Progress on Modeling the CDX-U Sawtooth with M3D

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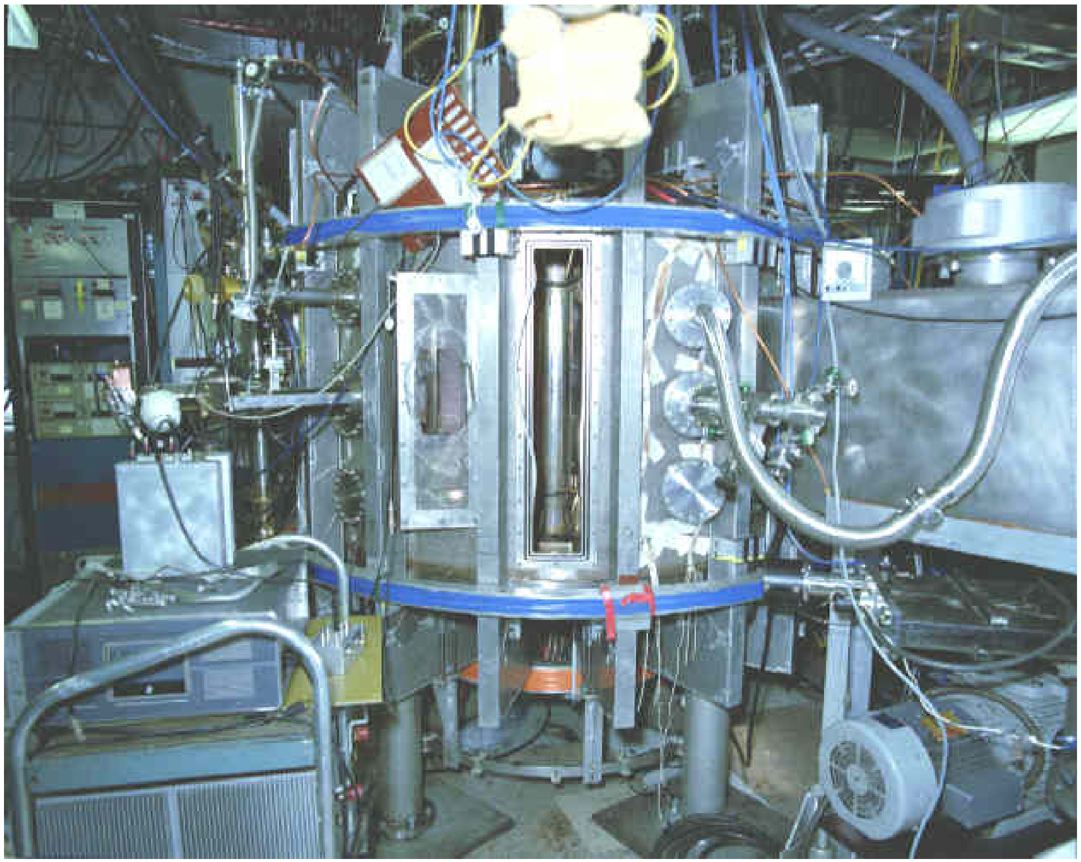
Motivation

- Model resistive MHD events in a tokamak plasma using realistic physical values to make quantitative predictions.

  - Large tokamaks have large disparities in spatial and temporal scales to be resolved.
    - Resistive MHD: Current sheet thickness ~ $S^{-1/2}$
    - Two-fluid MHD: ion skin depth ~ $n^{-1/2}$

  - Small tokamaks operate in regimes accessible to present-day codes.

Characteristics of the Current Drive Experiment Upgrade (CDX-U)

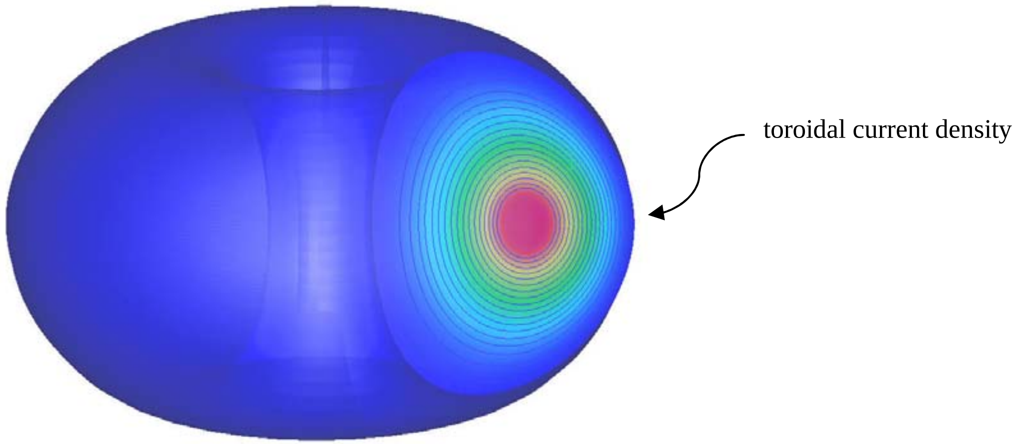


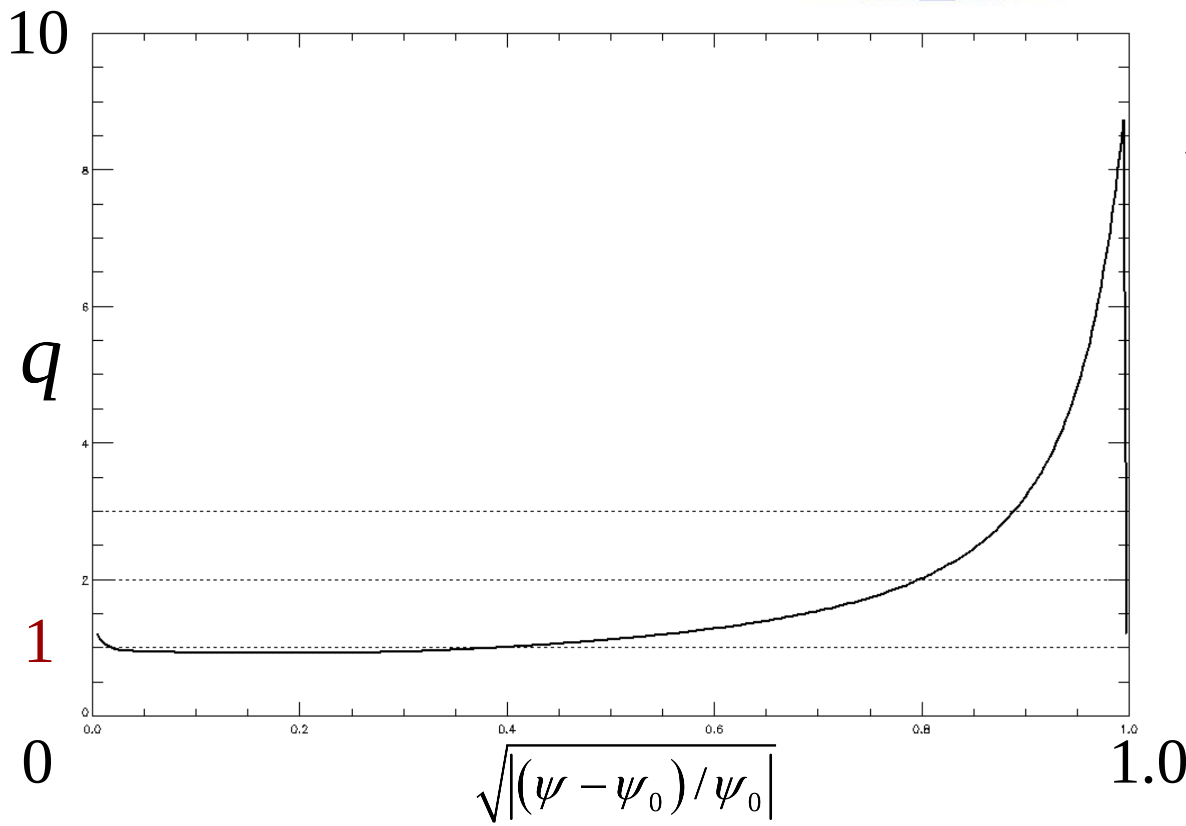
- Low aspect ratio tokamak (R_0/a = 1.4 – 1.5)
- Small (R_0 = 33.5 cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $I_p \sim 70$ kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV $\rightarrow$ S ~ 10^4
- Discharge time ~ 12 ms

- Soft X-ray signals from typical discharges indicate two predominant types of low-n MHD activity:
  - sawteeth
  - “snakes”

Equilibrium: $q_0 < 1$

- Equilibrium taken from a TSC sequence (Jsolver file).
- $q_{\min} \approx 0.922$
- $q(a) \sim 9$





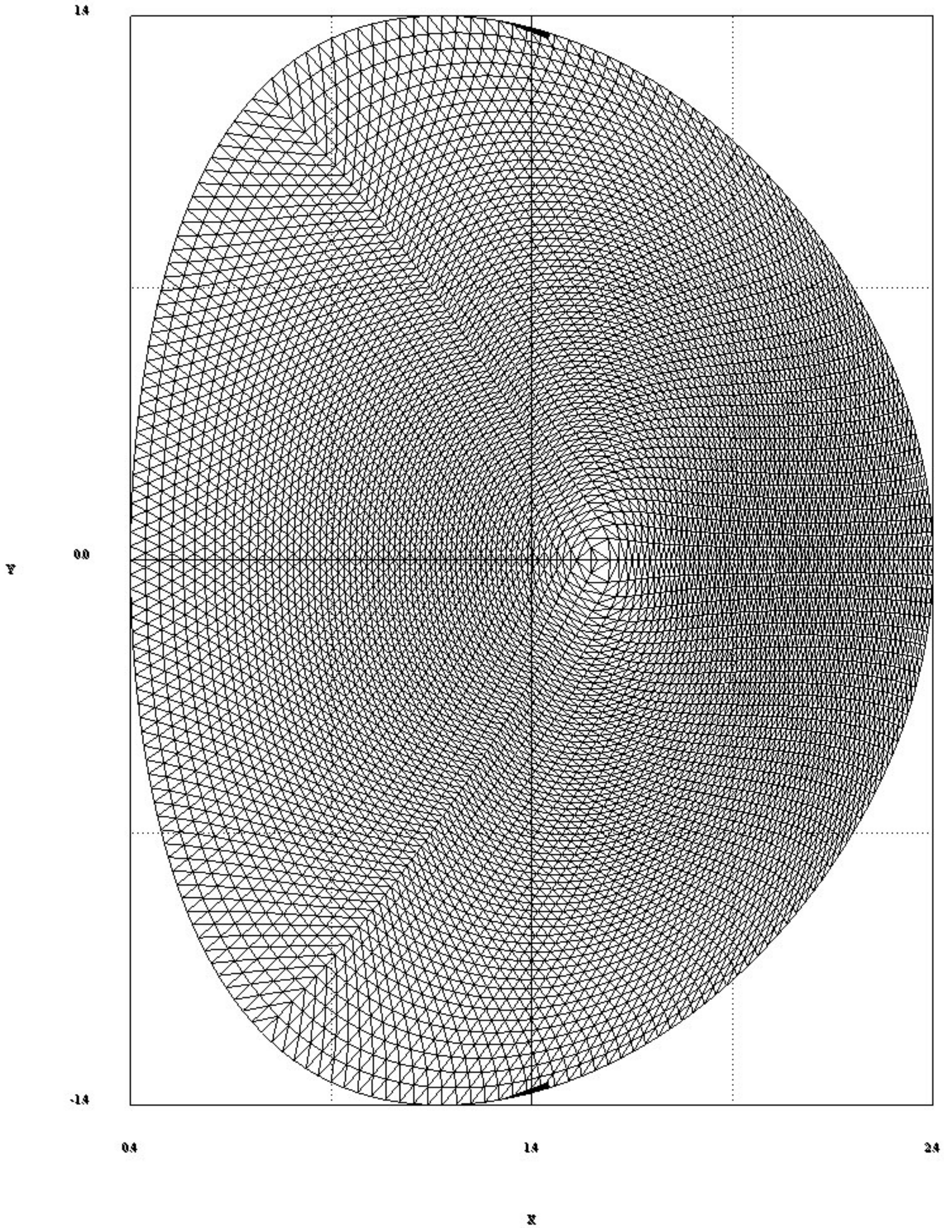
Questions to investigate:

- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Baseline Parameters for CDX

Lundquist Number S	~2×10^4 on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number Pr	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	Original study: 0 Followup study: 200 m^2/s (measured value)
Parallel thermal conduction (sound wave)	Original study: 0 Followup study: $V_{Te} = 6\ V_A$
Peak Plasma β	~ 3×10^{-2} (low-beta).
Density Evolution	Turned on for nonlinear phase.
Nonlinear initialization	Pure n=1 perturbation such that $\frac{\max(B^1_{\text{pol}})}{\max(B^0_\phi)} = 10^{-4}$

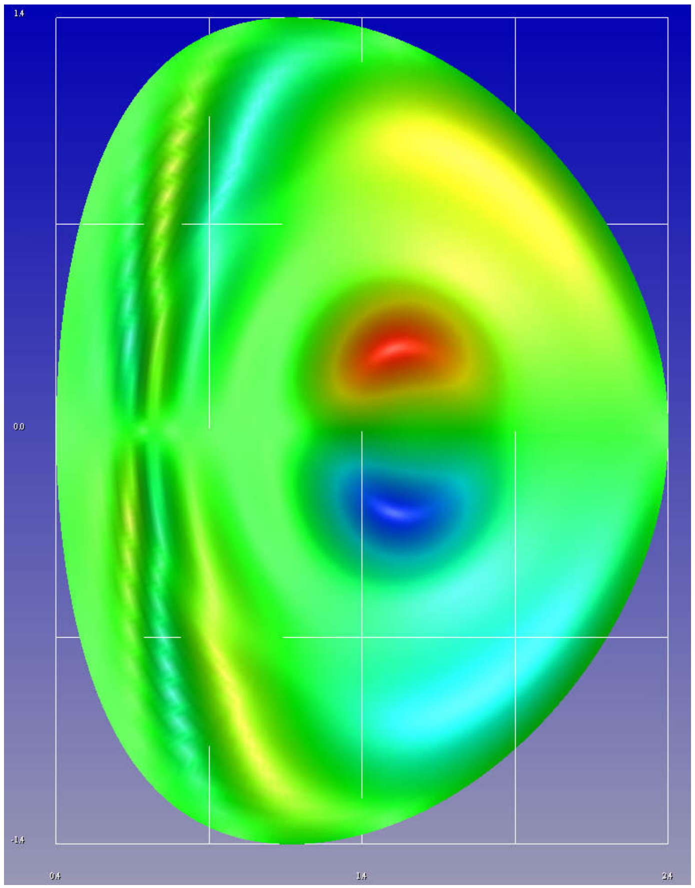
Poloidal Mesh for CDX



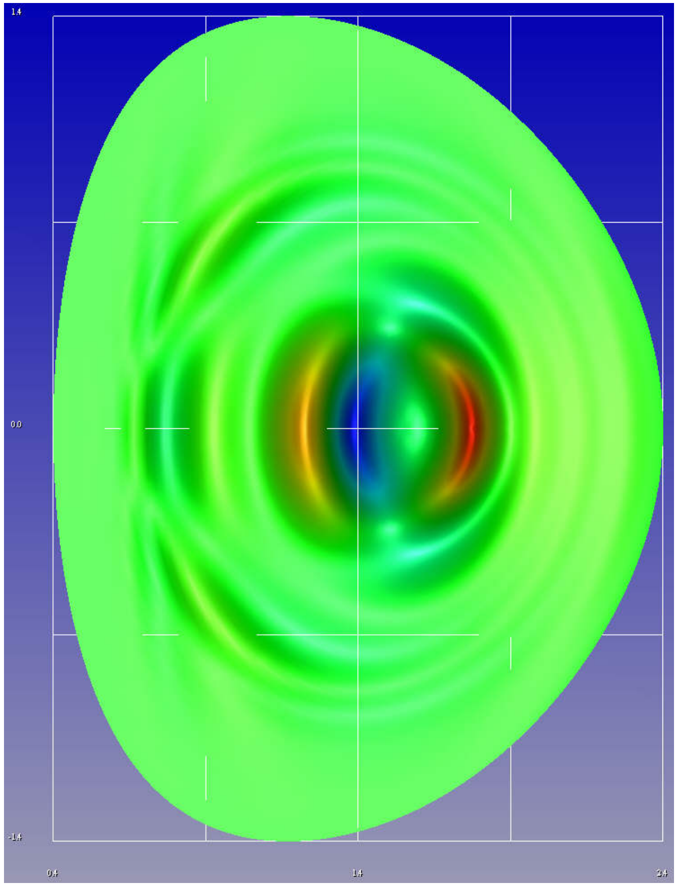
- 89 radial zones, up to 267 in θ in unstructured mesh
- Linear basis functions on triangular elements
- Conducting wall; current drive applied by adding a source term in Ohm's law.
- Finite differences toroidally; 24 planes

n=1 Eigenmode

Incompressible velocity
stream function U



Toroidal current density
J_ϕ

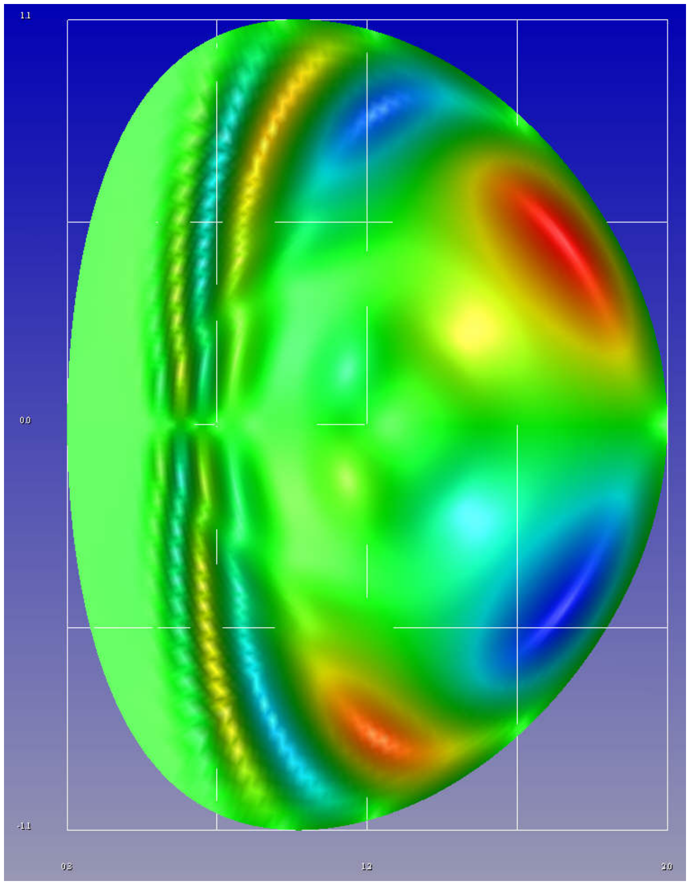


$\gamma\,\tau_A = 8.61 \times 10^{-3} \rightarrow$ growth time = 116 τ_A

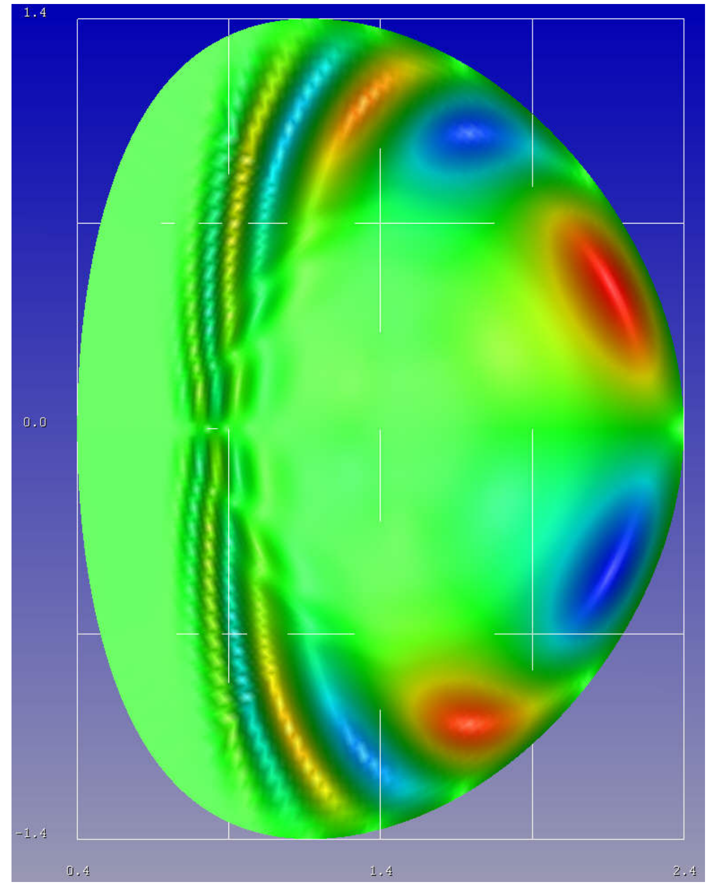
Higher *n* Eigenmodes

Incompressible velocity
stream function *U*

$n = 2$



$n = 3$



. . .

m ≥ 5
$\gamma \tau_A = 1.28 \times 10^{-2}$

m ≥ 7
$\gamma \tau_A = 1.71 \times 10^{-2}$

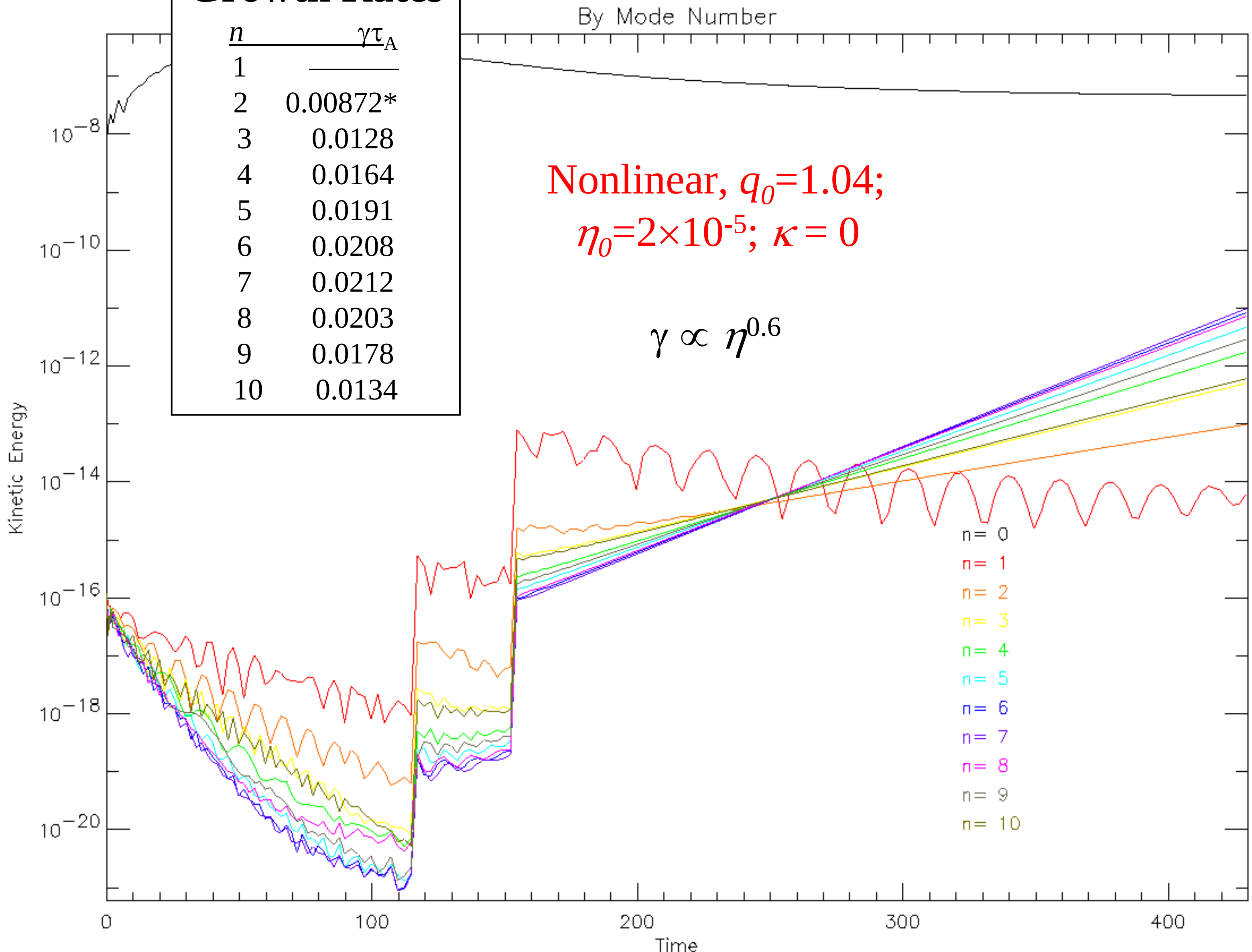
In Absence of Heat Conduction, Higher *n* Resistive Ballooning Modes are More Unstable than Internal Kink

Growth Rates

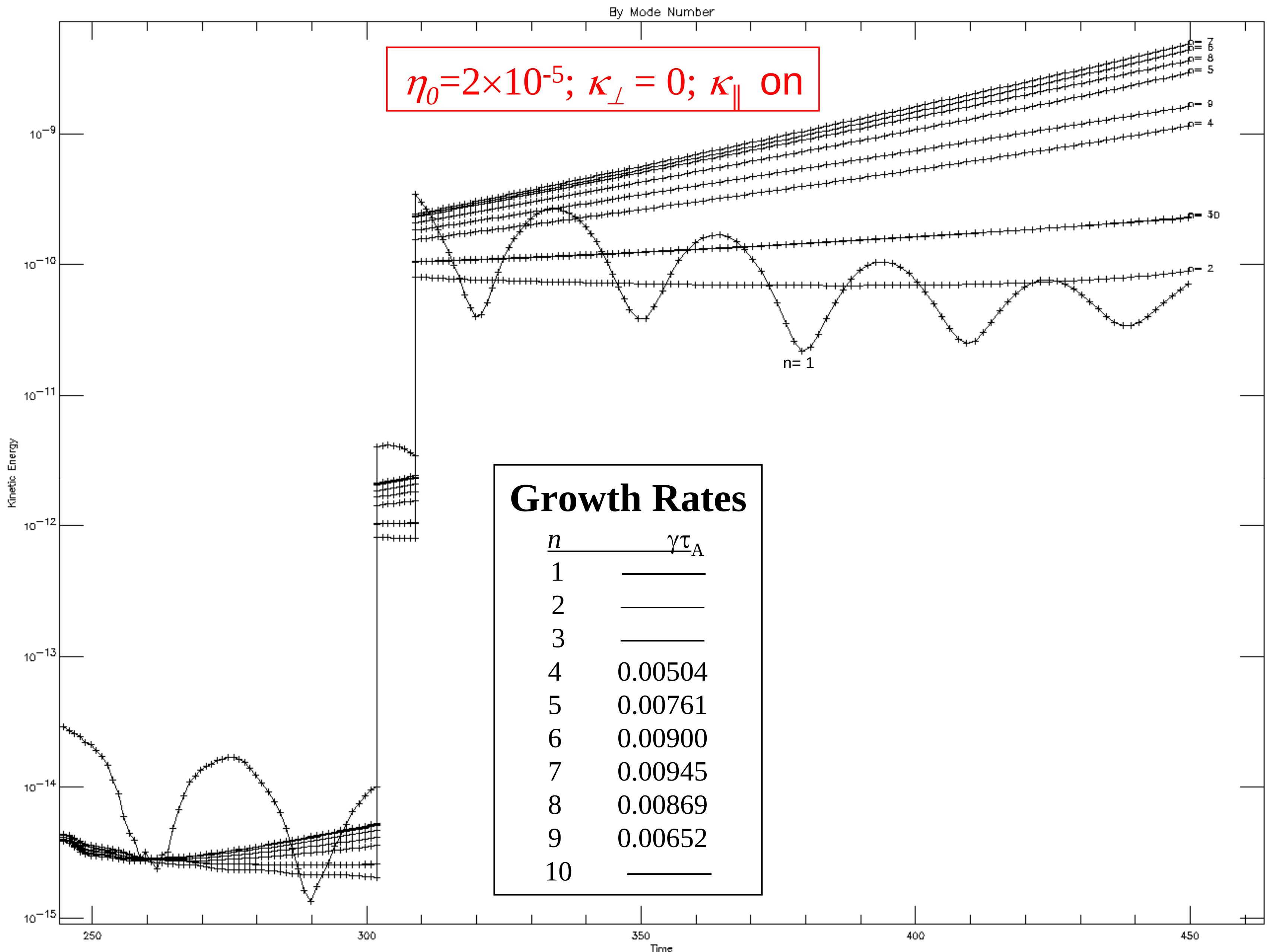
n	$\gamma\tau_A$
1	——
2	0.00872*
3	0.0128
4	0.0164
5	0.0191
6	0.0208
7	0.0212
8	0.0203
9	0.0178
10	0.0134

Nonlinear, q_0=1.04; η_0=2×10^{-5}; κ = 0

$\gamma \propto \eta^{0.6}$



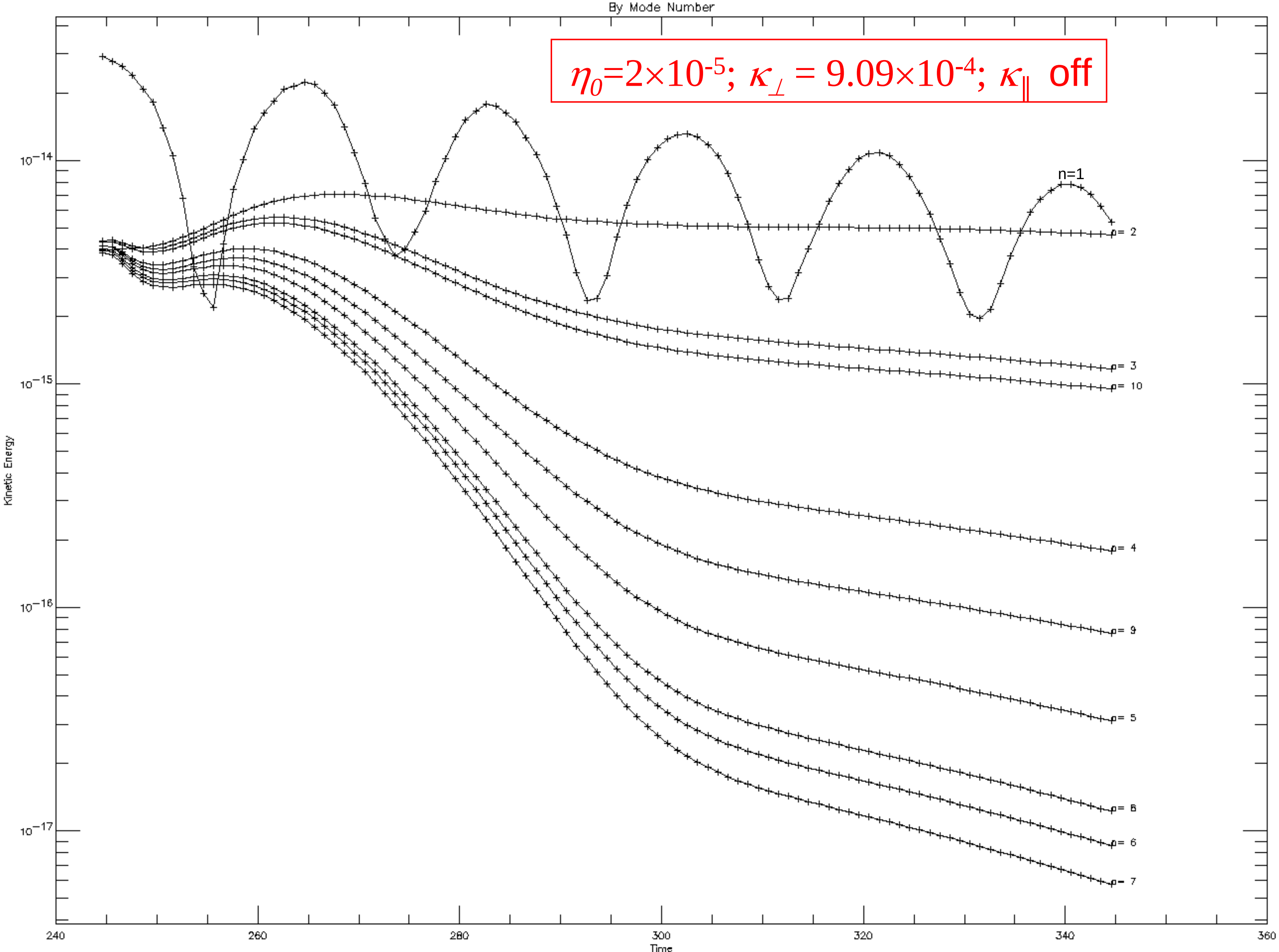
Parallel Heat Conduction Reduces Growth Rates But Does Not Stabilize the Ballooning Modes



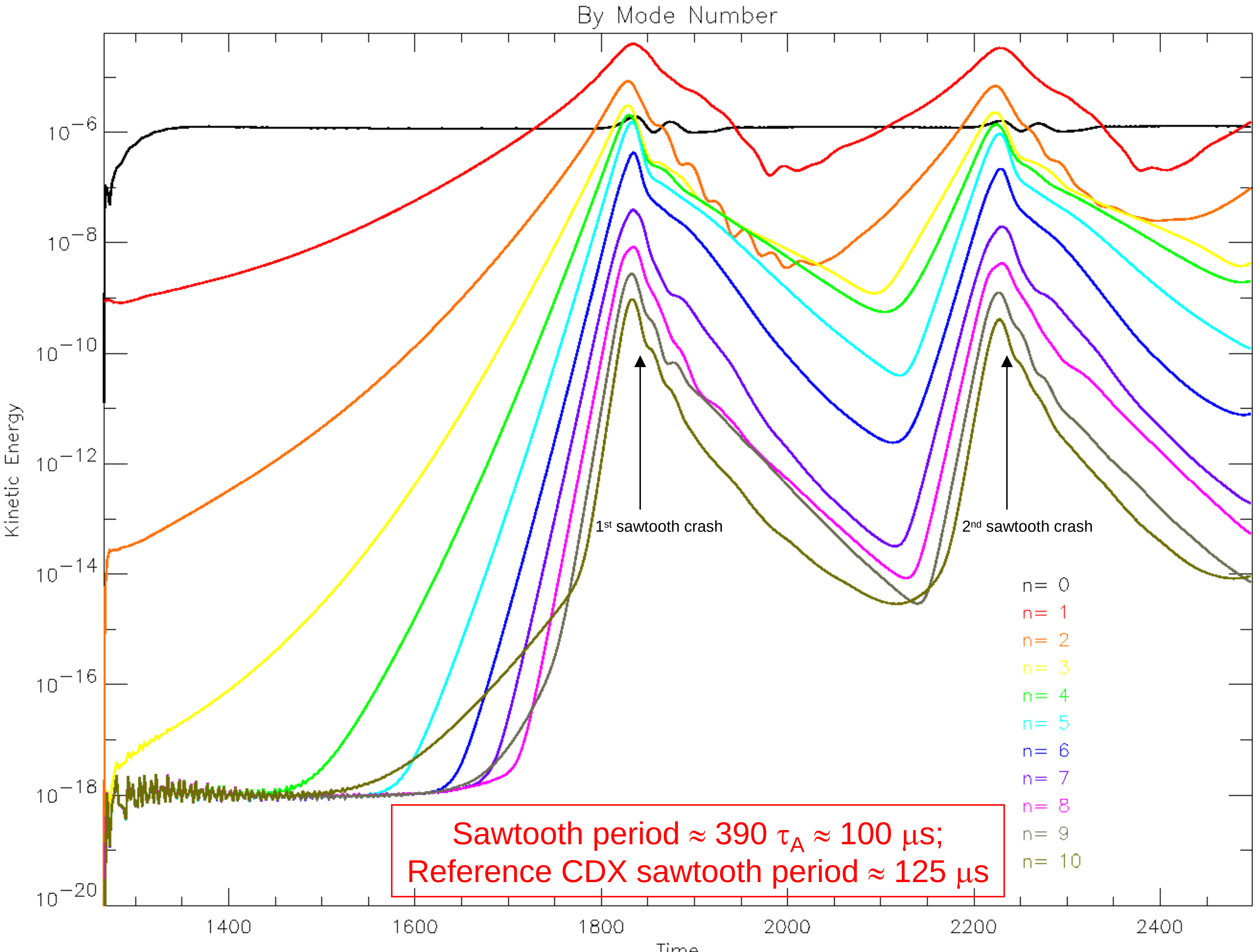
Growth Rates

n	$\gamma\tau_A$
1	———
2	———
3	———
4	0.00504
5	0.00761
6	0.00900
7	0.00945
8	0.00869
9	0.00652
10	———

High Perpendicular Heat Conduction Stabilizes All Ballooning Modes

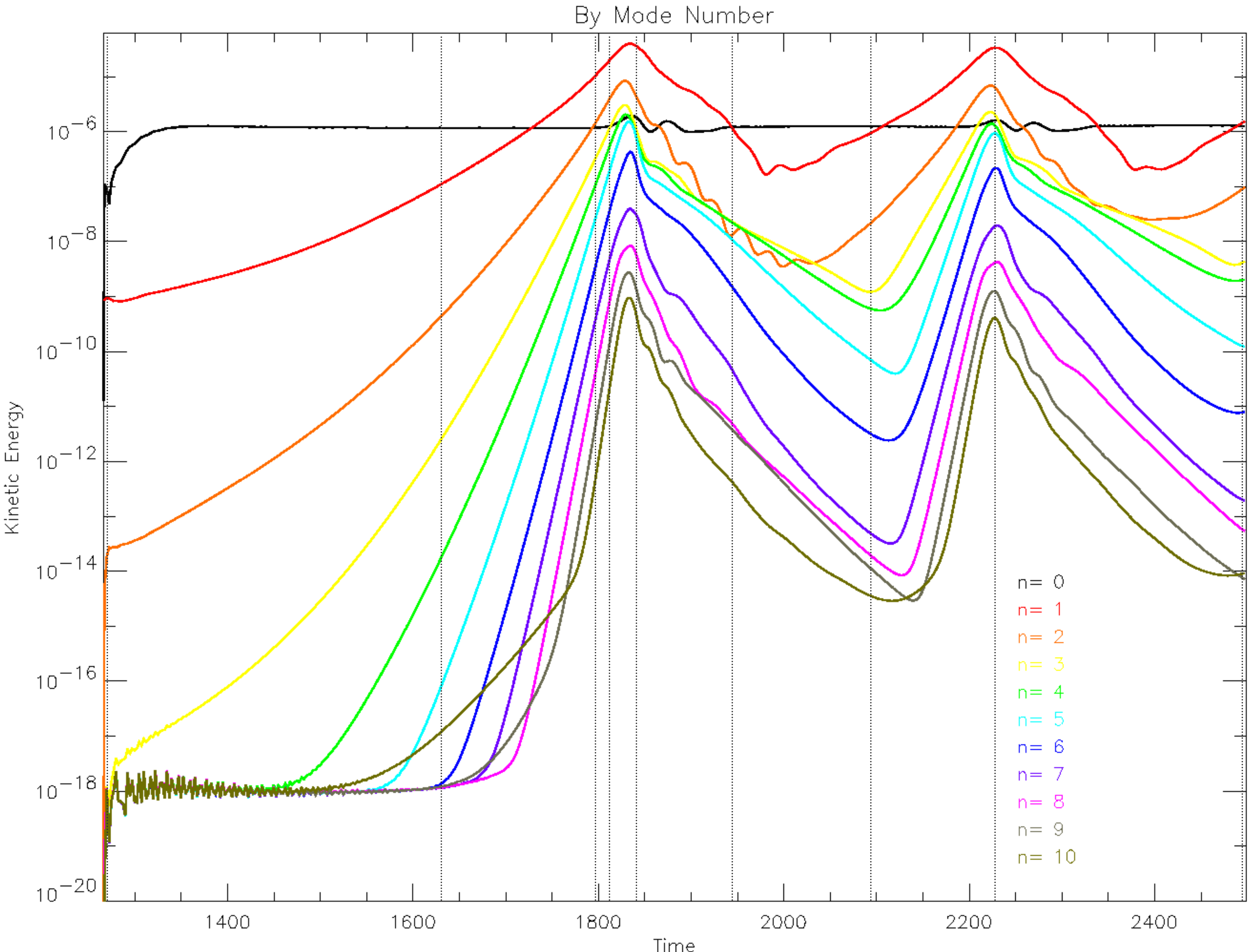


Nonlinear Evolution, Heat Conduction On



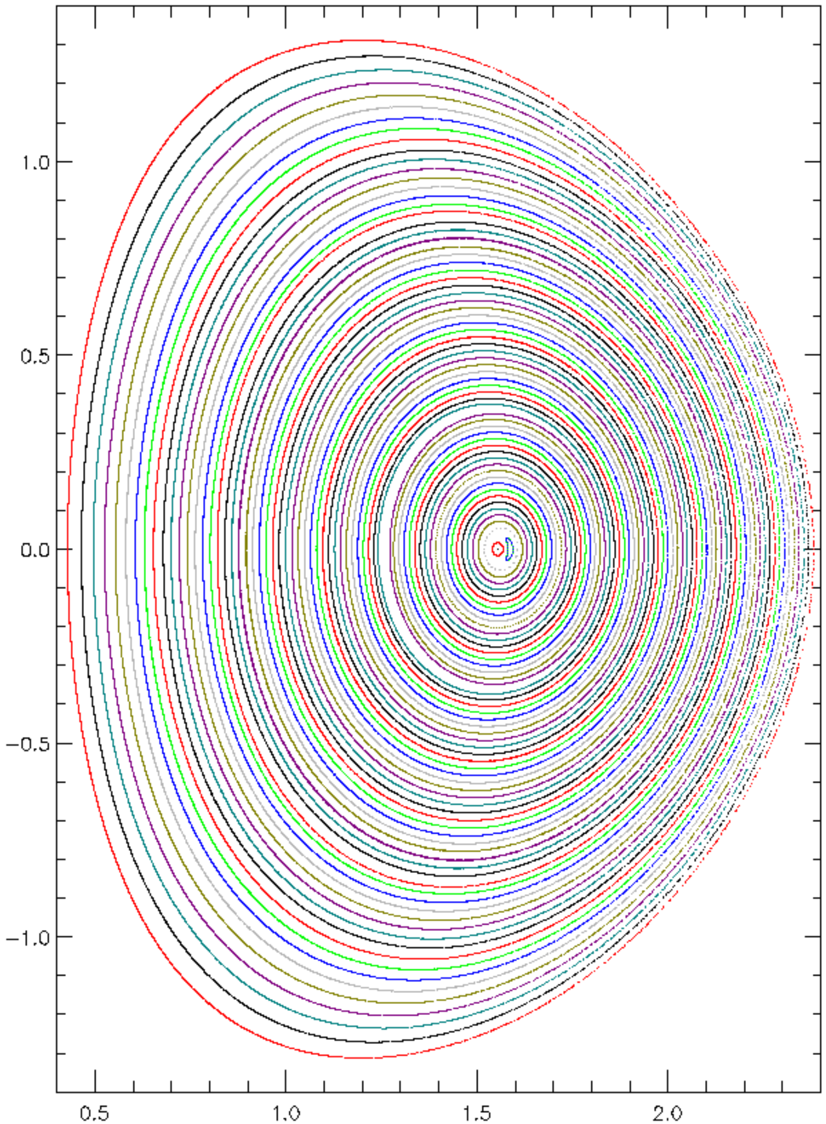
Sawtooth period ≈ 390 τ_A ≈ 100 μs;
Reference CDX sawtooth period ≈ 125 μs

Nonlinear Evolution, Heat Conduction On

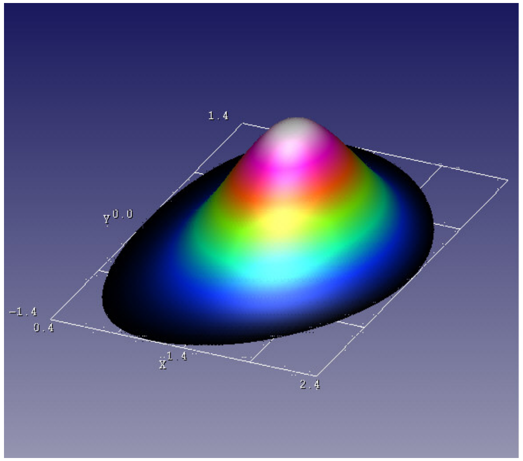


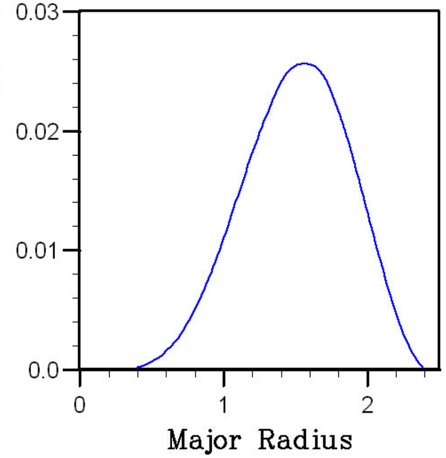
Initial state: t = 1266.17

Poincaré plot

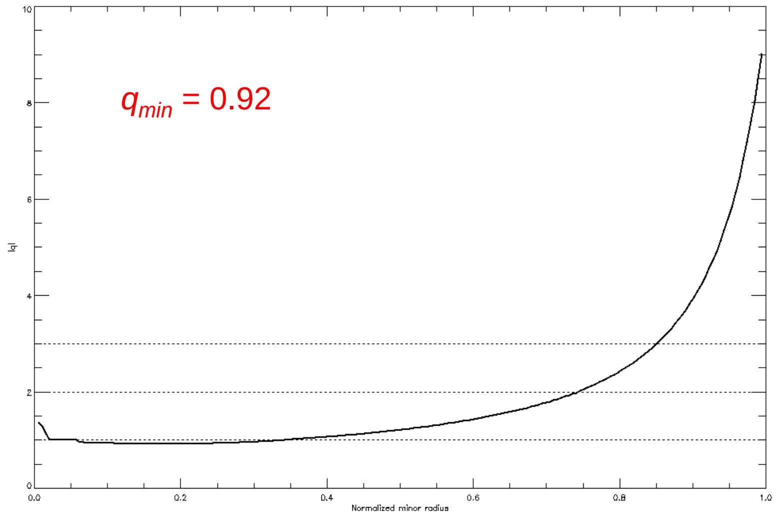


Temperature profile



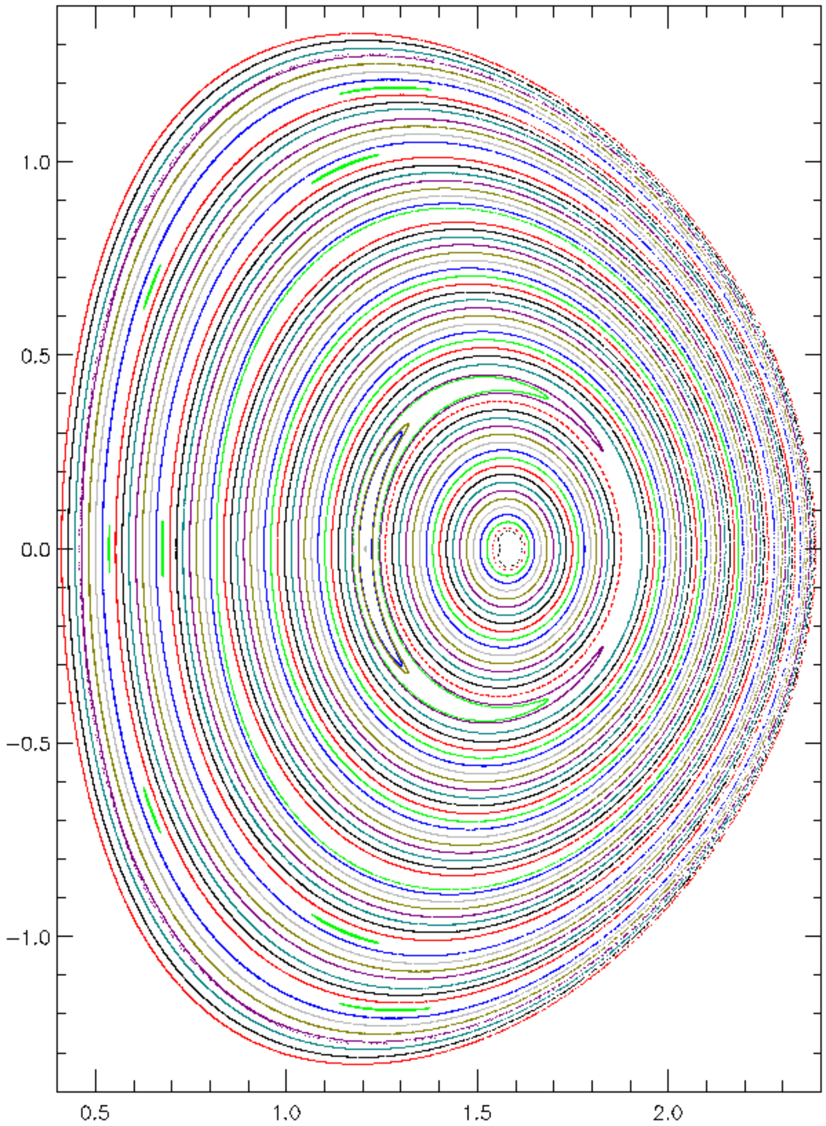


q profile

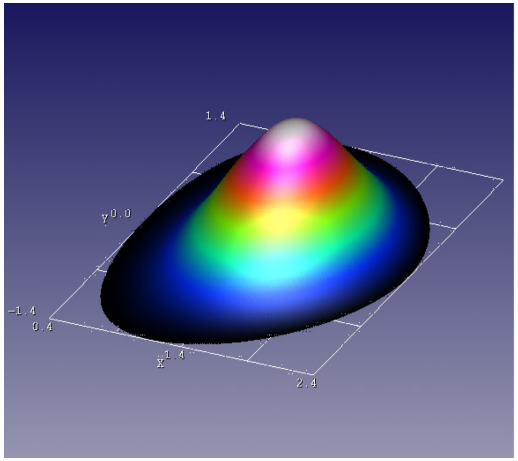


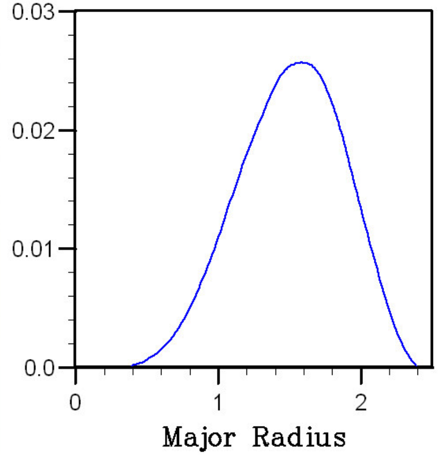
Late in linear phase: t = 1630.64

Poincaré plot



Temperature profile





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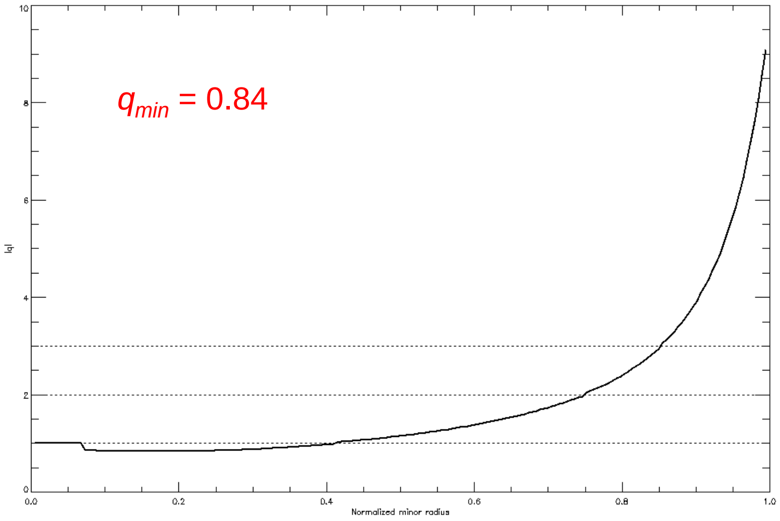
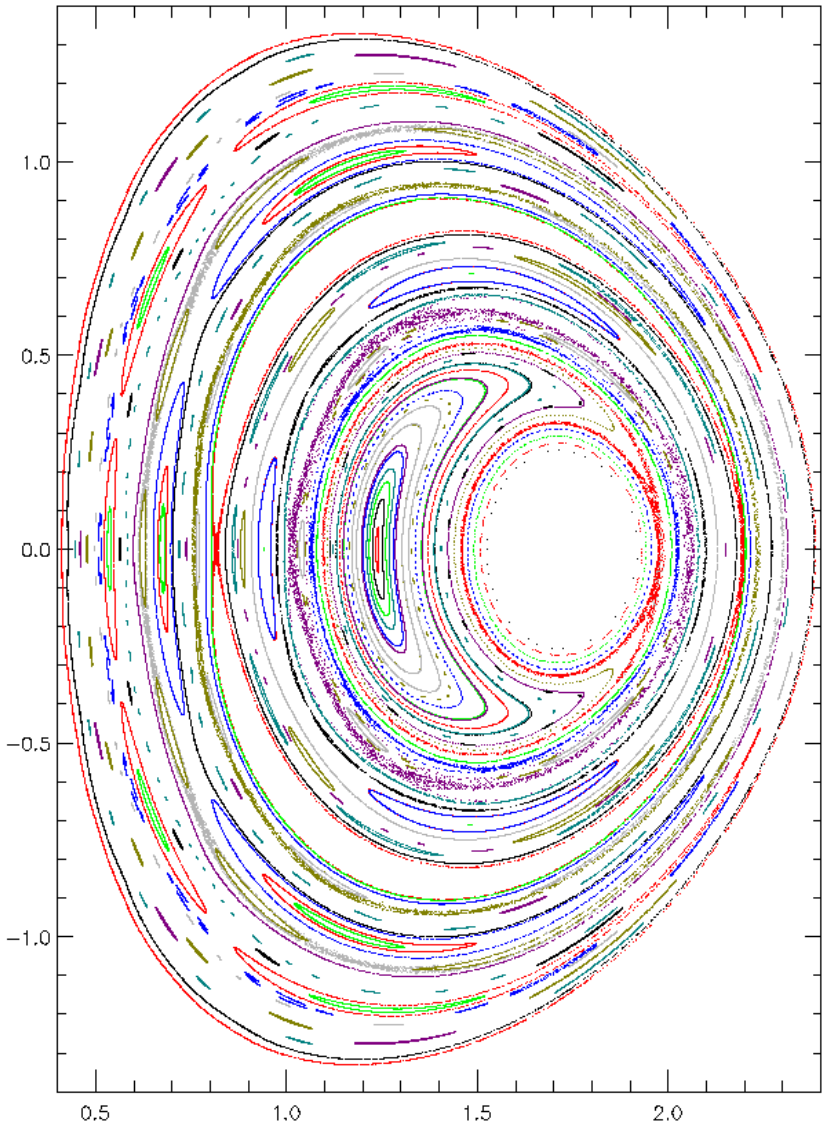
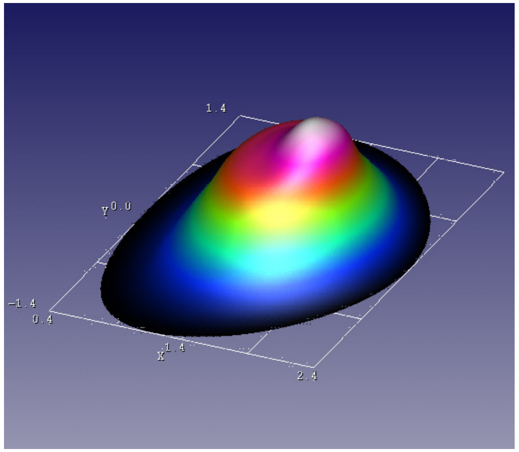


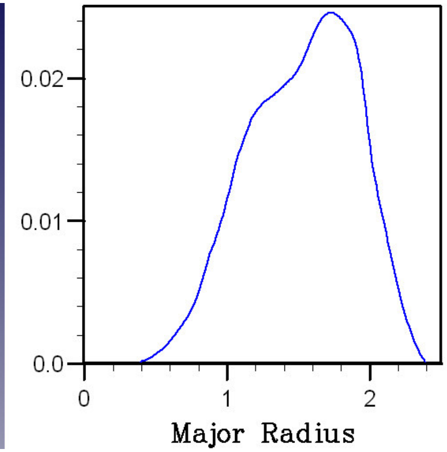
Nonlinear phase: t = 1795.61

Poincaré plot

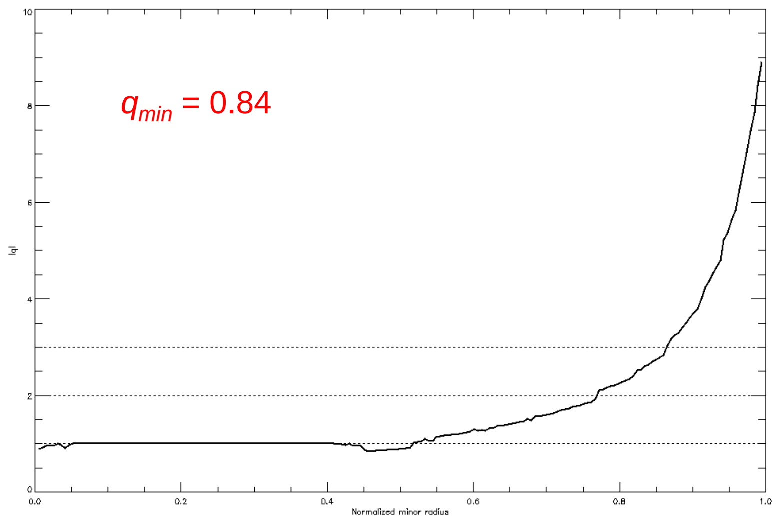


Temperature profile



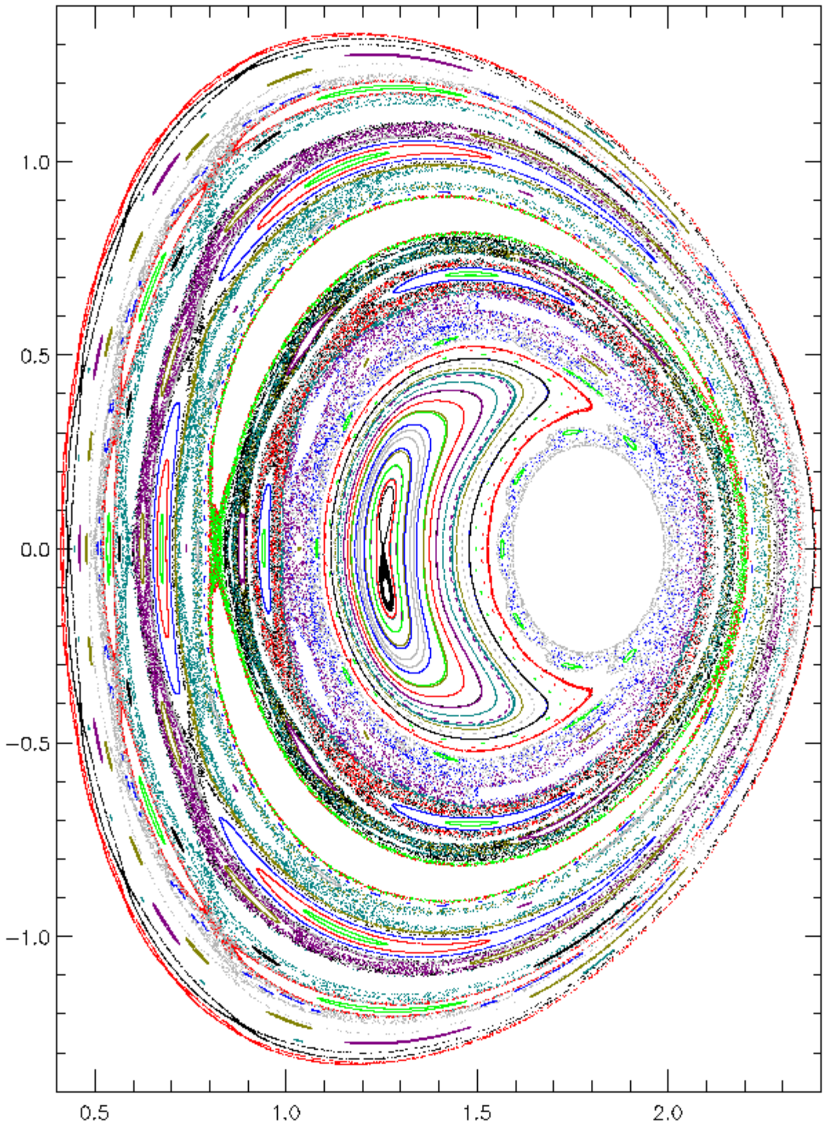


q profile

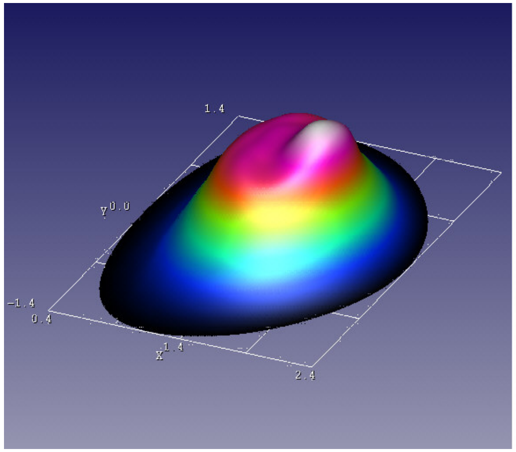


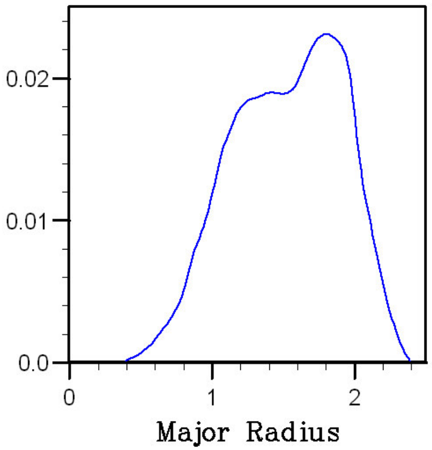
During 1st Crash: t = 1810.51

Poincaré plot

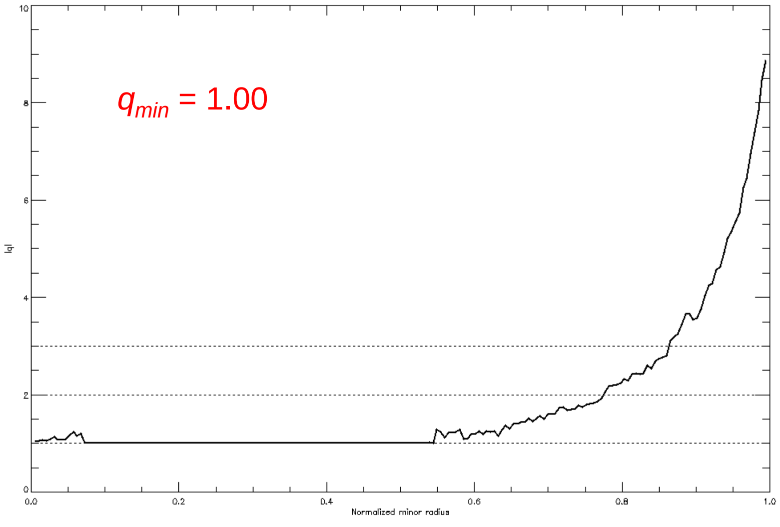


Temperature profile



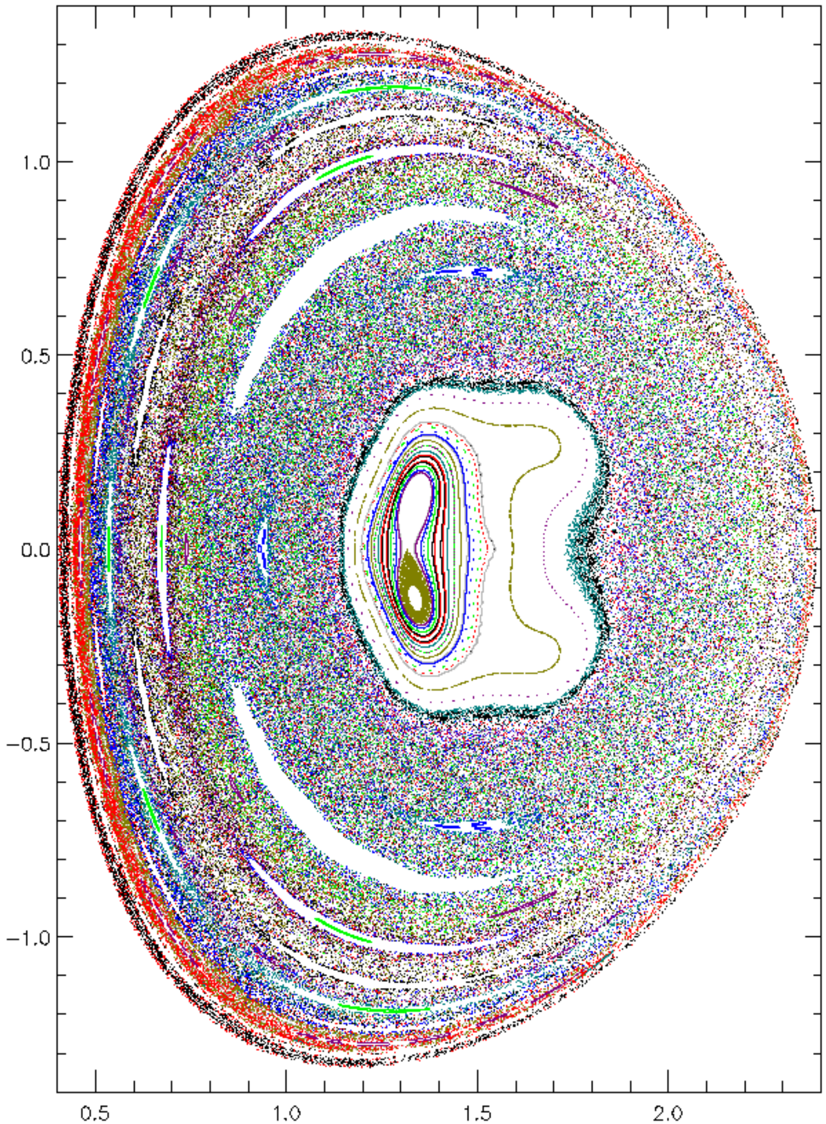


q profile

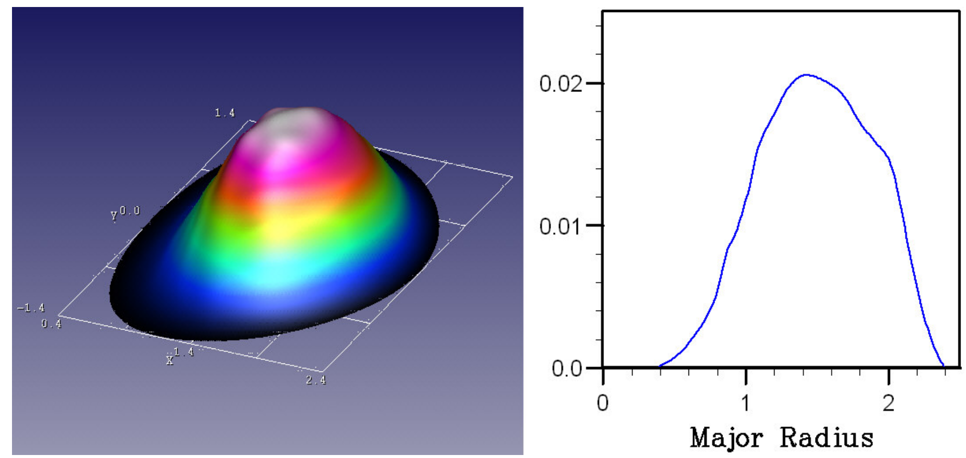


After 1st Crash: t = 1839.86

Poincaré plot



Temperature profile



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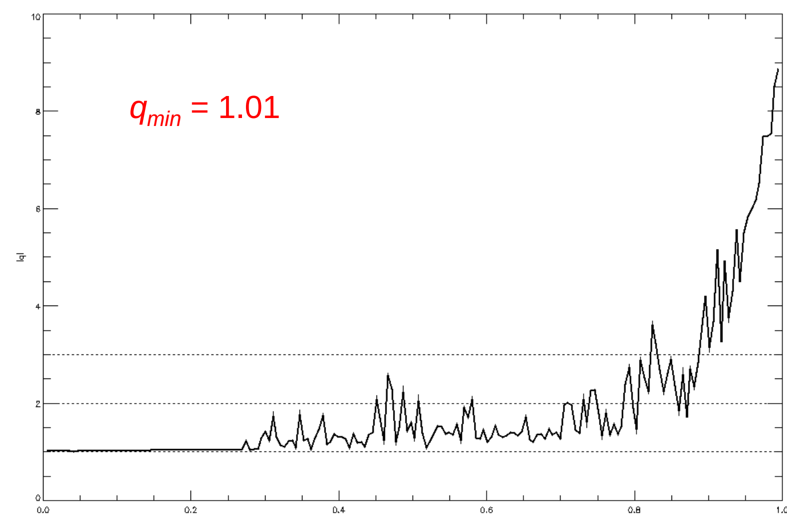
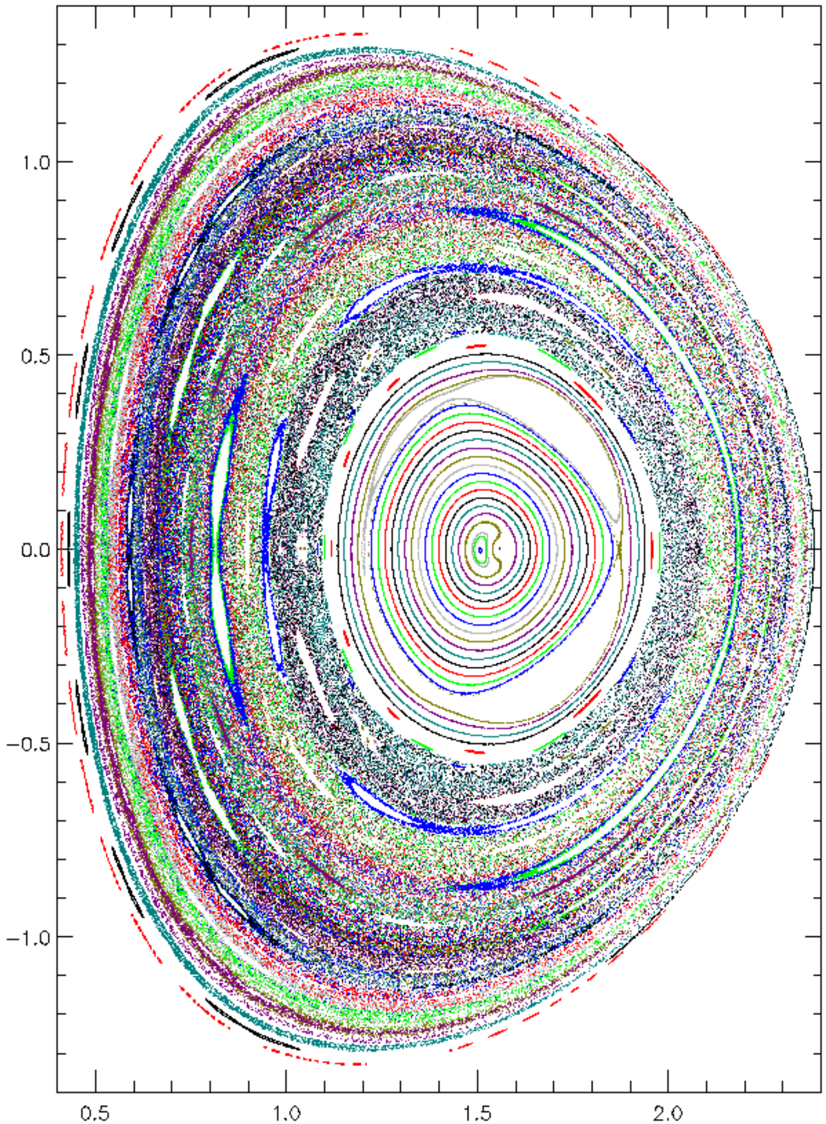
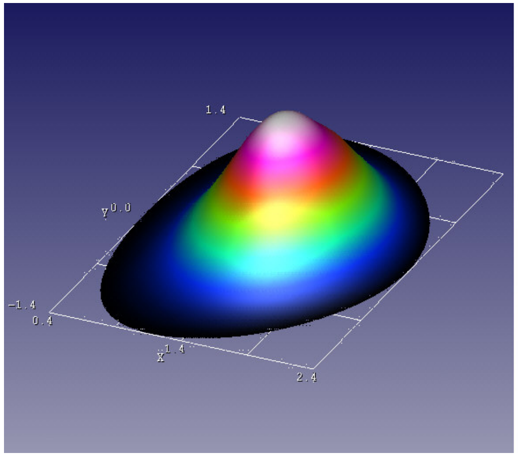


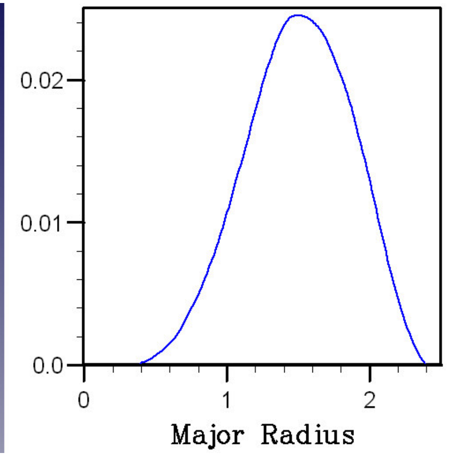
Stochasticity healing: t = 1944.27

Poincaré plot



Temperature profile





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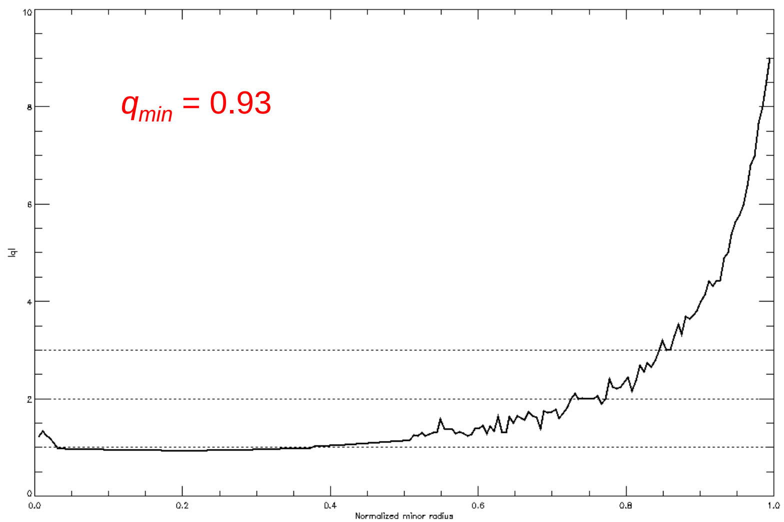
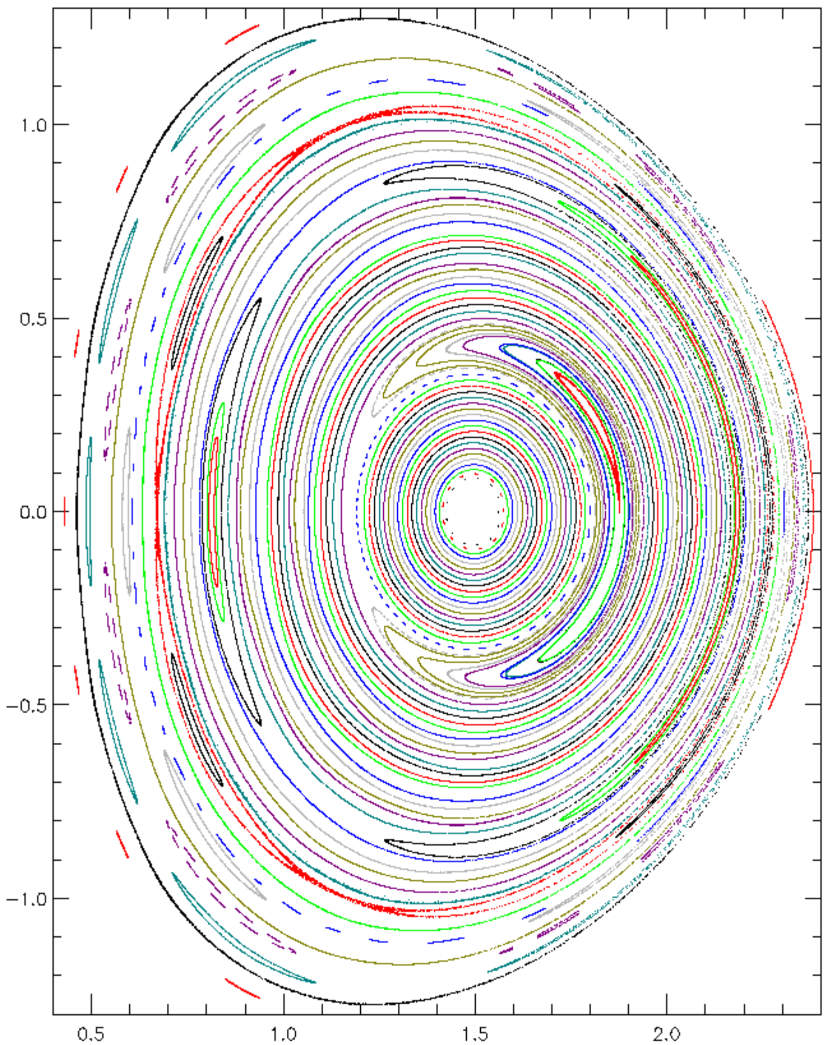
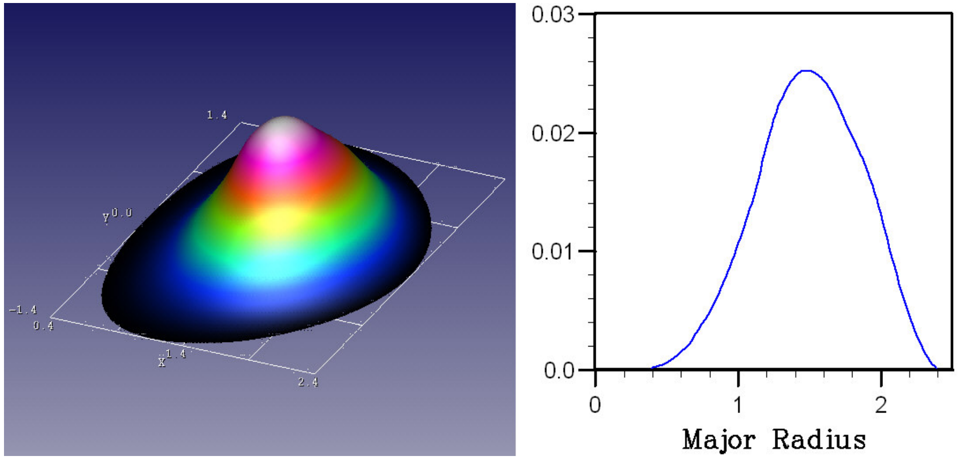


Flux surfaces recovered: t = 2094.08

Poincaré plot



Temperature profile



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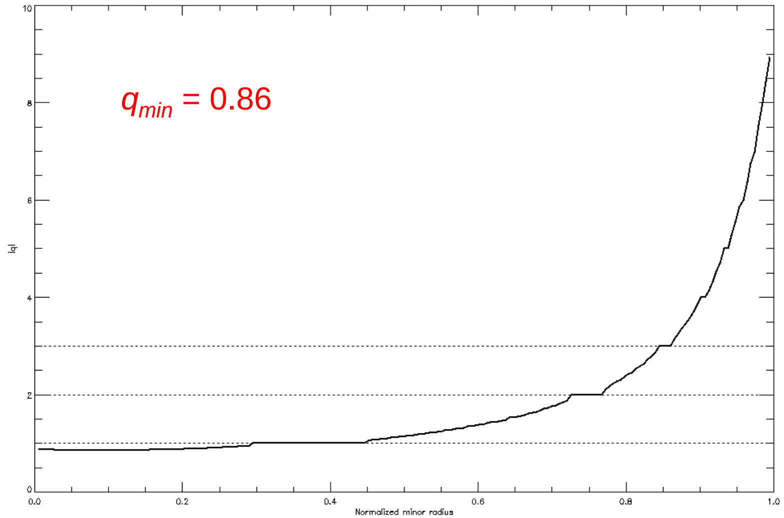
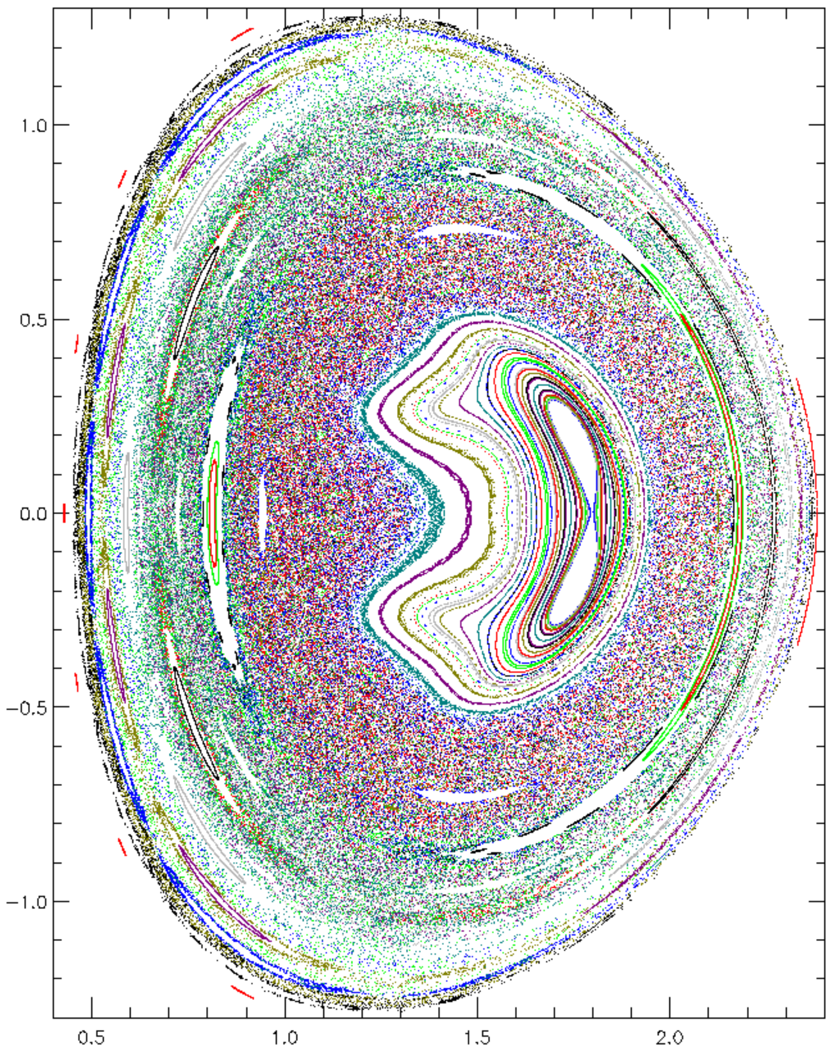
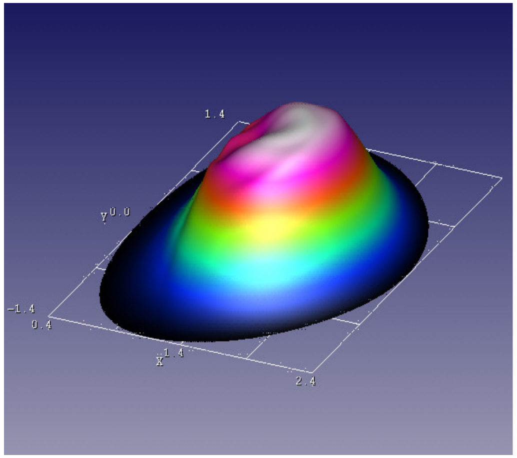


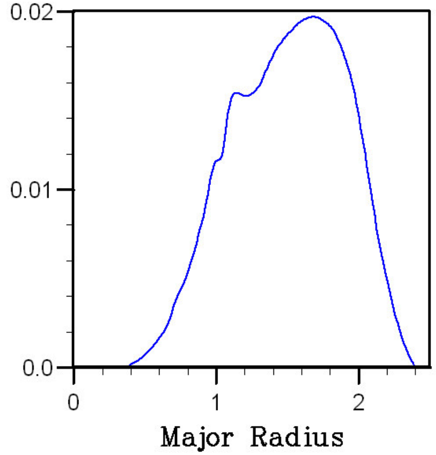
After 2nd Crash: t = 2228.62

Poincaré plot

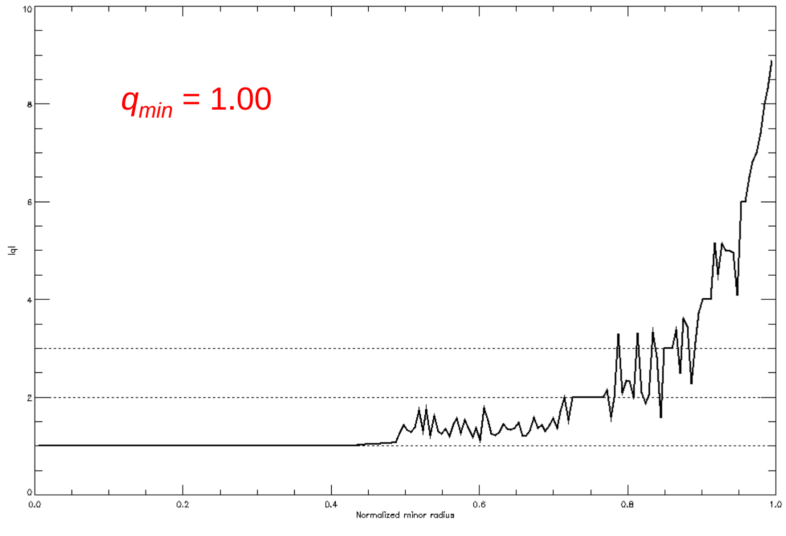


Temperature profile



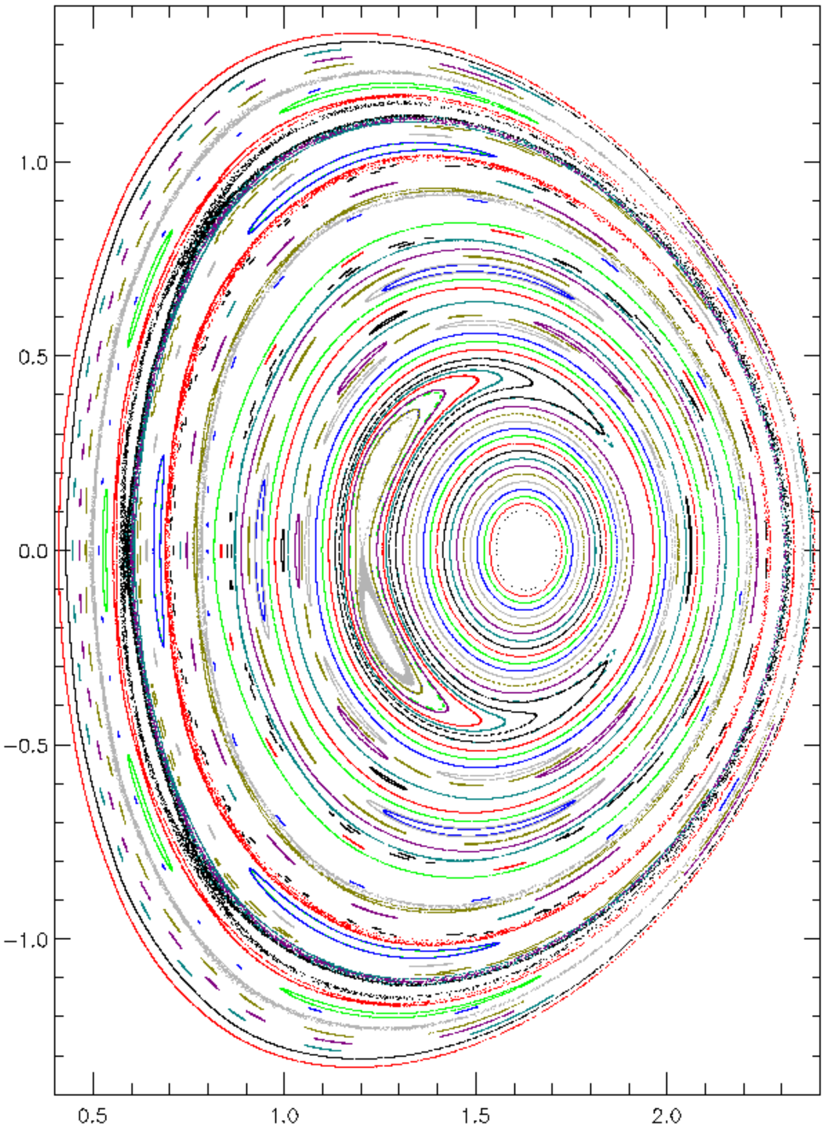


q profile

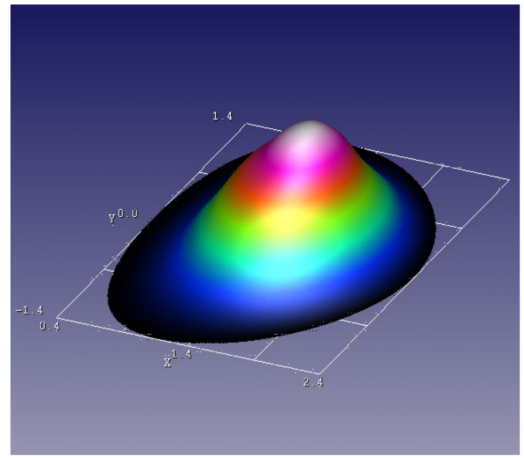


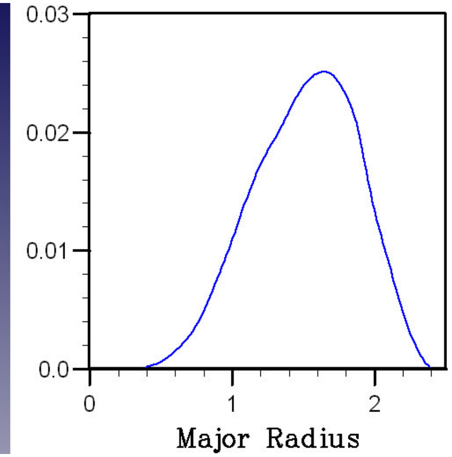
After 2nd recovery: t = 2498.25

Poincaré plot



Temperature profile





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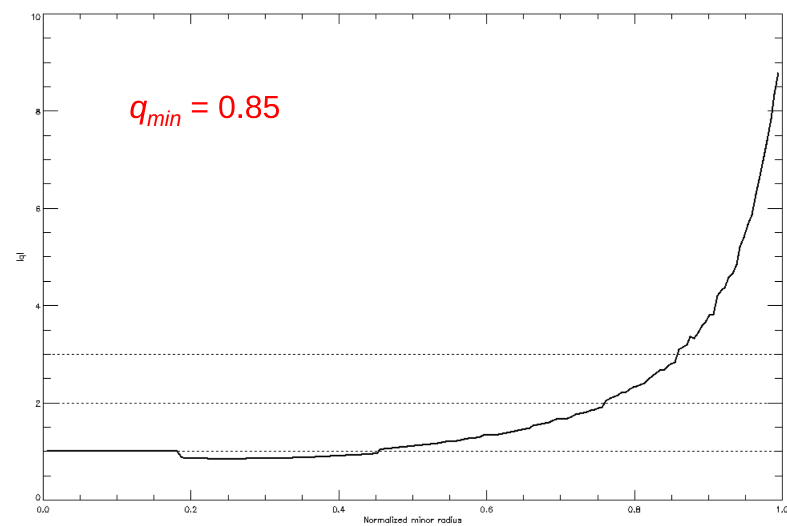


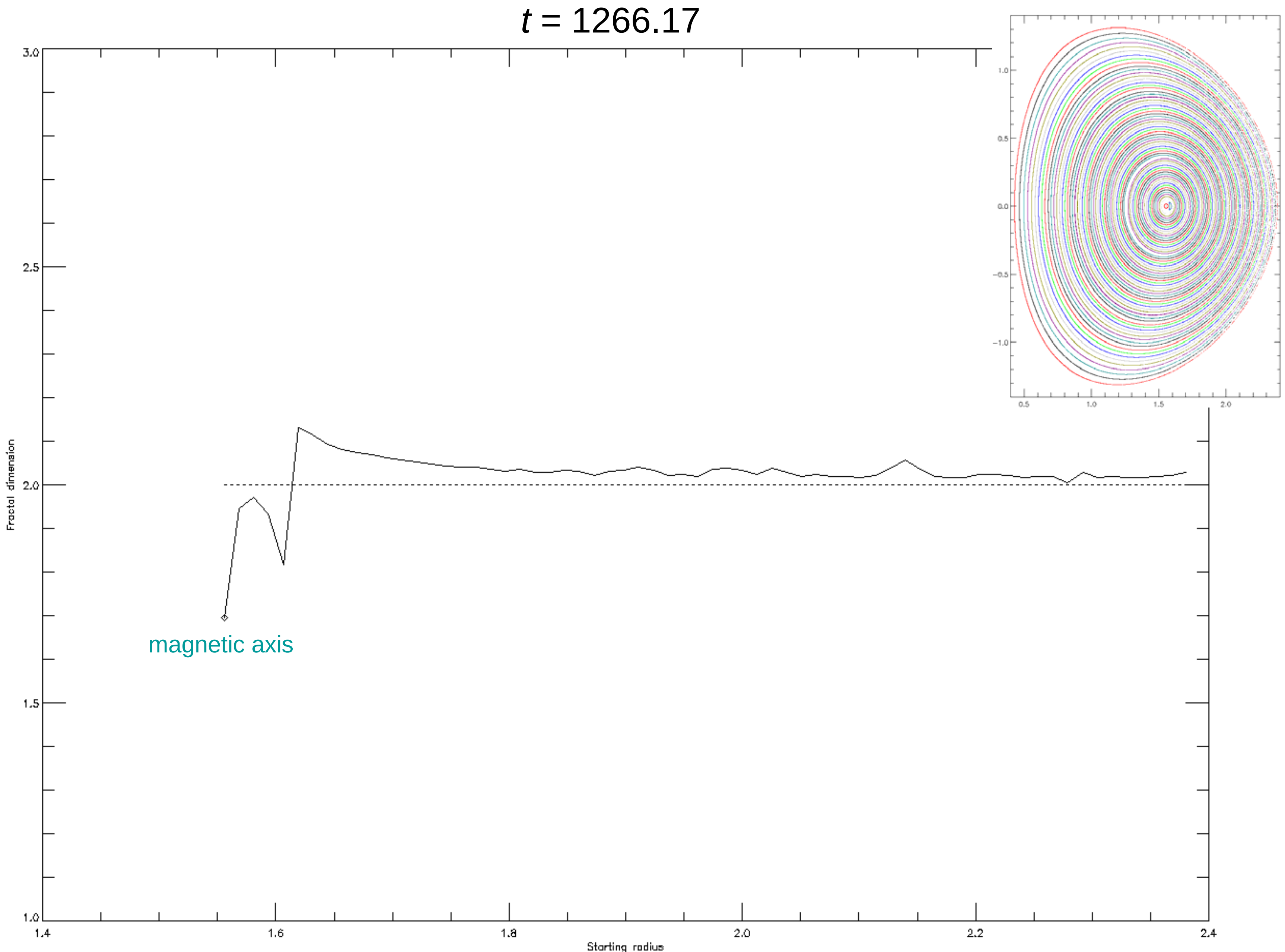
Characterizing Field Line Structure with Fractal Dimension

- The dimensionality of a field line inside the separatrix of a tokamak provides information relevant to confinement.
  - Lines tracing out irrational surfaces are two-dimensional.
  - Lines tracing out rational surfaces are one-dimensional.
  - Stochastic field lines are space-filling and potentially three-dimensional.
- The extent to which stochastic lines fill space may give an indication of the effect of parallel heat conduction on radial transport.
- A measure of non-integer dimensions in data sets is provided by the Hausdorff-Besicovitch fractal dimension

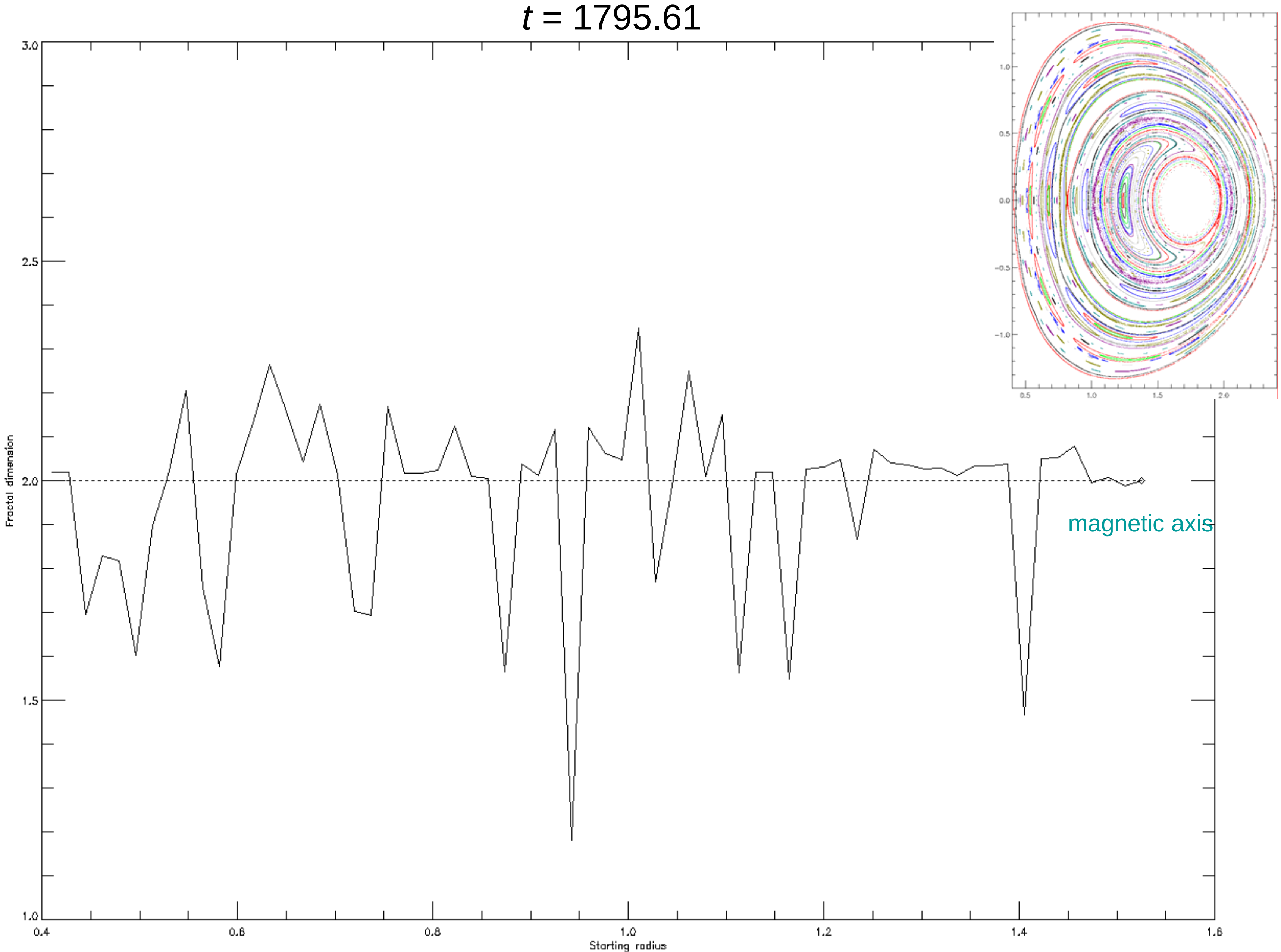
$$D = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln(1/\varepsilon)}$$

where $N(\varepsilon)$ is the minimum number of hypercubes of linear size ε necessary to cover all points in the set.

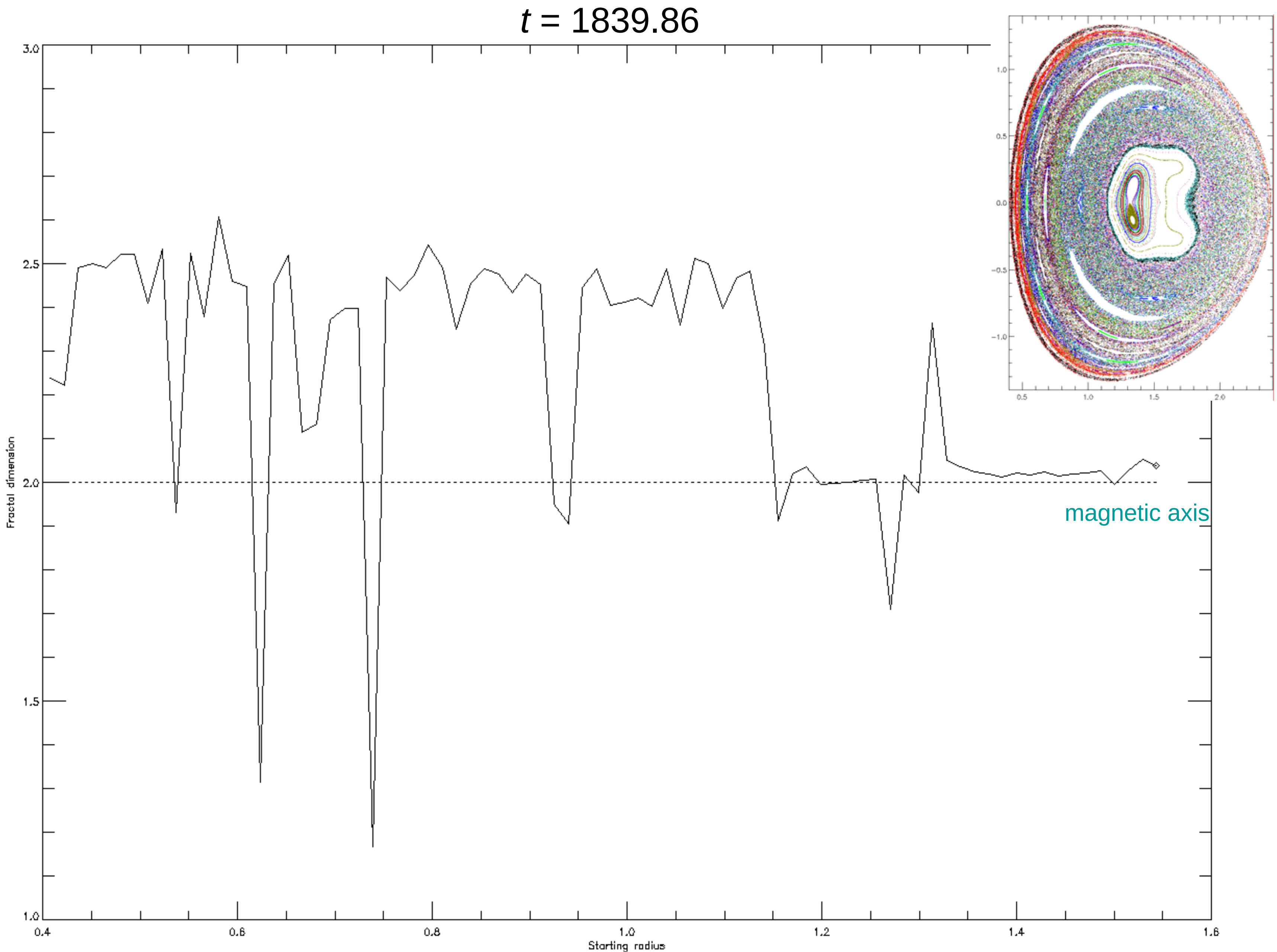
Fractal Dimension: Good Flux Surfaces



Fractal Dimension: Large Islands

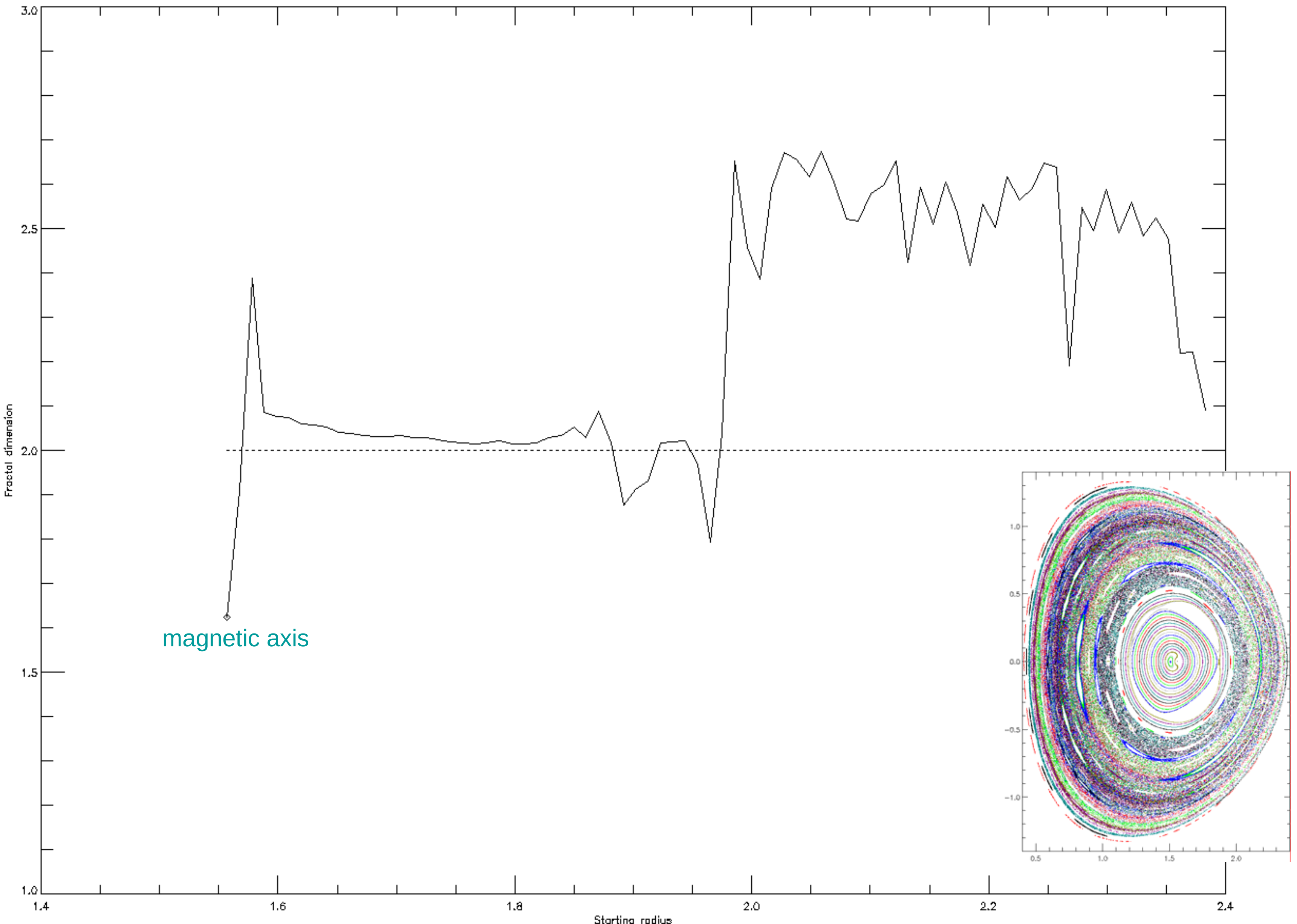


Fractal Dimension: High Stochasticity



Fractal Dimension: Moderate Stochasticity

t = 1944.27



Conclusions

- Nonlinear MHD simulation with actual device parameters is capable of tracking evolution through repeated sawtooth reconnection cycles.

- The fractal dimension diagnostic reliably identifies different field line types, but must demonstrate greater sensitivity to degrees of stochasticity if it is to prove more useful than simple inspection of Poincaré plots. Other diagnostics should be considered.

- Quantitative comparisons with experimental data will first require more careful attention to assumptions of the model.
  - Loop voltage (Ohmic) current drive in device vs. current source term in code.
  - Self-consistent Ohmic heating and evolving resistivity profile must be implemented.
  - Inclusion of two-fluid terms is likely to alter time and space scales of the sawtooth reconnection events.