Implicit PIC Studies of MHD Instabilities

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Sherwood 06

April 24, 2006

Overview

We are developing implicit, δf PIC methods for Hall MHD calculations. This work follows Parker and Peoble and adds implicit time differencing. Initial tests are done in 2D for the gravitational instability of Roberts and Taylor.*

Goal (short term): Understand algorithm

(long term): Implement in NIMROD

*Jim T. Peoble and Scott E. Parker, *Bull. Am. Phys. Soc.* **49**, 183 (2004).

Model

- Ions are particles, electrons are fluid

$$\dot{\mathbf{x}} = \mathbf{v}$$

$$\dot{\mathbf{v}} = \frac{Q}{M}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

$$\delta \dot{f} = -\frac{D\, f_0}{Dt}$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \bullet n_0 \mathbf{u} = 0$$

$$\frac{\partial\, \tilde{P}_e}{\partial t} + \mathbf{u} \bullet \nabla P_{e0} + \Gamma P_{e0} \nabla \bullet \mathbf{u} = 0$$

$$\frac{\partial \tilde{\mathbf{B}}}{\partial t} = -\nabla \times \tilde{\mathbf{E}}$$

$$\tilde{\mathbf{J}} = \frac{1}{\mu_0} \nabla \times \tilde{\mathbf{B}}$$

Progress

- δf formulation for arbitrary background equilibrium
- Proposed 3D solution method
- 2D ($\perp$ **B**) implementation w. orbit averaging
- 2D tests

From Moments to δf

- Assume “equilibrium” state described in fluid terms

$$n, T, \mathbf{u}, \mathbf{B}, \mathbf{E}, \mathbf{g}$$

- Flow not yet included

From Moments to δf

- Case: no ∇T

$$f_0 = f_M$$

$$f_M = n_0 \left(\frac{1}{2\pi v_T^2} \right)^{3/2} e^{-v^2/2v_T^2}$$

From Moments to δf

- Case: ∇T “small”

$$f_0 = f_M + \varepsilon f_{01} + \ldots$$

$$f_M = n_0 \left(\frac{1}{2\pi v_T^2} \right)^{3/2} e^{-v^2/2v_T^2}$$

$$\varepsilon = \ell / L_T : \; L_T = T / \left| \nabla T \right|, \;\; \ell \text{ is orbit size}$$

From Moments to δf

- If ρ_i/L_T finite (and small) as $\rho_i \to \infty$

$$f_0 = f_M + \left(\frac{v^2}{v_T^2} - 5\right)\frac{\mathbf{b} \times \nabla T_0 \bullet \mathbf{v}}{2\Omega T_0} f_M$$

From Moments to δf

$$w = \delta f \,/\, f_0$$

$$\frac{1}{1+w}\frac{Dw}{Dt} = -\frac{Q}{k_B T_0}\left\{\tilde{\mathbf{E}} \bullet \left[-\mathbf{v}\left(1+\frac{\mathbf{b}\times\nabla T_0 \bullet \mathbf{v}}{\Omega T_0}\right) + \left(\frac{v^2}{v_T^2}-5\right)\frac{k_B \mathbf{b}\times\nabla T_0}{2Q B_0}\right] - \left(\frac{v^2}{v_T^2}-5\right)\frac{k_B \mathbf{b}\times\nabla T_0}{2Q B_0}\times\tilde{\mathbf{B}} \bullet \mathbf{v}\right\}$$

Implicit HMHD PIC Method

- 2-fluid
- Electrons are fluid
- Ions are δf particles
- Unknown is new time $\mathbf{E}$, error is Ohm's law

$$en\left(\mathbf{E} + \overline{\mathbf{V}} \times \mathbf{B}\right) = \mathbf{J} \times \mathbf{B} - \nabla P_e$$

From particle ions

3D Solution Technique

- Use predictor/corrector
- Corrector adjusts $\mathbf{E}$
  - Assume $\delta E_{||}$ electrostatic and associated with ion acoustic
  - $\delta E_{\perp}$ represented in terms of "displacement" ξ

3D Parallel Solution

$$\mathbf{V}*_e = \overline{\mathbf{V}}* - \frac{\nabla \times \mathbf{B}*}{en\mu_0}$$

$$P*_e = P_e^{n+1/2} - \frac{\Delta t}{2}\left(\mathbf{V}*_e \bullet \nabla P_e^{n+1/2} + \Gamma_e P*_e \nabla \bullet \mathbf{V}*_e\right)$$

$$P_e^{n+3/2} = P_e^{n+1/2} - \Delta t\left\{\mathbf{V}*_e \bullet \nabla P*_e - \right.$$

$$\left. + \Gamma_e P*_e \nabla \bullet \left[\mathbf{V}*_e - \frac{\Delta t}{2Mn}\nabla_{\|}\left(P_e^{n+3/2} - P_e^{n+1/2}\right)\right]\right\}$$

$$\mathbf{E}_{\|}^{n+1} = -\frac{\nabla_{\|}\left(P_e^{n+3/2} + P_e^{n+1/2}\right)}{2en}$$

$$P_e^{n+1} = \frac{P_e^{n+3/2} + P_e^{n+1/2}}{2}, \delta\mathbf{E}_{\|} = \mathbf{E}_{\|}^{n+1} - \mathbf{E}*$$

3D Perpendicular Solution

$$M\, n_0 \boldsymbol{\xi} - \frac{\Delta t^2}{4\mu_0} \nabla \times \nabla \times \boldsymbol{\xi} \times \mathbf{B}_0 \times \mathbf{B}_0 - \frac{\Delta t^2}{4} \mathbf{J}_0 \times \nabla \times \boldsymbol{\xi} \times \mathbf{B}_0$$

$$- \frac{\Delta t^2}{4} \nabla\left(\boldsymbol{\xi} \bullet \nabla P_{e0} + \Gamma P_{e0} \nabla \bullet \boldsymbol{\xi}\right) - \frac{\Delta t^2}{4} e\, \mathbf{E}_0 \nabla\left(\boldsymbol{\xi} \bullet \nabla n_0 + n_0 \nabla \bullet \boldsymbol{\xi}\right) = \boldsymbol{\varepsilon}$$

$$\tilde{\mathbf{E}}^{n+1} = \tilde{\mathbf{E}}^{*} - \Delta t \boldsymbol{\xi} \times \mathbf{B}_0 : \;\; \boldsymbol{\Theta} = Q \Delta t\, \mathbf{B}_0 / 2M$$

These are the linear terms only. Also, it is assumed that $\Theta >> 1$

3D Perpendicular Solution

- Using electron equilibrium (momentum) equation, this is a symmetric operator
- Essentially $\delta W + \nabla n$ term
- Can use CG to invert?

2D Prototype Code (IMP2)

- Assumes $\mathbf{B} \perp$ to simulation plane, $\mathbf{E}$ in plane
- $\mathbf{g}$ added to give Roberts-Taylor mode
- Full Lorentz force orbits
- Orbit averaging
- Ions also can be treated as fluid (moment)

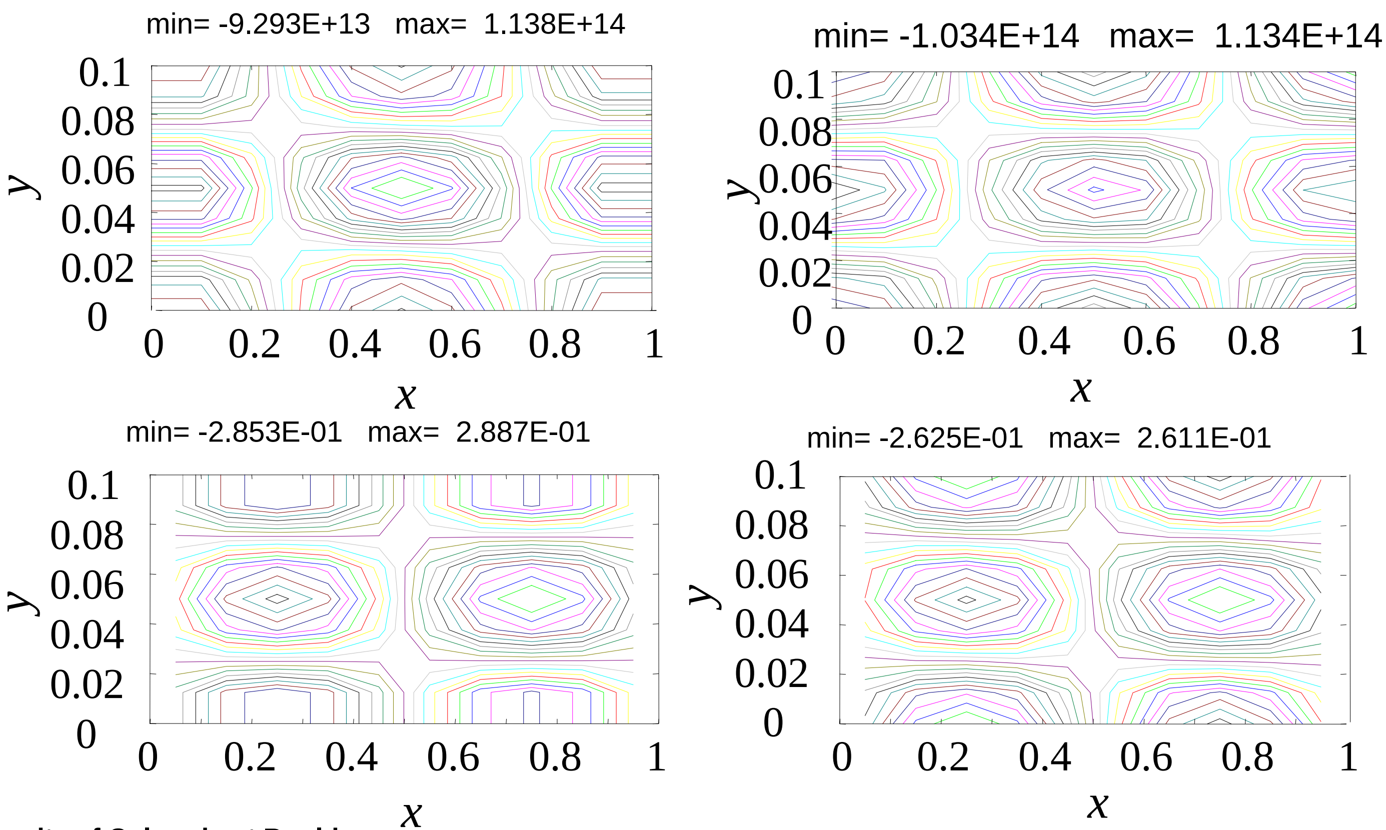
2D Tests With IMP2

Moment | 225 Particles/cell

Density

Drive with “open loop” E

E x B Velocity



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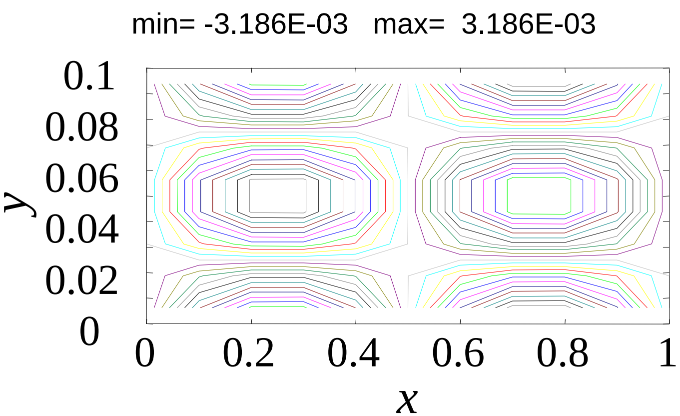
2D Tests With IMP2

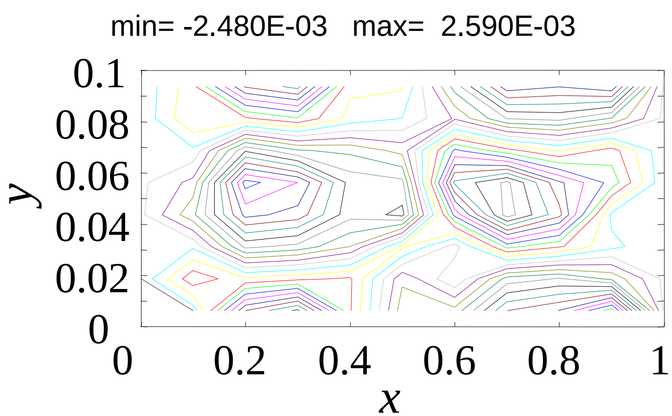
Polarization Velocity

Drive with "open loop" E

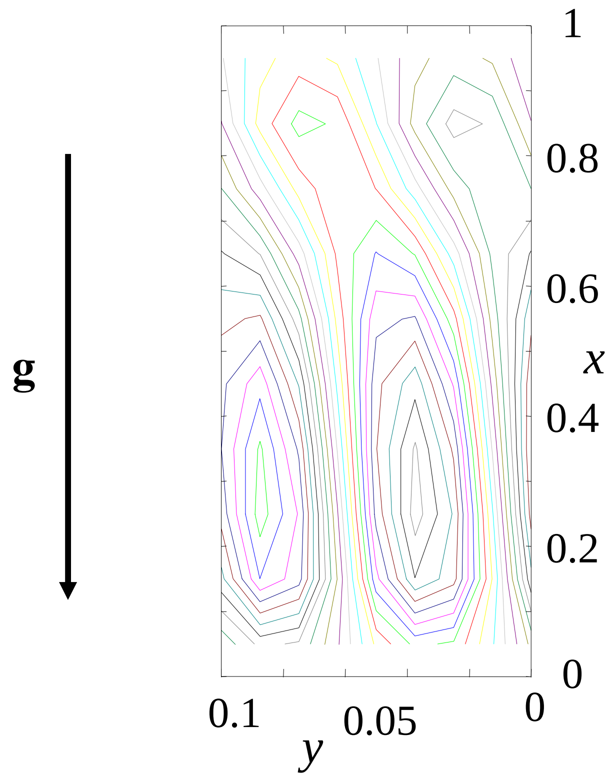
Moment



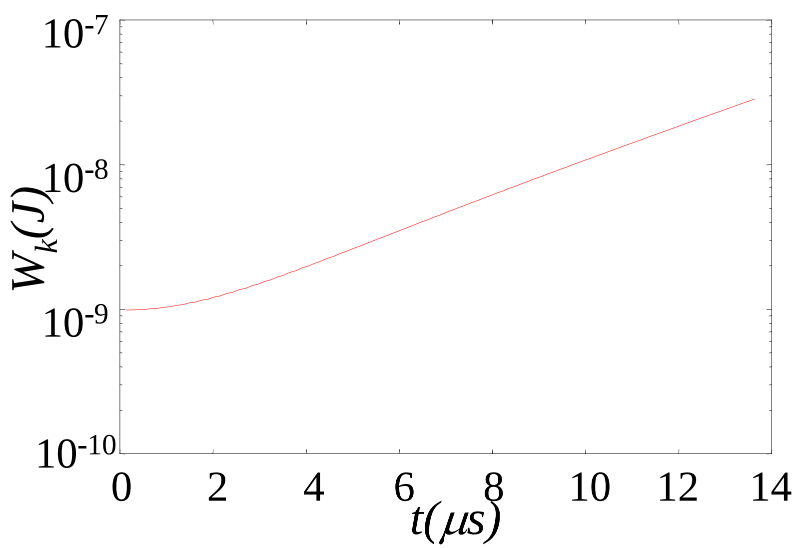
225 Particles/cell



g-mode Tests (moment)

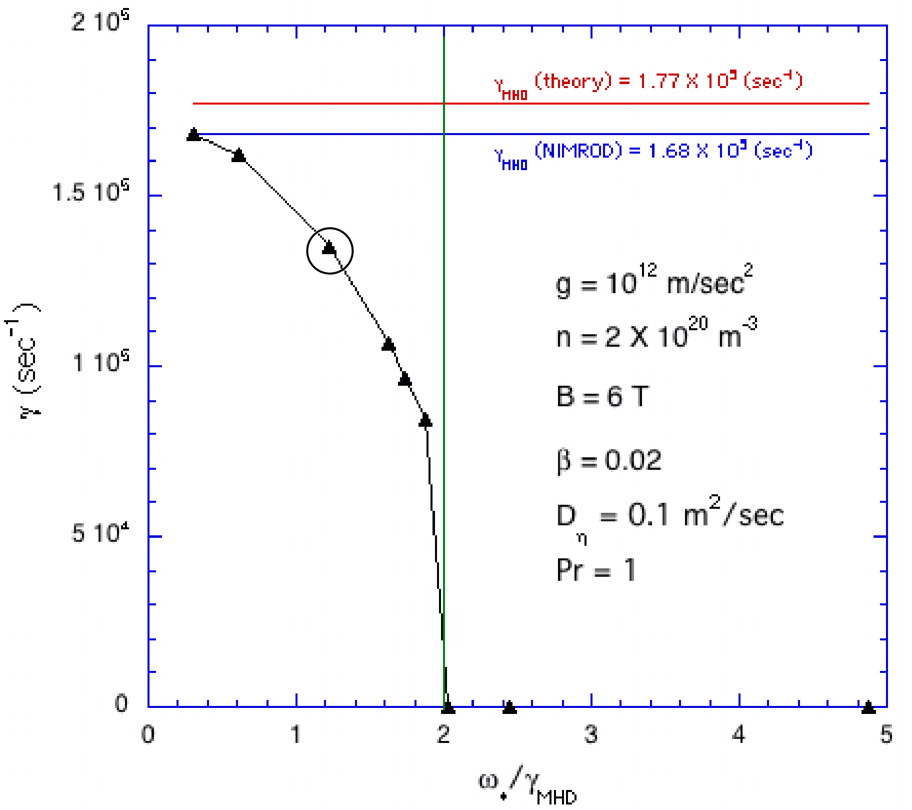


x-velocity



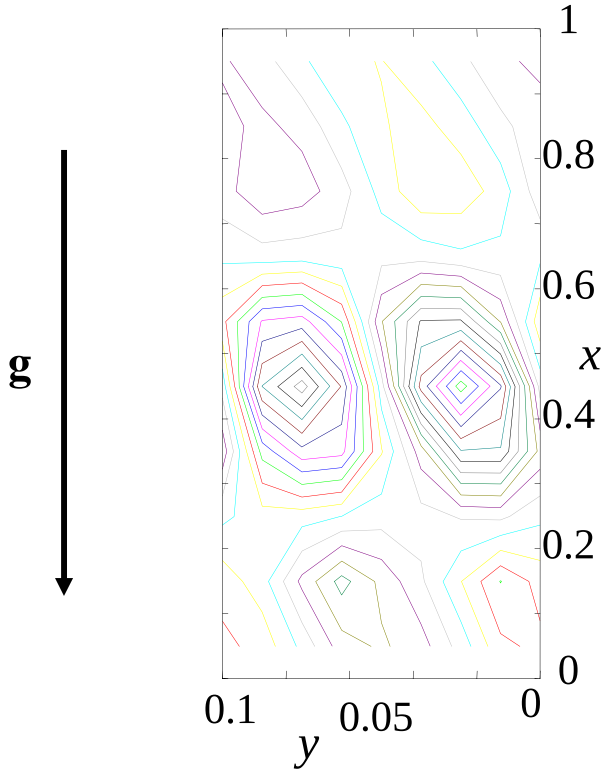
Kinetic energy: $\gamma = 1.37 \times 10^5\ s^{-1}$

g-mode Tests

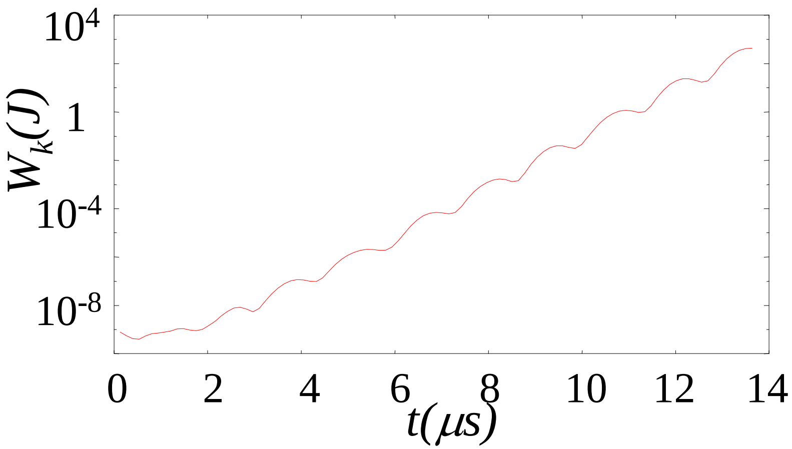


D. D. Schnack and D. C. Barnes (2006)

g-mode Tests (25particles/cell)

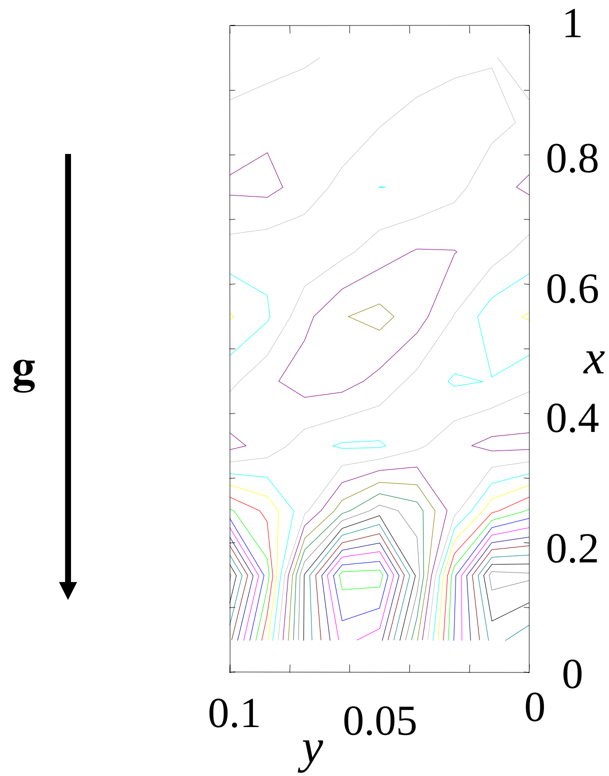


x-velocity

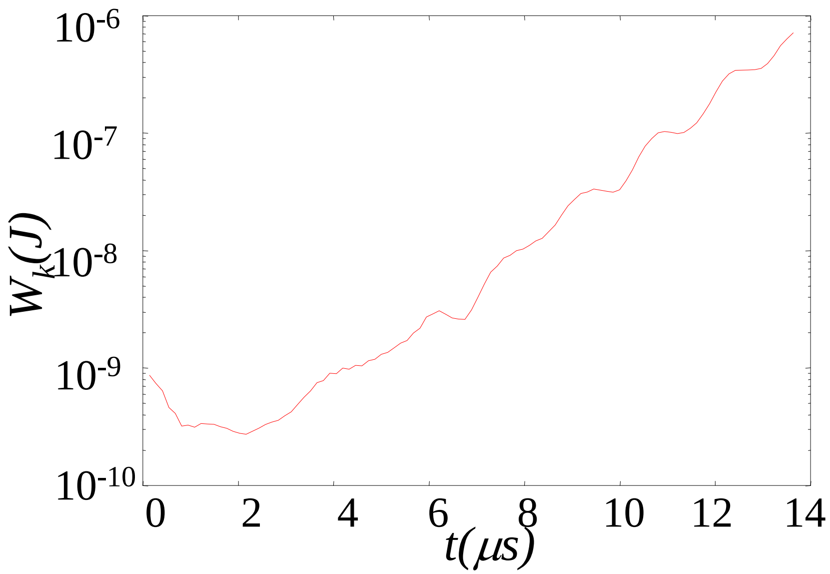


Kinetic energy: $\gamma = 1.14 \times 10^6\ s^{-1}$

g-mode Tests (100particles/cell)



x-velocity



Kinetic energy: $\gamma = 3.83 \times 10^5\ s^{-1}$

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Conclusions and Future

- g-mode verified with moment ions
- Implicit field solver seems to work for E ⊥ B
- Need many more particles or improve method
- 3D implementation and tests
- NIMROD implementation