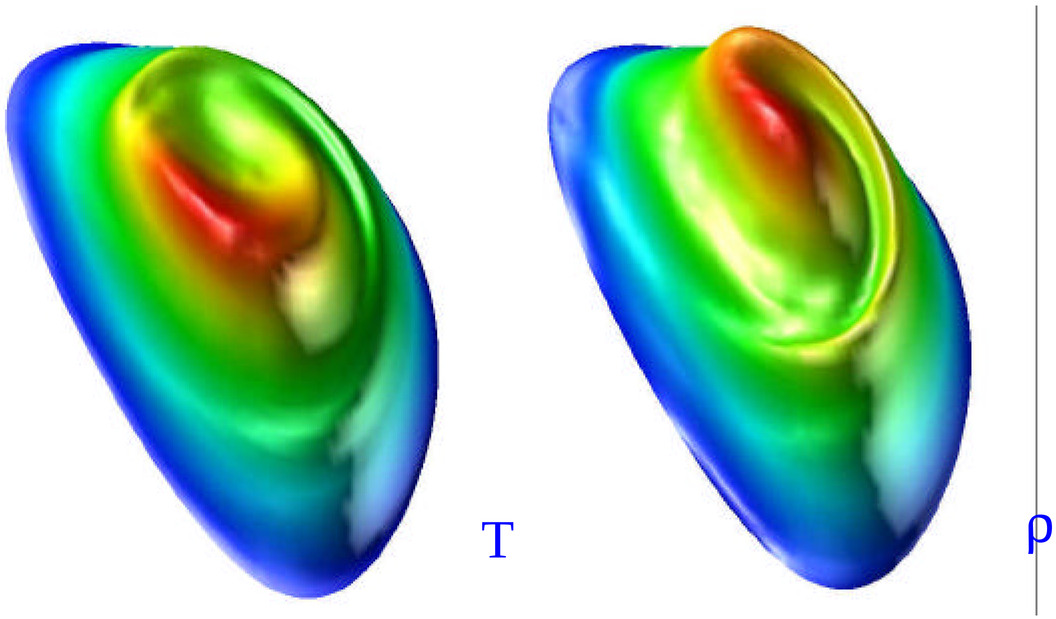


Vϕ profile evolves with reconnection

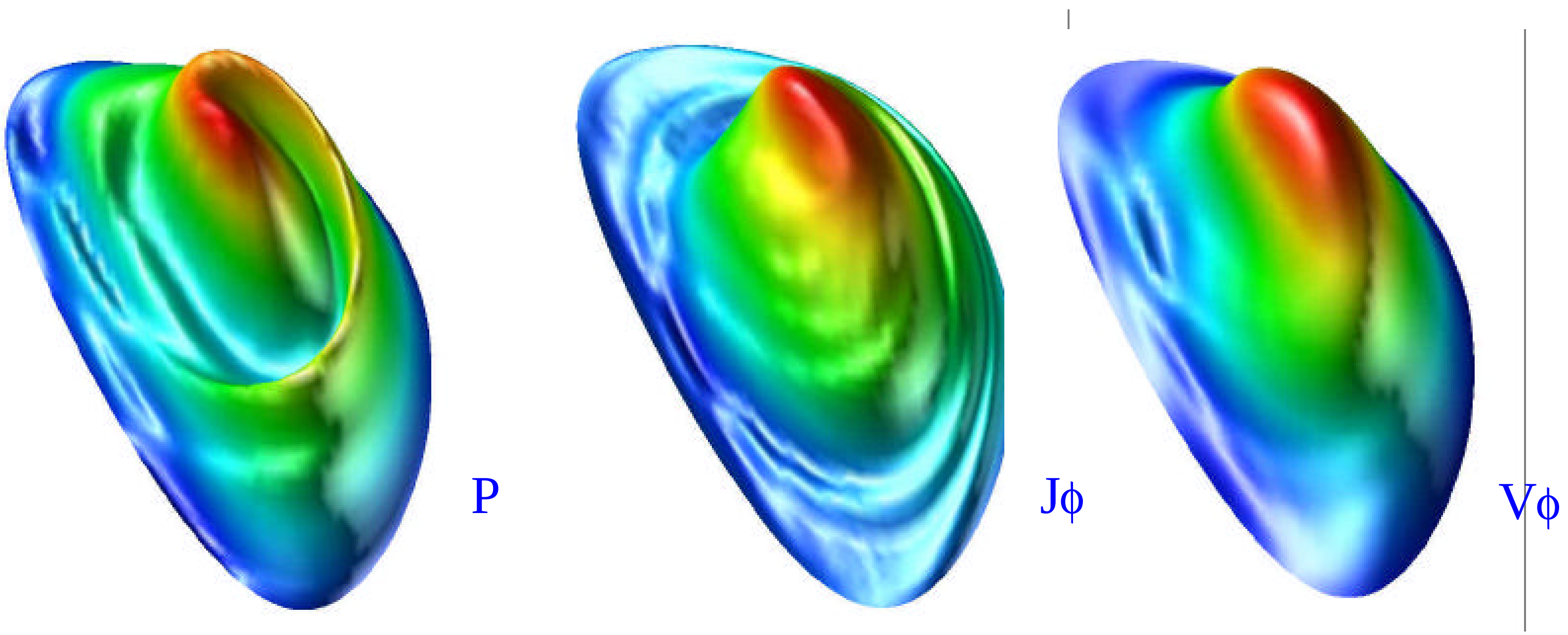


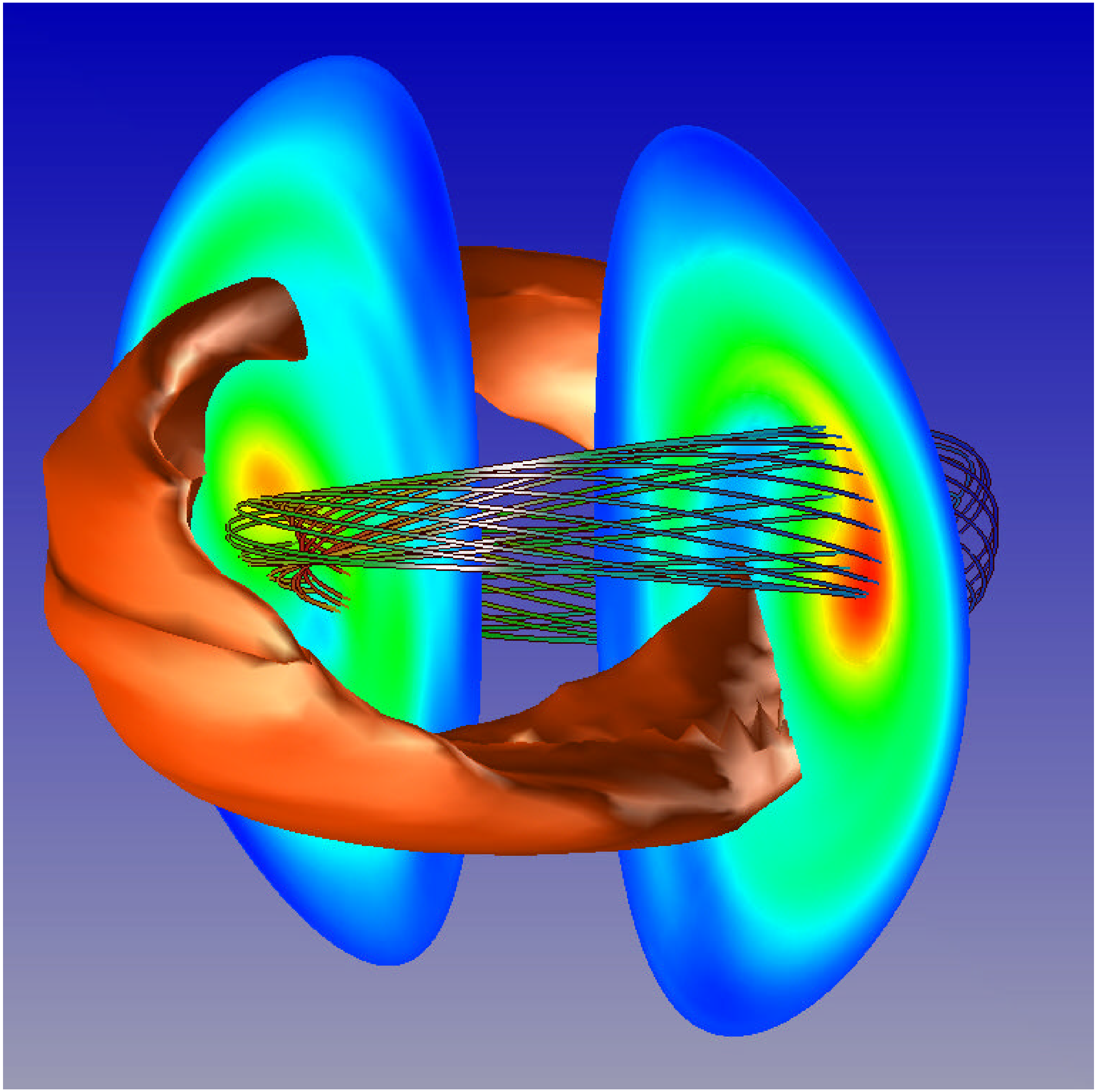

T
ρ
2nd crash phase
P
Jϕ
Vϕ

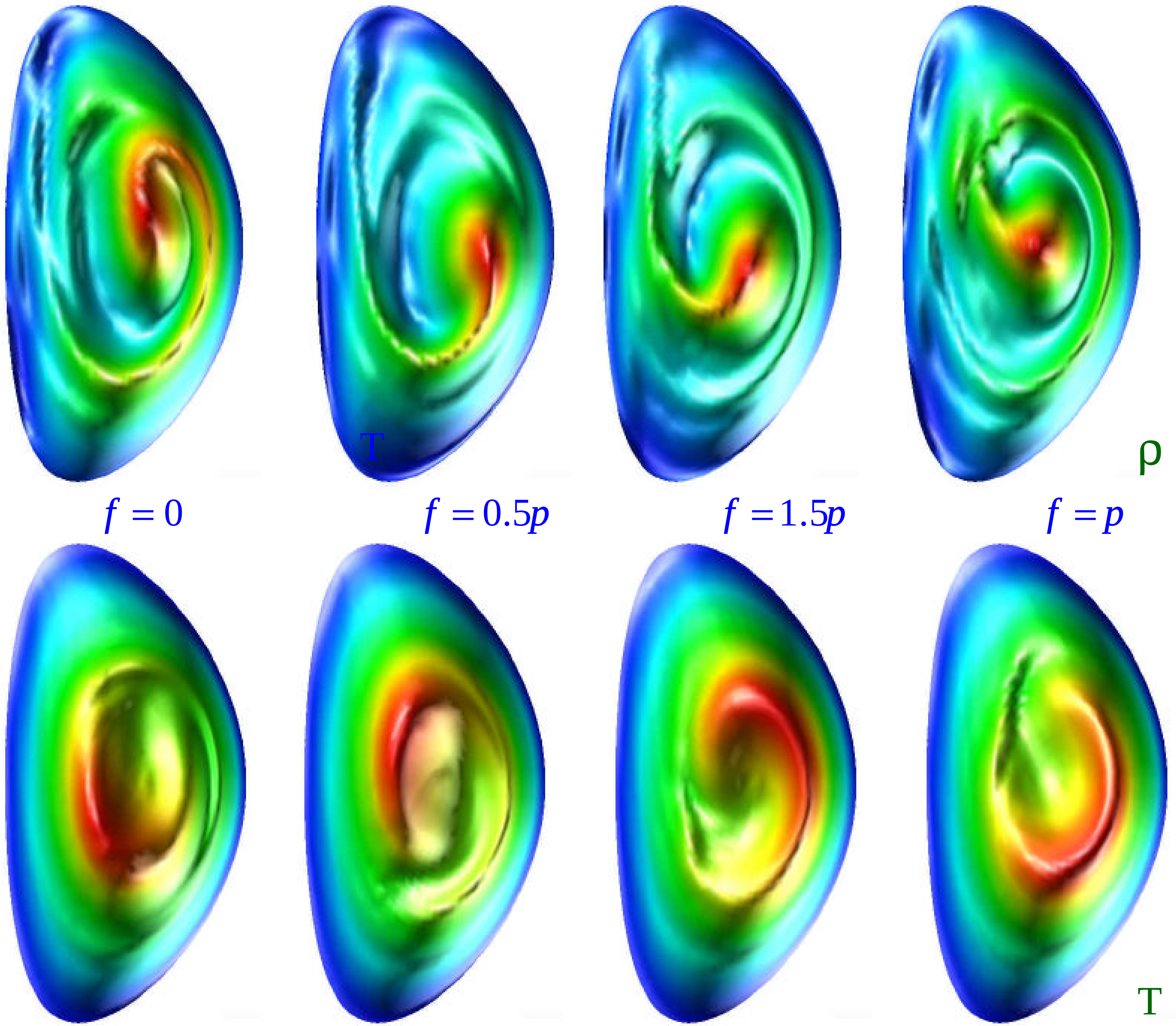

T
ρ
ρ and P peak in the island
P
Jϕ
Vϕ



Probable saturated steady state



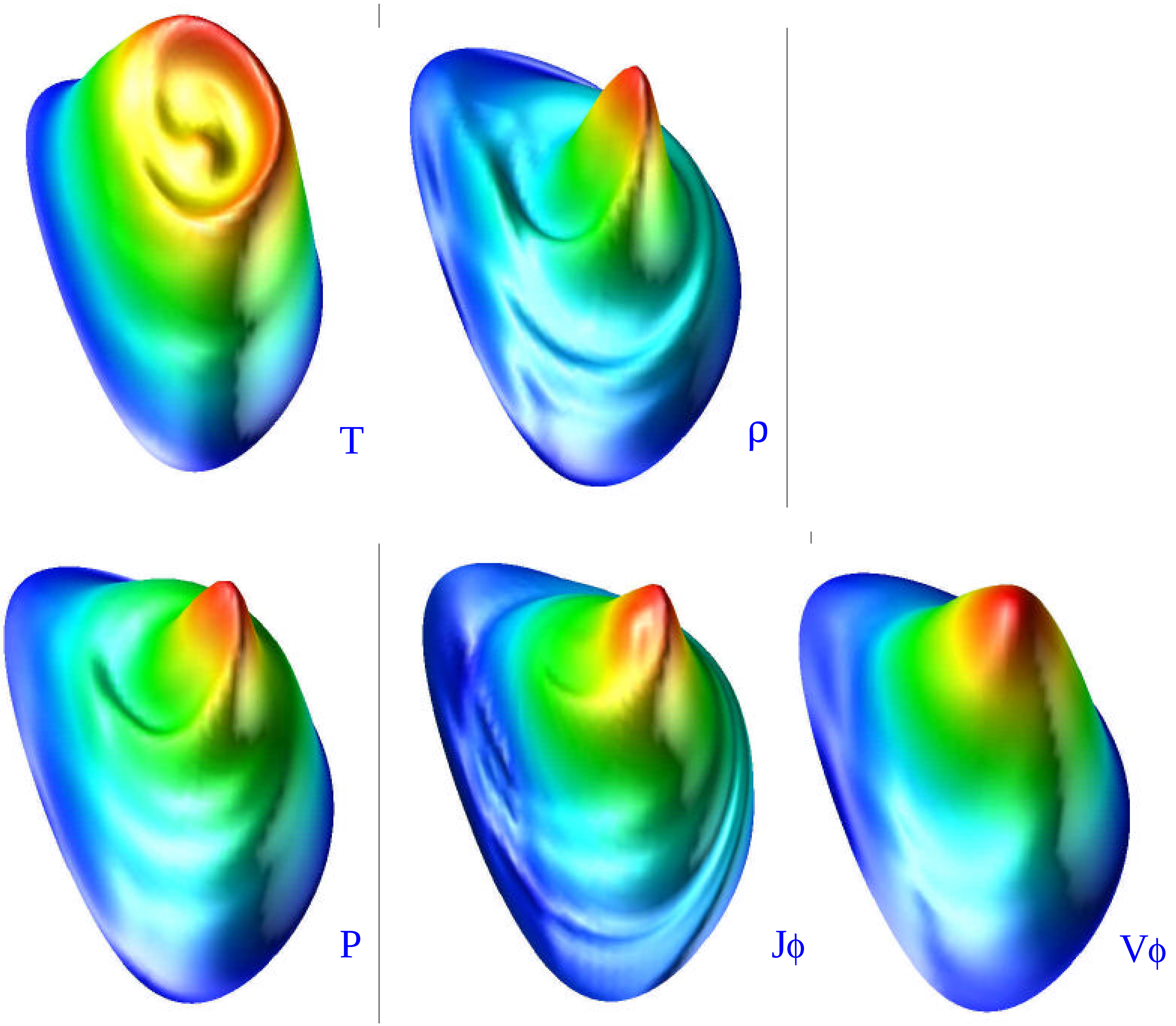




T
ρ
f = 0
f = 0.5p
f = 1.5p
f = p
T

Summary

- MHD, Two-fluids, and Hybrid levels of M3D are used for simulation studies of NSTX.
- Non-MHD effects do not cause major changes to 2D steady states with flow in the cases studied.
- The relative density shift relation holds both in the simulation and experiment, with the centrifugal force of the hot component included.
- Toroidal sheared rotation reduces linear growth of internal kink, moves the mode inboard, and give mode rotation comparable to the peak plasma rotation.
- Toroidal shear rotation can saturate the internal kink, except that the rotation profile itself evolves with reconnection. Therefore the flow source rate is an important factor.
- Another way toroidal sheared rotation can give saturation of internal kink is by causing density peak inside the island.
- IRE:Disruption can occur at least in two ways; due to stochasticity, and due to localized steepening of pressure driven modes.



T
ρ
P
Jϕ
Vϕ