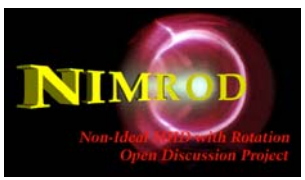
Two-fluid FRC Calculations with NIMROD

D. C. Barnes for the NIMROD team



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Time-centered implicit 2-fluid NIMROD

- First-order in time system

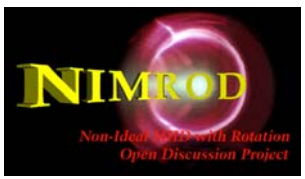
$$\frac{d}{dt}\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix} = F\left[\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix}, \frac{d}{dt}\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix}, t\right] \Rightarrow \frac{d}{dt}\delta\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix} = F_0 + \begin{bmatrix}0 & \mathcal{P}U\\ \mathcal{P}L & 0\end{bmatrix}\delta\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix}$$

$$\frac{1}{\Delta t}\delta\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix} = F_0 + \begin{bmatrix}0 & \mathcal{P}U\\ \mathcal{P}L & 0\end{bmatrix}\delta\begin{pmatrix}\mathbf{B}\\ P\\ \mathbf{v}\end{pmatrix}$$

- Make **B** and P “slaves” to **v**

$$\frac{\delta\mathbf{v}}{\Delta t} = F_{\mathbf{v}0} + \mathcal{P}L\mathcal{P}'U\delta\mathbf{v} \approx F_{\mathbf{v}0} + \mathcal{P}LU\delta\mathbf{v}$$

- High k requires iteration



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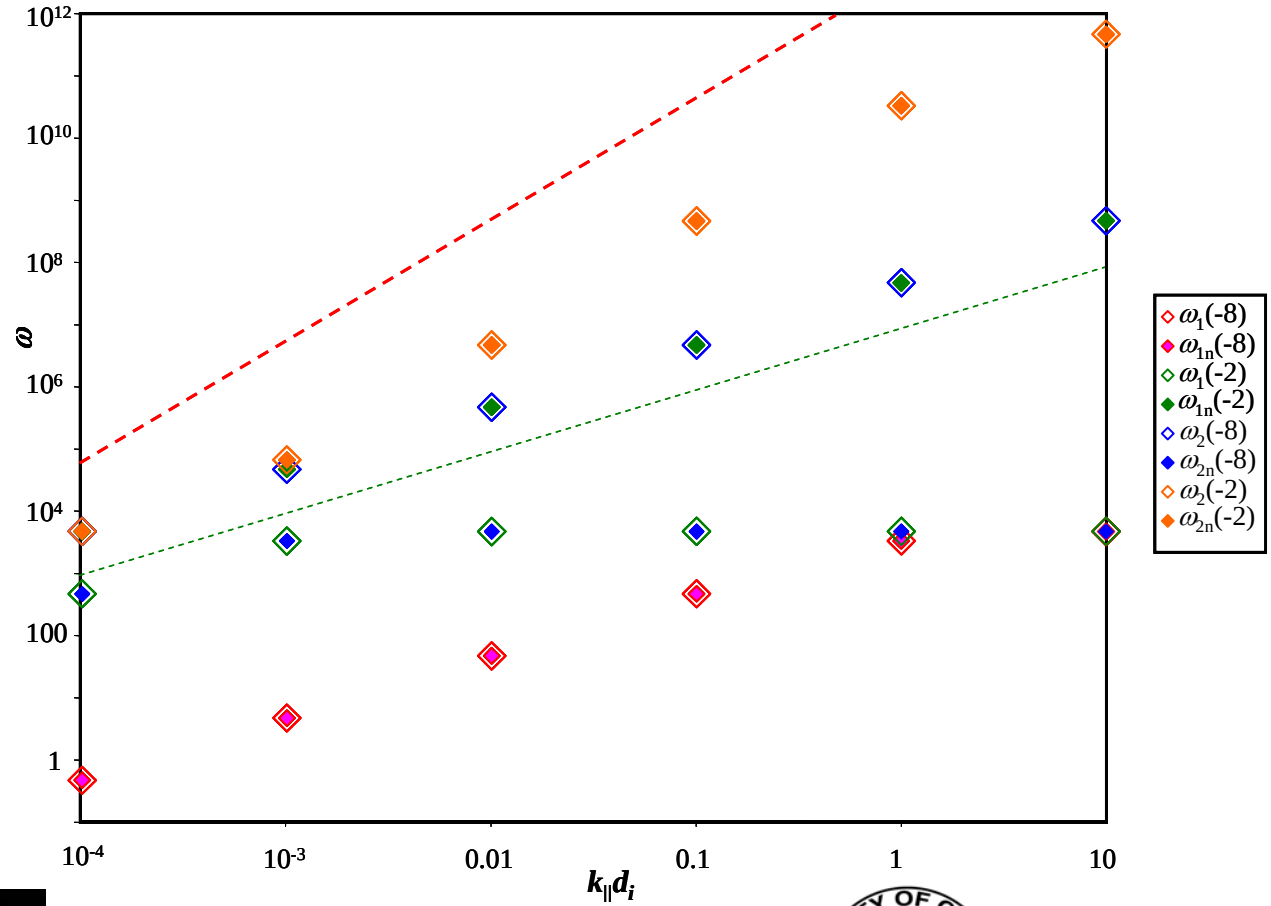
Some details

- Use 3 time levels on right to get damping
  - Similar to BDF2 scheme
- Get asymmetric matrix
  - Neglect toroidal harmonic coupling
  - Use direct inversion (SuperLU)
- Accelerate convergence

Status

- NIMROD 2F implicit implementation completed
  - All linear terms
  - Most nonlinear terms
- QGMRES implemented for convergence acceleration
- FRC results obtained which differ somewhat from previous numerical results

(Last time…) Waves in box show good Dispersion



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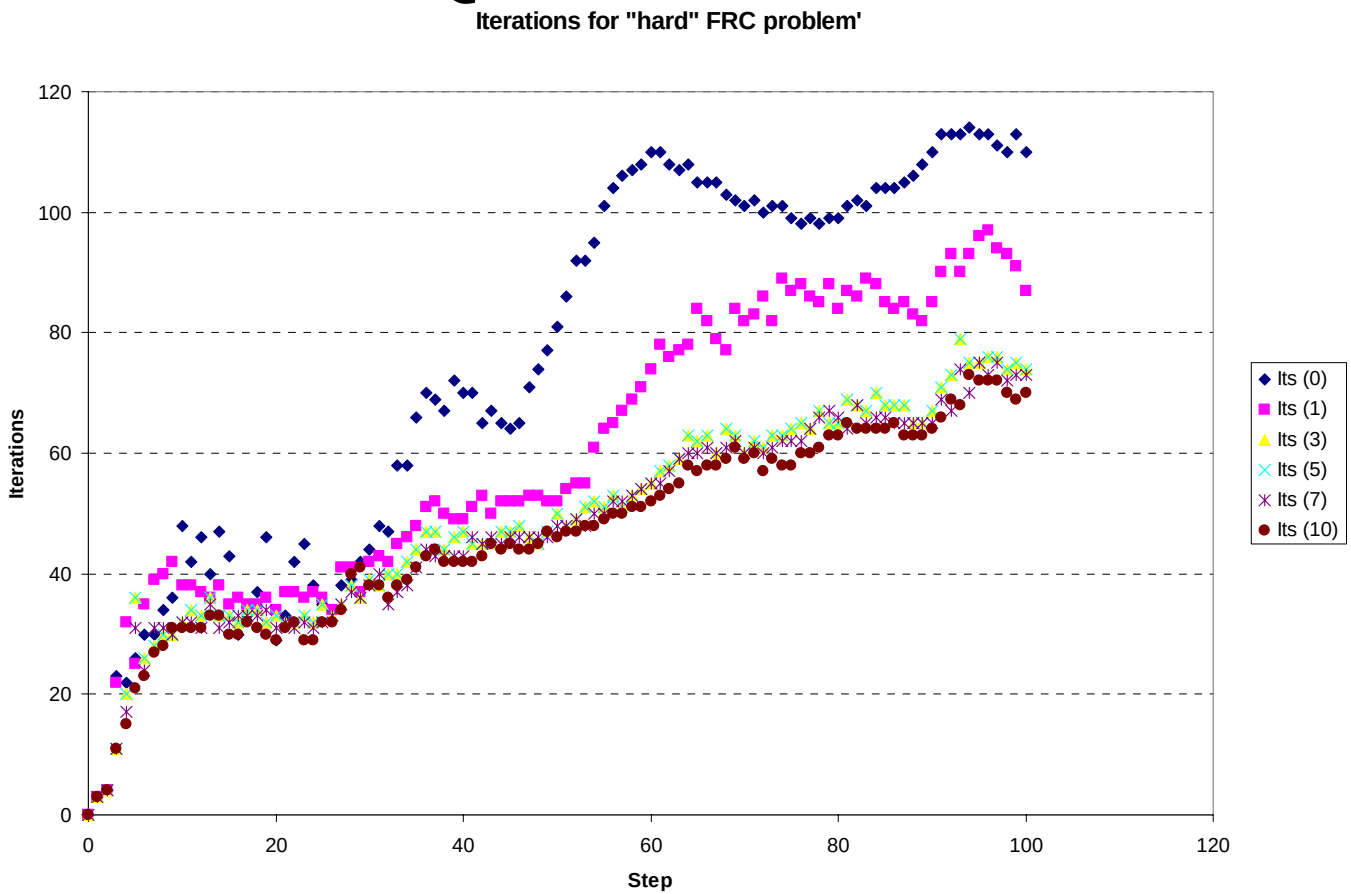
Convergence acceleration *via* QGMRES



Increasing number of past time levels stored accelerates convergence

> 5 levels give diminishing returns, so used 5 for production

Previous FRC tilt work

- Previous numerical work on Hall MHD
  - Stabilization of "fundamental" (rigid shift per ψ surface)
  - "Higher" parallel structure modes with fraction of MHD growth rate
- Long-thin analysis shows
  - "Perfect" agreement with MHD and HMHD fundamental
  - *No* higher modes
- To resolve dichotomy, need
  - Better codes (improve NIMROD 2-fluid to allow big Δt)
  - Better theory (has to agree with codes)

FRC Results

- Use Barnes long-thin equilibrium
- MHD baseline
- 2-fluid with varying d_i

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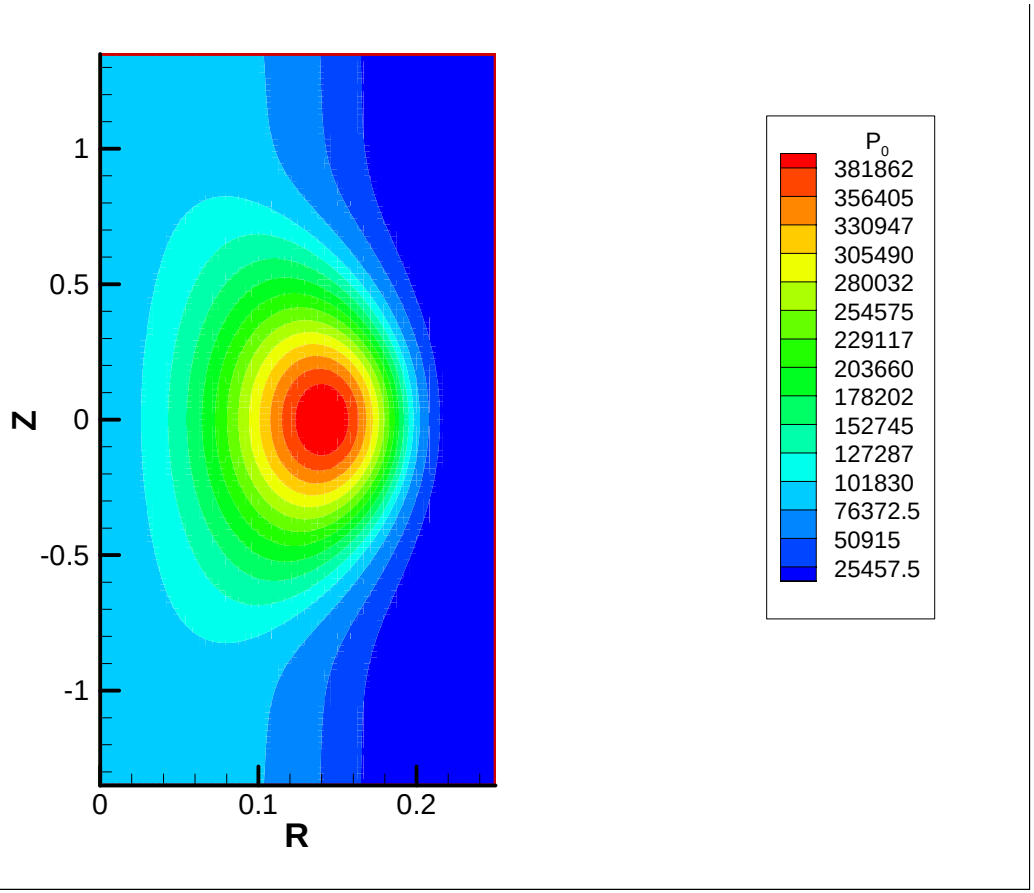
Barnes long-thin FRC Equilibrium (6.25:1)



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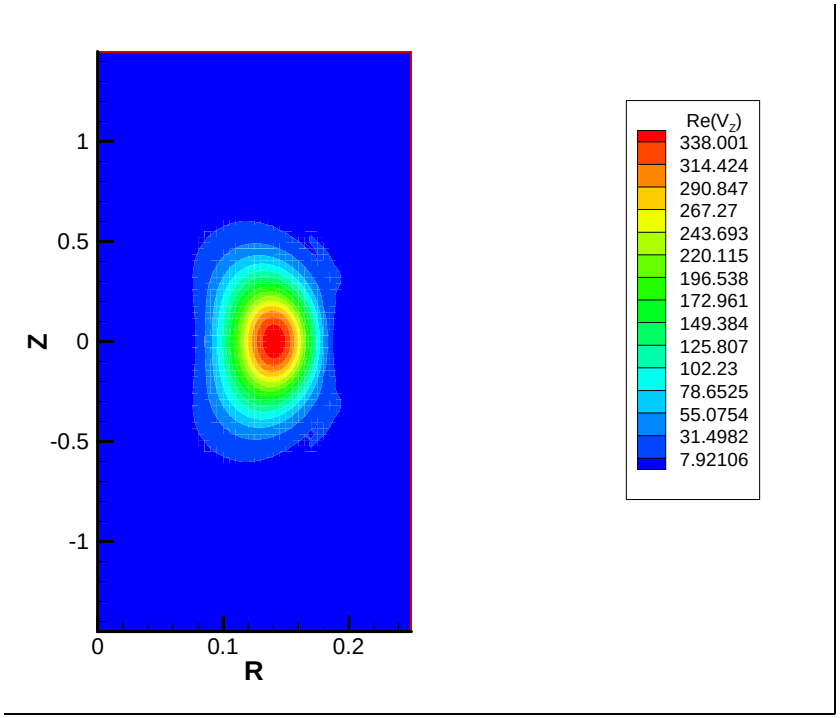
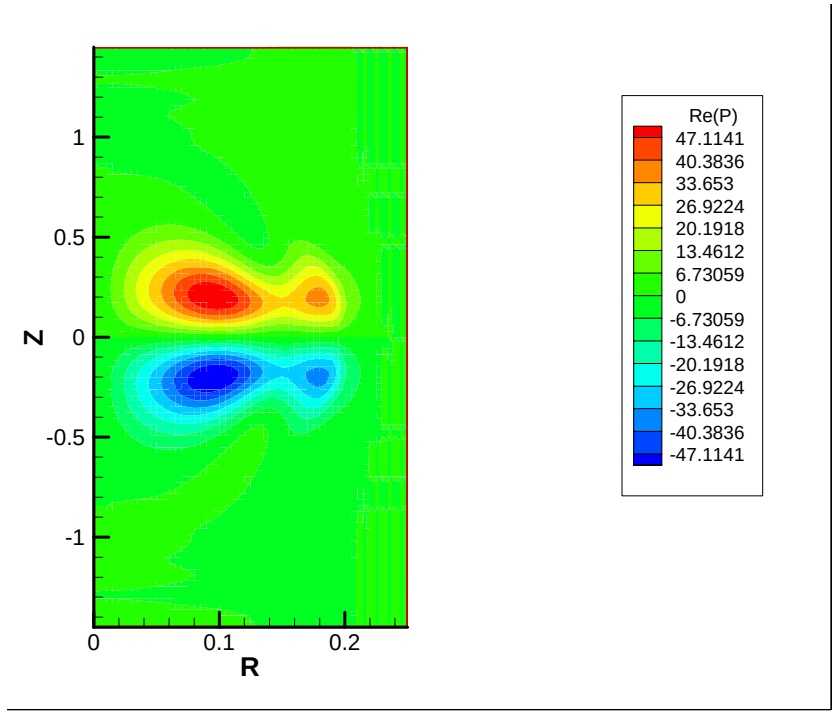
NIMROD 1&2-fluid Reproduces Previous Numerical FRC Result



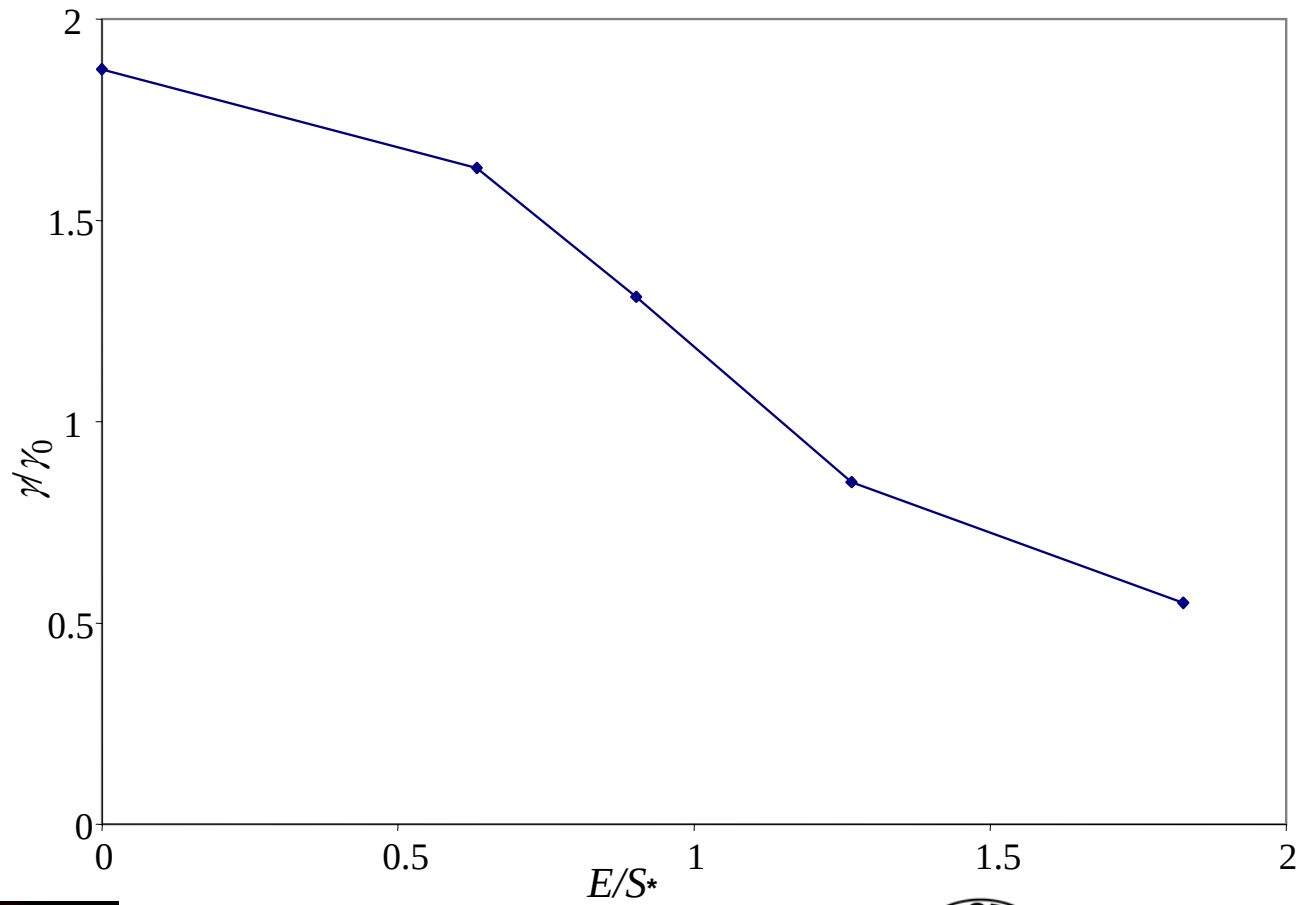


Z-velocity Pressure

Single fluid (MHD): S = 2. x 10^6, Pm = 10, $\gamma = 1.88\ \gamma_0$

MHD mode is rigid shift of each ψ surface with growth $\gamma \sim L_z/2\ v_A$

Growth rate vs. kineticity



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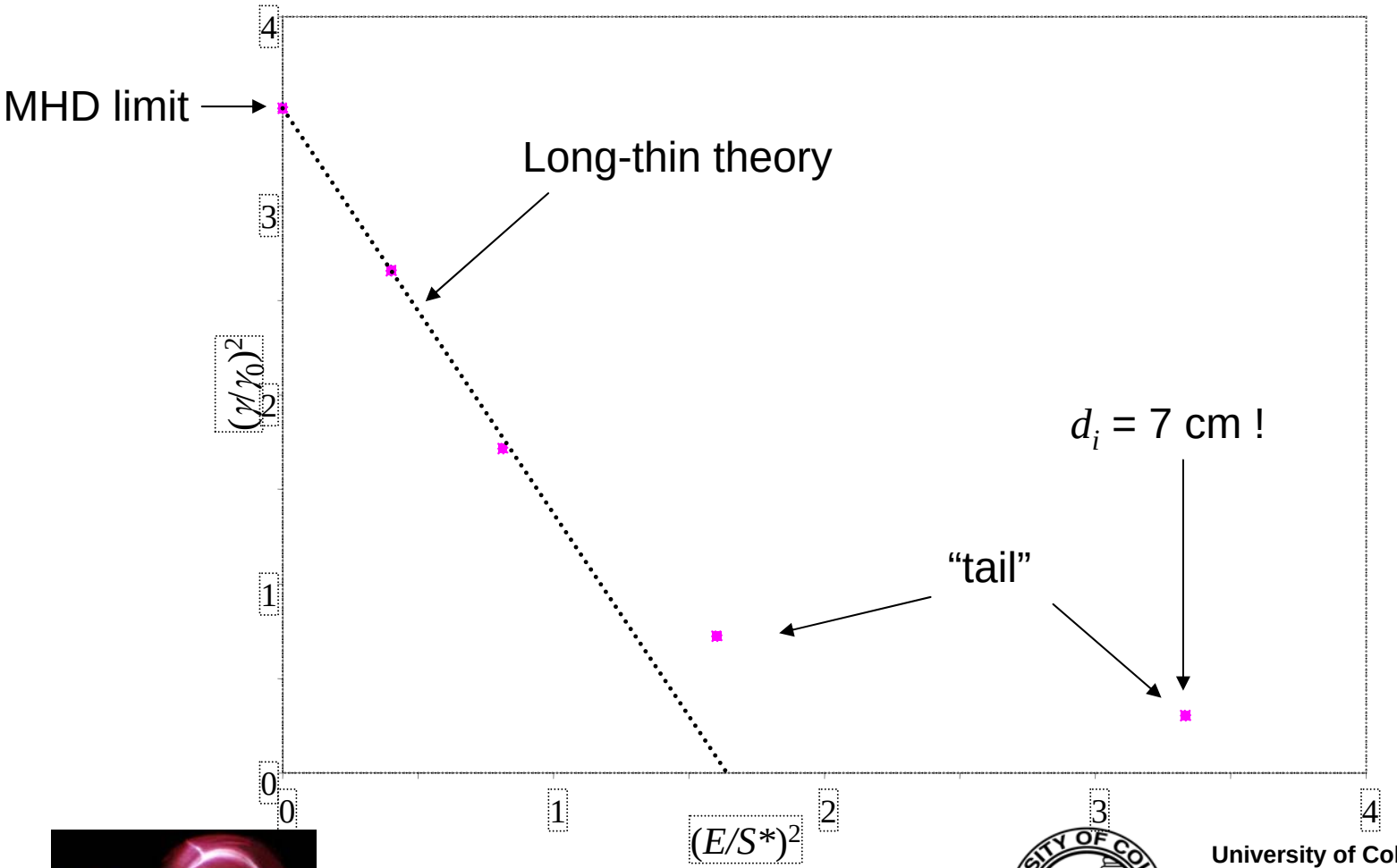
Growth rate vs. kineticity



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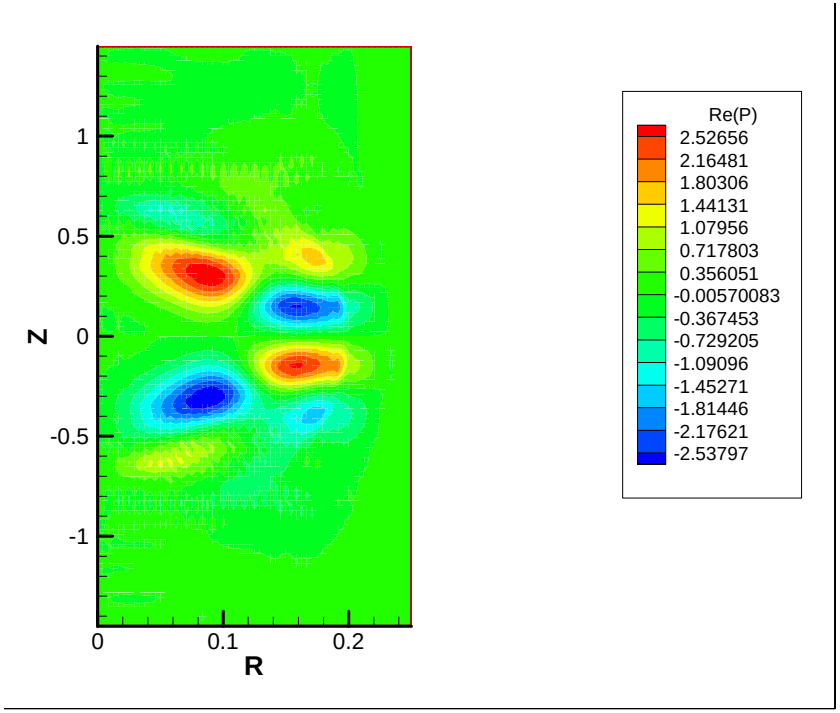
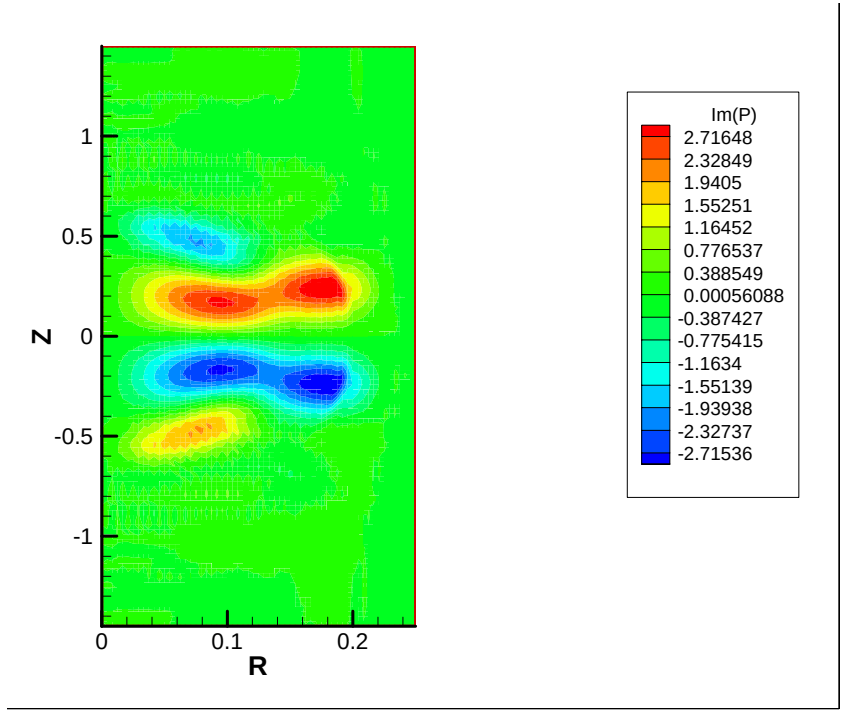
Transition case shows z structure in P





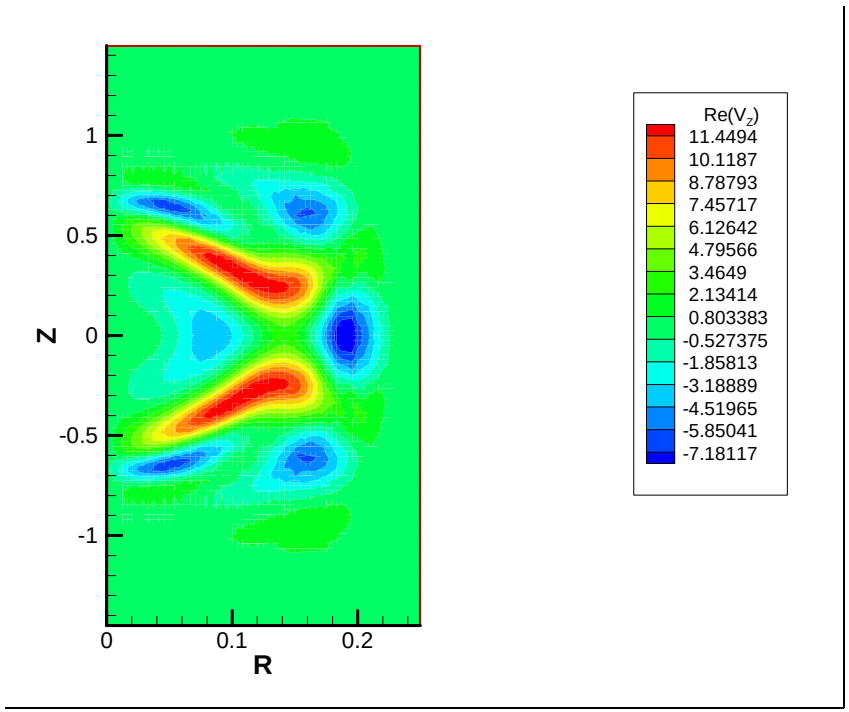
Re Pressure

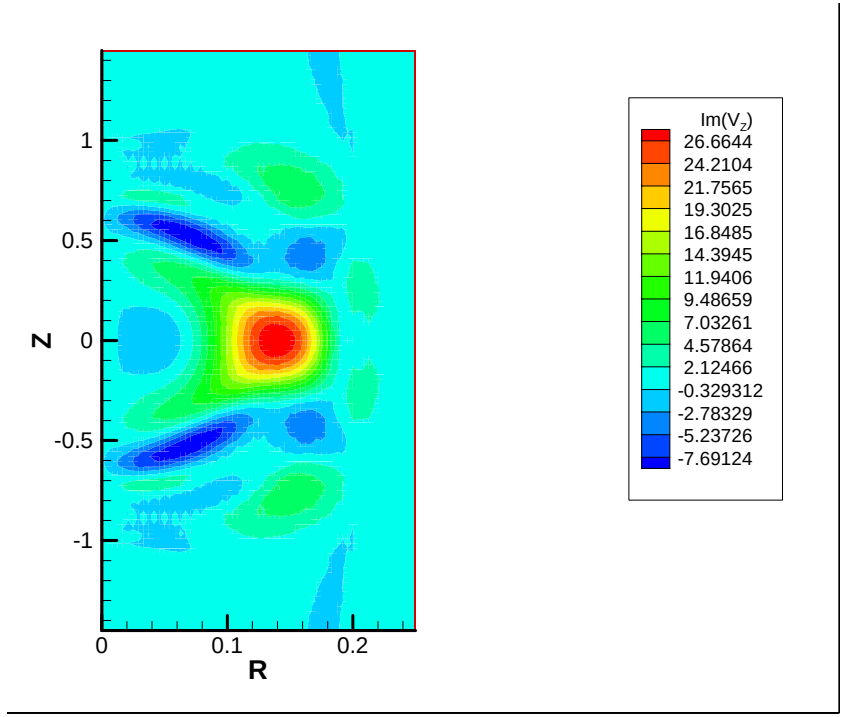
Im Pressure

Two fluid: $S = 2 \times 10^6$, $Pm = 10$, $r_s/d_i = 4.94$ $\gamma = 0.85\ \gamma_0$

Two-fluid mode is modification of single fluid mode

… and in v_z?



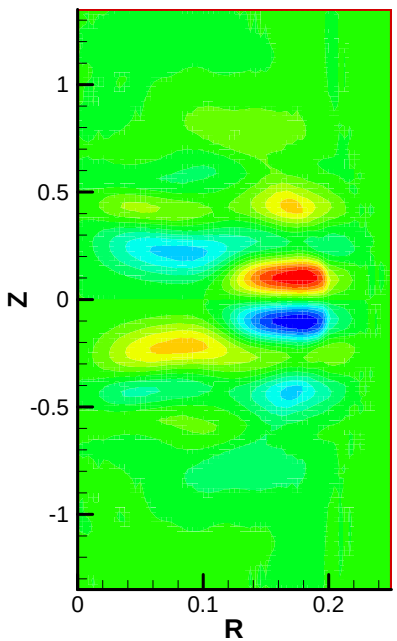


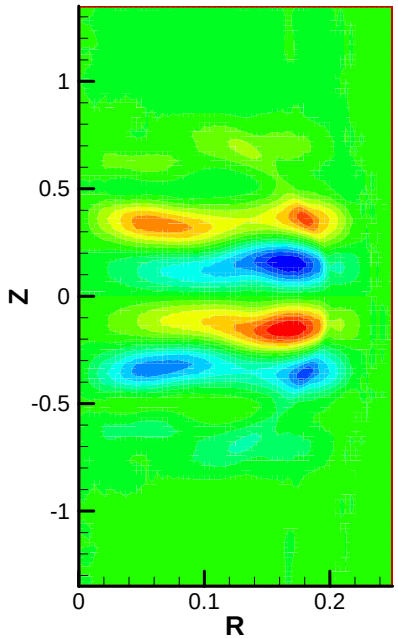
Re v_z Im vz

Two fluid: S = 2 x 10^6, Pm = 10, r_s/d_i = 4.94 $\gamma = 0.85\ \gamma_0$

Two-fluid mode is modification of single fluid mode

Most kinetic case shows higher structure in P





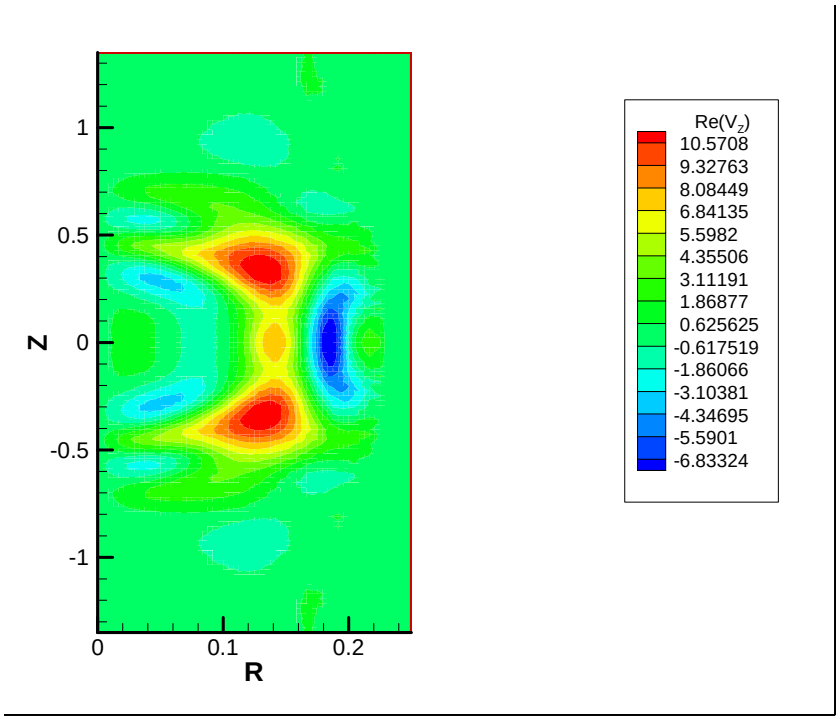
Re Pressure

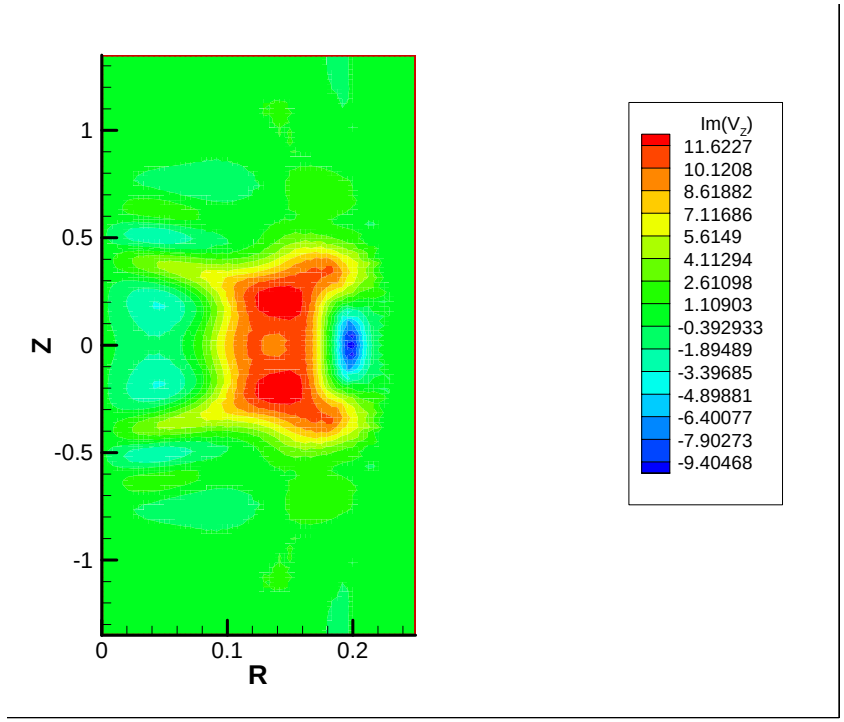
Im Pressure

Two fluid: S = 2 x 10^6, Pm = 10, r_s/d_i = 3.44 γ = 0.55 γ_0

Two-fluid mode is on outer ψ surfaces and has parallel structure

… and in v_z?





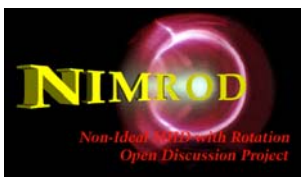
Re v_z Im vz

Two fluid: S = 2 x 10^6, Pm = 10, r_s/d_i = 3.44 γ = 0.55 γ_0

Two-fluid mode is on outer ψ surfaces and has parallel structure

Summary

- NIMROD 2F time-centered implicit operational with effective Δt (same as 1-fluid, just resolve mode time scale)
- Higher parallel modes seen by NIMROD
- Mode structure not so different (in comparison with past results) from MHD
  - Maybe negative energy mode destabilized by dissipation?



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Future work

- Q205
  - Updated, optimized 2F version available
  - Nonlinear implemented and tested
  - Tokamak application?
- Q305
  - Direct solve of all linear time-centered equations implemented (iteration for nonlinearity only)