Neoclassical Tearing Modes Simulations with Integral Heat Flow Closures [1]

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April 2003

Sherwood Fusion Theory Conference
Corpus Christi, TX

[1]Research supported by U. S. DoE under grant no. DE-FG03-01ER54618

Abstract

The rapid thermal motion of electrons and ions along magnetic field lines represents a dominant process in astrophysical and terrestrial plasmas. In this work, we describe the incorporation of this dominant physics in toroidal plasma fluid simulations of neoclassical tearing modes (NTMs). Pressure flattening across magnetic islands is responsible for the bootstrap current perturbation that drives NTMs. This pressure evolution is dominated by the large anisotropy in the parallel and perpendicular heat transport. The higher-order-element formulation of the NIMROD code [2] is able to account for anisotropies greater than $\kappa_{\|}/\kappa_{\perp} \sim 10^{10}$ in the form of a diffusive operator which relates the parallel heat flows to local thermodynamic gradients. Using this formulation, we present numerical convergence studies of a NIMROD NTM simulation of DIII-D discharge 86144. In this discharge a 3/2 NTM is presumably excited above the threshold island width via coupling to a 1/1 mode. Once above the threshold, the NTM in the experiment grows and saturates at an island width of about 1/10 the minor radius. Comparative simulations are also performed using a quantitative, integral form for the parallel heat flow closure that incorporates the kinetic effects of free-streaming, pitch-angle scattering and particle trapping [3]. This closure has recently been implemented in NIMROD's ion and electron temperature equations. In plasmas where different components of the magnetic spectrum overlap radially, temperature variations along magnetic field lines possess multiple scale lengths. In such instances, the integral closure predicts comparable heat flows from all scale lengths for nearly collisionless plasmas of interest. In contrast, because the diffusive form for the closure relies on the local parallel gradient, it underestimates temperature flattening over longer scale lengths. This result is borne out in converged simulations using the diffusive closure where pressure flattening is only able to drive the 3/2 island to about a $2cm$ saturated width. Results showing the enhanced NTM drive provided by the integral form for the parallel heat flow closure are presented.

[2] C. R. Sovinec, et al, UW CPTC Report 02-5 (2003).

[3] E. D. Held, J. D. Callen, C. C. Hegna and C. R. Sovinec, Phys. Plasmas **8**, 1171 (2001).

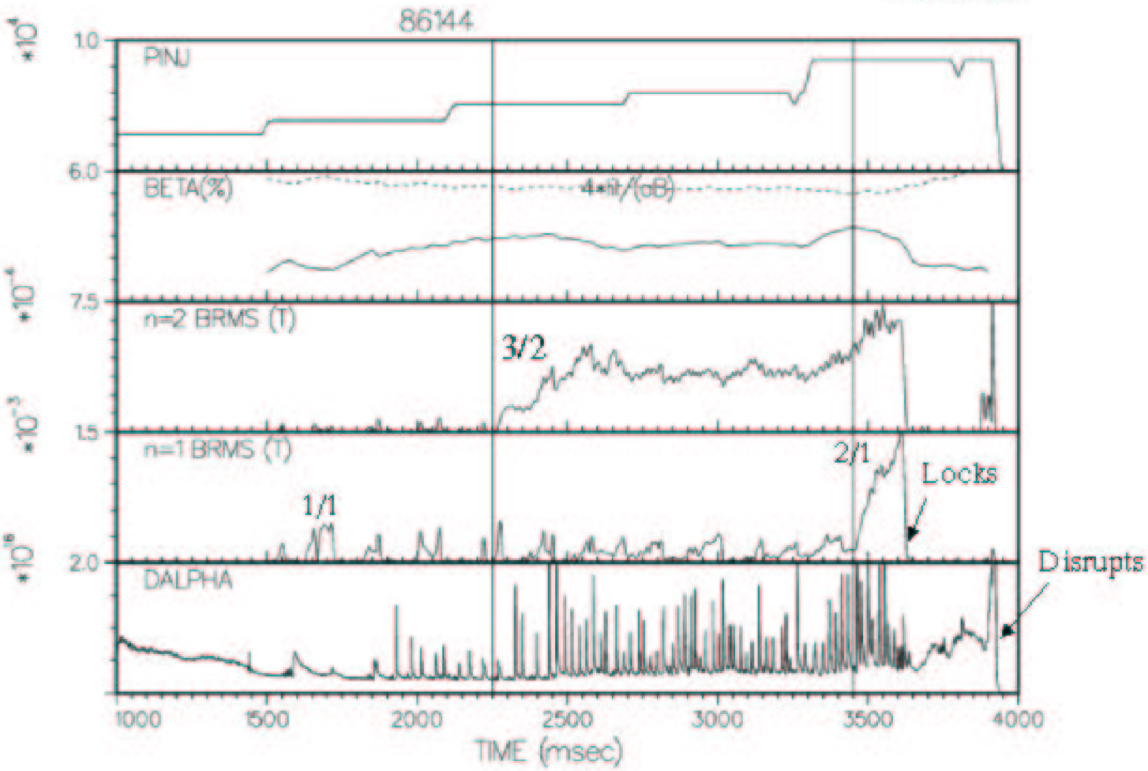
Thesis

- **Previous neoclassical tearing mode simulations of D3D Shot 86144 lack grid resolution and integral heat flow closures.**

Outline

- **Secondary Island Formation from internal kink.**
- **Nonlinear island evolution equation.**
- **Nonlinear threshold for neoclassical tearing modes.**
- **Lack of resolution for pressure dynamics.**
- **Integral versus diffusive forms for heat flow closure.**

Modern Tokamak Discharges Have Rich Magnetic Behavior



Courtesy of R. LaHaye, General A

NIMROD Simulations are based on poloidal flow damping closure form.

- **The suggested form for $\vec{\nabla} \cdot \Pi_\alpha$ is** [4]

$$\vec{\nabla} \cdot \Pi_\alpha = \rho_\alpha \mu_\alpha \left\langle B^2 \right\rangle \frac{\vec{V}_\alpha \cdot \vec{e}_\Theta}{\left(\vec{B} \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta,$$

  - μ_α **is the viscous damping frequency for each species** α,
  - **Depends on the collisionality regime.**
  - $\vec{e}_\Theta = \mathcal{J} \vec{\nabla} \zeta \times \vec{\nabla} \psi$ **and** ζ **is the axisymmetric toroidal angle,**
  - ψ **is the poloidal flux,**
  - $\mathcal{J}$ **is the Jacobian of the coordinate system.**
  - **The form can be shown to be dissipative.**
  - **Linear layer analysis yields bootstrap current, flow damping, and neoclassical enhancement of the polarization current.**

- **Presently, NIMROD simulations are using simplified diamagnetic form for electron flow damping:**

$$\vec{V}_e \approx \vec{J} \approx \frac{\vec{B}_0 \times \vec{\nabla} P}{B^2}.$$

[4] T. A. Gianakon, S. E. Kruger, and C. C. Hegna, *Phys Plasmas*, **9**, 536 (2002)

Nonlinear Rutherford island evolution equation predicts a stability boundary.

$$\frac{k_0}{\eta^*}\frac{dW}{dt} = \Delta^* + \frac{W}{W^2 + W_d^2}\left(D_{nc} + \frac{D_R}{\alpha_s - H}\right) + \ldots$$

where W is the full-width of the island.

D_{nc} is the measure of neoclassical tearing mode stability.

$D_R = E + F + H^2$ is the resistive interchange parameter.

α_S and α_R are the small and large Mercier index.

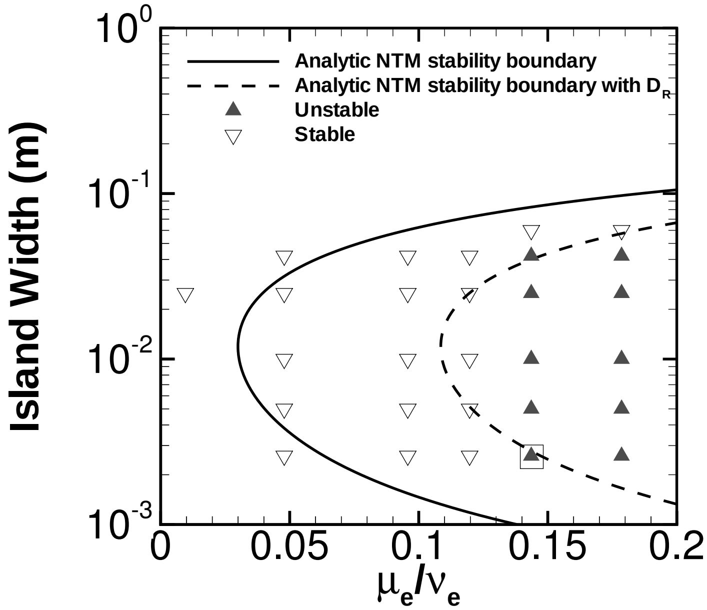
η^* is the resistive diffusion coefficient in flux space.

$\Delta^* = \Delta' |W/2|^{-2\alpha_l} \sqrt{-4D_i}$.

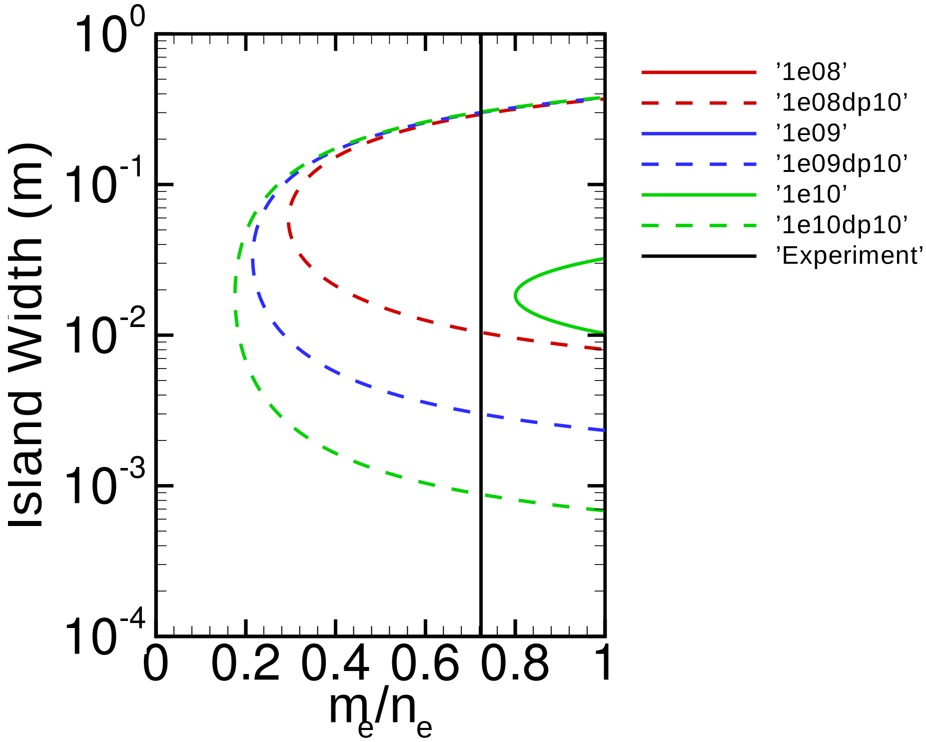
- May be additional effects such as FLR, NEPC.
- Δ' is typically stabilizing.
- D_{nc} is typically destabilizing.
- D_R is typically stabilizing and the anisotropic thermal diffusion may take a different form.

Neoclassical Tearing Mode Stability Boundary agrees with analytics.

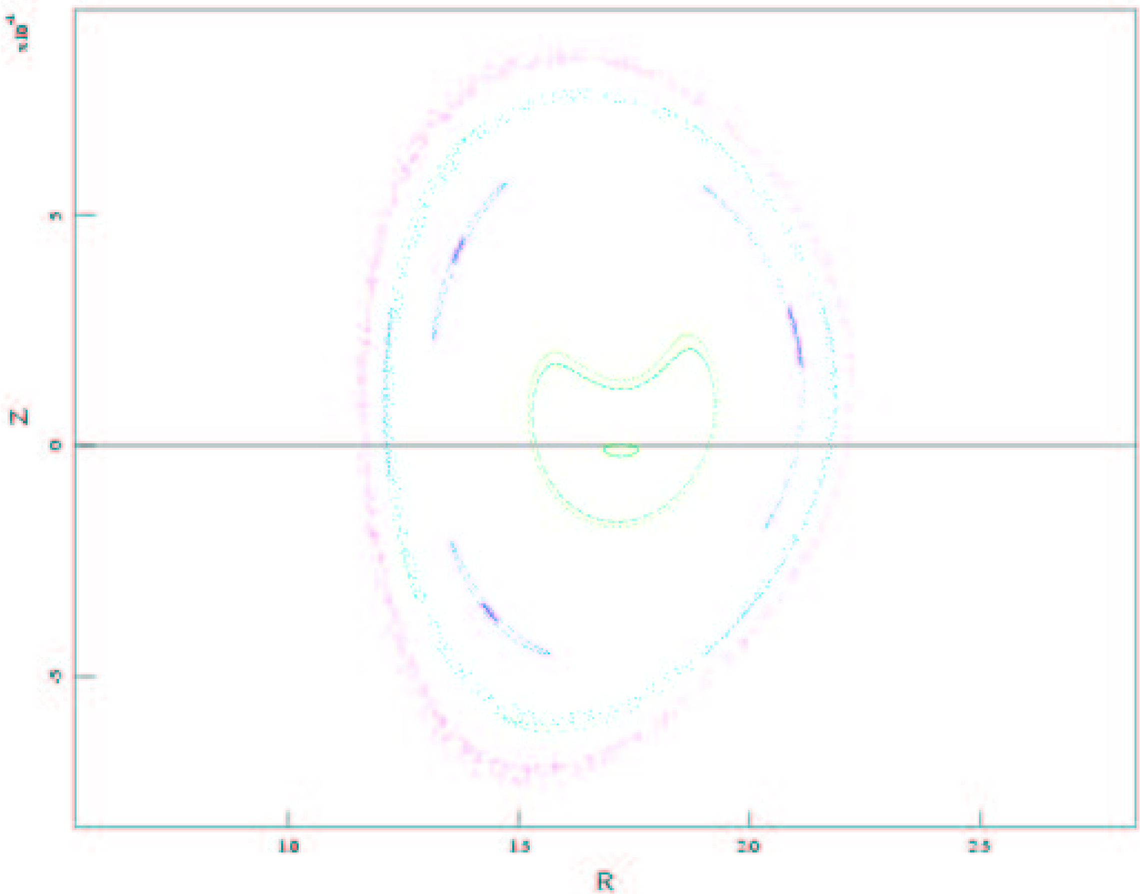
- **Here, μ_e/ν_e parameterizes the bootstrap current, $D_{nc} \propto \mu_e/\nu_e/(1+\mu_e/\nu_e)$.**
- **Stability boundary requires inclusion of D_R.**
- **Discrepancy exists at samll μ_e/ν_e.**



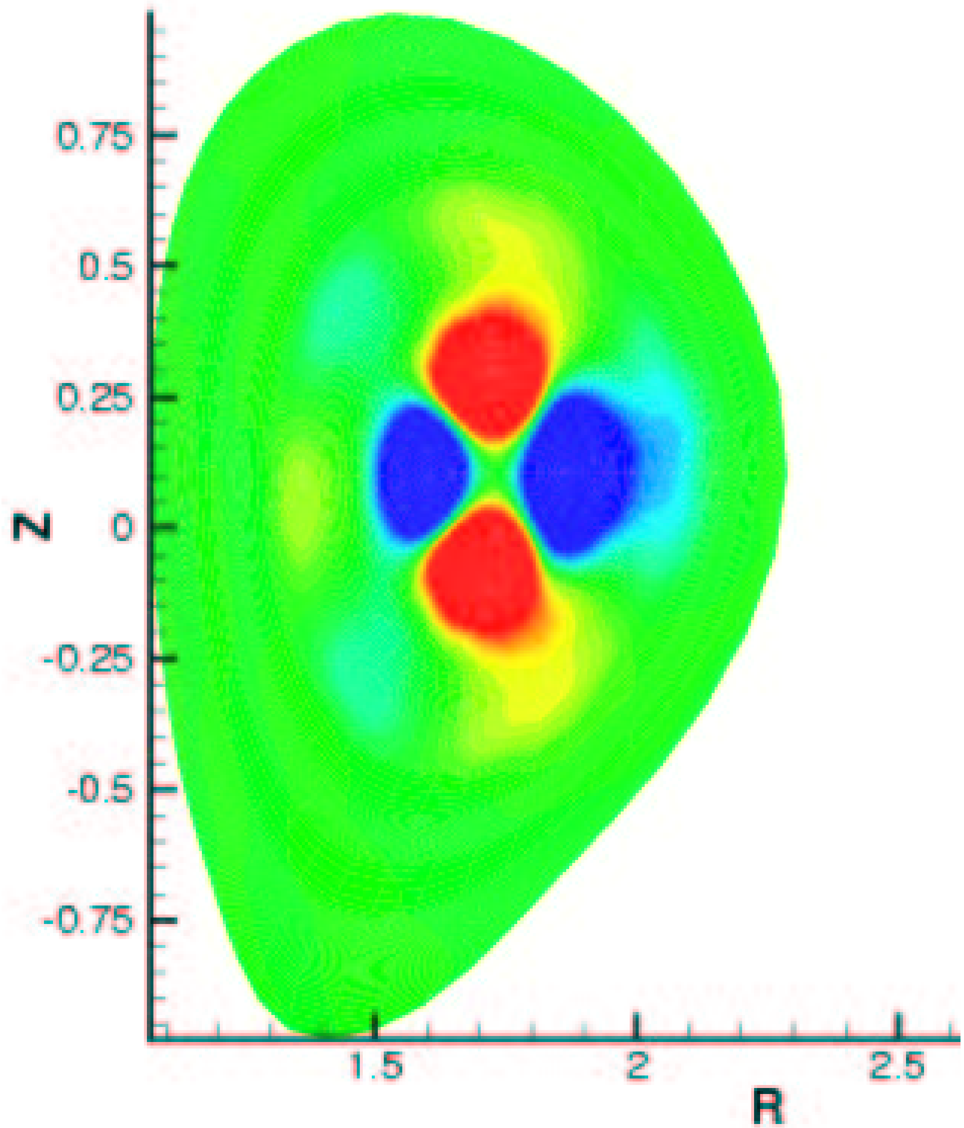
The 3/2 NTM tearing mode should be unstable.



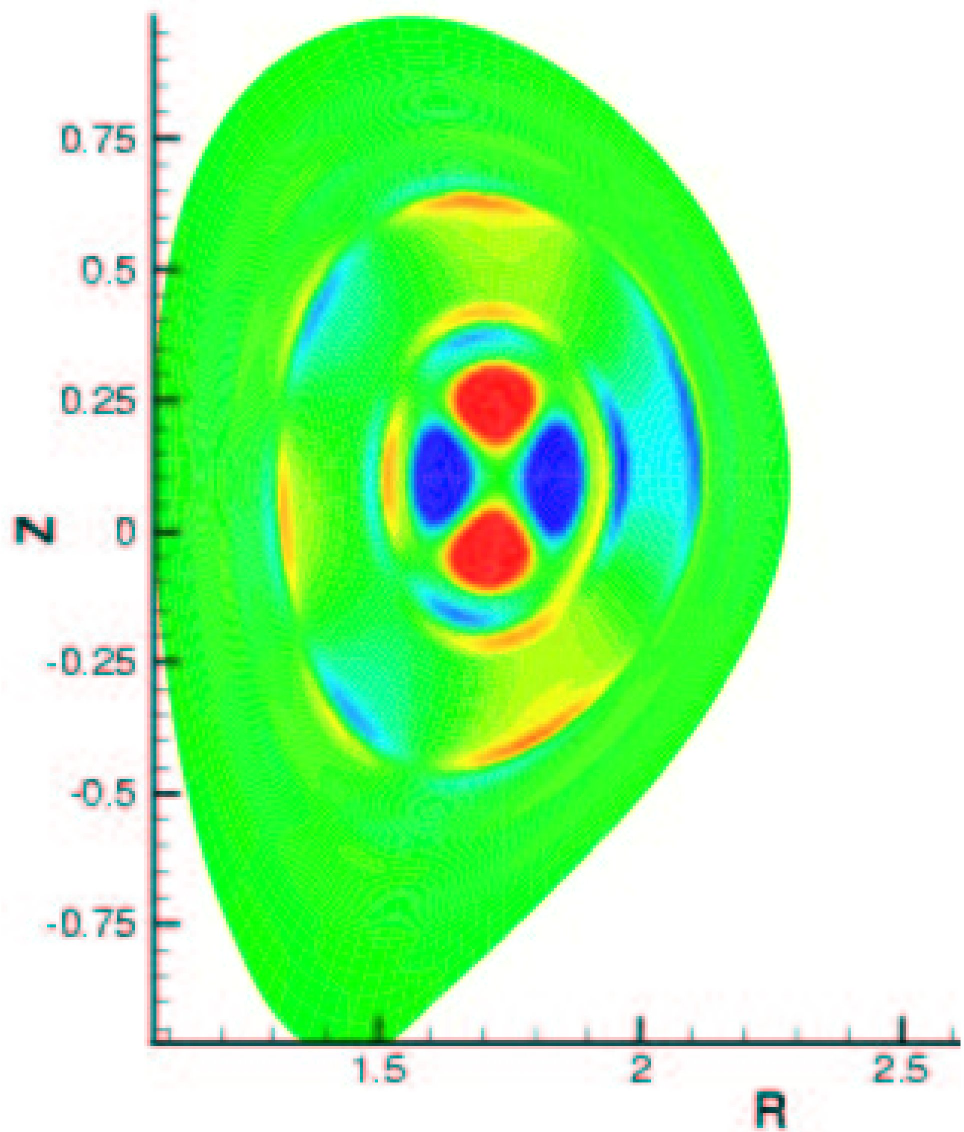
Nonlinear coupling to 1/1 drives 3/2 island above threshold.



Lack of poloidal resolution (my = 16 with cubic polynomials) unable to resolve pressure flattening inside 3/2 island.

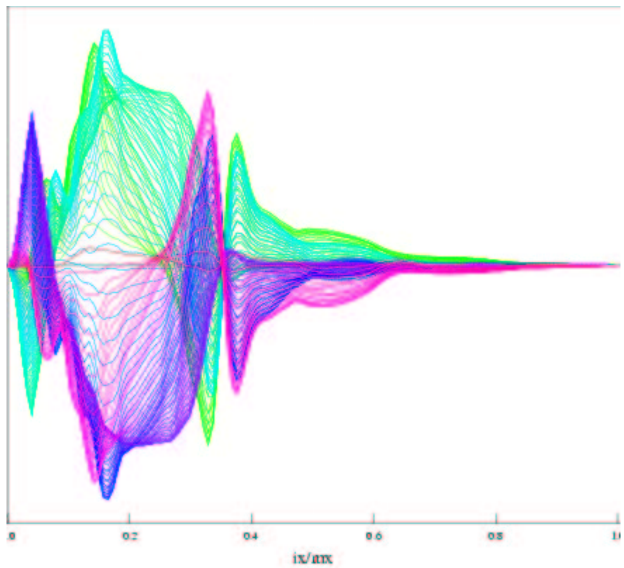


Additional poloidal resolution (my = 32 with quartic polynomials) shows desired pressure response inside island.

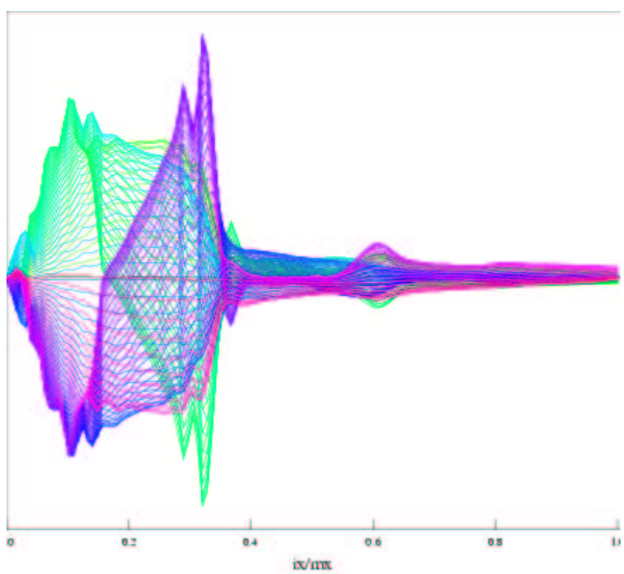


Enhanced resolution of pressure gradients reflected in neoclassical flow damping and current drive.

- V_Z structure:



- B_R structure:



Integral heat flow closure may provide enhanced pressure flattening.

- Integral closure retains free-streaming and collisional effects [5],

$$q_{\|}(L') = \int^{L'} dL \, [T(L'-L) - T(L'+L)] \, \frac{\partial K}{\partial(\ln L)},$$

where

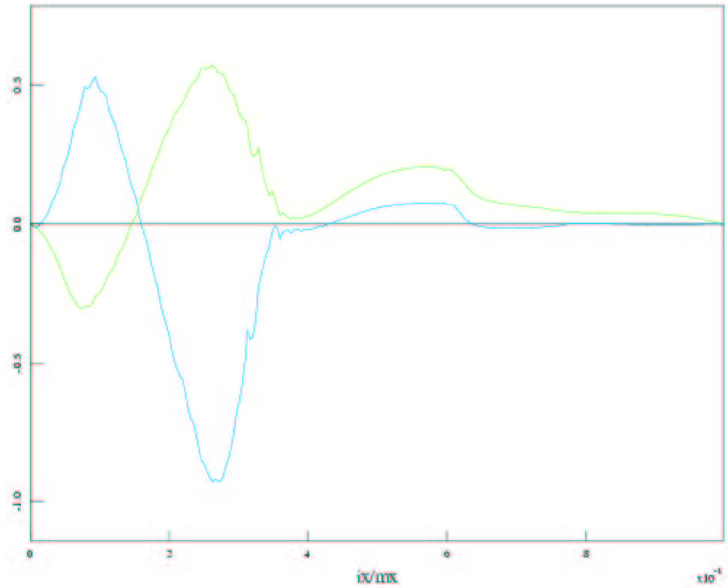
$$K(L) = n_0 v_{th} s^3 \left(s^2 - \frac{5}{2}\right)^2 \sum_{i=0}^{N} a_i \exp\left(-s^2 - \bar{k}_i L\right).$$

Here L_{mfp} is the collisional mean free path, and the parameters a_i and $\bar{k}_i(s, L_{mfp})$ are generated in the solution of the drift kinetic equation.

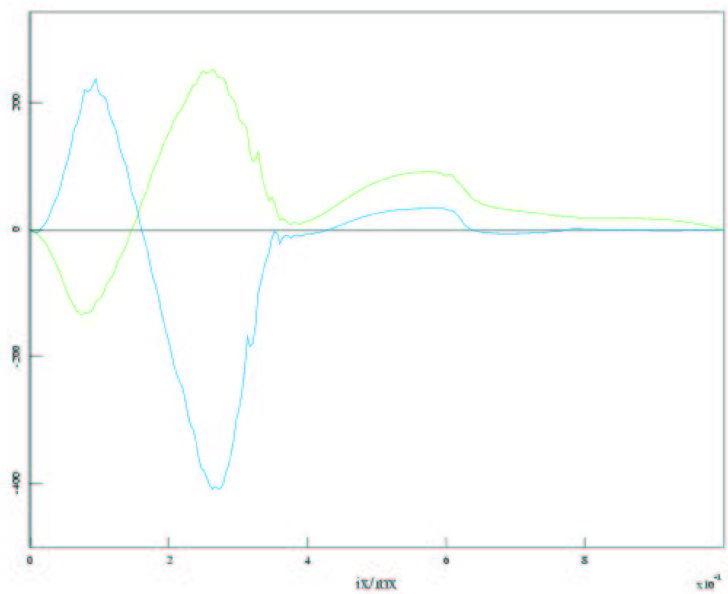
[5] E. D. Held, J. D. Callen, C. C. Hegna and C. R. Sovinec, Phys. Plasmas **8**, 1171 (2001).

Diffusive and highly collisional integral heat flows rely on local temperature gradient.

- **Diffusive closure:**

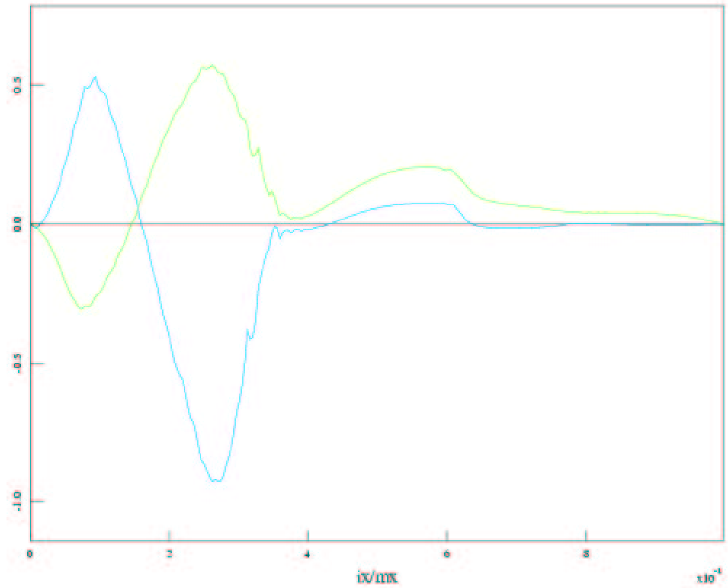


- **Integral closure with $T = 0.1$eV:**

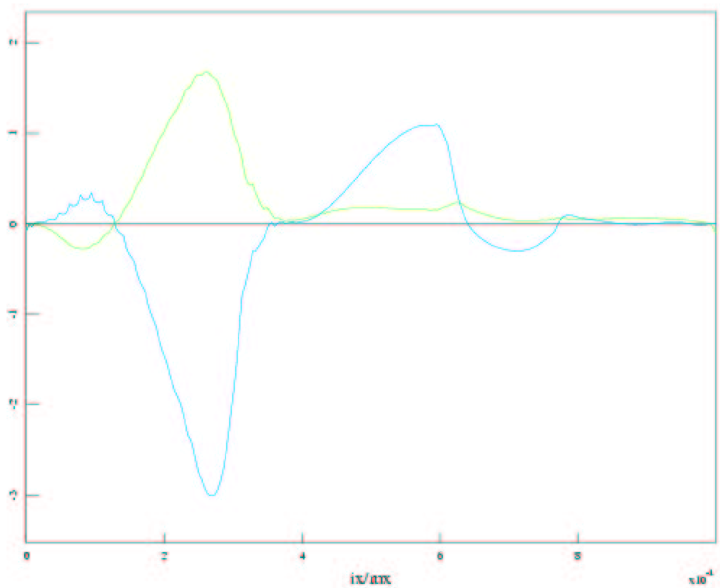


Nonlocal dependence of integral form for heat flow evident at 100 eV.

- **Diffusive closure:**

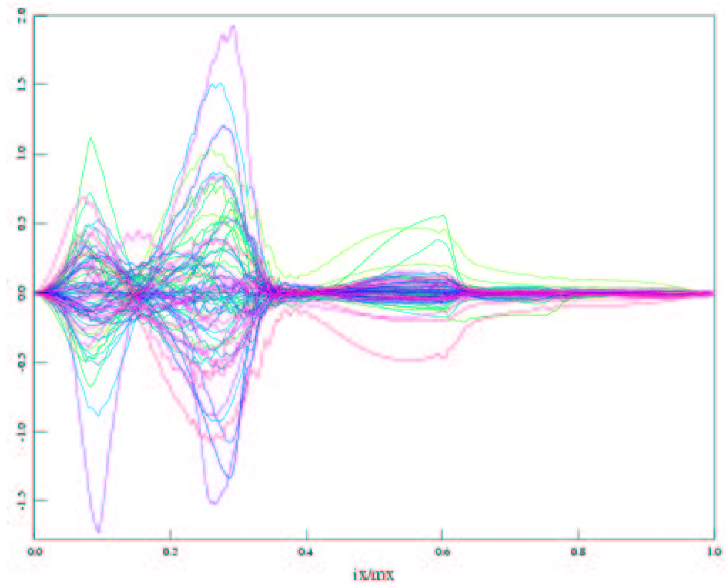


- **Integral closure with $T = 100$eV:**

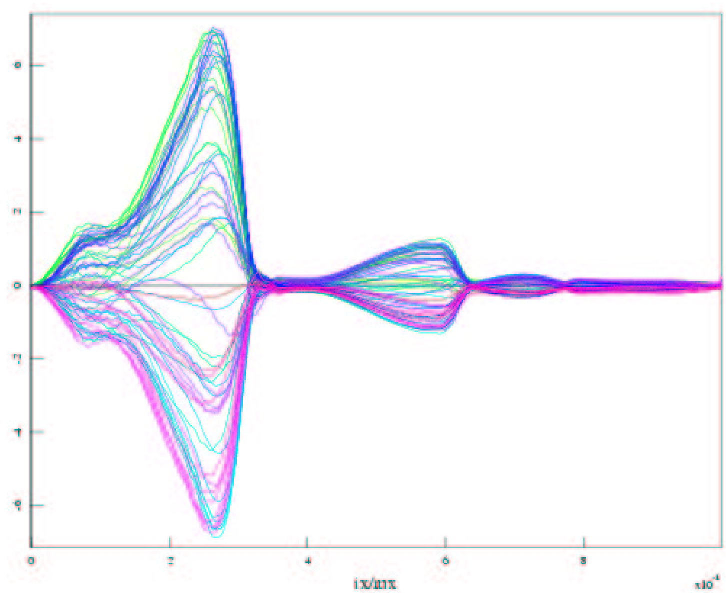


Integral closure predicts different heat flow response.

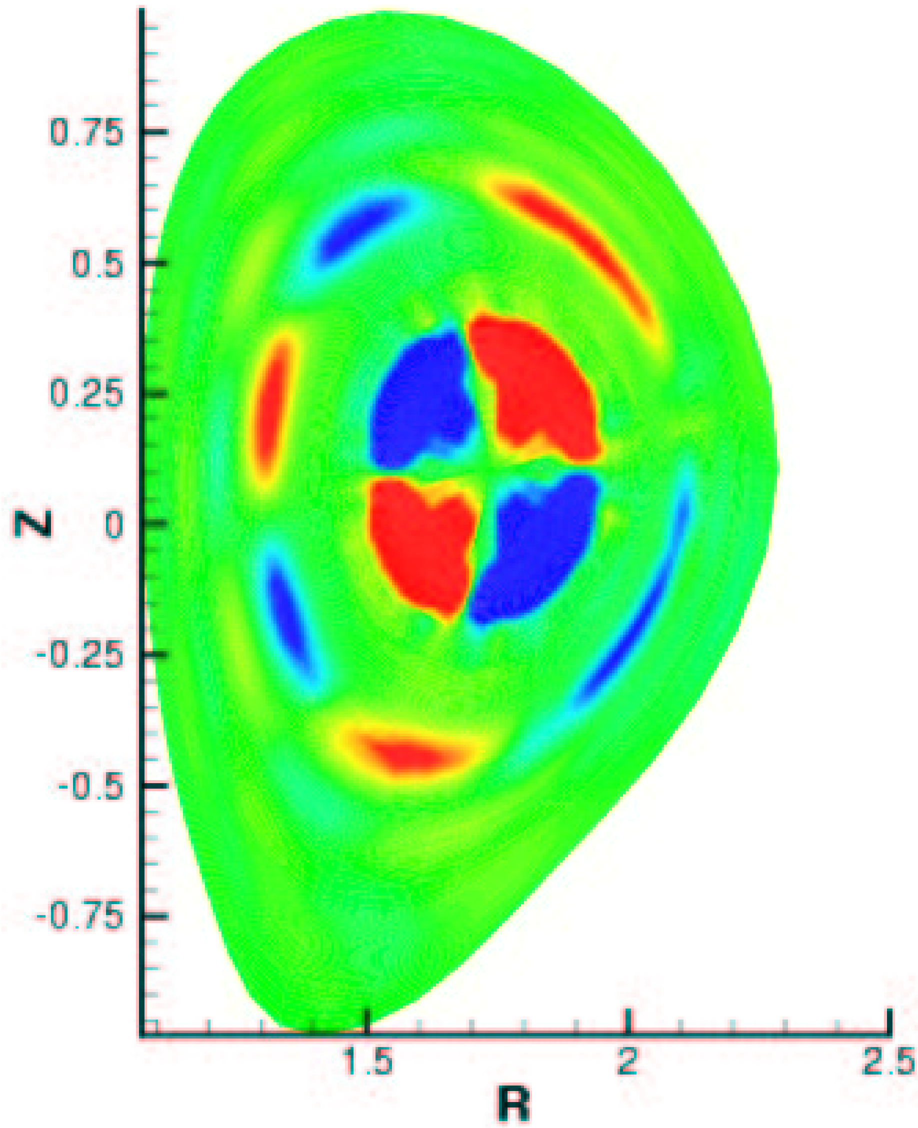
- **Diffusive closure:**



- **Integral closure with $T = 100$eV:**

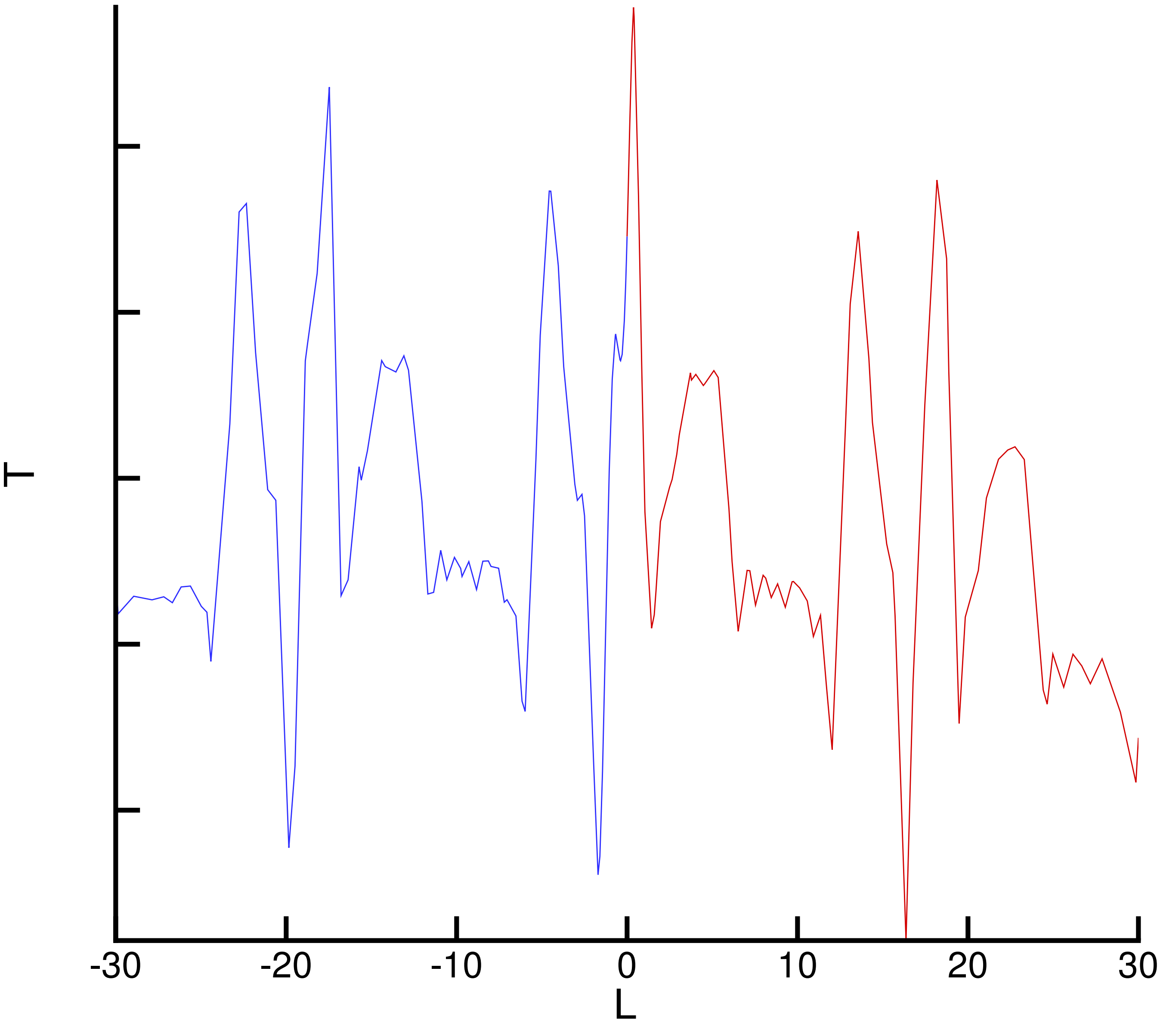


Parallel heat flow contours for k=2 show 3/2 island and response to 1/1 perturbation.

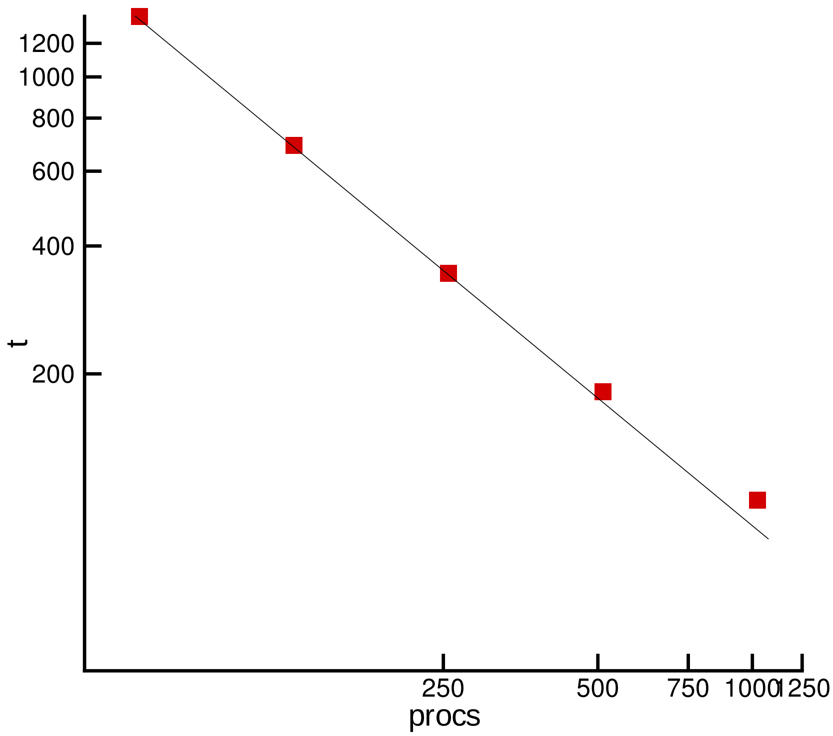


Nonlocal dependence of temperature perturbations evident.

- **Heat flow converges after 30m for $L_{mfp} \approx 1.0m$ at $R = 2.15$ and $Z = 0.0$.**



Integral closure calculation scales well with number of processors.



Conclusions

- **An effective form of the neoclassical-viscous stress tensor has been implemented and tested.**
- **Analytic theory predicts the experimental observation of 2/1 NTM stability and 3/2 tearing instability.**
- **Previous NIMROD NTM simulations of 86144 lack resolution in poloidal direction.**
- **Enhanced resolution of pressure gradients reflected in neoclassical terms.**
- **Integral and diffusive forms for heat flow closure disagree in long mean free path regimes.**
- **Massively parallel implementation of integral heat flow closure may hold key to enhanced pressure flattening and stronger NTM drive.**