The Hybrid Kinetic-MHD Equations[a]

- in the limit $n_h \ll n_0$, $\beta_h \sim \beta_0$, quasi neutrality, only modification of MHD equations is addition of the hot particle pressure tensor in the momentum equation:

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\underline{\mathbf{p}}_b - \nabla\cdot\underline{\mathbf{p}}_h$$

  the subscripts b, h denote the bulk plasma and hot particles

- assume CGL-like form $\delta\underline{\mathbf{p}}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix}$

[a]C.Z.Cheng,"A Kinetic MHD Model for Low Frequency Phenomena",*J. Geophys. Rev*, **96**, 1991

- evaluate pressure moment using δf[b] at a position $\mathbf{x}$ is

$$\begin{pmatrix} \delta p_\perp \\ \delta p_\parallel \end{pmatrix} = \sum_i \begin{pmatrix} \mu B \\ m v_\parallel^2 \end{pmatrix} \delta f$$

$$\dot{\mathbf{x}} = v_\parallel \hat{\mathbf{b}} + \frac{m}{eB^4}\left(u^2 + \frac{v_\perp^2}{2}\right)\left(\mathbf{B} \times \nabla \frac{B^2}{2}\right) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp + \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

$$m\dot{v}_\parallel = -\hat{\mathbf{b}} \cdot (\mu \nabla B - e\mathbf{E})$$

$$\dot{\delta f} = f_{eq}\left\{\frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B} \cdot \nabla B - \mu_0 v_\parallel \mathbf{J} \cdot \mathbf{E}\right] + \frac{\delta\mathbf{v} \cdot \nabla\psi_p}{\psi_0} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_0^{3/2}}\mathbf{v}_D \cdot \mathbf{E}\right\}$$

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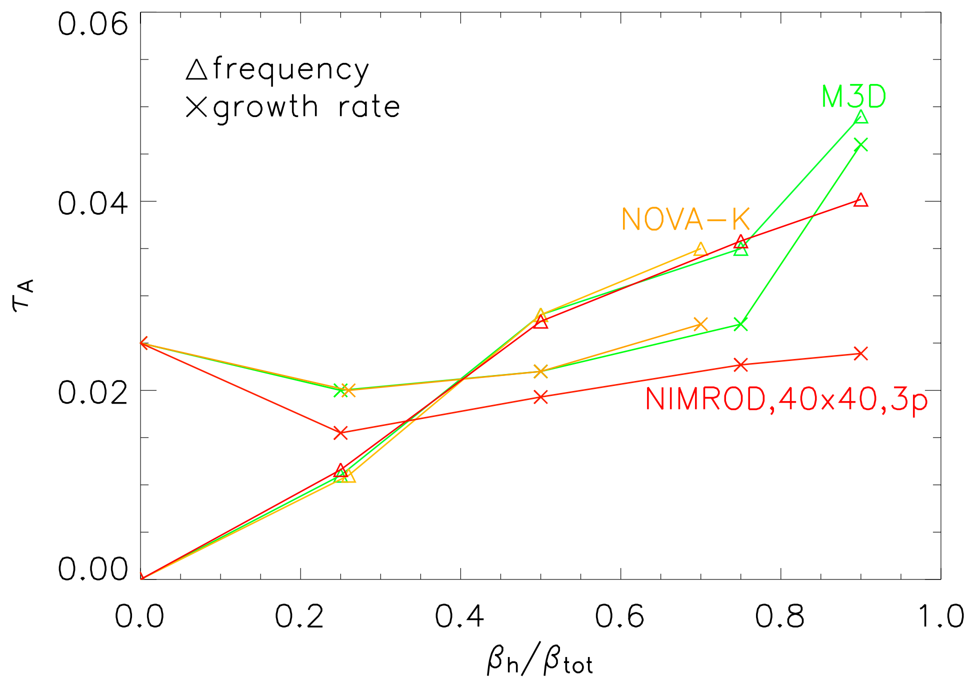
[b] S. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

$(1,1)$ **Benchmark with M3D**

- transition from internal kink mode to fishbone[a]
- monotonic q, $q_0 = .6$, $q_a = 2.5$, $\beta_0 = .08$, circular tokamak R/a=2.76
- dt=1e-7, $\tau_A = 1.e6$



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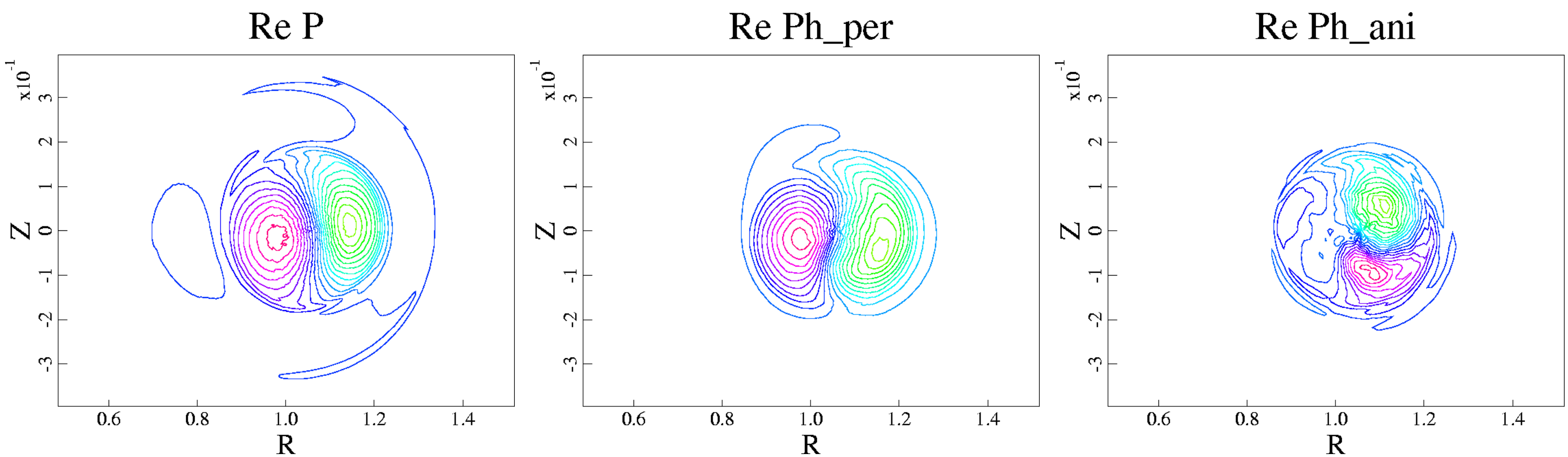
[a] F. Porcelli, "Fast Particle Stabilisation", *Plasma Physics and Controlled Fusion*, **33**, 1991

Ideal Hot Particle eigenmodes $\beta_c = \beta_h$

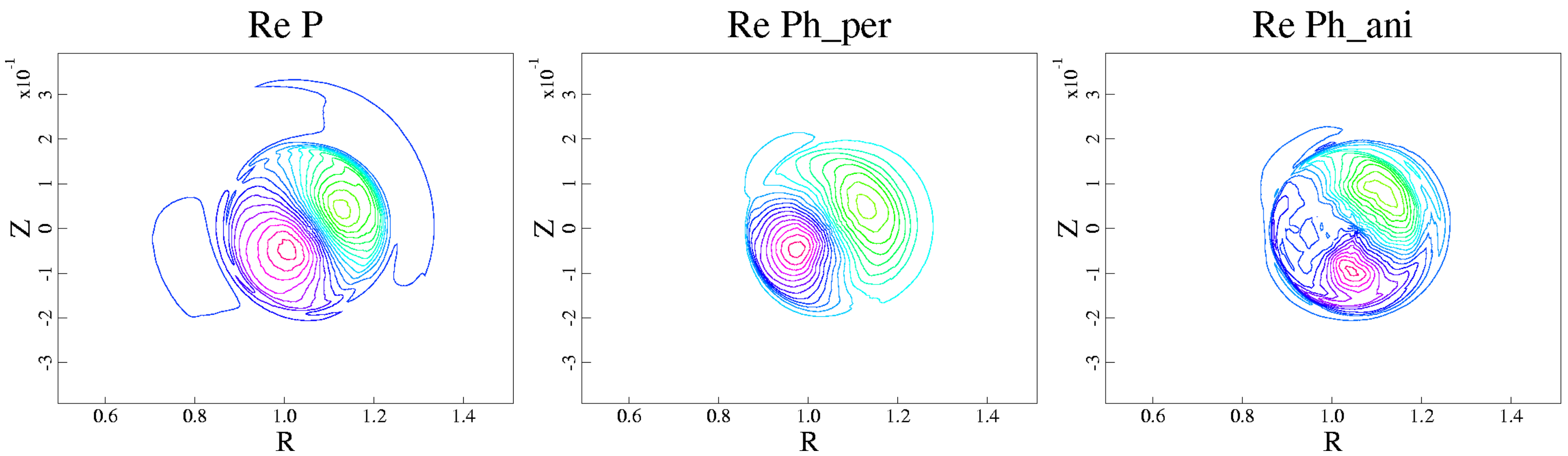
- anisotropic hot particle pressure induces real frequency



- $\gamma\tau_A = .019(.025), \omega\tau_A = .027$

Resistive Hot Particle eigenmodes $\beta_c = \beta_h$

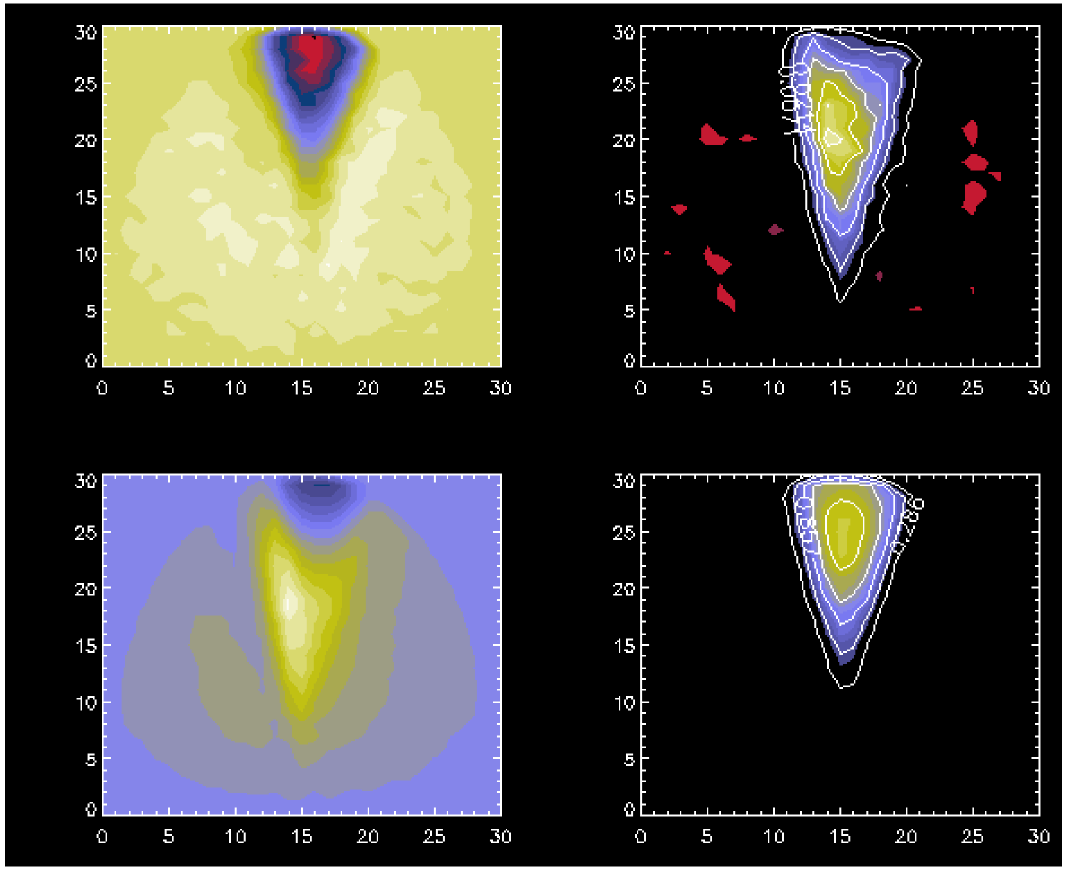
- stabilization effect comparable to ideal case
- real frequency significantly reduced
- note increased activity near resonant layer



- resistive $S = 1e5, \gamma\tau_A = .017(.028), \omega\tau_A = .011$

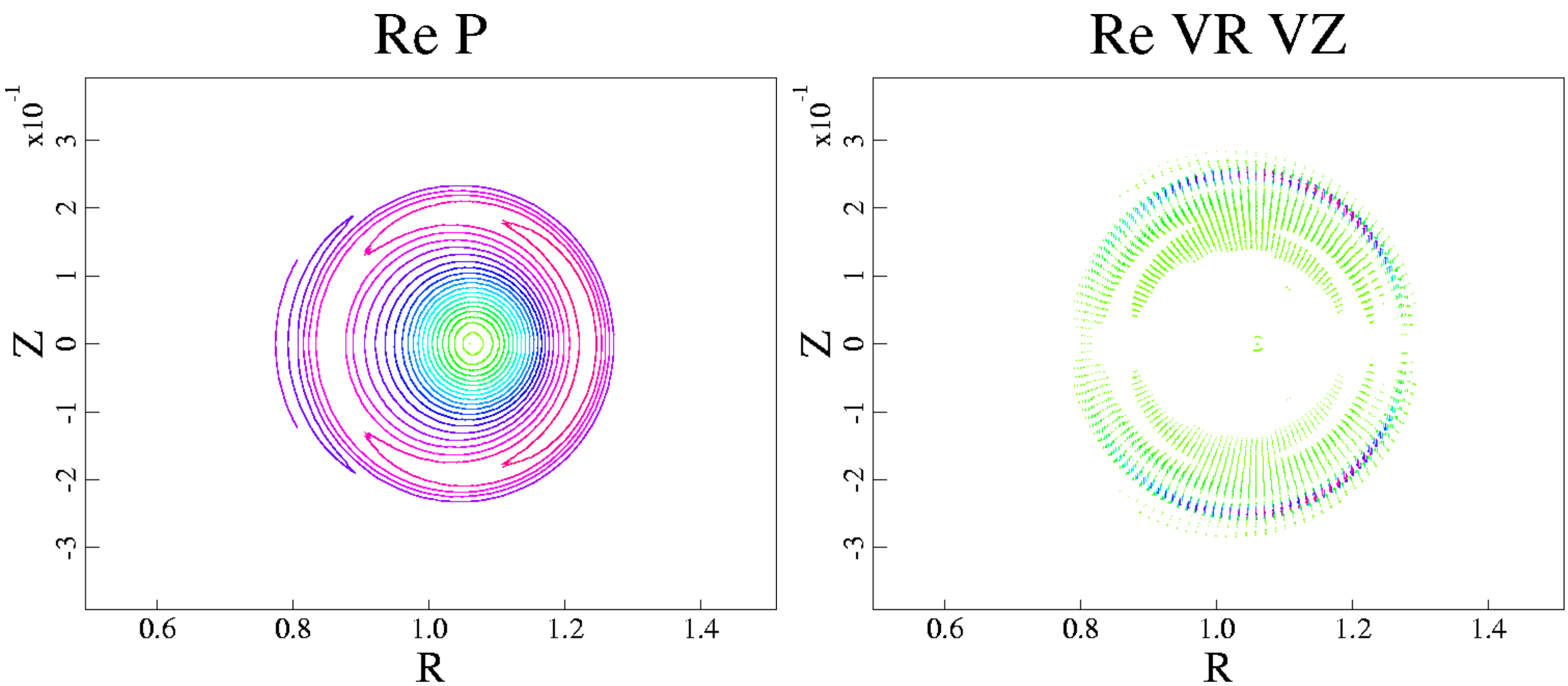
Hot Particle $(v_{\parallel}, v_{\perp})$ $\beta_c = \beta_h$

- hot particles effects entirely due to trapped particles



Vanilla (Resistive) n=1 reconnection

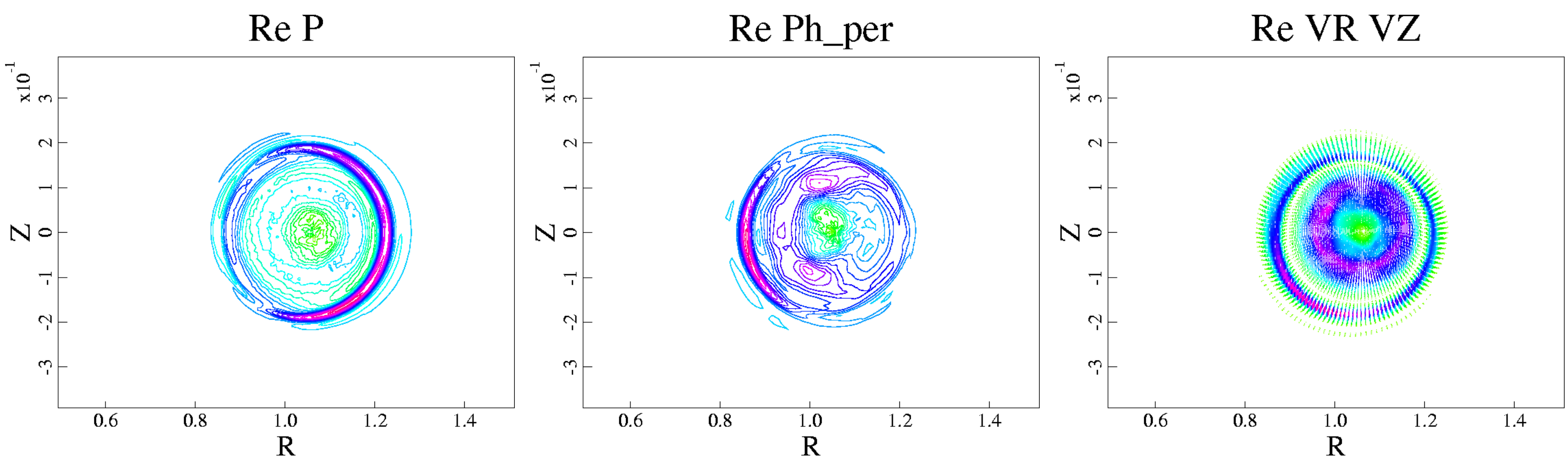
- $n = 0$ perturbation flattens pressure, flow predominantly around rational surface



- $S = 1e5, Pr = 1$

Nonlinear resistive n=1 w/ hot particles

- $\beta_h/\beta = .5$ $\gamma\tau_A = .017(.019)$ of $n = 1$ $\omega\tau_A = .008(.027)$

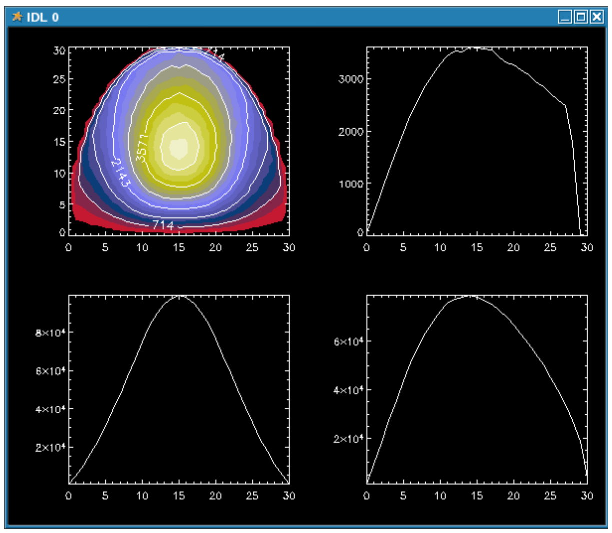


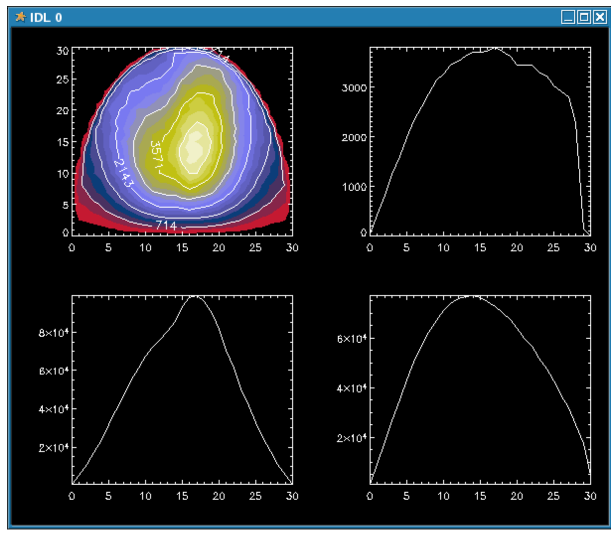
- substantial $n = 0$ flow in core

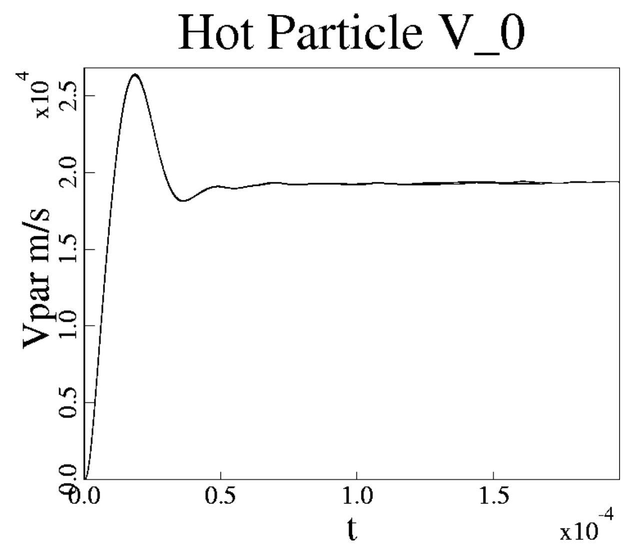
- should hot particle momentum be taken into account?

Slowing Down Distribution

- symmetry of distribution is broken within passing time







- hot particle momentum is comparable to $n = 0$ bulk momentum

Possible excitation of Alfven waves by hot particles?

- with β_h fixed, broaden range of $v_h = [v_A, 1.2v_A, 1.3v_A, 1.4v_A]$
- observed that γ drops but ω fixed until $v_h = 1.4v_A$ $\omega\tau_A = .25$