Wave Propagation Tests with AMRMHD Code

Ravi Samtaney

Computational Plasma Physics Group

Princeton Plasma Physics Laboratory

Princeton University

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Outline

- Linear wave propagation tests
  - *Formulation*
  - *Phase velocity diagram*
  - *Error analysis*
- Other verifications
- Unsplit Godunov algorithm
- Status of AMRMHD Code

Linear Waves: Formulation

- Ideal MHD equations written in quasilinear form in 2D

$$\frac{\partial V}{\partial t} + A_x(V)\frac{\partial V}{\partial x} + A_y(V)\frac{\partial V}{\partial y} = 0,$$
$$V = \{\rho, u, v, w, B_x, B_y, B_z, p\}^T.$$

- Linearize about a constant equilibrium state V_0

$$\frac{\partial V_1}{\partial t} + A_x(V_0)\frac{\partial V_1}{\partial x} + A_y(V_0)\frac{\partial V_1}{\partial y} = 0,$$

- Wave propagation along a direction ξ

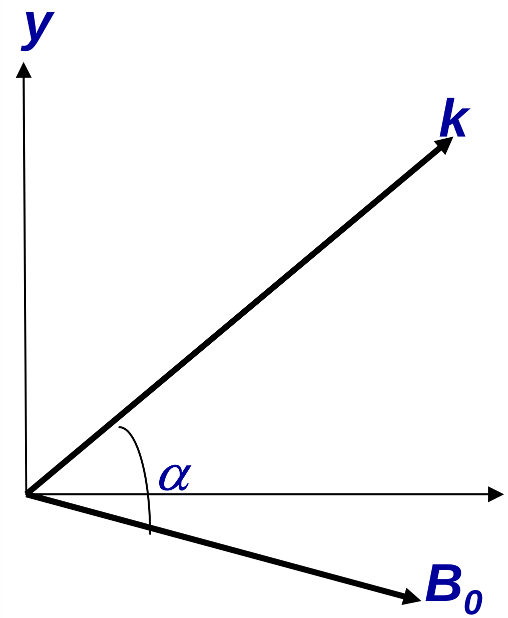
$$\frac{\partial V_1}{\partial t} + A_\xi(V_0)\frac{\partial V_1}{\partial \xi} = 0,$$

- Rewrite in characteristic variables

$$\frac{\partial W_k}{\partial t} + \lambda_k(V_0, \xi)\frac{\partial W_k}{\partial \xi} = 0,$$
$$\lambda_k = \{0, \pm a_\xi, \pm c_f, \pm c_s\}.$$

- Initial conditions to set up the l-th wave

$$W_k(x, y, 0) = W_{0,k}(V_0) + \delta_{kl}\ \epsilon \cos(k_x x + k_y y)$$



Linear Waves : Formulation

- Domain [0:2]x[0:2]
- Wavenumber vector: $k_x = n\,\pi$, $k_y = m\, k_x$
  - *(n,m)=(1,1), (1,2), (1,3)*
- Angle between wave direction and $\mathbf{B_0}$ varied from 0 to $\pi/2$
- Amplitude of waves $\varepsilon = 10^{-5}$
- Equilibrium state: $\{\rho_0, 0, 0, 0, B_{x,0}, B_{y,0}, p_0\}$
  - $\rho_0 = 1$, $p_0 = 0.1$
  - $|B_0| = 1$
- t_{end}¼ 2
- Computed with nonlinear code (nonlinearities ~ $O(\varepsilon^2)$)

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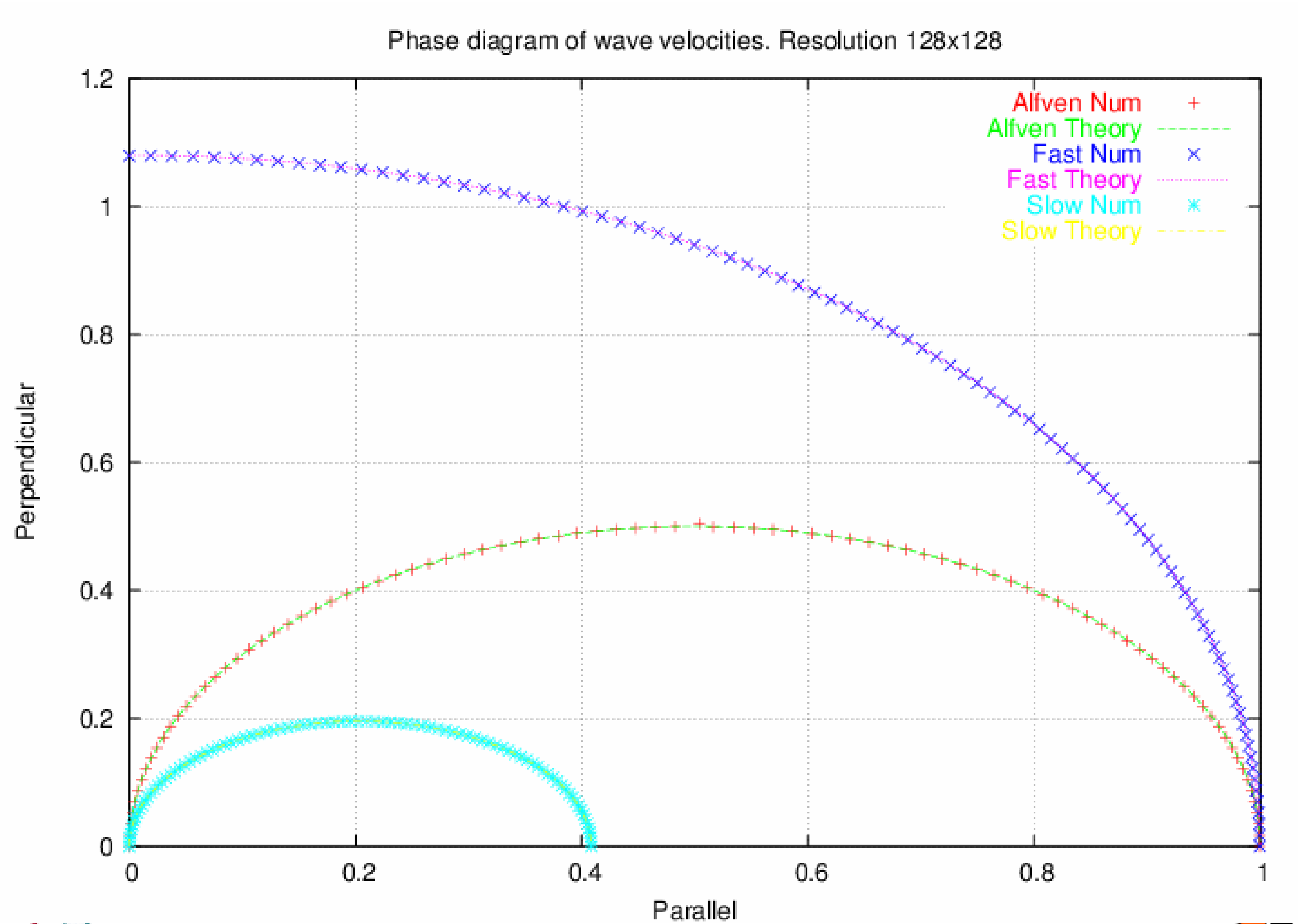
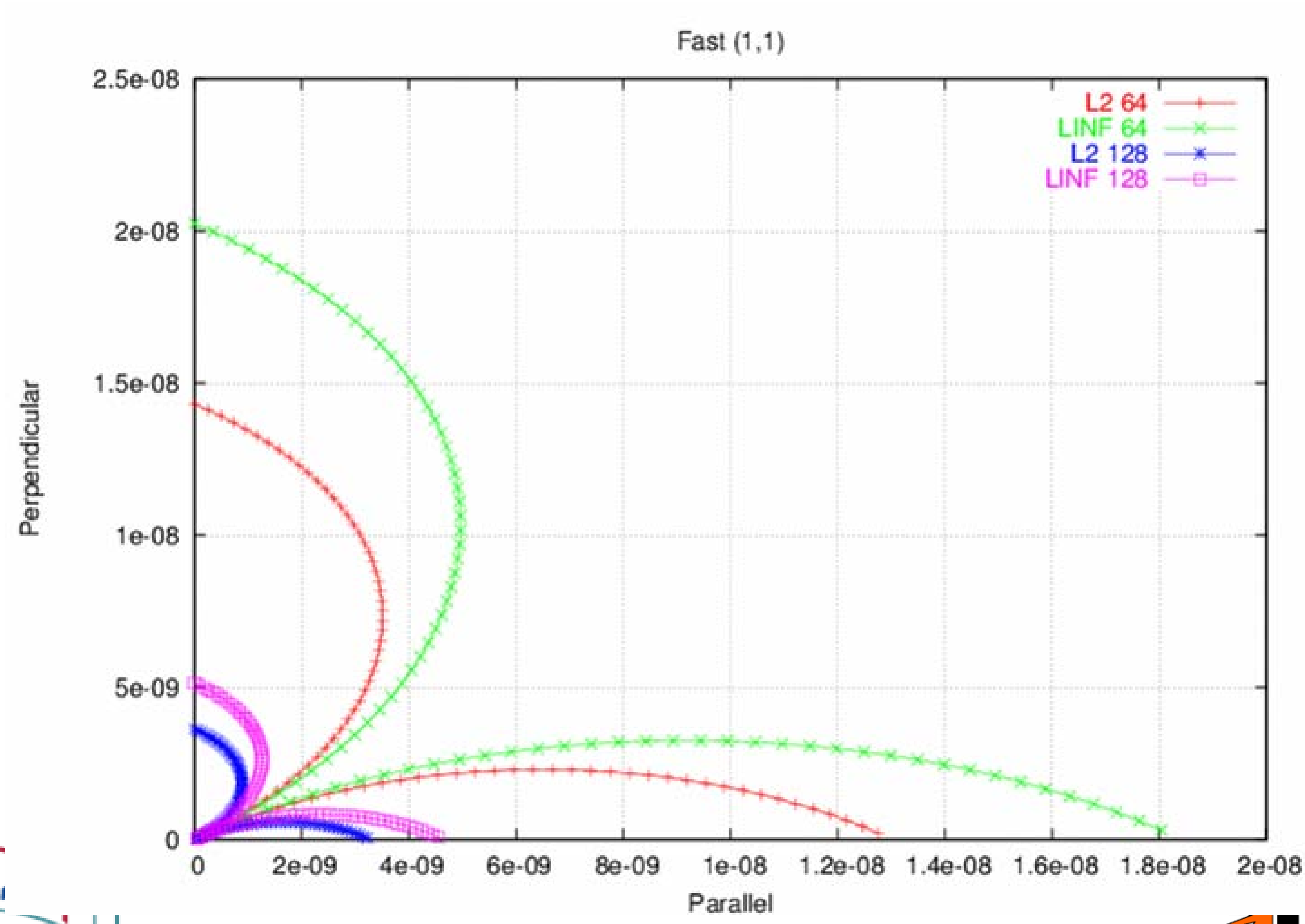
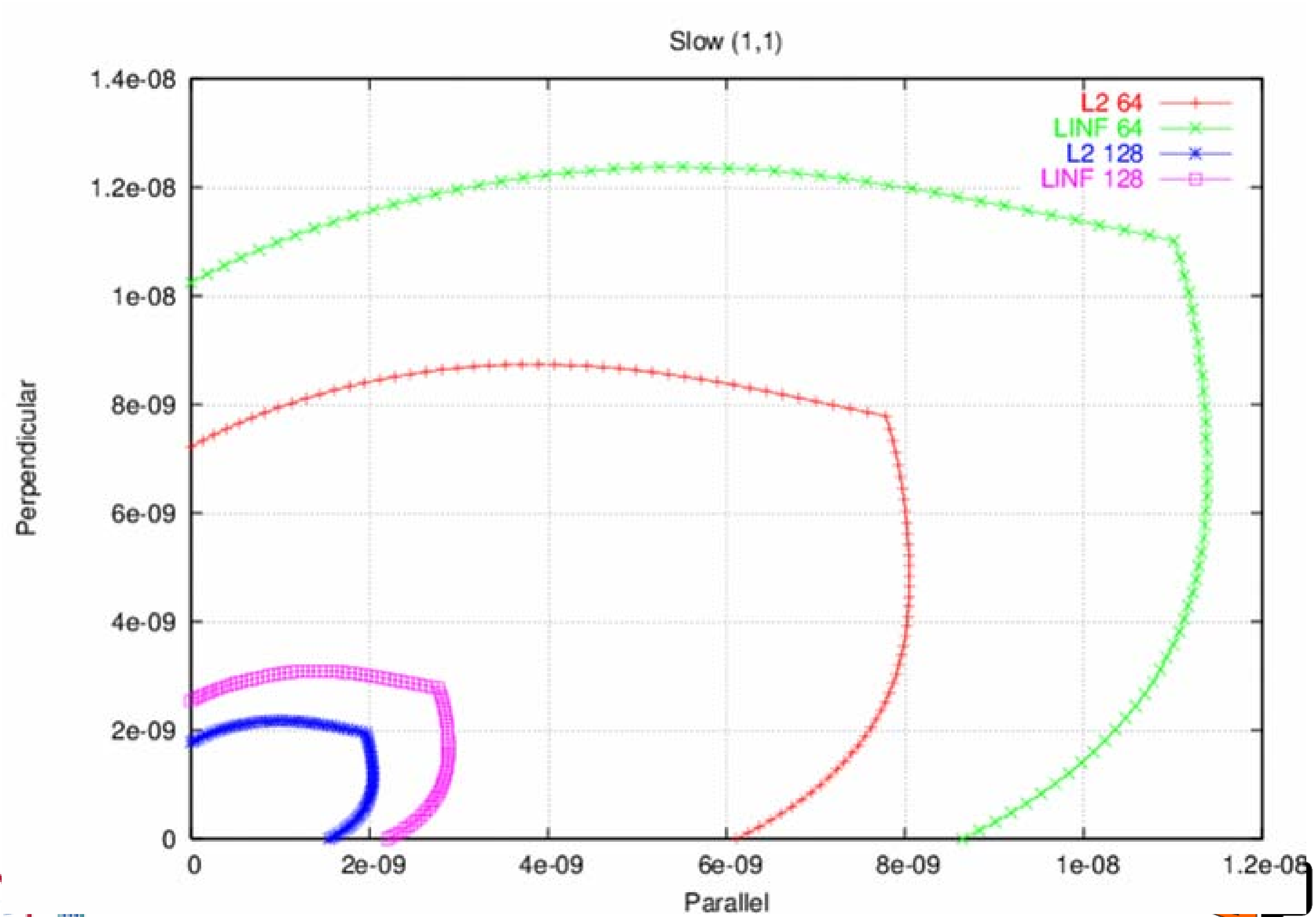
Linear Waves: Wave Phase Velocity



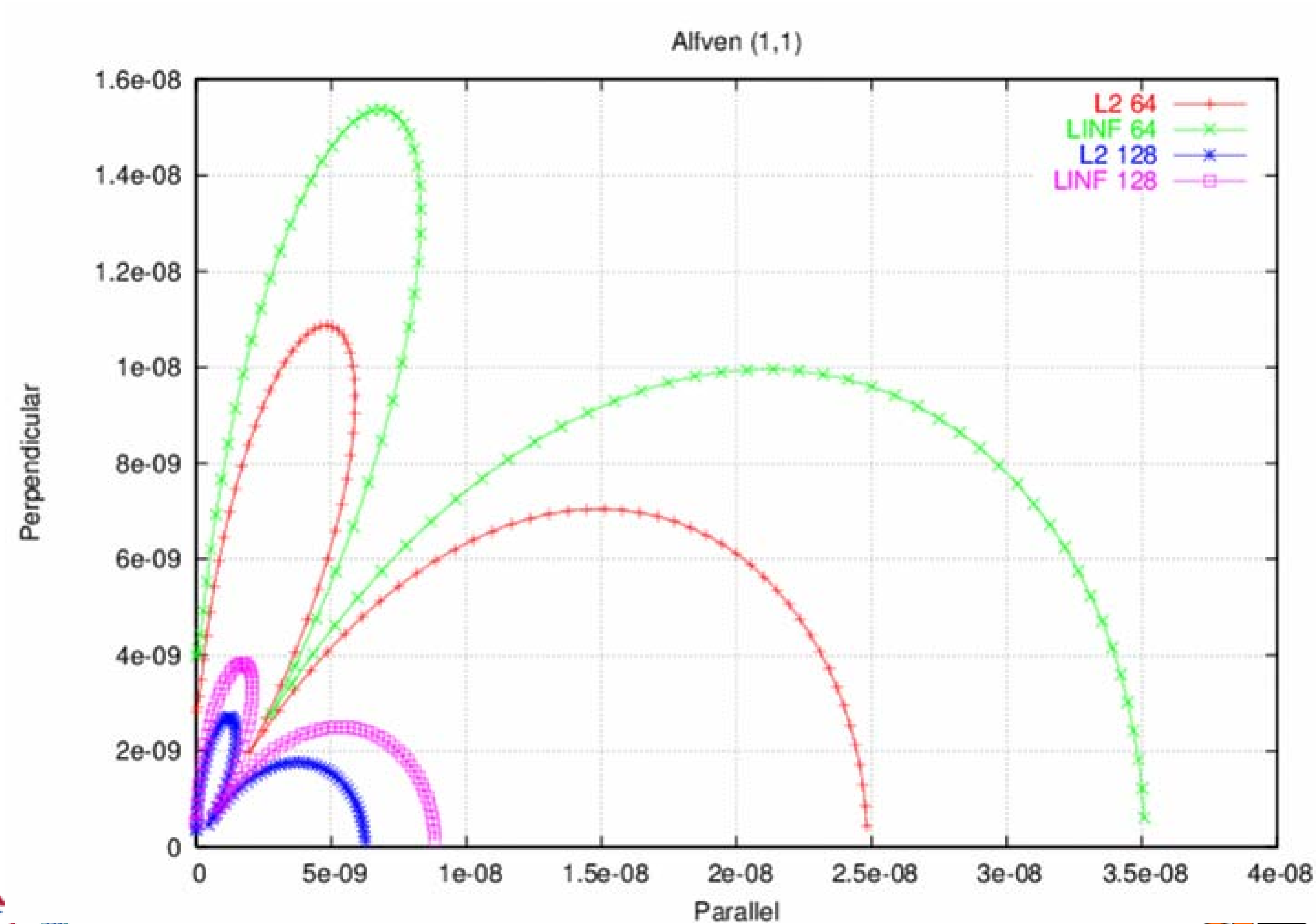
Linear Waves: Error Fast Wave



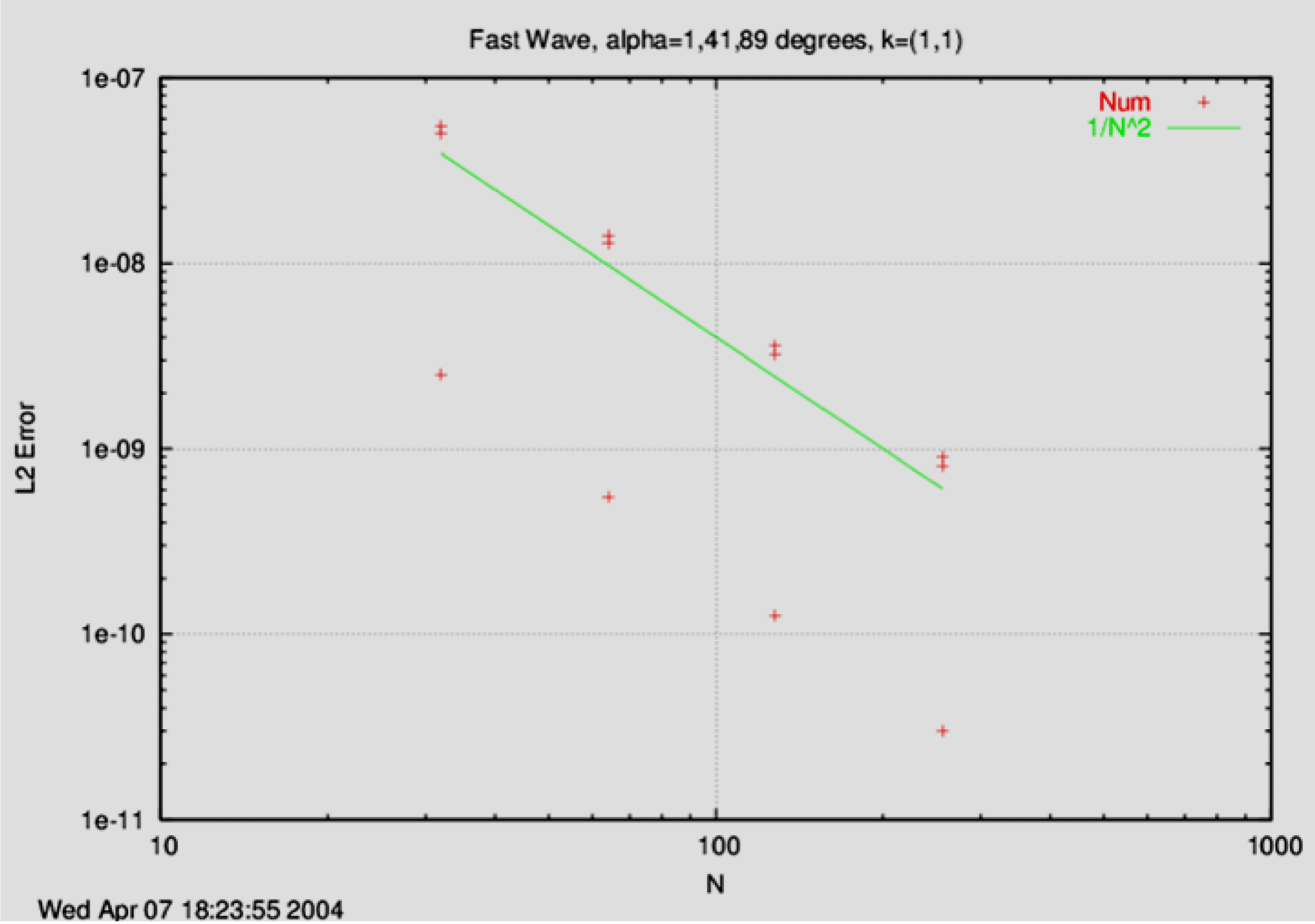
Linear Waves: Error Slow Wave



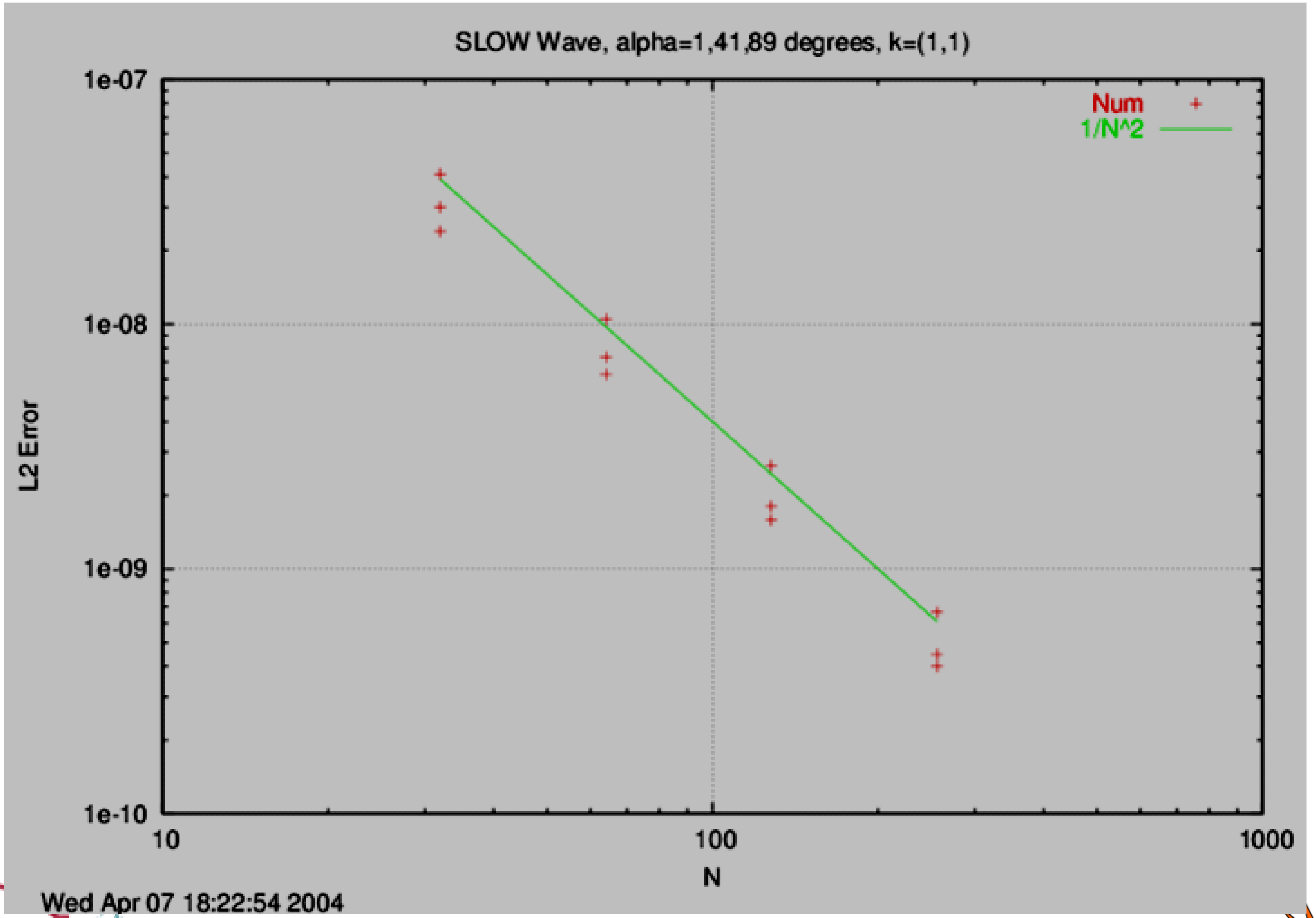
Linear Waves: Error Alfven Wave



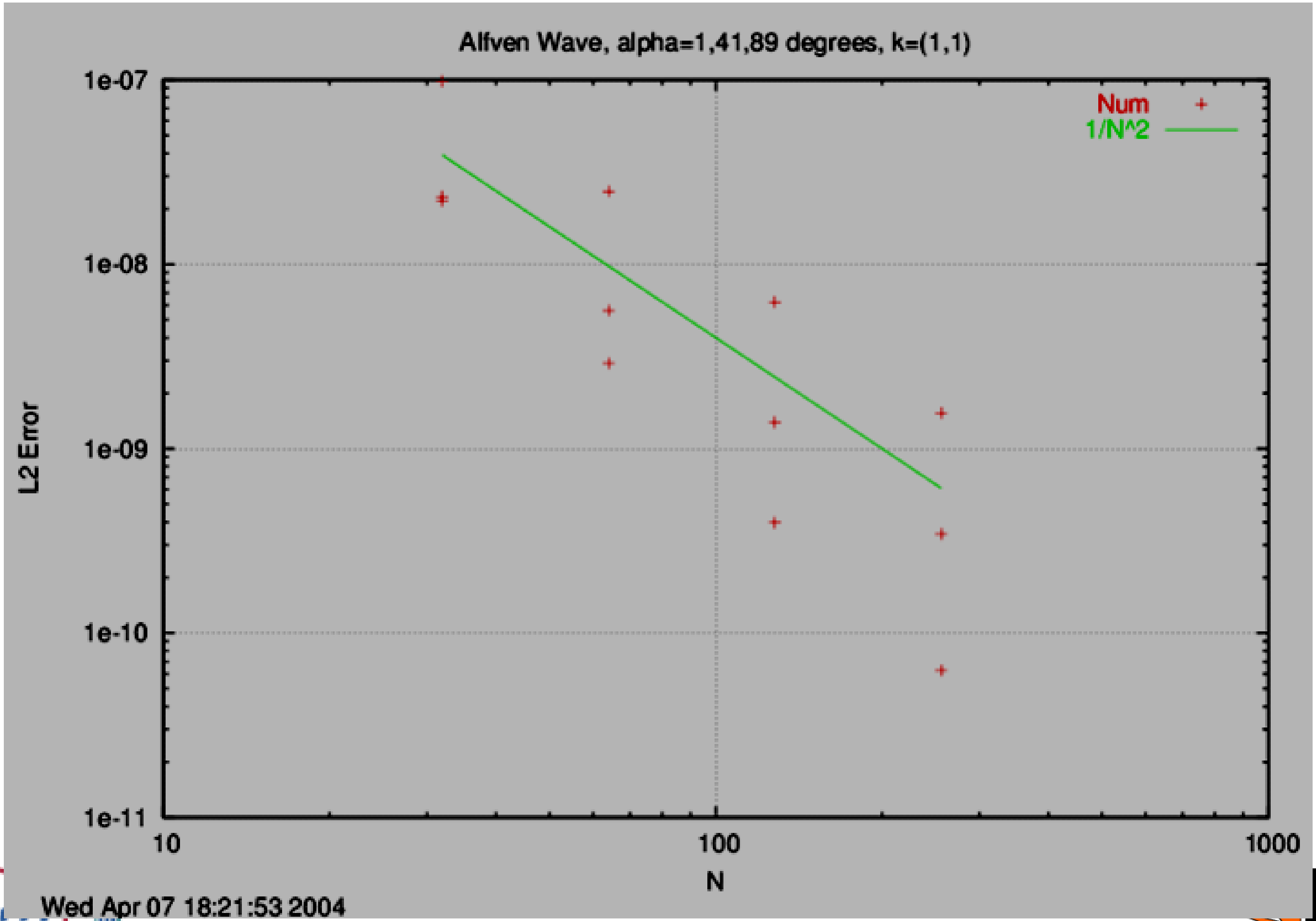
Linear Waves: Accuracy



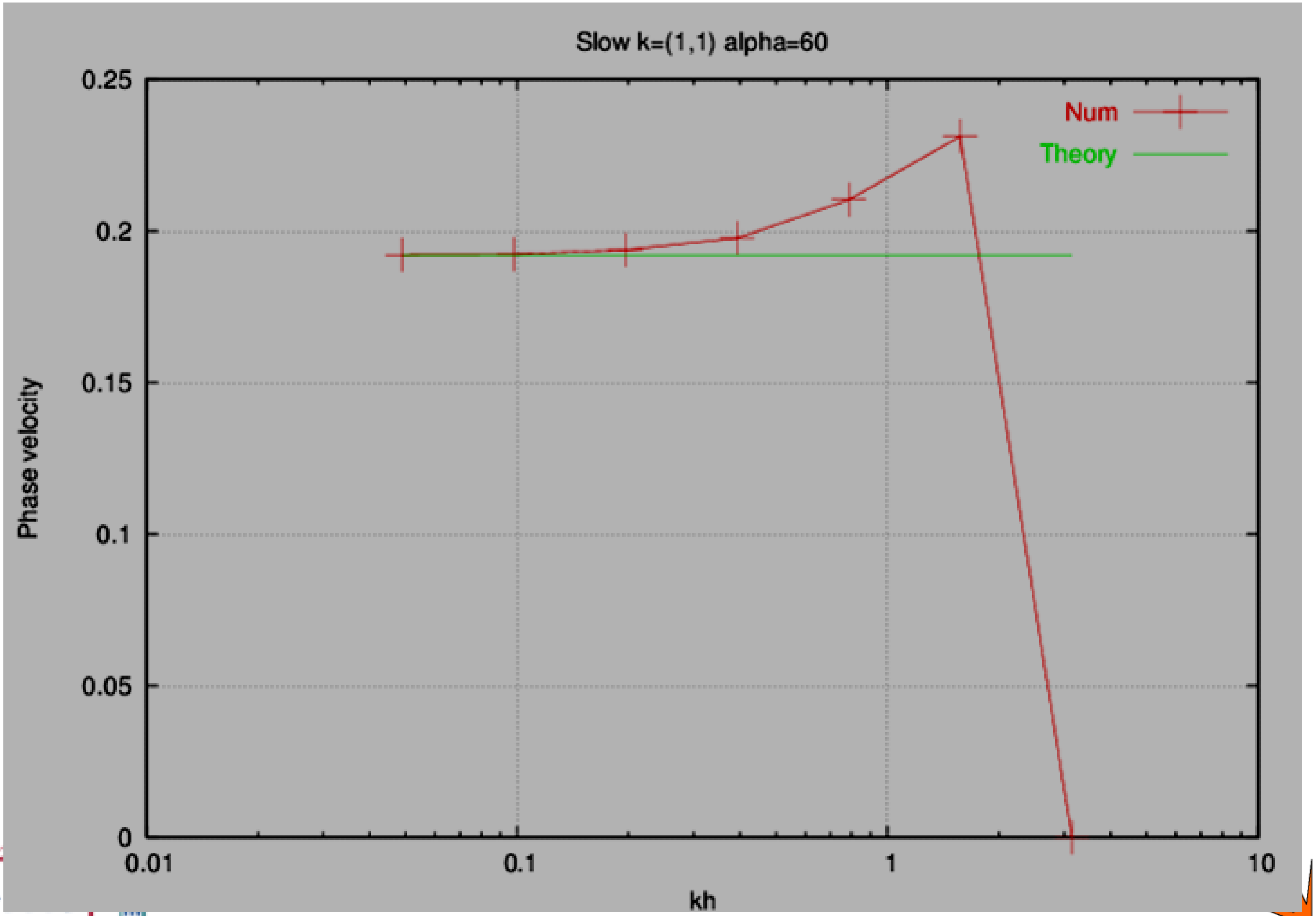
Linear Waves: Accuracy



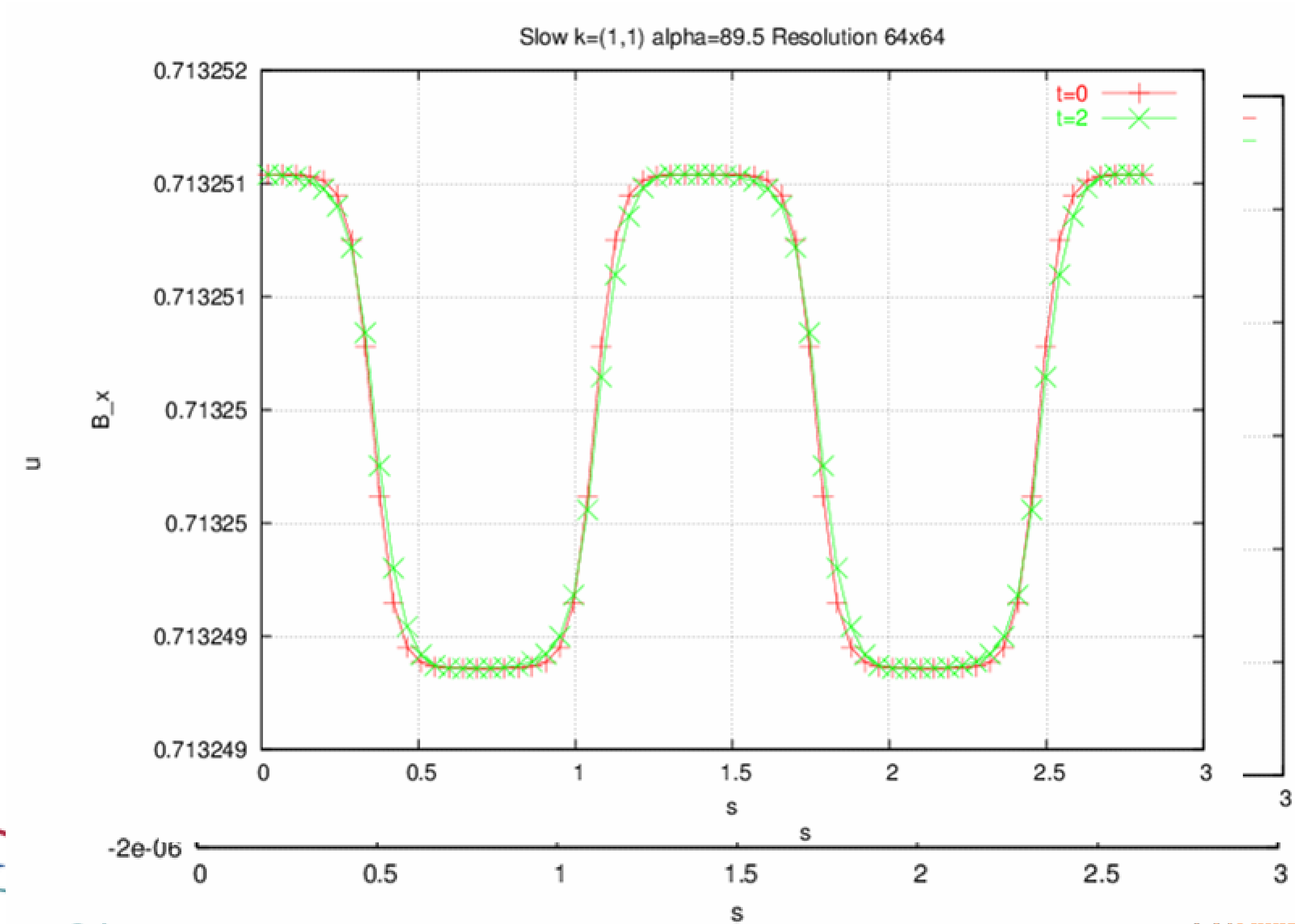
Linear Waves: Accuracy



Linear Waves: Wave number dependence



Linear Waves: Square Wave

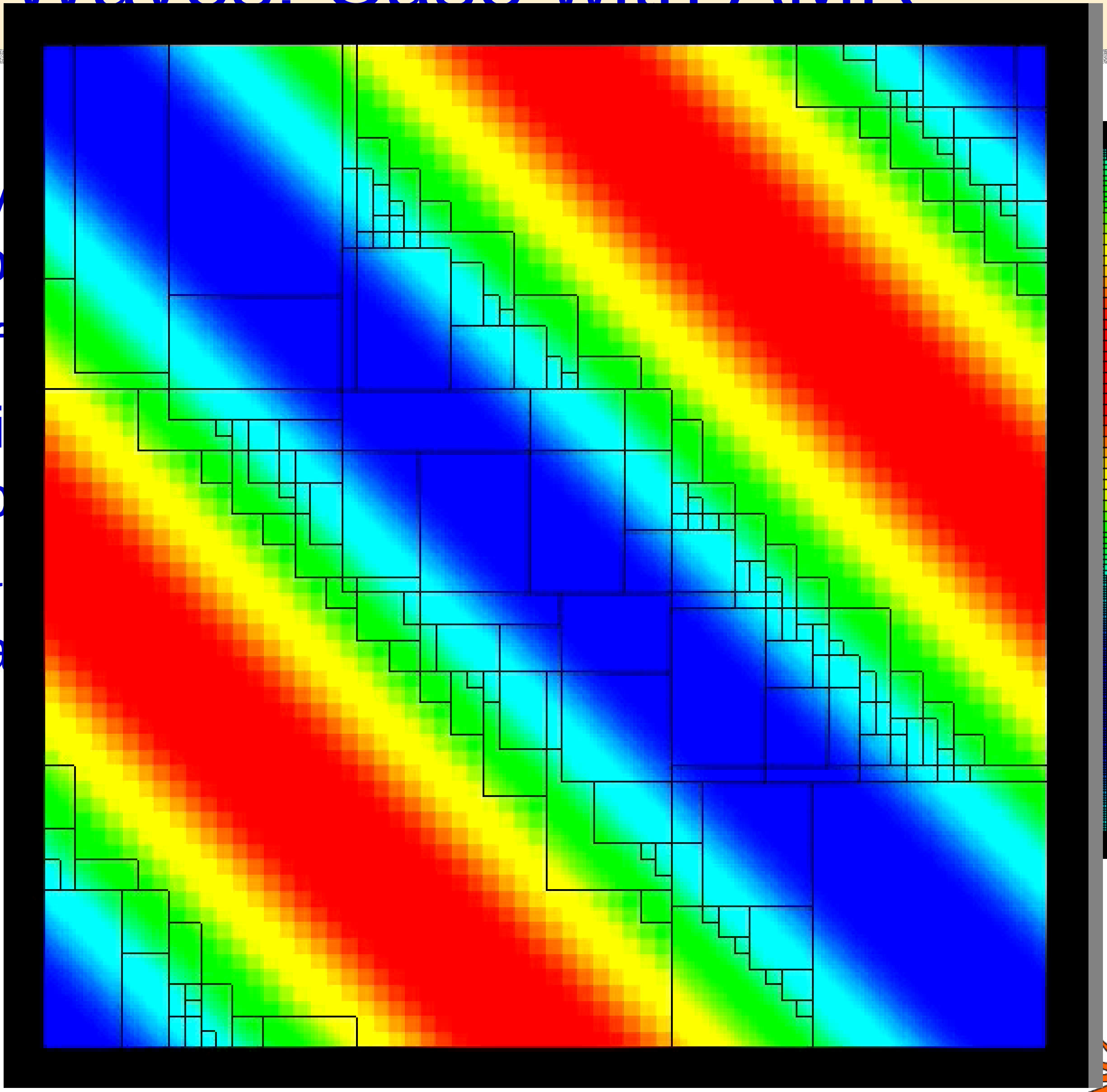


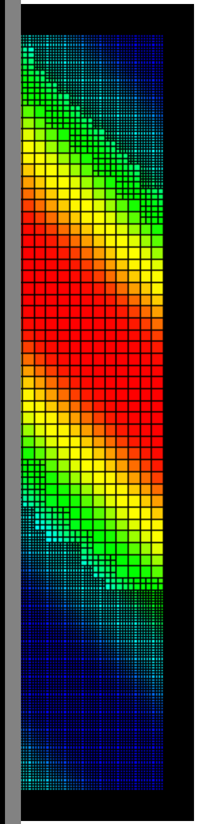
Linear Waves: Summary

- Reproduced the theoretical phase diagram
- Error is O(h^2)
- Similar results for wave vector k=(1,2) and (1,3)
- Energy and mass is conserved to machine precision

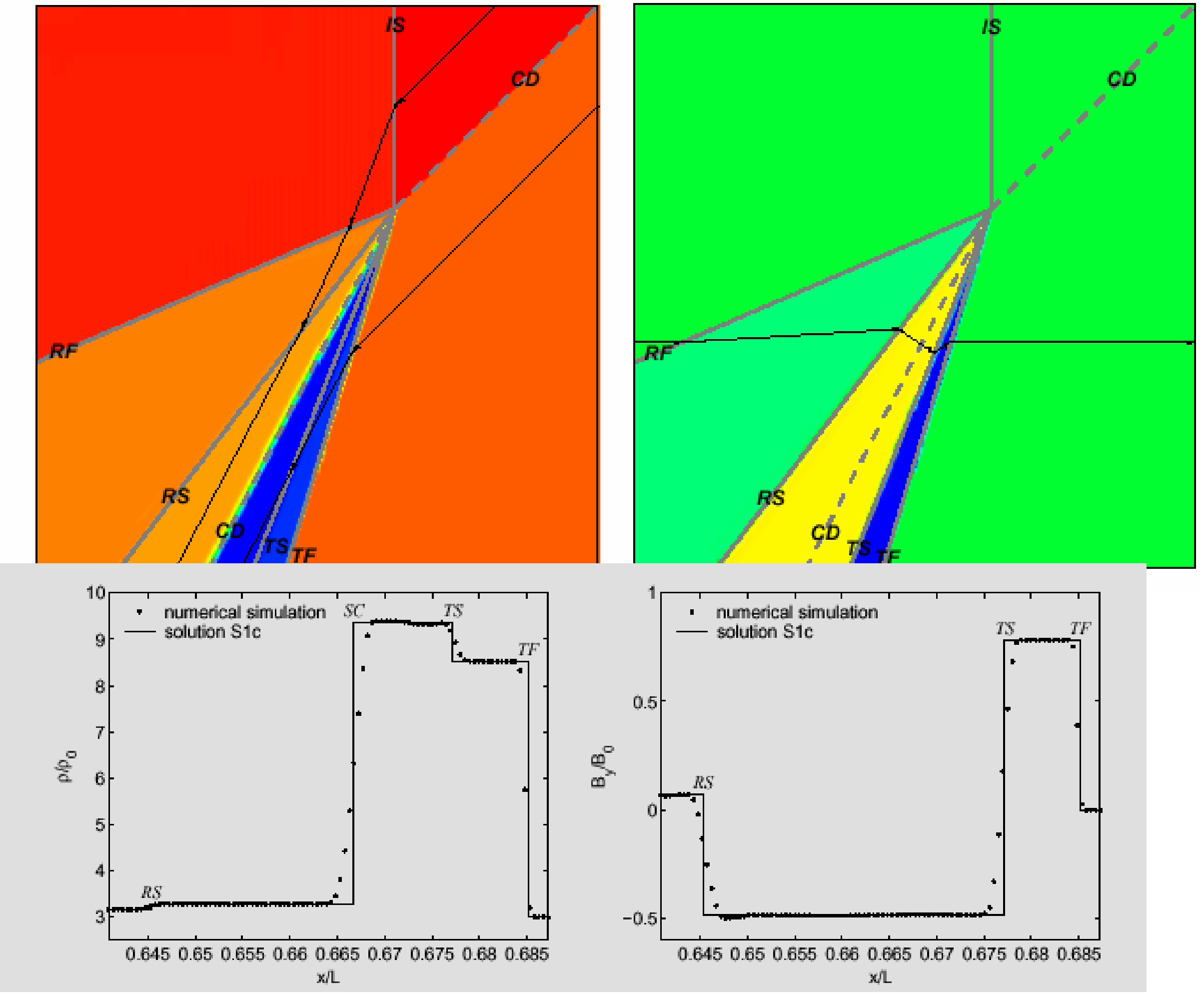
Linear Waves: Case with AMR

- Energy conserv precisio
- Flux-ref
- No obvi reflectio bounda
- Fast wa



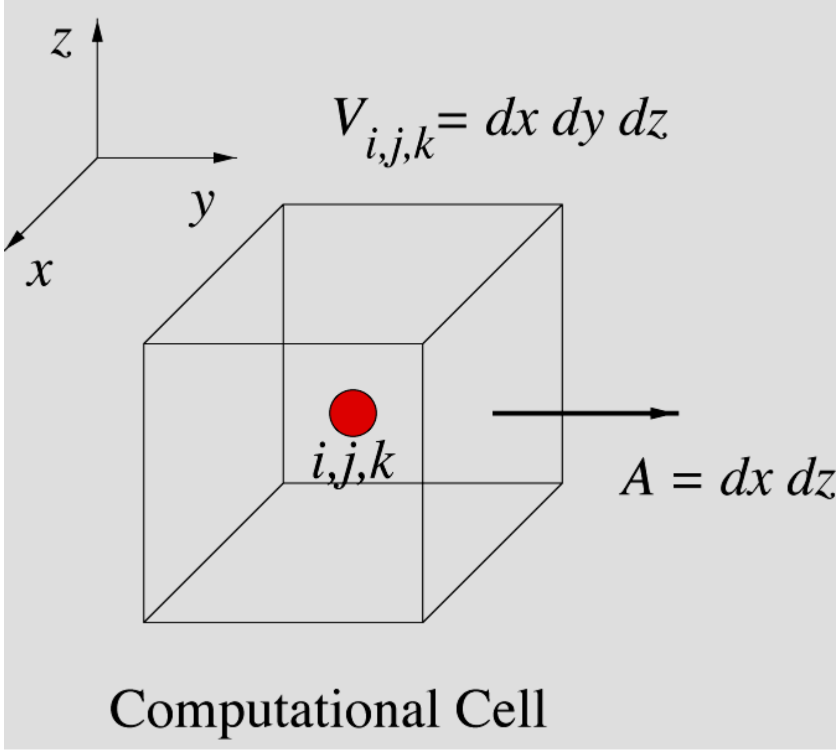


Code Verification: MHD Shock Refraction



- For regular refraction at the contact discontinuity, in a small neighborhood of the point where all discontinuities meet, the MHD PDEs can be reduced to algebraic equations.
- TF and RF are fast shocks
- Local analysis shows that the RS is a slow shock, while shock TS is a 2-4 intermediate shock. (V. Wheatley, D. I. Pullin, R. Samtaney, Journal of Fluid Mechanics)

Numerical Method: Finite Volume Approach



- Conservative (divergence) form of conservation laws:

$$\frac{dU}{dt} + \nabla \cdot F = S$$

- Volume integral for computational cell:

$$\frac{dU_{i,j,k}}{dt} = -\sum_{faces} A \cdot F + S_{i,j,k}$$

- Fluxes of mass, momentum, energy and magnetic field entering from one cell to another through cell interfaces.
- This is a Riemann problem.

Numerical Method

- Hyperbolic fluxes determined using the unsplit upwinding method (Colella, J. Comput. Phys., Vol 87, 1990)
  - *Predictor-corrector (2nd order in time)*
  - *Fluxes obtained by solving Riemann problem*
  - *Good phase error properties due to corner coupling terms*
  - *Modification:* $B_n^{i\S\frac{1}{2}}=B_n^{i}$,

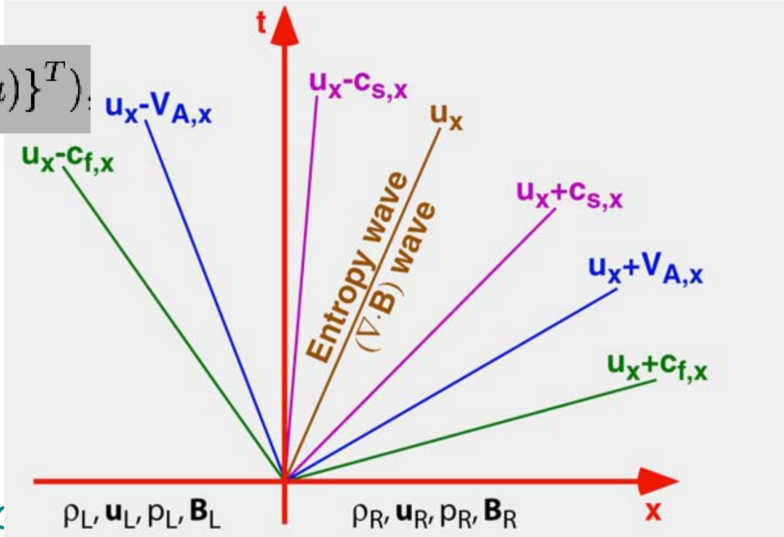
$$F^{n+\frac{1}{2}}_{\boldsymbol{i}+\frac{1}{2}\boldsymbol{e}^d} = R(W^{n+\frac{1}{2}}_{\boldsymbol{i},+,d}, W^{n+\frac{1}{2}}_{\boldsymbol{i}+\boldsymbol{e}^d,-,d}, d)$$

$$U^{n+1}_{\boldsymbol{i}} = U^n_{\boldsymbol{i}} - \frac{\Delta t}{h}\sum_{d=0}^{\mathbf{D}-1}(F^{n+\frac{1}{2}}_{\boldsymbol{i}+\frac{1}{2}\boldsymbol{e}^d} - F^{n+\frac{1}{2}}_{\boldsymbol{i}-\frac{1}{2}\boldsymbol{e}^d})$$

- MHD Equations written in symmetrizable near-conservative form (Godunov, Numerical Methods for Mechanics of Continuum Media, 1, 1972, Powell et al., J. Comput. Phys., vol 154, 1999).

$$S_{\nabla\cdot\mathbf{B}}(U) = -\nabla\cdot\mathbf{B}(\{0, B_R, B_\phi, B_z, u_R, u_z, u_\phi, u_z, (B\cdot u)\}^T).$$

- The symmetrizable MHD equations lead to the 8-wave method.
  - *The fluid velocity advects both the entropy and div(**B**)*
  - *Modification: RP returns average of left/right states for* B_n
  - *Method may be viewed as 7-wave + Stone correc*
    *because in the final corrector step we have projected out **r.B***



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$\nabla\cdot\mathbf{B}=0$ by Projection

- Compute the estimates to the fluxes $F^{n+1/2}_{i+1/2,j}$ using the unsplit formulation
- Use face-centered values of B to compute $\nabla\cdot$ B.
- Solve the Poisson equation $\nabla^2\phi = \nabla\cdot B$
- Correct B at faces: $B=B-\nabla\phi$
- Correct the fluxes $F^{n+1/2}_{i+1/2,j}$ with projected values of B
- Update conservative variables using the fluxes
- Poisson equation solved using multigrid using GSRB for smoothing and BiCGStab as the bottom smoother

Status of AMRMHD Code

- Solves single fluid resistive MHD equations written in conservation form
  - *Used to study reconnection*
  - *Both explicit and implicit treatment of resistive/viscous term*
  - *Implicit treatment requires variable coefficient elliptic solvers with AMR*
    - *Elliptic solvers require solve of the full hierarchy for during synchronization of coarse-fine boundary fluxes*
    - *Handling nonlinear properties still not implemented*
- Study of pellet injection in tokamaks
  - *Source terms to handle toroidal geometry (See poster 1C33 Monday 10AM-Noon)*
  - *Differences in HFS and LFS pellet lauches*
- Work in progress towards fully nonlinearly implicit Jacobian Free Newton-Krylov implementation (with D. Reynolds and C. Woodwards, TOPS Center, LLNL)
  - *Works for simple test problems for compressible MHD equations without preconditioning and without mesh adaptivity*

Future Directions

- 3D wave propagation tests
- Mapped grids (flux tube coordinates) for pellet injection simulations
- Higher order (fourth order) to better handle anisotropy

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