Status of Time-split Semi-implicit Hall Implementation 4/24/04

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- We are implementing a semi-implicit advance as part of two-fluid development.
- The Hall si operator is similar to the Harned-Mikic operator.

$$\Delta\mathbf{B} - \nabla\times\left(\frac{\Delta t}{\mu_0 ne}\left(\nabla\times\nabla\times\left(\frac{\Delta t}{\mu_0 ne}(\nabla\times\Delta\mathbf{B})\times\mathbf{B}_0\right)\right)\times\mathbf{B}_0\right) = rhs$$

- Basis functions with C^0 continuity require an auxiliary field.
- We are using SuperLU to solve the nonsymmetric 6-vector linear system.

Implementation Status

- A 6-vector Hall advance was functioning before APS '03 for Hall-only periodic systems.

$$\int d\mathbf{x}\,\mathbf{A}\cdot\Delta\mathbf{B}+\int d\mathbf{x}\,\nabla\times\mathbf{f}\cdot\left(\frac{\Delta t}{\mu_0 ne}(\nabla\times\mathbf{A})\times\mathbf{B}_0\right)=rhs$$

$$\int d\mathbf{x}\,\mathbf{g}\cdot\mathbf{f}\quad-\int d\mathbf{x}\,\nabla\times\mathbf{g}\cdot\left(\frac{\Delta t}{\mu_0 ne}(\nabla\times\Delta\mathbf{B})\times\mathbf{B}_0\right)=\mathbf{0}$$

- Bounded systems produced numerical instabilities.
- The "integrand" routine was re-written with temporary arrays representing

$$\nabla\times\vec{\alpha}_{\nu i}\qquad\qquad(\nabla\times\vec{\alpha}_{\nu i})\times\vec{B}_0$$

in matrix form at Gaussian quadrature points.

array(res-vec index , ν-index, i-index, hori-cell, vert-cell)

Implementation Status (cont.)

- With unsplit div(B) diffusion, this version is numerically stable for Hall-only bounded systems.

$$\int d\mathbf{x}\,\mathbf{A}\cdot\Delta\mathbf{B}+\Delta t\kappa_b\int d\mathbf{x}\nabla\cdot\mathbf{A}\nabla\cdot\Delta\mathbf{B}+\int d\mathbf{x}\nabla\times\mathbf{f}\cdot\left(\frac{\Delta t}{\mu_0 ne}(\nabla\times\mathbf{A})\times\mathbf{B}_0\right)=rhs-\Delta t\kappa_b\int d\mathbf{x}\nabla\cdot\mathbf{A}\nabla\cdot\mathbf{B}$$

$$\int d\mathbf{x}\,\mathbf{g}\cdot\mathbf{f}\quad-\int d\mathbf{x}\nabla\times\mathbf{g}\cdot\left(\frac{\Delta t}{\mu_0 ne}(\nabla\times\Delta\mathbf{B})\times\mathbf{B}_0\right)=\mathbf{0}$$

- Successful Hall-only tests include:
  - box domains with all sides, two sides, and only Fourier direction periodic with arbitrary $\mathbf{k}$, $\mathbf{B}_0$
  - cylindrical domains with & without $\mathbf{J}_0$
- *Extreme* large-density limit is numerically unstable.
- Even simple periodic boxes are unstable with Hall and MHD.
  - diagnosing simplest case should be relatively easy
- See Hao's poster Tuesday morning for more implementation information.