Numerical Simulation of Magnetic Reconnection using AMR

Ravi Samtaney, Steve Jardin
Computational Plasma Physics Group
Princeton Plasma Physics Laboratory
Princeton University

Phillip Colella, Terry Ligocki
Applied Numerical Algorithms Group
Lawrence Berkeley National Laboratory
University of California

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Single-fluid resistive MHD Equations

- Equations in conservation form

$$\frac{\partial U}{\partial t}+\boxed{\frac{\partial F_j(U)}{\partial x_j}}=\boxed{\frac{\partial \tilde{F}_j(U)}{\partial x_j}}$$

Parabolic

Hyperbolic

$$U=\{\rho,\rho u_i,B_i,e\}^T$$

$$F_j(U)=\left\{\begin{array}{c}\rho u_j\\ \rho u_i u_j+p\delta_{ij}+\frac{1}{2}B_kB_k\delta_{ij}-B_iB_j\\ u_jB_i-B_ju_i\\ \left(e+p+\frac{1}{2}B_kB_k\right)u_j-B_iu_iB_j\end{array}\right\}$$

$$\tilde{F}_j(U)=\left\{\begin{array}{c}0\\ Re^{-1}\tau_{ij}\\ S^{-1}\eta\left(\frac{\partial B_i}{\partial x_j}+\frac{\partial B_j}{\partial x_i}\right)\\ S^{-1}\eta\left(\frac{1}{2}\frac{\partial B_iB_i}{\partial x_j}-B_i\frac{\partial B_j}{\partial x_i}\right)+Re^{-1}\tau_{ij}u_i+Pe^{-1}\kappa\frac{\partial T}{\partial x_j}\end{array}\right\}$$

Reynolds no.

Lundquist no.

Peclet no.

$$\tau_{ij}=\rho\nu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right)$$

$$e=\frac{p}{\gamma-1}+\frac{1}{2}\rho u_iu_i+\frac{1}{2}B_iB_i$$

Vector Potential Equations

$$B_i=\epsilon_{ijk}\nabla_jA_k$$

$$A_i=\epsilon_{ijk}\delta_{j3}\nabla_k\Omega+\psi\delta_{i3}$$

$$B_i=\epsilon_{ijk}\delta_{j3}\nabla_k\psi+B_i\delta_{i3}$$

$$B_3=\nabla_i\nabla_i\Omega$$

$$\frac{\partial\psi}{\partial t}+u_i\frac{\partial\psi}{\partial x_i}=S^{-1}\eta\frac{\partial^2\psi}{\partial x_i^2}$$

$$B_1=\frac{\partial\psi}{\partial x_2}\qquad B_2=-\frac{\partial\psi}{\partial x_1}$$

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Numerical Method

- Combination of generalized upwinding (8-wave formulation by Powell et al. JCP vol 154, 284-309, 1999) and vector potential
- Hyperbolic flux at cell interfaces given by

$$F(U_L, U_R) = \frac{1}{2}\left(F(U_L) + F(U_R)\right) + \frac{1}{2}\sum_{k=1}^{8} L_k(U_R - U_L)|\lambda_k|R_k$$

where $L_k \frac{\partial F}{\partial U} = \lambda_k L_k$ and $\frac{\partial F}{\partial U} R_k = \lambda_k R_k$

The eigenvalues are

$$\lambda = \{u, u, u + c_a, u - c_a, u + c_f, u - c_f, u + c_s, u - c_s\}$$

  - *The fluid velocity advects both the entropy and div(**B**) in the 8-wave formulation*

- The left and right states at a cell interface are obtained by fitting linear profiles and performing slope-limiting to the variables projected on to the local characteristic space

Numerical Method

- Vector potential ψ evolved using central differences
- At end of each stage in time integration replace x and y components of **B** using ψ
  - *Central difference approximation of div(**B**) is zero*
  - *Non-conservative source in 8-wave formulation is not required*
- Correct total energy using newer values of **B**
  - *Total energy conservation is not maintained*
  - *Tests indicate that loss of conservation is small*

$$\left| \frac{\int e(t)dV}{\int e(0)dV} - 1 \right| < 0.02$$

Adaptive Mesh Refinement with Chombo

- ***Chombo*** is a collection of C++ libraries for implementing block-structured adaptive mesh refinement (AMR) finite difference calculations (http://www.seesar.lbl.gov/ANAG/chombo)
- Mixed language model
  - *C++ for higher-level data structures*
  - *FORTRAN for regular single grid calculations*
- Reusable components. Component design based on mathematical abstractions to classes
- Based on public-domain standards
  - *MPI, HDF5*
- Chombovis: visualization package based on VTK, HDF5
- AMR Parameters for magnetic reconnection in 2D
  - *4-5 AMR levels with refinement ratio of 2*
  - *clustering efficiency of 0.85*
  - *cluster buffer width of 3, remeshing every two time steps*
  - *refinement criterion: Current density J > 20/(L+1), where L=AMR level*

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Initial and Boundary Conditions

- Initial conditions on domain [-1:1]x[0:1]

$$
\begin{aligned}
\rho(x,y,0) &= 1 \\
u_i(x,y,0) &= 0 \\
p(x,y,0) &= 0.2 \\
\psi(x,y,0) &= -\cos k_x x \sin k_y y \\
B_z(x,y,0) &= -(k_x^2 + k_y^2)^{\frac{1}{2}} \cos k_x x \sin k_y y
\end{aligned}
$$

$$k_x = \frac{3\pi}{2}, \quad k_y = 2\pi$$

- Boundary conditions

$$\vec{u} \cdot \hat{n} = 0 \quad \vec{B} \cdot \hat{n} = 0$$

$$\nabla(\psi) \cdot \hat{n} = 0 \quad \vec{E} \cdot \hat{t} = 0$$

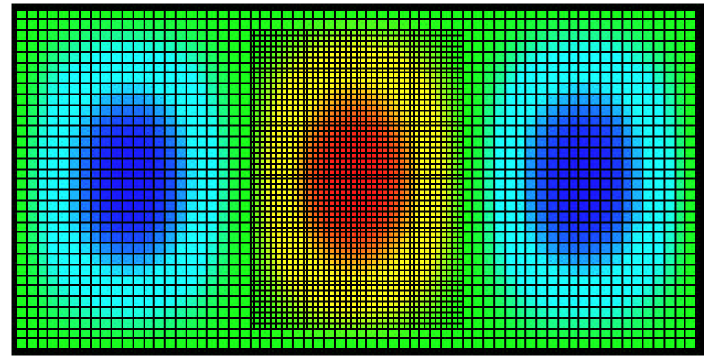
$$\nabla(T) \cdot \hat{n} = 0$$

- Other parameters: Re=10^3, Pe= 10^3
  Dimensionless conductivity and viscosity set to unity
- Resitivity to annihilate middle island

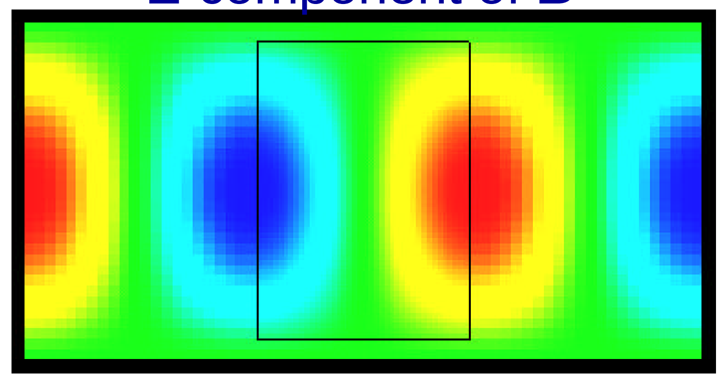
$$\eta = \eta^- + (\eta^+ - \eta^-)\left[1 - \exp(-177.69\psi^2)\right] \times max(0,\ -sign(\psi))$$

$$\eta^- = 1$$

$$\eta^+ = 0.1/S$$



Z-component of **B**



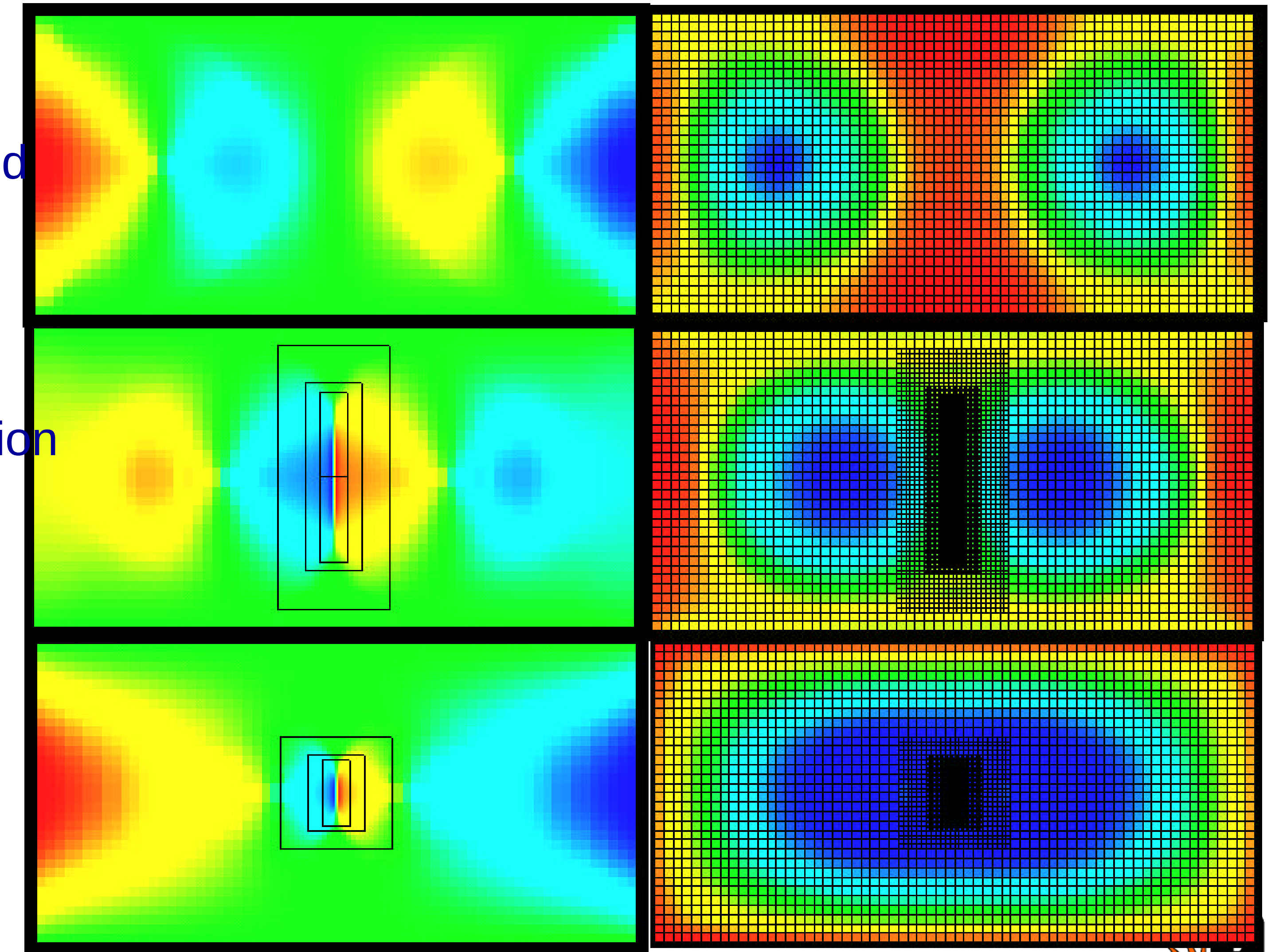
Y-component of **B**

Results for $S=10^4$

Stage 1
Middle island
decays

Stage 2
Reconnection

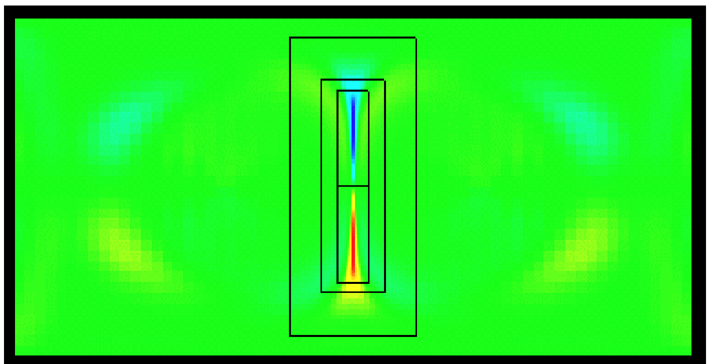
Stage 3
Decay

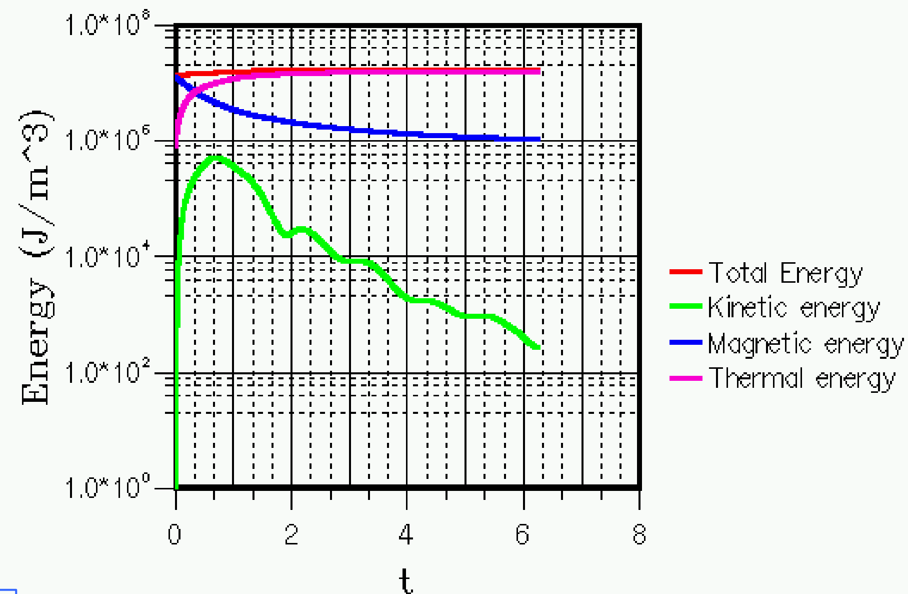


Y-component of **B**

Z-component of **B**

Results for $S= 10^4$ (cont'd)





Y-momentum at t=1.86 shows plasma squeezed out with large equal and opposite velocities in a narrow region above and below the X-point of reconnection

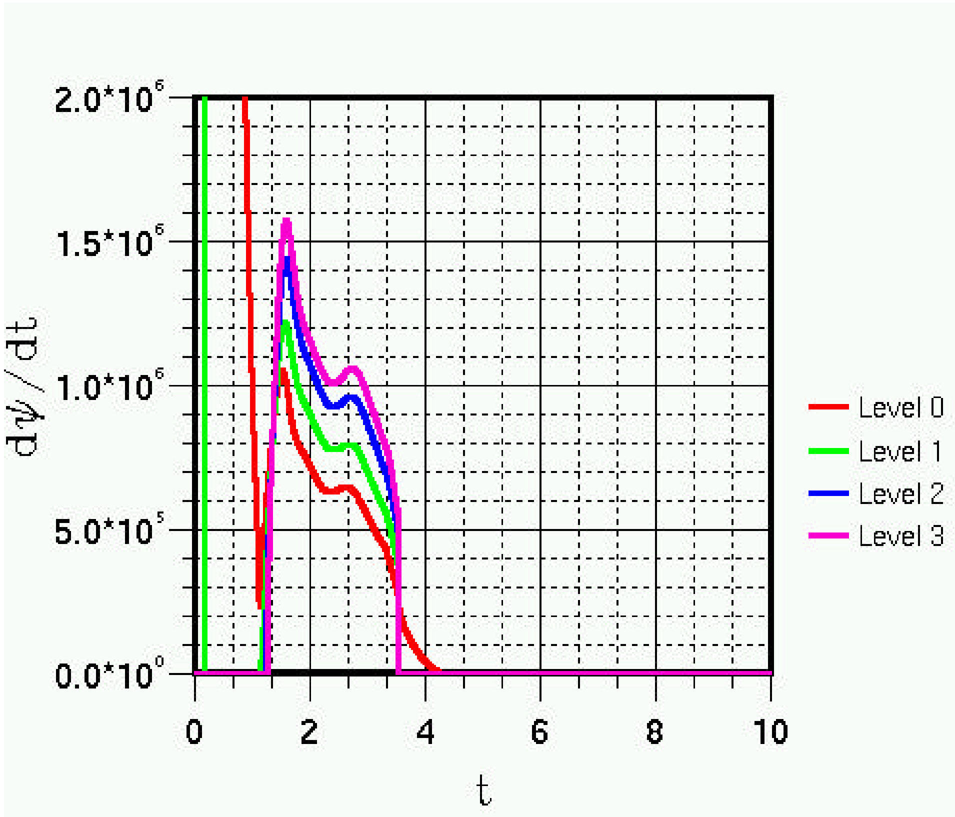
Boxes indicate meshes at various refinement levels

Energy budget for $S=10^4$

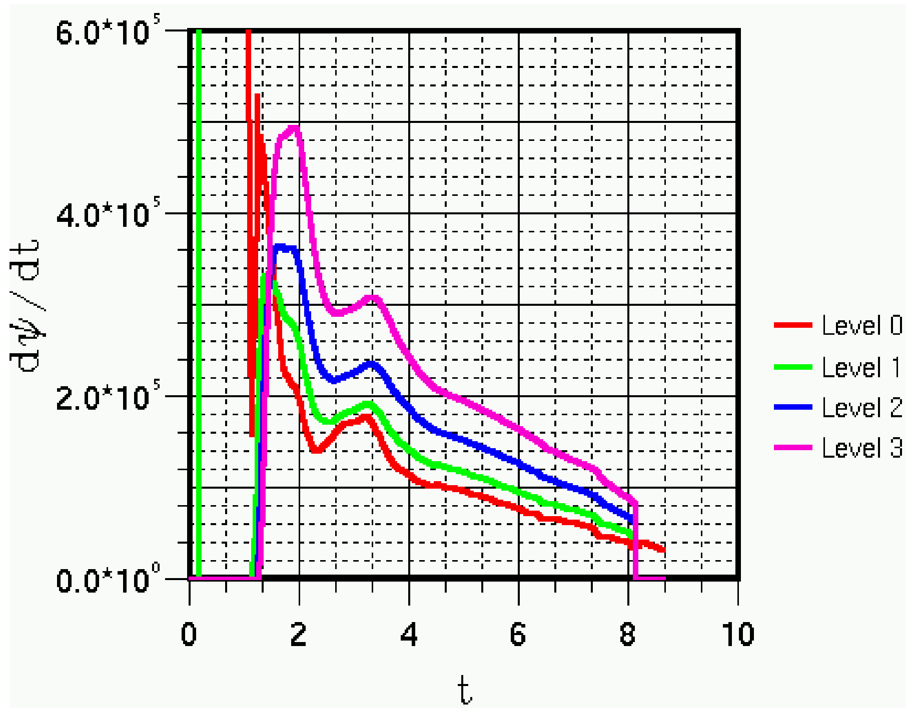
•Energy exchange between magnetic and thermal energy during transient phase when the middle island is annihilated.

•Kinetic energy, though small, indicates "bouncing" during reconnection

Results: $\dot{\psi}$ at X-point of reconnection



S= 10³ (well-resolved)



S= 10⁴ (marginally resolved)

Level 0 is the coarsest mesh while Level 3 is the finest mesh

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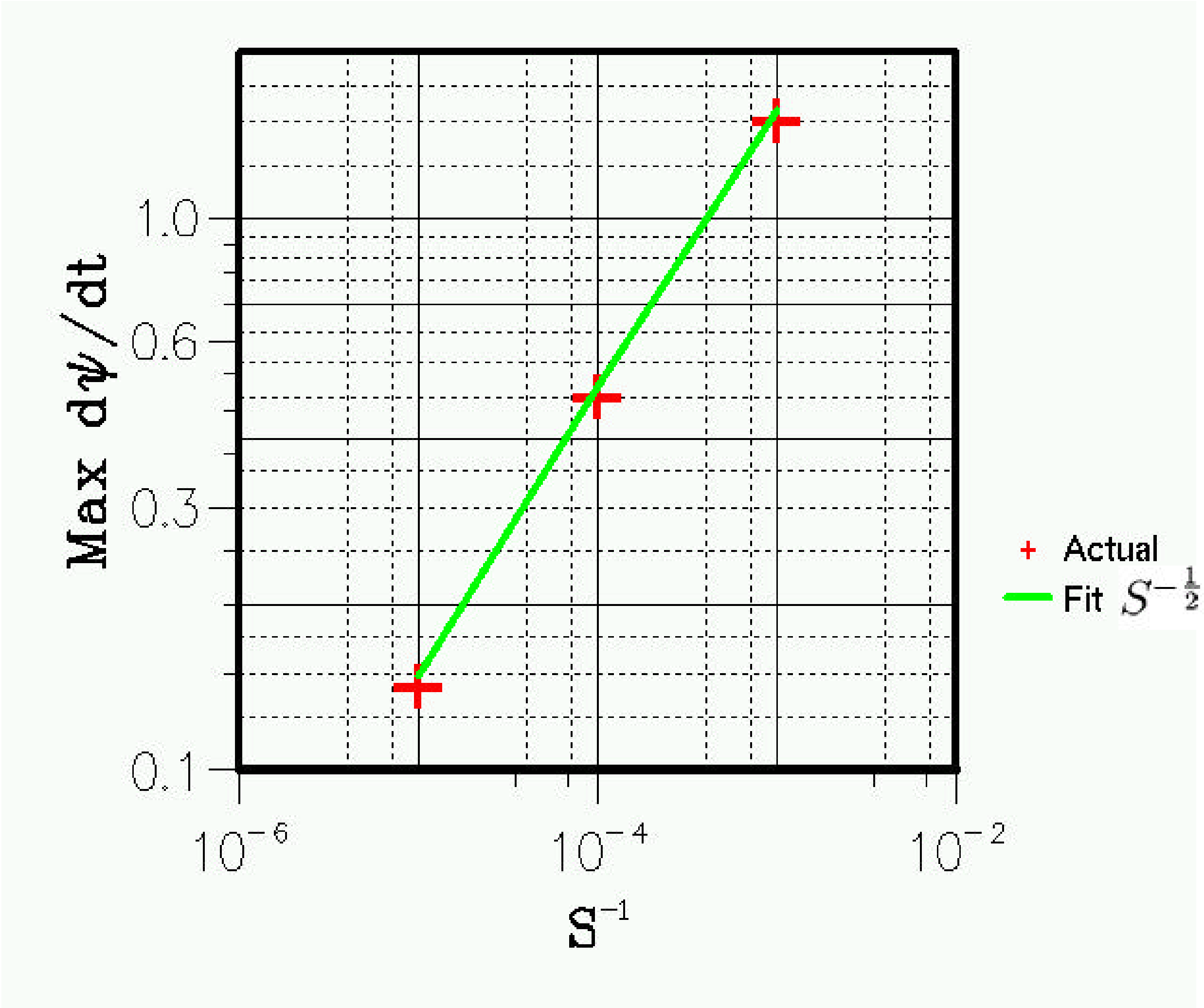
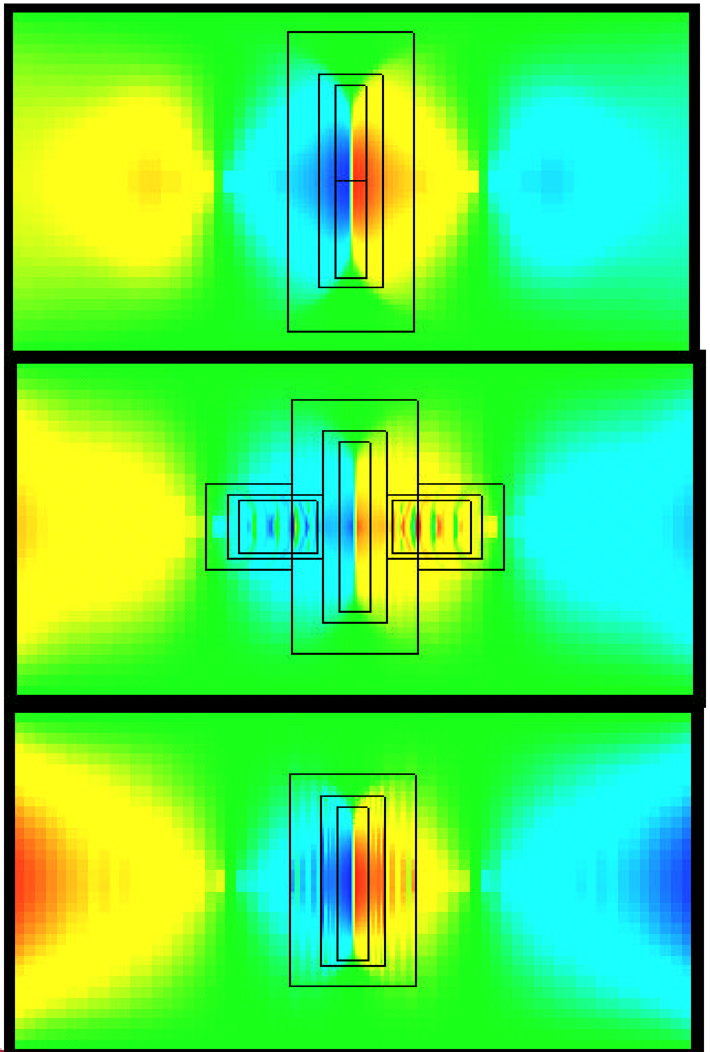
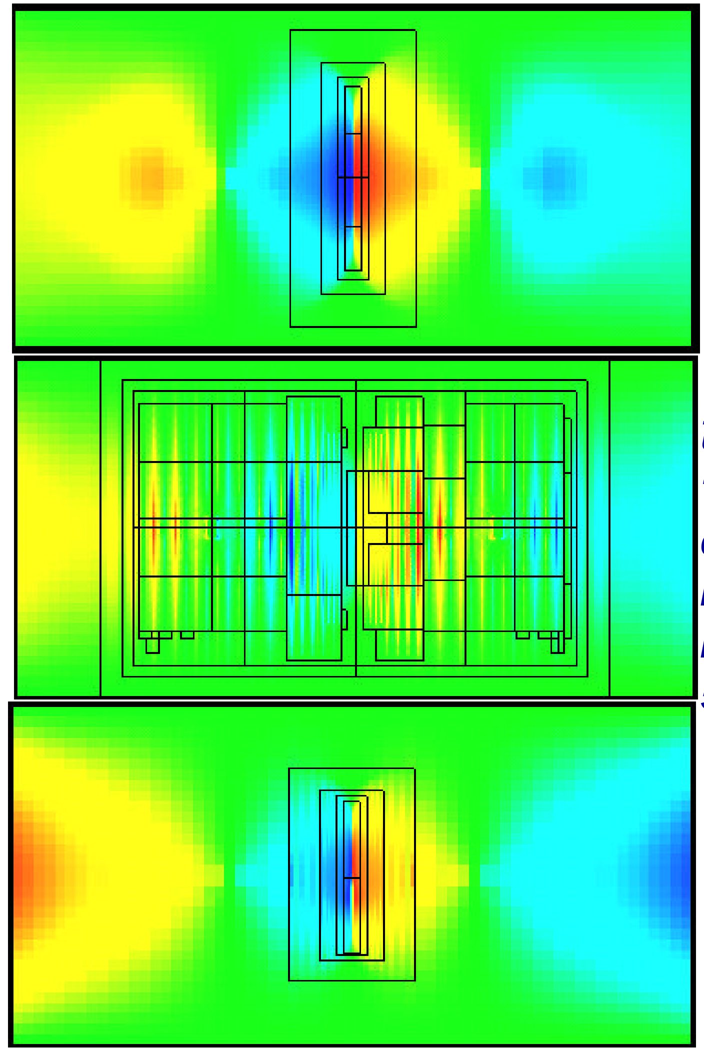
Results:Max $\dot{\psi}$ scaling with S



Results for S= 10^5



Y-component of **B** with 4 AMR Levels



t=1.59

t=3.07

"Intermittent" event with nearly ubiquitous refinement in the 5 level simulation

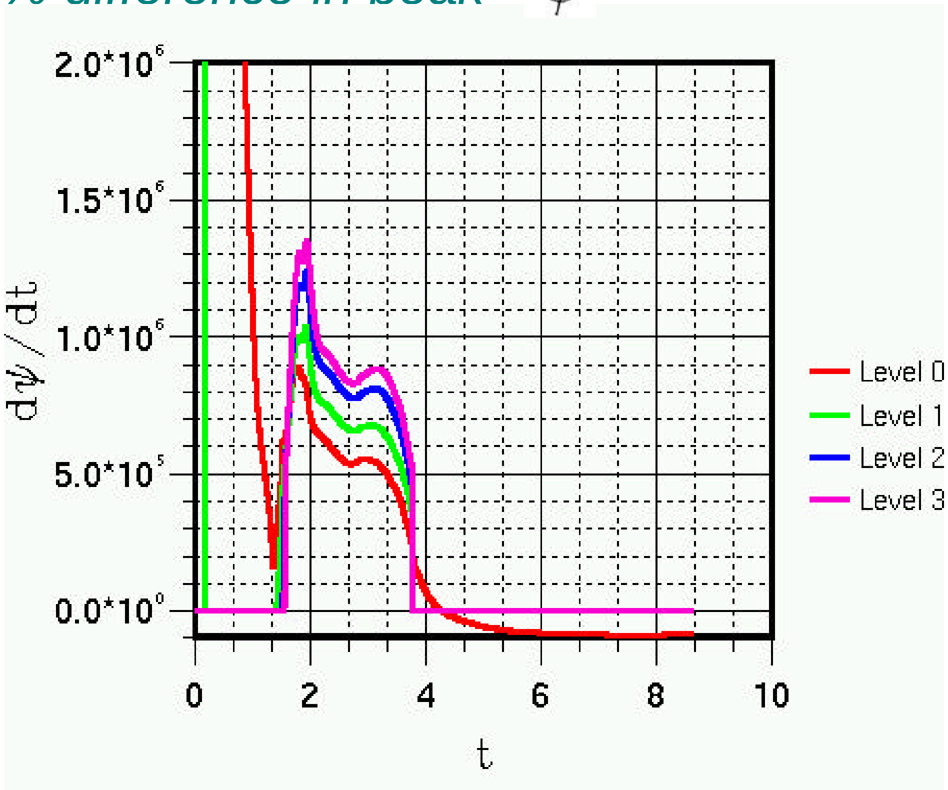
t=8.49

Note: Simulation may be under-resolved

Y-component of **B** with 5 AMR Levels

Alternative formulations -Entropy

- Using entropy instead of total energy
  - *parabolic part cannot be expressed in conservation form*
- Results for S= 10^3 comparable to total energy formulation
  - *14 % difference in peak* $\dot{\psi}$

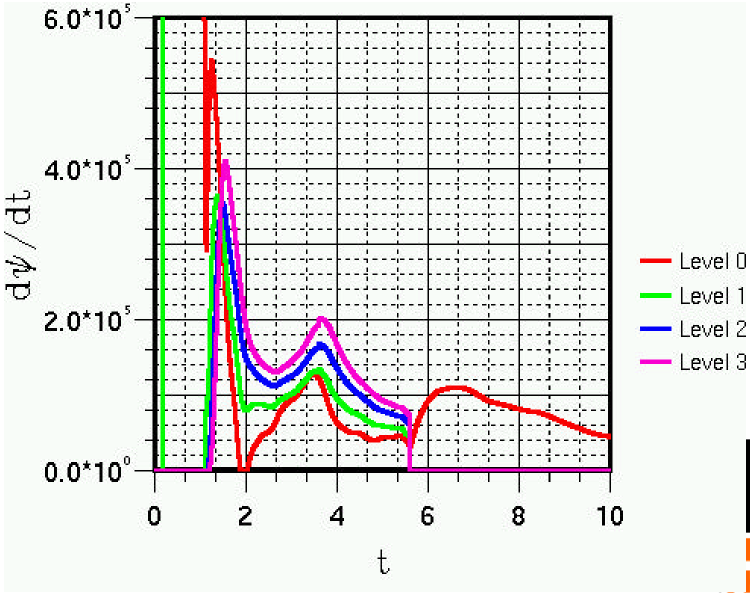


Alternative formulations- diffusing div(**B**)

- Use the 8-wave formulation modified for stability
  - *Vector potential is not used*
  - *Requires the non-conservative source term* $\frac{\partial B_k}{\partial x_k}\{0, B_i, u_i, B_j u_j\}^T$
- Central difference evaluation of $\nabla \cdot \vec{B} = 0$ should be $O(h^2)$
- At the end of each time step change **B** using

$$\vec{B} = \vec{B} + \lambda \nabla (\nabla \cdot \vec{B})$$

  - *This is equivalent to diffusing* $\nabla \cdot \vec{B}$
  - *The diffusion coefficient is* $\lambda = O(h^2)$
- This method is stable for the reconnection problem
- Results shown for S=10^4 show significant differences compared to the upwinding + vector potential formulation



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Conclusion and Future Work

- This preliminary study indicates that AMR is a viable approach to *efficiently* resolve the near-singular current sheet in high Lundquist magnetic reconnection

S	Levels	Speedup
10^3	3	8
10^3	4	31
10^4	3	6
10^4	4	18
10^5	4	9
10^5	5	15

Speedup is defined as ratio of total simulation time taken by a unimesh calculation at the finest resolution to the total AMR simulation time

Note: this is based on wall-clock time

- A numerical method was developed which combines 8-wave upwinding formulation with a vector potential to preserve the solenoidal property of the magnetic field
- Future work
  - *unsplit corner transport upwinding for better phase-error properties*
  - *implicit treatment of resistive and viscous terms*
  - *two-fluid MHD with Hall effect*
  - *Implicit treatment of fast wave*
  - *Projection to ensure div(**B**)=0*
  - *3D magnetic reconnection*