# TWO-FLUID EFFECTS ON STABILITY AND BETA LIMITS IN 3D CONFIGURATIONS


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# ABSTRACT

**Two-fluid plasmas, with independently evolving fluid electron and ion species connected through quasineutrality, can generate steady state flows in non-axisymmetric configurations, sustained by the equilibrium pressure gradients, but smaller than the diamagnetic drifts. These flows can have stabilizing effects on the pressure-driven instabilities that set one important limit on the maximum plasma beta. Starting from an ideal MHD equilibrium configuration (eg, calculated by the VMEC code) with zero plasma mass flow $\mathrm{v}_i = 0$, two-fluid flows develop rapidly on an MHD time scale, some few tens of shear Alfvén times, in the region of strong equilibrium pressure gradient. They persist over times long compared to the scales of MHD and the relaxation of the initial configuration Two fluid effects can favorably influence the stability of high and moderate mode number localized ballooning instabilities through shear flow stabilization. They can also affect the rate of magnetic reconnection occurring at mode rational surfaces, depending on the ratio of the electron and ion pressures and plasmas with higher ratios of the electron to ion pressure may have larger steady state islands (in the absence of neoclassical effects). Illustrations are given for stellarators, using the nonlinear two-fluid model in the massively parallel M3D code [1]. In the high beta, quasi-axisymmetric NCSX design, ballooning stability limits may be raised significantly above the nominal design value calculated from linear stability codes.** [1] **L.E. Sugiyama and W. Park, Phys. Plasmas 7 4644 (2000).**

# INTRODUCTION:
# STELLARATOR STEADY STATES AND BETA LIMITS

- A consistent two-fluid picture of the basic properties of stellarator steady states and possible beta limits is starting to emerge from studies with the M3D 3D initial value code.

- Prediction of 3D steady state configurations and beta limits for stellarators is less well understood and less tested against experiment than for axisymmetric confined plasmas.

  - Much previous design work has been carried out at the level of simple theories such as MHD, because of the complexity of the geometry and the lack of sophisticated tools designed for it.

- The additional physics introduced by two-fluid processes relative to MHD, still in a fluid picture, are found to have important consequences for high beta stellarators and to be crucial to understanding them. They affect key questions regarding fusion performance:

- **Guaranteeing the existence of well-constrained, contained flux surfaces is a more difficult problem for helical than axisymmetric configurations. Magnetic reconnection and island saturation are important.**

- **High beta is important for fusion stellarators, both for good fusion yield and for good plasma properties and control. The beta limit is an important open question.**

- **The two-fluid plasma model is also an approximation. Other non-MHD processes, such as neoclassical healing of magnetic islands, may also be important.**

# QUASI-AXISYMMETRIC STELLARATOR

- **The high beta, quasi-axisymmetric stellarator NCSX has been designed primarily from the standpoint of ideal MHD equilibrium and stability, with contributions from bootstrap current effects and confinement considerations.**
  **The reference design at $\beta = 4\%$ is marginally stable to major instabilities (kink, vertical, Mercier/ballooning).**

- **Relatively broad pressure and density profiles with strong gradients near the plasma edge (assumed profile), where bootstrap current is large on outboard side.**

- **The M3D analysis used the NCSX reference case li383 at 4.2% beta, obtaining the equilibrium configuration from the VMEC code.**
  **Higher beta cases used the same toroidal current profile (including the estimated bootstrap current for the reference case) and the same pressure profile shape, multiplied by an overall factor.**

- **Numerical parameters:**
  **Lundquist number $S = 10^5$, ion viscosity $\mu = 5 \times 10^{-4}$,**

Accelerated equilibration of $T_e$ and $T_i$ along the magnetic field, electrons faster than ions ('artificial sound' method of M3D).
Fixed plasma boundary, perfectly conducting.

- Resistivity of $S \geq 10^5$ behaves fairly closely to ideal MHD for the MHD ballooning instability in the linear and nonlinear cases (APS-DPP 2002, IAEA 2002).

- The two-fluid model uses a nonlinear drift approximation, $v \sim (\rho_i/L)v_{thi} \sim \delta v_{thi}$, where the ion Larmor radius $\rho_i$ is assumed small compared to the plasma scale length $L$. It includes the ion gyroviscous stress tensor, the Hall term and electron pressure gradient in the Ohm's law, independent electron and ion temperature evolution with perpendicular and parallel 'thermal conduction,' and full plasma compressibility.
Two-fluid parameter $c/(\omega_{pi}R) \simeq$0.01–0.02, expected value.

# MHD RESULTS:
# INITIAL ISLAND HEALING

- **7% beta case was used for most of the analysis. The reference 4.2% case was analyzed previously with M3D (IAEA 2002).**
- **Magnetic islands exist in the initial ideal MHD equilibrium due to the singular toroidal current density calculated in the VMEC equilibrium code.**
  **These islands relax resistively on a short, MHD time-scale to smaller size (25 $\tau_A$) (see Fig. 1).**

# MHD BETA LIMIT – BALLOONING

- **Beta of 4.2% is marginally stable to ideal MHD ballooning with infinite toroidal mode number $n$. In M3D with moderate spatial resolution, ideal MHD instability is seen at approximately 6% beta, with $m, n$ ~10–20.**

- **Thus, at 7% beta, MHD ballooning modes (with contributions from the resistive interchange) develop over a slightly longer time than the initial island healing (80–90 $\tau_A$) (see Fig. 2).**

# TWO-FLUID RESULTS: STEADY STATE POLOIDAL ION FLOW

- **Two-fluid effects produce a steady state poloidal rotation in the $\omega_{*i}$-direction that has magnitude significantly smaller than $|(1/2)\mathrm{v}_*|$, where $\mathrm{v}_*$ is the diamagnetic drift based on the total pressure. (see Fig. 3, 7% beta)**

- **The strongest ion poloidal flow is localized in the region of strong pressure gradient and diamagnetic drift near the plasma edge, where its magnitude is relatively independent of the equilibrium ratio $p_i/p_e$. (Fig. 4 shows toroidal current and e- diamagnetic drift.)**

- **In the interior, the ion flow increases in magnitude with increasing $p_e/p_i$. Away from the region of strong pressure gradient, $v_{*i}$ is smaller and no longer dominates the momentum balance.**

- **The ion poloidal flow develops rapidly, on an MHD time scale, (eg, within 30 $\tau_A$) from an initial stationary MHD equilibrium, then remains relatively unchanged, showing that it is a steady state effect.**

- Writing $\mathrm{v}_i = \mathrm{v} + \mathrm{v}_{di}$, where $\mathrm{v}_{di} \equiv \mathrm{J}_{\perp B}/en + \mathrm{v}_{*e}$ is a generalized ion diamagnetic frequency and $\mathrm{v}$ the common part of the electron and ion motion perpendicular to $\mathrm{B}$, the equation for the common part of the vorticity $w \equiv -(R_o/R)\hat{\phi}\cdot\nabla\times\mathrm{v}$, from the total momentum equation, is

$$\begin{aligned} \frac{\partial w}{\partial t} \;=\; & -(\mathrm{B}\cdot\nabla)\left(\frac{R_o J_\phi}{nR}\right) - (\mathrm{J}\cdot\nabla)\left(\frac{R_o^2 I}{nR^2}\right) + \mu\nabla_\perp^2(w + w_{di}) \\ & + \frac{R_o^2}{R}\nabla_\perp\frac{1}{n}\times\nabla_\perp p\cdot\hat{\phi} \\ & - \frac{R_o}{R}\hat{\phi}\cdot\nabla\times\left[\left(\left(\mathrm{v}+\mathrm{v}_{di}\right)\cdot\nabla\right)\mathrm{v}_{\perp B} + \left(\mathrm{v}\cdot\nabla\right)\mathrm{v}_{\parallel B}\right]. \end{aligned} \tag{1}$$

The first two terms on the RHS come from $\mathrm{J}\times\mathrm{B}$, the third from the ion collisonal viscosity, and the fourth from the pressure gradient. The last term comes from the combination of advection and the IGV stress. Neglecting the ion viscosity, the only feedback on $w$ comes from the last line. We assume $v_{i\theta} \simeq 0$ and $v_{i\phi} \simeq 0$, from neoclassical momentum damping, while the radial component must also be small, so writing $\mathrm{v} = -\mathrm{v}_{*i} + \delta\mathrm{v}$ or $\mathrm{v} \simeq \delta\mathrm{v}$ near steady state, the last line becomes approximately

$$-\frac{R_o}{R}\hat{\phi}\cdot\nabla\times\left[\left(\delta\mathrm{v}\cdot\nabla\right)\left(-\mathrm{v}_{*i}+\delta\mathrm{v}_{\perp B}\right) + \left(\left(-\mathrm{v}_{*i}+\delta\mathrm{v}\right)\cdot\nabla\right)\delta\mathrm{v}_{\parallel B}\right] \tag{2}$$

Since $\mathrm{v}_{*i}$ changes relatively little, this modifies the common fluid velocity $\delta\mathrm{v}$. In particular, where $\mathrm{v}_{*i}$ is large, it makes $\delta\mathrm{v}$ a function of the radial variable only and tries to reduce the magnitude of $|(-\mathrm{v}_{*i} + \delta\mathrm{v})|$, causing a $\delta\mathrm{v}_{\perp B}$ in the $\omega_{*i}$-direction, as observed.

When $p_i = 0$, the term reduces to MHD advection, which does not produce plasma flow in MHD, but in the two-fluid case, $\delta\mathrm{v}$ is modified by existence of a nonzero radial electric field related to $(\mathrm{v}_{*e} + \mathrm{v}_{*i})$.

- The neoclassical parallel viscous stress contributes a term to the RHS

$$+\frac{R_o}{R}\left[NRJ_\phi - \nabla_\perp N \cdot \nabla\psi + \nabla_\perp N \times \nabla_\perp F \cdot \hat{\phi}\right], \tag{3}$$

where $N = \langle \mathrm{B} \cdot \nabla \cdot \Pi_i^{neo} \rangle / (n m_i B^2) \simeq (1/\tau_{ii} B^2)(\mu_0 V_{i\theta} + \mu_1 Q_{i\theta})$. The neoclassical 'poloidal momentum damping,' $\langle \mathrm{B} \cdot \nabla \cdot \Pi_i^{neo} \rangle \rightarrow 0$, may be modified by the two-fluid IGV stress.

# TWO-FLUID FLOWS STABILIZE THE BALLOONING MODE

- **The two-fluid poloidal flows stabilize the ballooning/resistive interchange mode (see Fig. 3, 7% beta).**
  - **Is it $\omega_{*i}$ stabilization through mode structure distortion, a plasma flow through the ballooning region, which is fixed in the lab (magnet) frame, or a flow shear effect? Or all three . . .**
- **Stabilization is robust:**
  **Ideal MHD unstable modes even at 8% beta !**
  **Larger-than-actual resistivity, which worsens resistive ballooning**
  **Realistic values of the two-fluid parameter $c/(\omega_{pi}R)$.**

# TWO-FLUID ELECTRON EFFECTS INCREASE GROWTH OF MAGNETIC ISLANDS

- Electron two-fluid effects increase the growth rate of magnetic islands at low rational-mode-number magnetic surfaces (see Fig. 5).
- In MHD, magnetic islands remain at their small, mostly healed size during the time that the outer plasma is destroyed by ballooning modes.
  During the same time, two-fluid islands can grow substantially.
- Electron effects: islands grow faster at higher $p_e/p_i$.
- Well-known that electron effects are linearly and nonlinearly destabilizing for magnetic reconnection, but stellarator case is difficult to analyze:
  Nonlinear
  Seed islands
  Not $\Delta'$ limited?
  Helical geometry means that some processes impt in axisymmetry are not important, eg, the $\nabla_{\parallel} p_e$ term in Ohm's law.

- **Only cylindrical analysis of $\Delta'$ for reconnecting modes is usually done in stellarators!**
- **Numerically, difficult to separate out the terms driving the mode . . .**
- **Question: Does the saturated size change? At $t = 153\tau_A$, islands still remain similar size in the high $p_i/p_e$ case.**

# BETA LIMIT

- **Magnetic islands also grow faster at higher beta (interchange term and $p_e$ are destabilizing).**
- **At 8% beta, the two-fluid plasma remains stable to edge ballooning modes, but rapidly develops large, overlapping interior islands with the low-rational-mode-number ones growing fastest initially (5/3, 6/3). Outer flux surfaces remain good, but the interior becomes mixed (see Fig. 6).**
- **This suggests that the practical beta limit may result from increasingly poor confinement rather than a catastrophic instability.**

# NEOCLASSICAL PARALLLEL VISCOUS STRESS

- **NCSX analysis (Physics Review 2001) suggested that neoclassical effects should reduce magnetic island growth in the reversed shear of NCSX ("anti-NTM" ).**
- **Preliminary results with M3D on the effects of the neoclassical parallel collisional viscous stresses, using the parallel part of the terms, $\langle \mathrm{B}\cdot\nabla\cdot\Pi_j^{neo}\rangle \mathrm{B}$, have effects localized to the outer half to one-third of the minor radius and strongest in the region of maximum pressure gradient.**

  **For NCSX, this covers the region of the fastest growing 5/3 mode, but possibly not the 6/3 and 7/3 modes (see Fig. 7).**

# SUMMARY

- **Numerical studies with the M3D code show that two-fluid processes have strong effects on the characteristic steady states and the resulting beta limits in stellarators. Two-fluid limits are quite different from MHD ones.**
- **Quasi-axisymmetric NCSX configurations may exist at betas significantly higher than the reference design, up to some 7%.**
  - **Ideal and resistive ballooning modes are robustly stabilized by two-fluid ion flow/diamagnetic effects at even higher beta. Similar effects probably also stabilize free boundary kink modes.**
- **Magnetic island growth at interior, low-mode-number rational flux surfaces, encouraged by electron fluid effects (higher $p_e/p_i$ at the same total beta), becomes an important consideration at high beta.**
  - **Hot ion plasmas may be better confined than hot electron ones.**
  - **The practical beta limit may be due to increasingly poor confinement caused by the loss of good interior magnetic flux surfaces, rather than catastrophic instability.**
- **Work is continuing on these effects and on the two-fluid model.**