Some Observations from the M3D CDX-U Sawtooth Study

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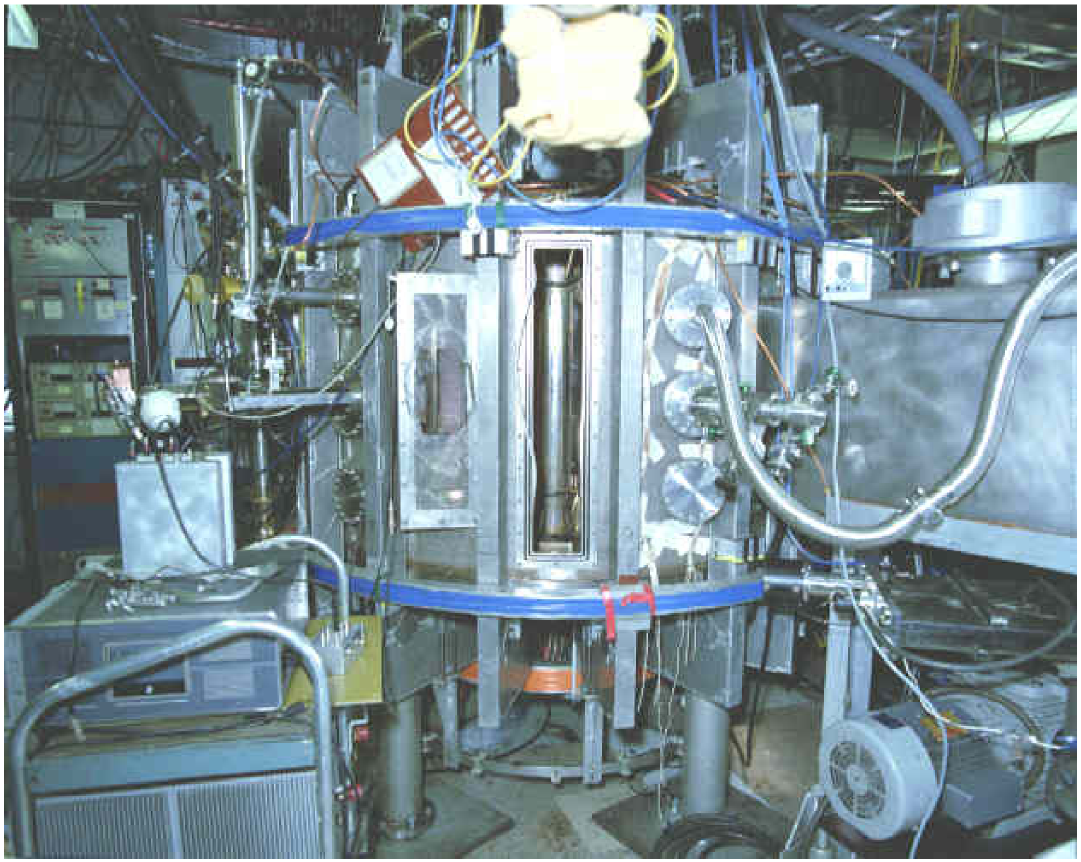
Motivation

- As a nonlinear code benchmark, model sawtooth events in a tokamak plasma with extended MHD models using realistic physical values to make quantitative predictions.

  - Large tokamaks have large disparities in spatial and temporal scales to be resolved.
    - Resistive MHD: Current sheet thickness ~ $S^{-1/2}$
    - Two-fluid MHD: ion skin depth ~ $n^{-1/2}$

  - Small tokamaks operate in regimes accessible to present-day codes.

Characteristics of the Current Drive Experiment Upgrade (CDX-U)

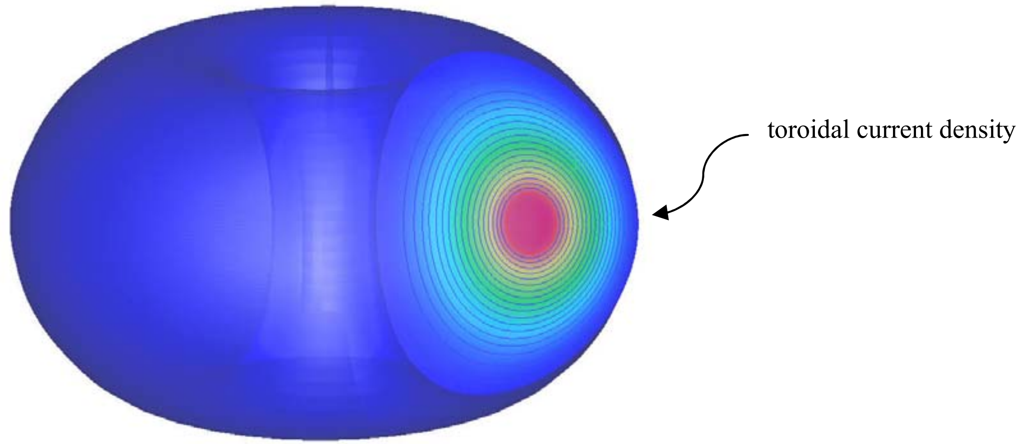


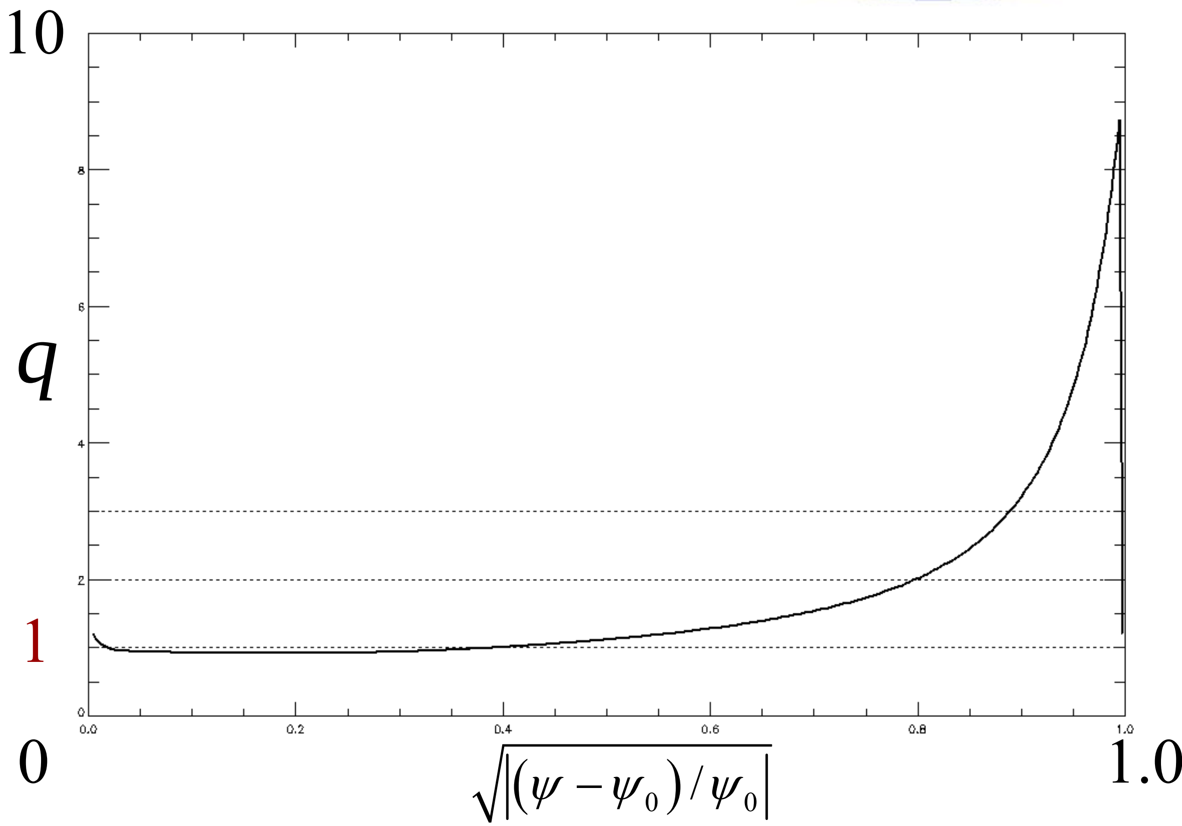
- Low aspect ratio tokamak ($R_0/a = 1.4 – 1.5$)
- Small ($R_0 = 33.5$ cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $I_p \sim 70$ kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV $\rightarrow$ S $\sim 10^4$
- Discharge time ~ 12 ms

- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  – sawteeth
  – "snakes"

Equilibrium: $q_0 < 1$

- Equilibrium taken from a TSC sequence (Jsolver file).
- $q_{min} \approx 0.922$
- $q(a) \sim 9$





Questions to investigate:

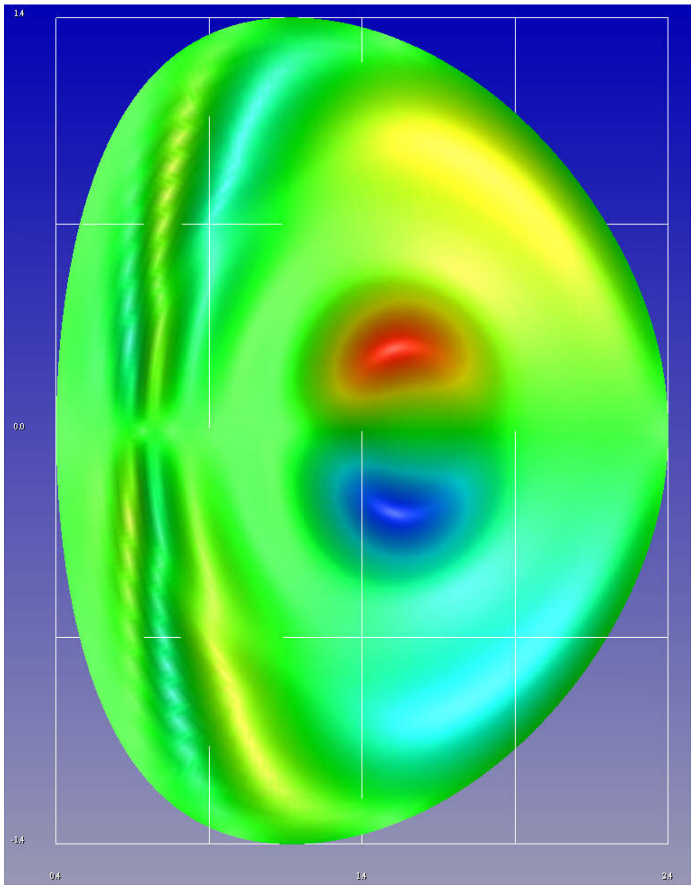
- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Baseline Parameters for CDX

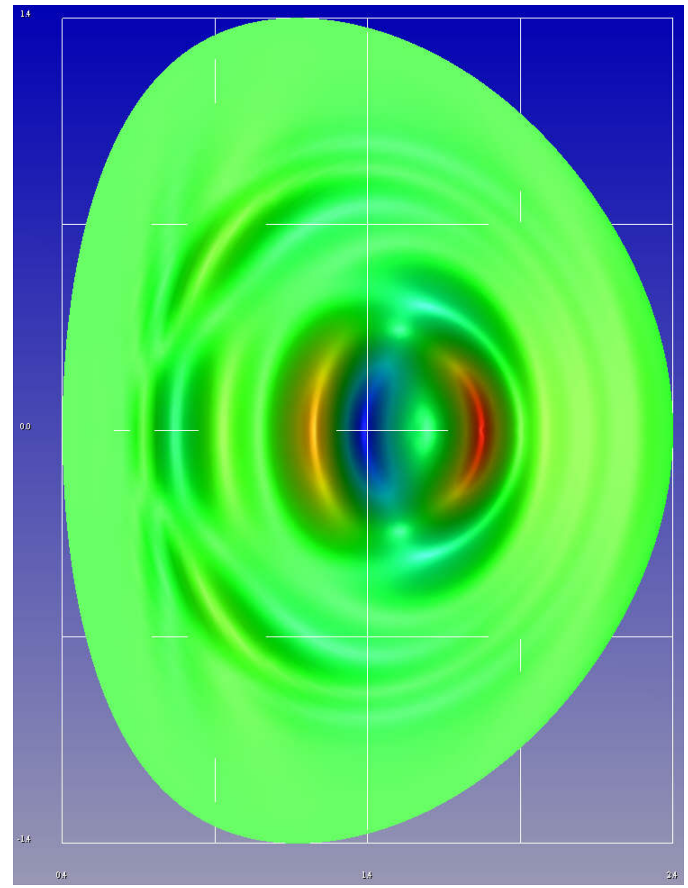
Lundquist Number S	~2×10^4 on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number Pr	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	200 m^2/s (measured value at edge)
Parallel thermal conduction (sound wave)	$V_{Te} = 6\ V_A$
Peak Plasma β	~ 3×10^{-2} (low-beta).
Density Evolution	Turned on for nonlinear phase.
Nonlinear initialization	Pure n=1 perturbation such that $\frac{\max(B^1_{pol})}{\max(B^0_\phi)} = 10^{-4}$

n=1 Eigenmode

Incompressible velocity
stream function U



Toroidal current density
J_ϕ

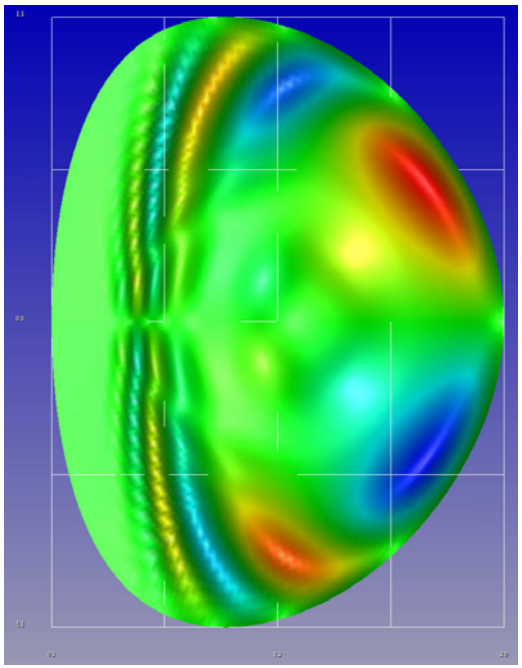


$\gamma \tau_A = 8.61 \times 10^{-3} \rightarrow$ growth time $= 116\ \tau_A$

Higher n Eigenmodes

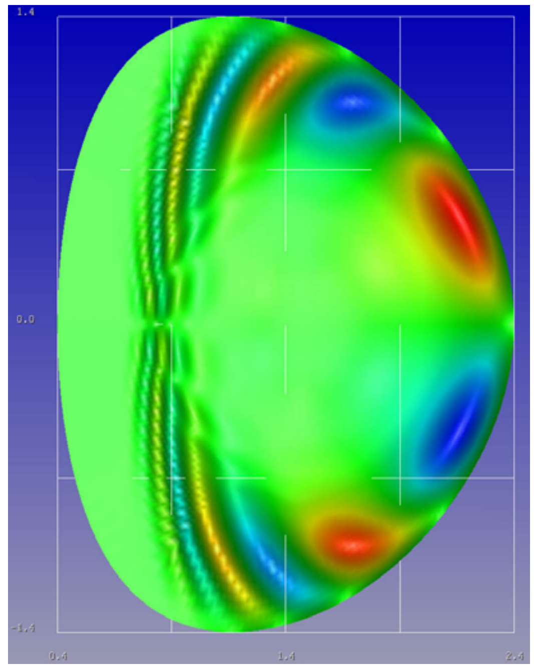
Incompressible velocity
stream function U

$n = 2$



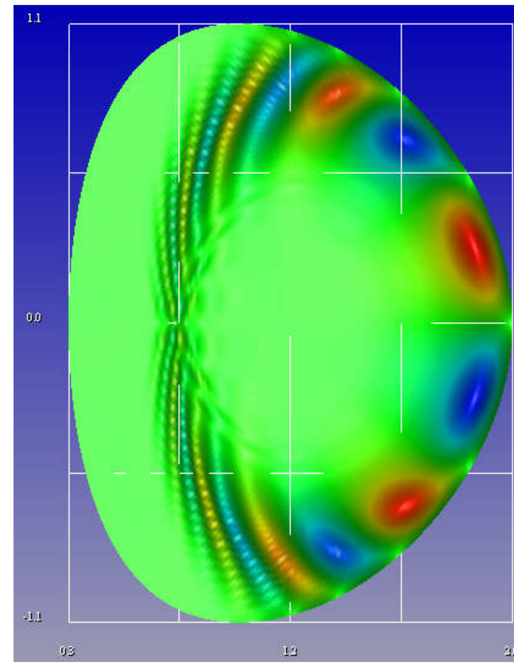
$m \geq 5$
$\gamma \tau_A = 1.28 \times 10^{-2}$

$n = 3$



$m \geq 7$
$\gamma \tau_A = 1.71 \times 10^{-2}$

$n = 4$



$m \approx 9$
$\gamma \tau_A = 1.87 \times 10^{-2}$

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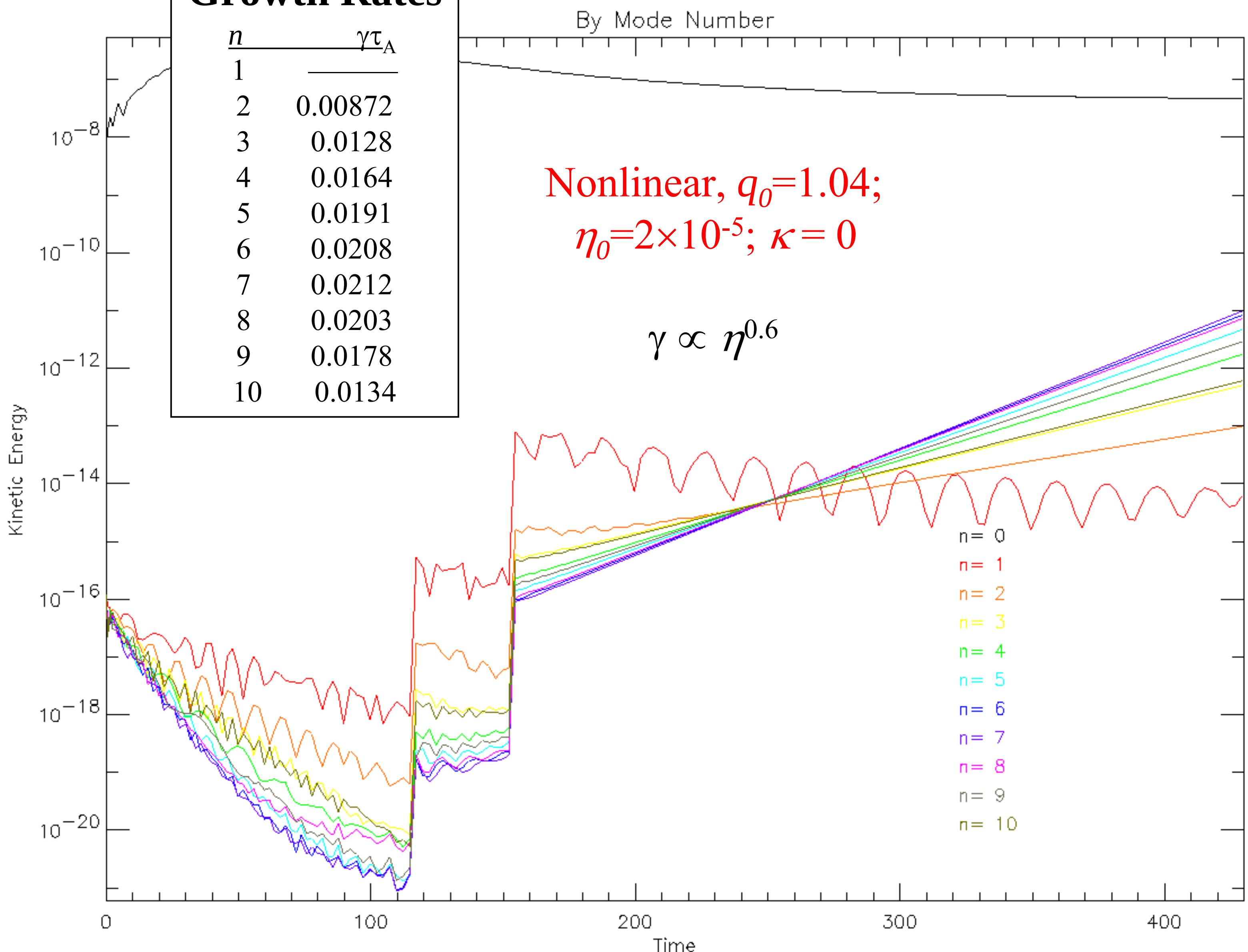
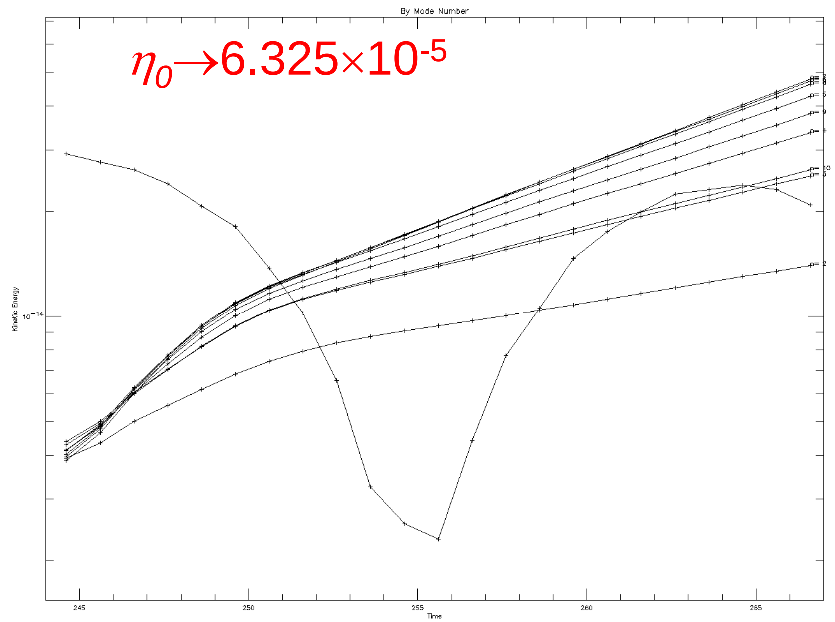
In Absence of Heat Conduction, Higher *n* Modes are More Unstable than Internal Kink

Growth Rates

n	$\gamma\tau_A$
1	—
2	0.00872
3	0.0128
4	0.0164
5	0.0191
6	0.0208
7	0.0212
8	0.0203
9	0.0178
10	0.0134

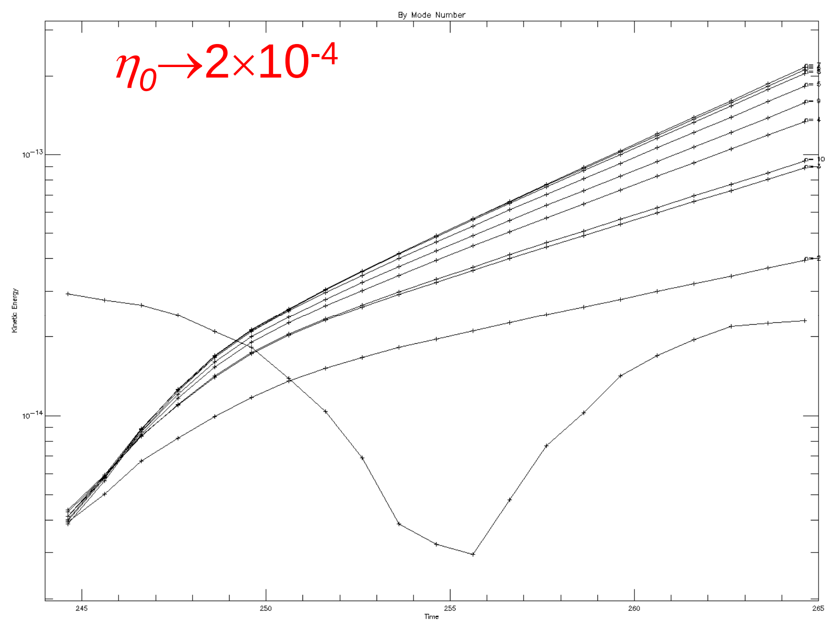


Resistivity Scaling is Consistent with low-n Resistive Ballooning Modes



Growth Rates
(gcut=12)

n	$\gamma\tau_A$
1	——
2	0.0184
3	0.0270
4	0.0337
5	0.0387
6	0.0416
7	0.0425
8	0.0409
9	0.0363
10	0.0281



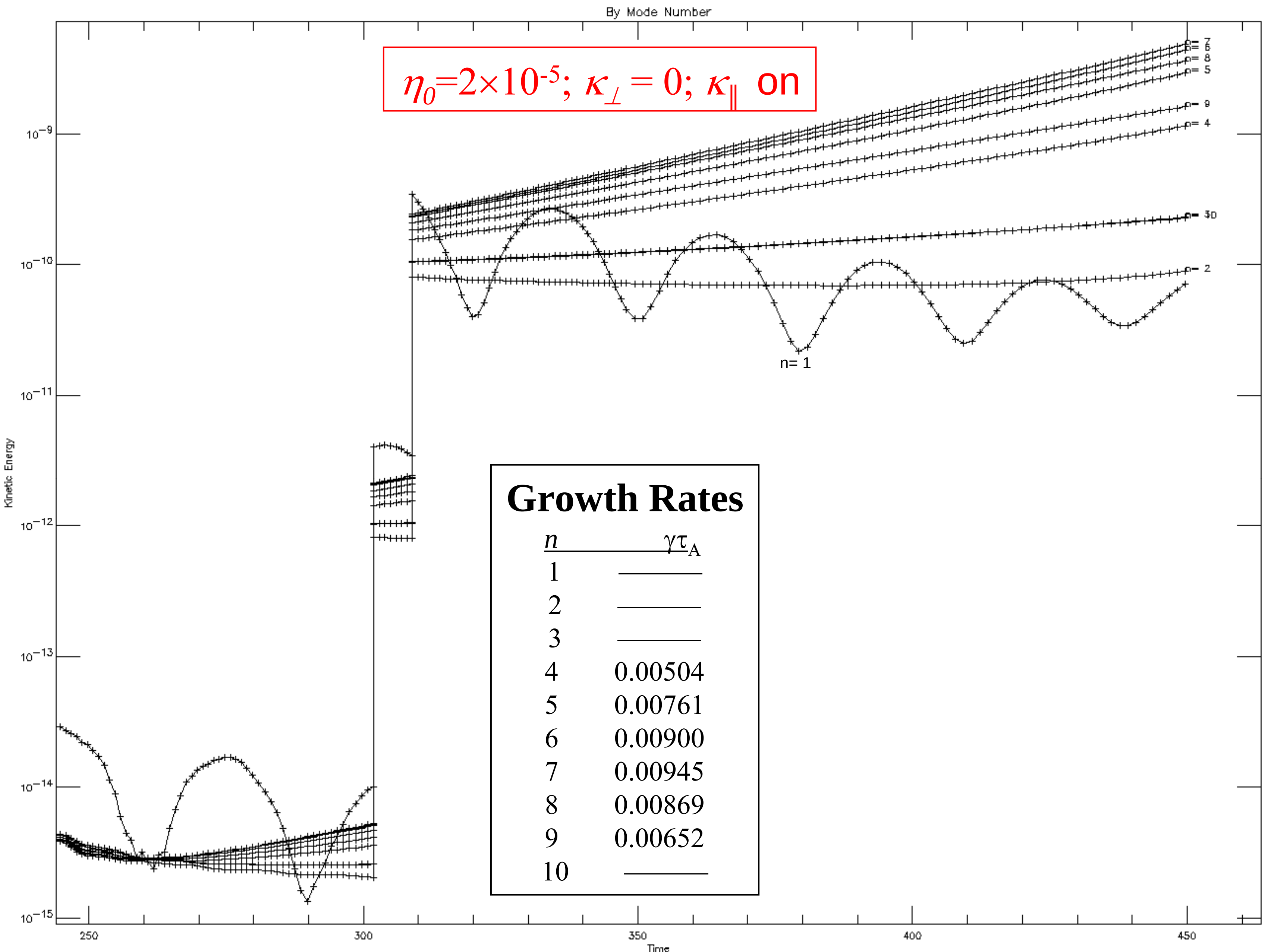
Growth Rates
(gcut=11)

n	$\gamma\tau_A$
1	——
2	0.0345
3	0.0498
4	0.0607
5	0.0682
6	0.0726
7	0.0738
8	0.0714
9	0.0646
10	0.0516

For each toroidal mode number n, the linear growth rate γ is found to be proportional to η^{α}:

n	α
2	0.597
3	0.590
4	0.568
5	0.553
6	0.543
7	0.542
8	0.546
9	0.560
10	0.586

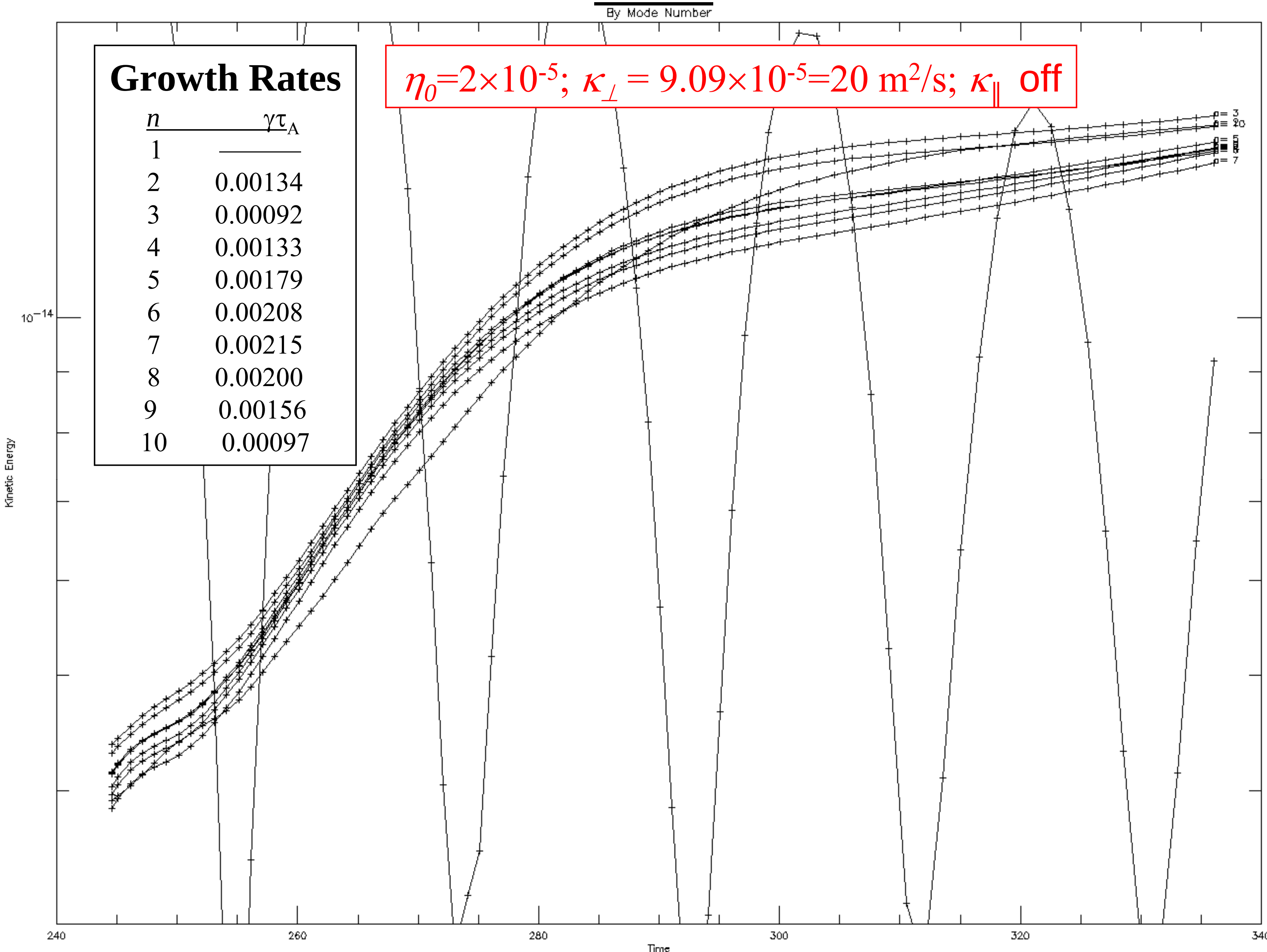
Parallel Heat Conduction Alone Stabilizes Some Modes, But Does Not Appear to Cause Saturation of Unstable Modes



Growth Rates

n	$\gamma\tau_A$
1	—
2	—
3	—
4	0.00504
5	0.00761
6	0.00900
7	0.00945
8	0.00869
9	0.00652
10	—

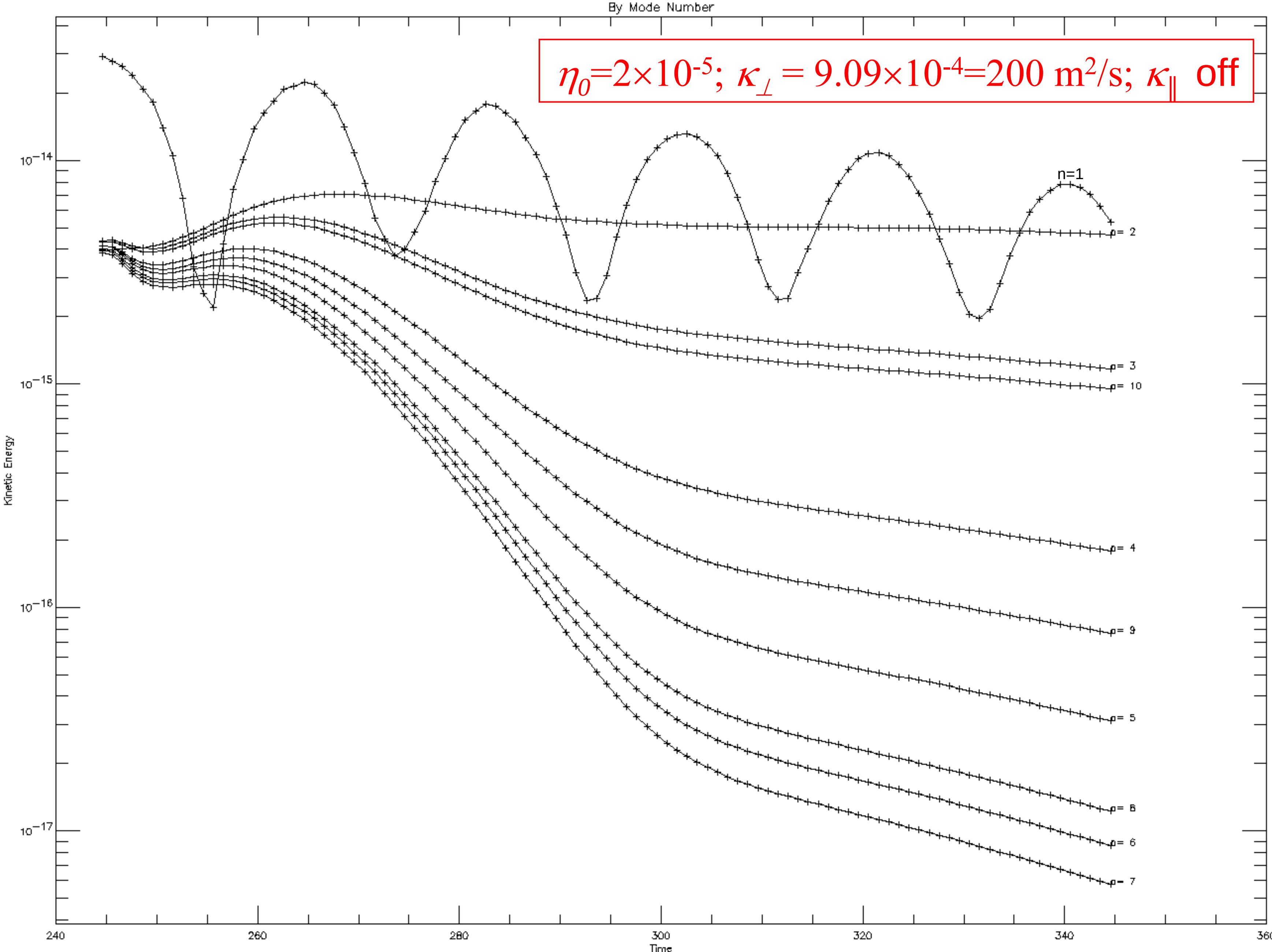
Moderate Isotropic Heat Conduction Has Stronger Stabilizing Effect



Growth Rates

n	$\gamma\tau_A$
1	———
2	0.00134
3	0.00092
4	0.00133
5	0.00179
6	0.00208
7	0.00215
8	0.00200
9	0.00156
10	0.00097

High Perpendicular Heat Conduction Stabilizes All Ballooning Modes



Heat Conduction in M3D

Isotropic component in resistive MHD energy equation

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \kappa_{\perp} \nabla \left(\frac{p}{\rho} - \frac{p_0}{\rho_0} \right) \qquad (1)$$

where p_0/ρ_0 is the equilibrium temperature.

Artificial sound wave model for $\kappa_{\parallel}$:

$$\frac{\partial T}{\partial t} = s \frac{\mathbf{B} \cdot \nabla u}{\rho} \qquad (2a)$$

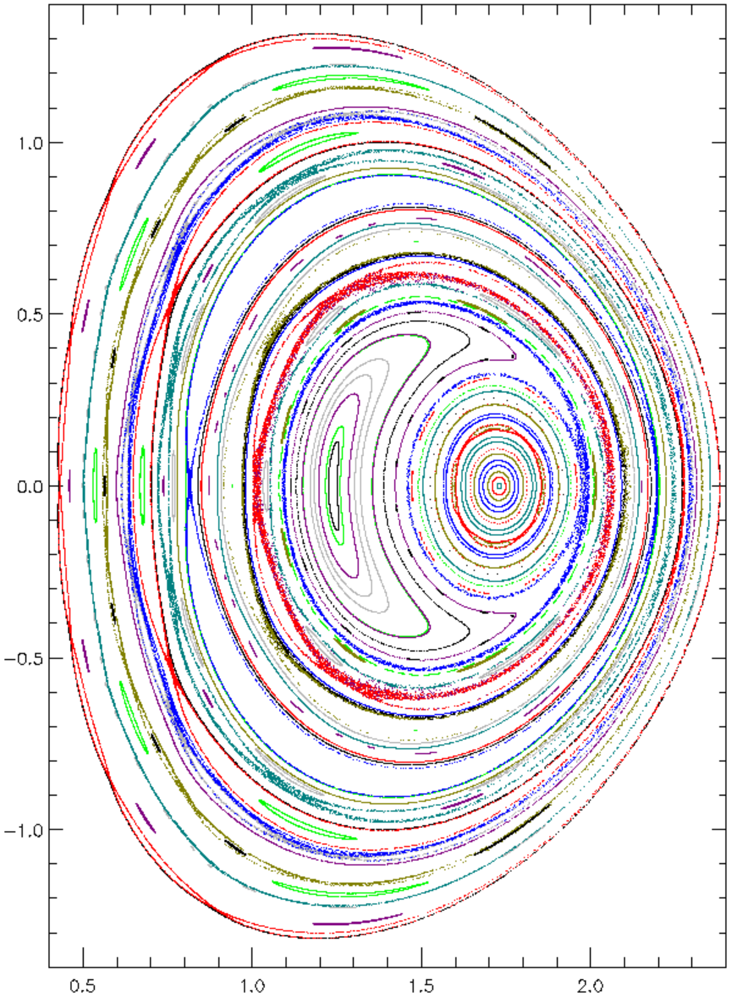
$$\frac{\partial u}{\partial t} = s \mathbf{B} \cdot \nabla T + \nu \nabla^2 u \qquad (2b)$$

where s is the electron sound speed.

Parallel Transport

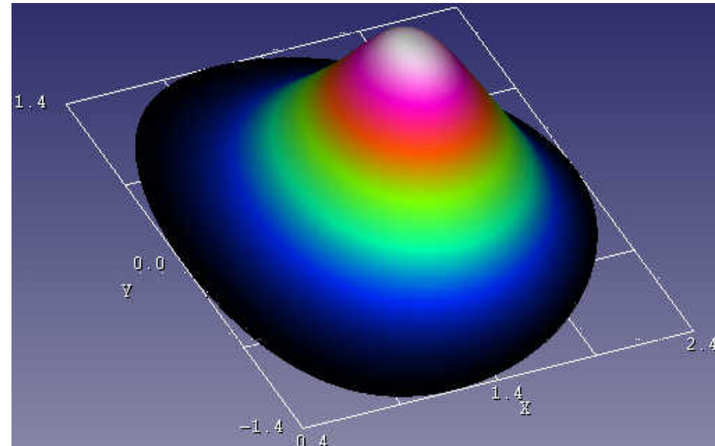
Solving (2) only, holding other quantities fixed, with $s = 6\ v_A$.
Start from equilibrium temperature distribution, with field as shown for $t = 1795.61$ below.

Poincaré plot

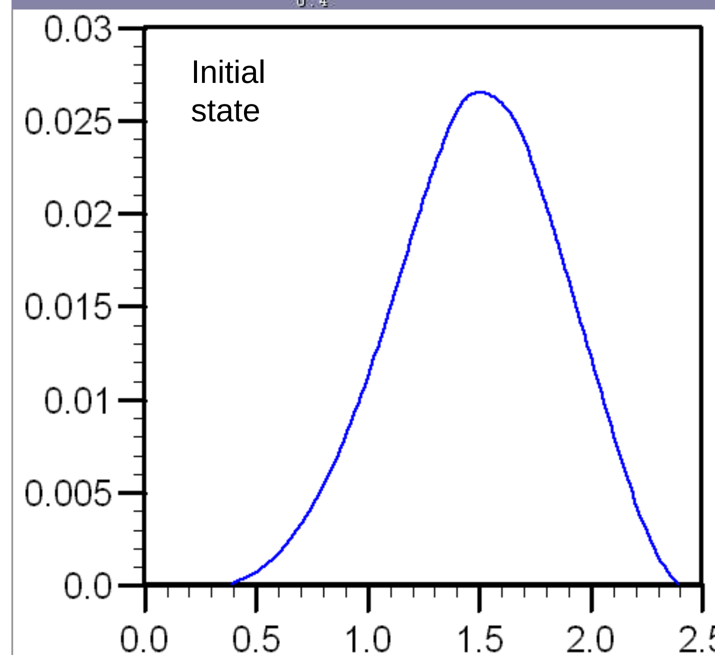


t = 1795.61

T, Surface plot, ϕ=0



T, midplane cross-section

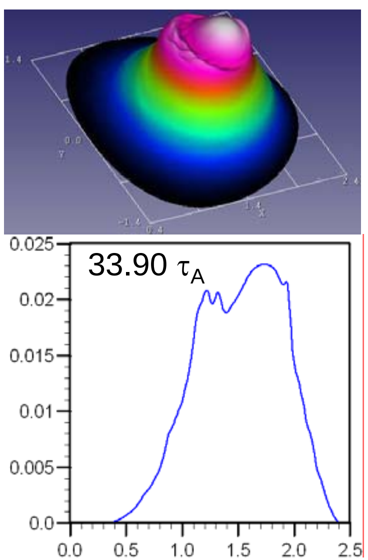


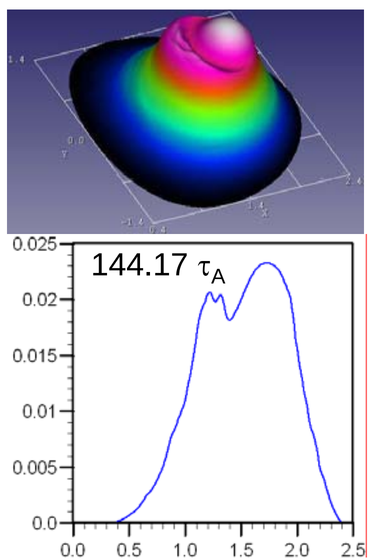
Parallel + Perpendicular Transport

• Parallel heat conduction alone results in very fast equilibration, but cannot rapidly smooth out all bumps where field is stochastic.

• From (1), the time scale for perpendicular equilibration is approximately $\lambda^2/4\pi^2\kappa_\perp$, where λ is the wavelength of the temperature fluctuation being smoothed. Adding some $\kappa_\perp$ and solving (1) as well as (2) can quickly smooth the smallest bumps:

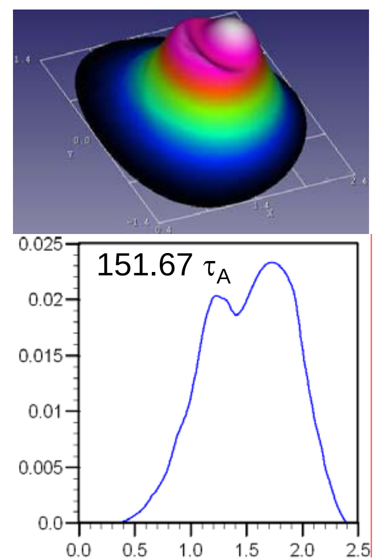
$\kappa_\perp = 10^{-5}$ (2.2 m^2/s) $\rightarrow \tau/\tau_A \sim 2500\ \lambda^2$:

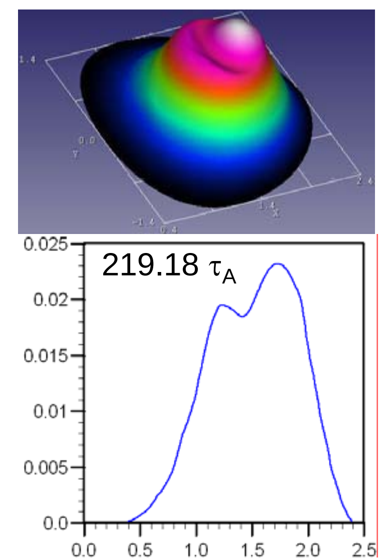




continue, setting

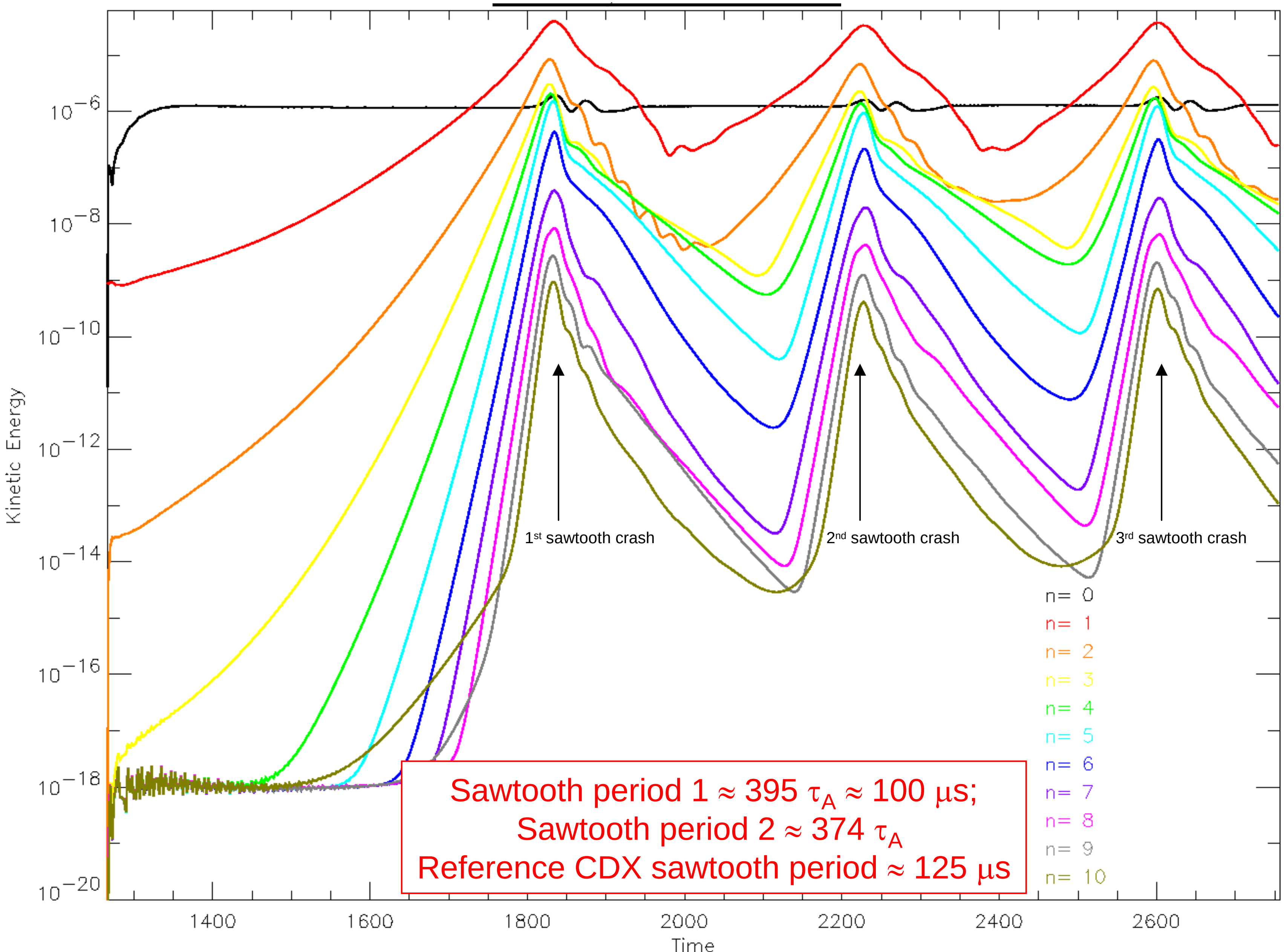
$\kappa_\perp = 10^{-4}$ (22 m^2/s) $\rightarrow \tau/\tau_A \sim 250\ \lambda^2$:



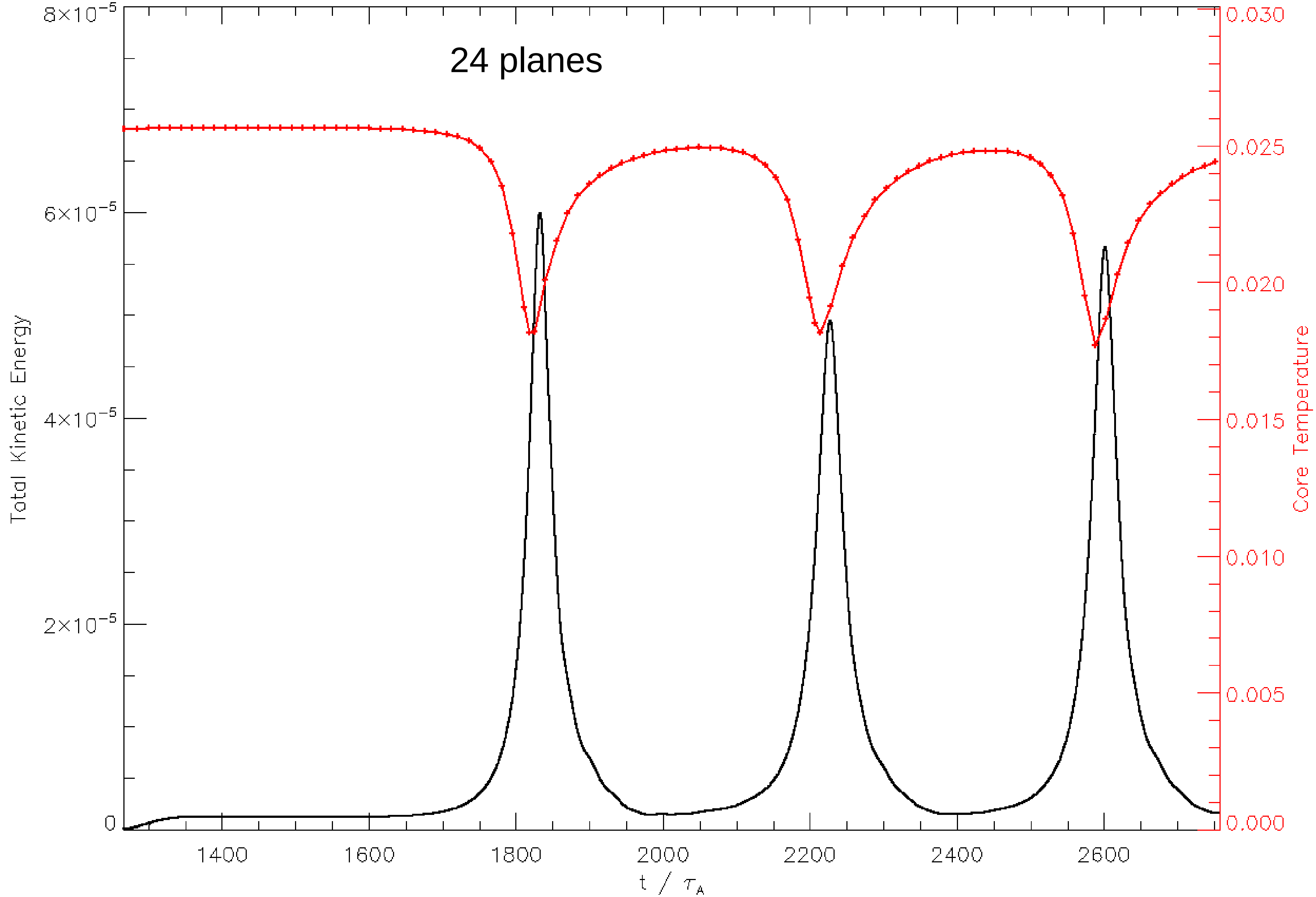


Nonlinear Sawtooth History

10 Modes Retained

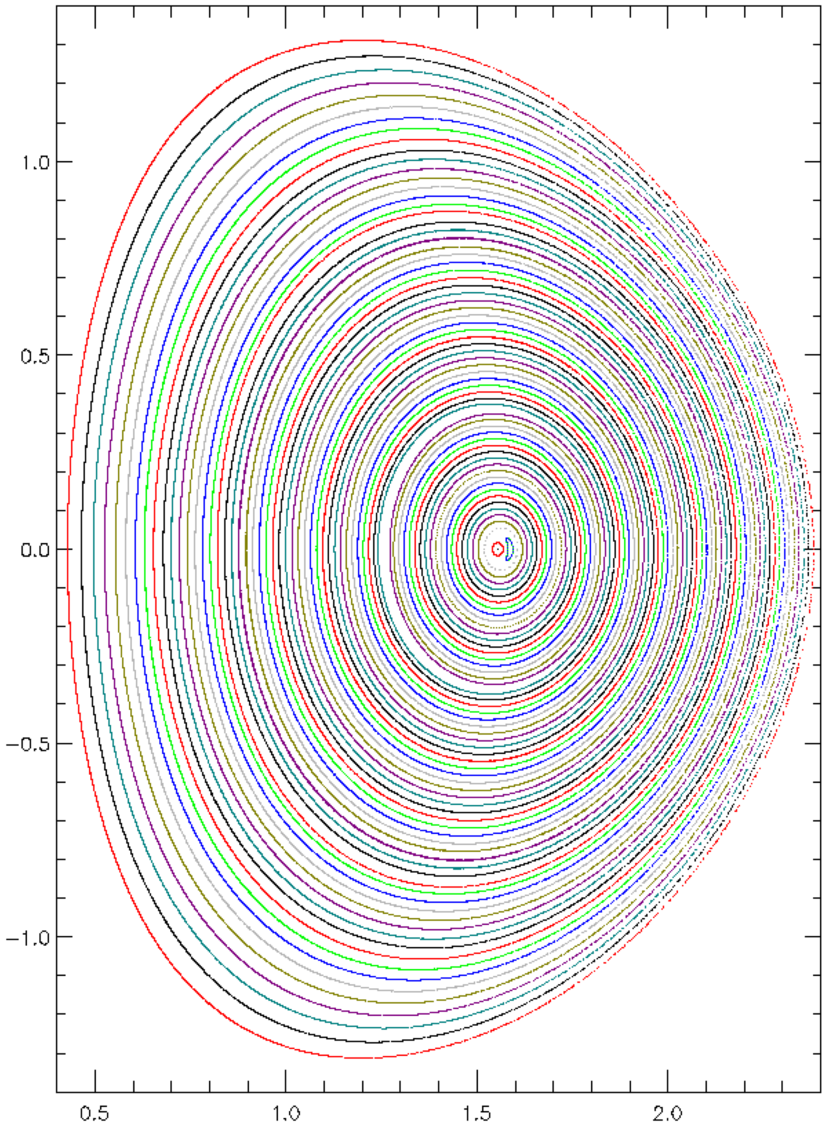


Total Energy and Core Temperature

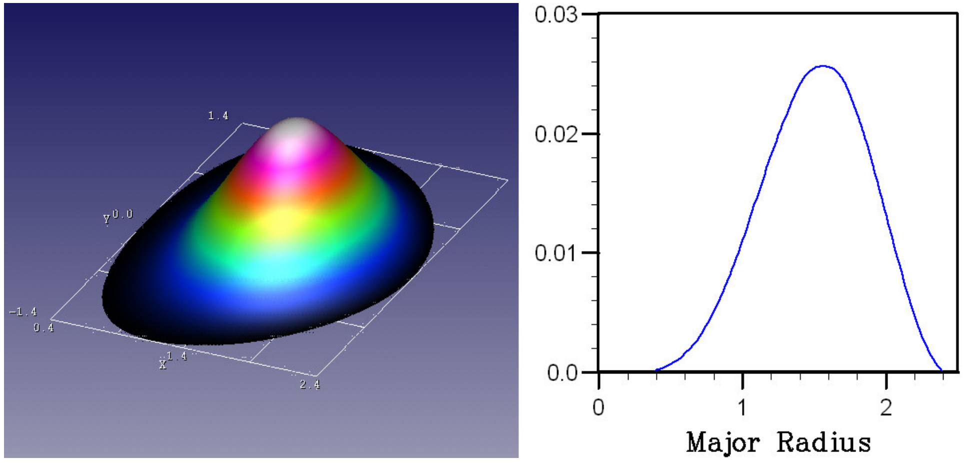


Initial state: t = 1266.17

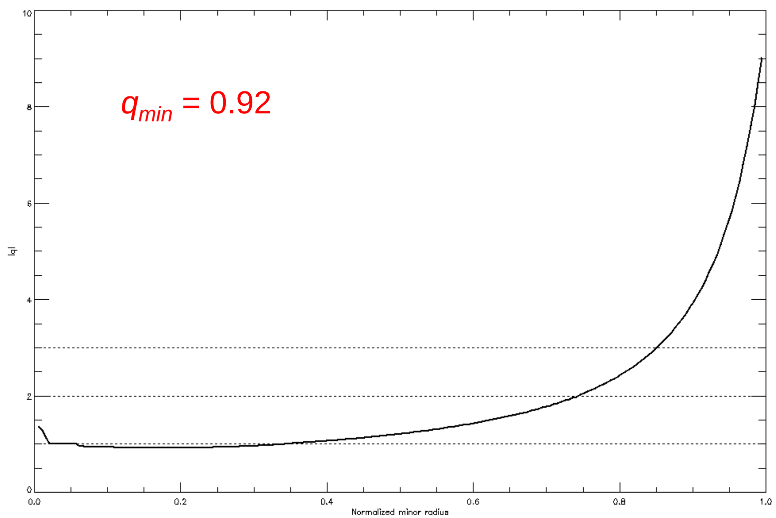
Poincaré plot



Temperature profile

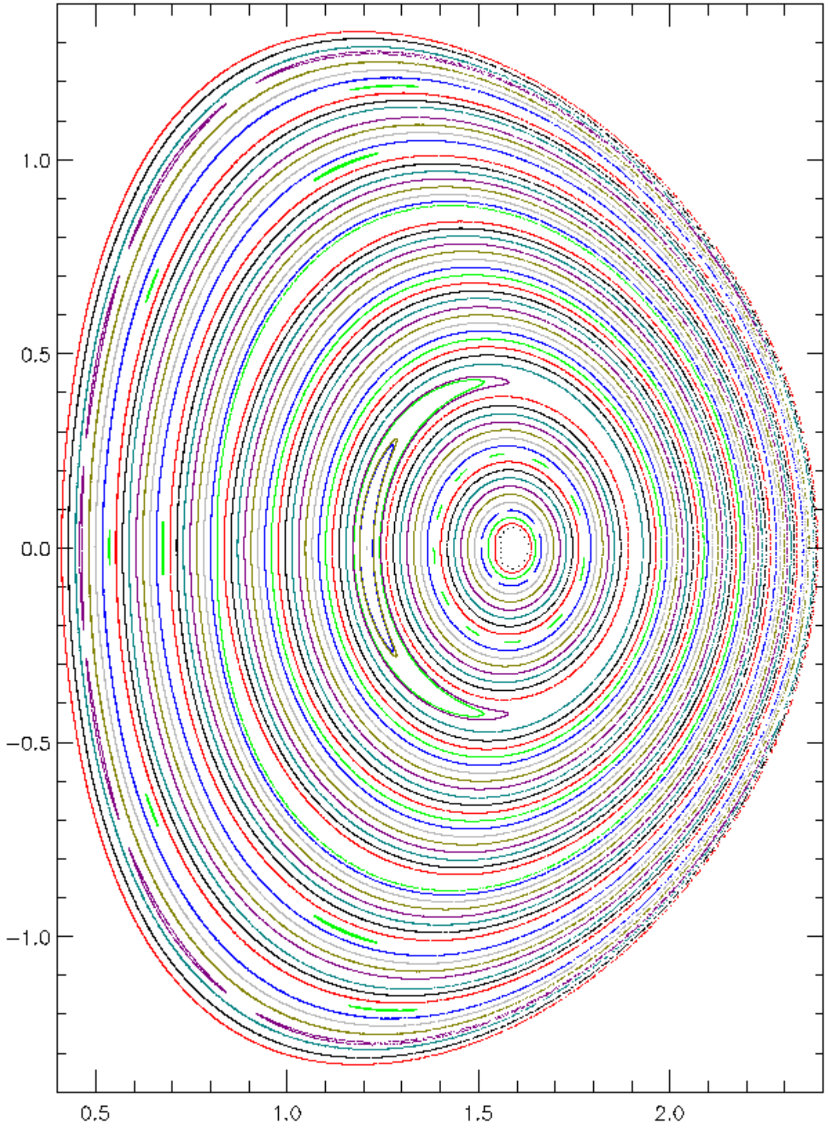


q profile

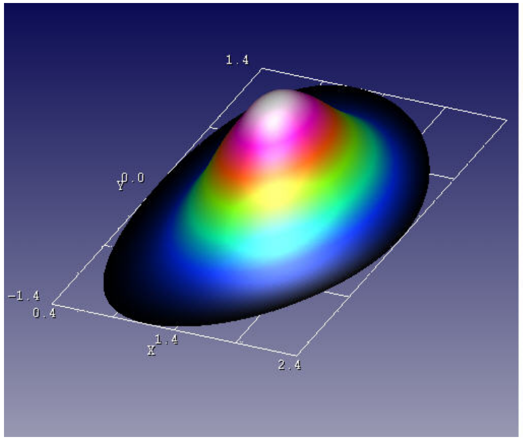


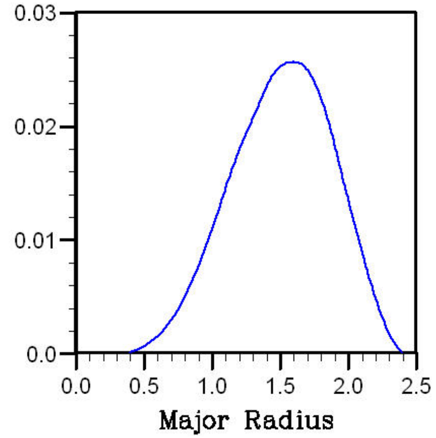
Island growing: t = 1660.70

Poincaré plot



Temperature profile





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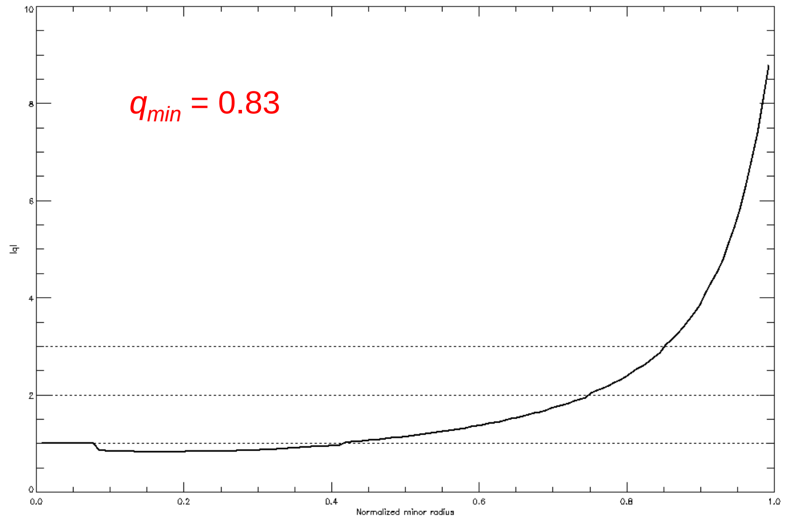
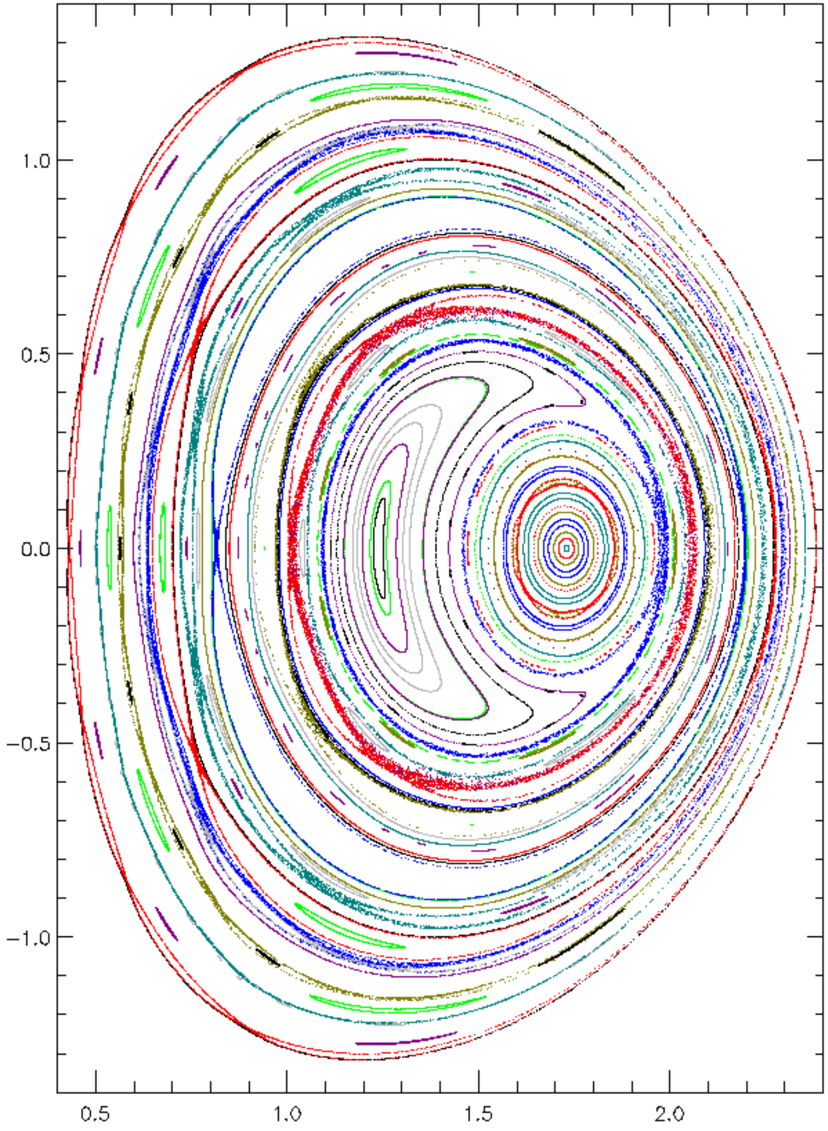
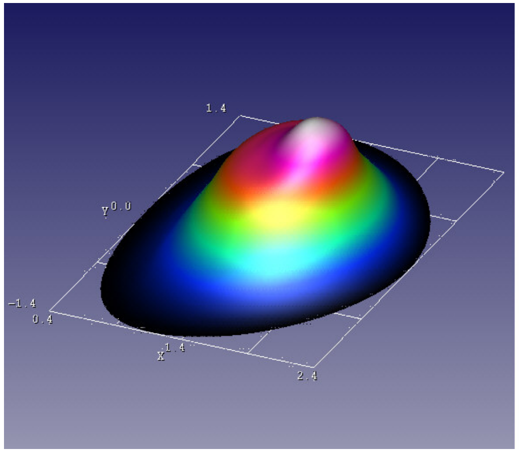


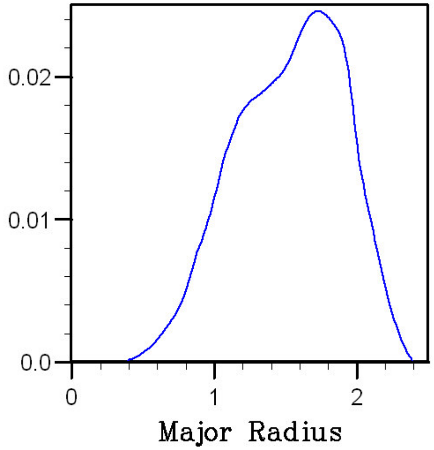
Nonlinear phase: t = 1795.61

Poincaré plot

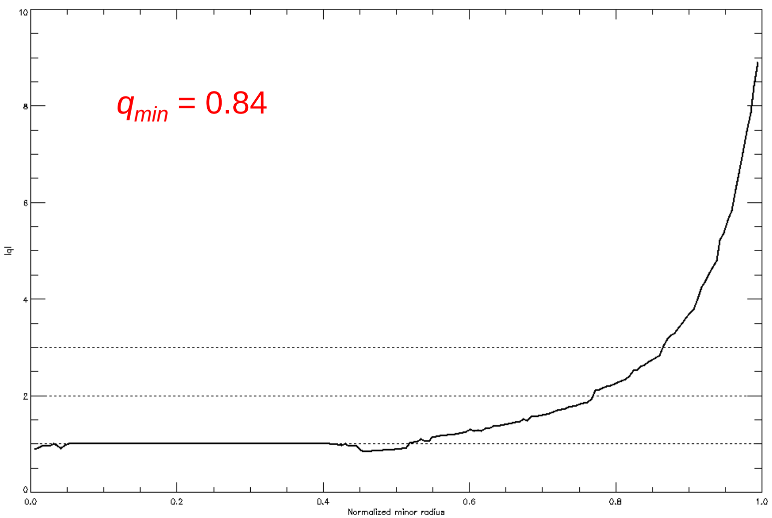


Temperature profile



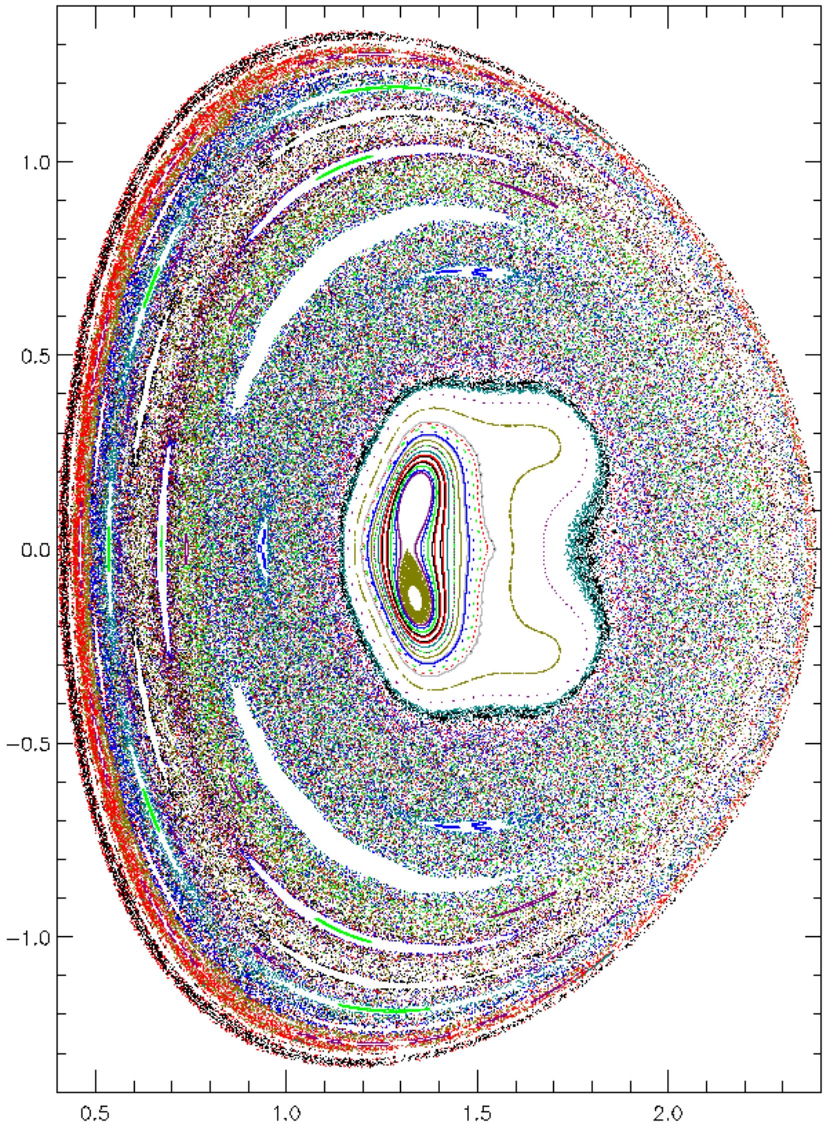


q profile

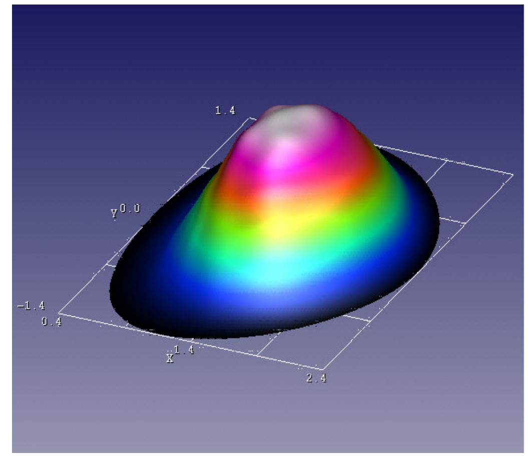


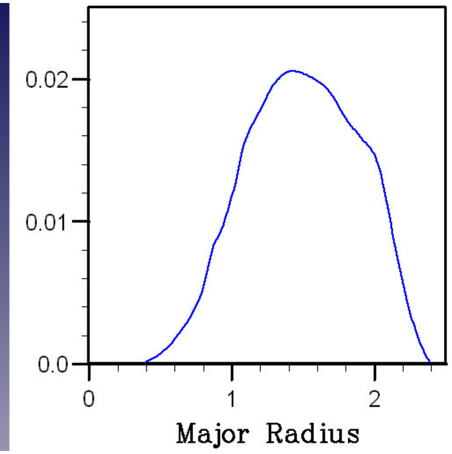
After 1st Crash: t = 1839.86

Poincaré plot

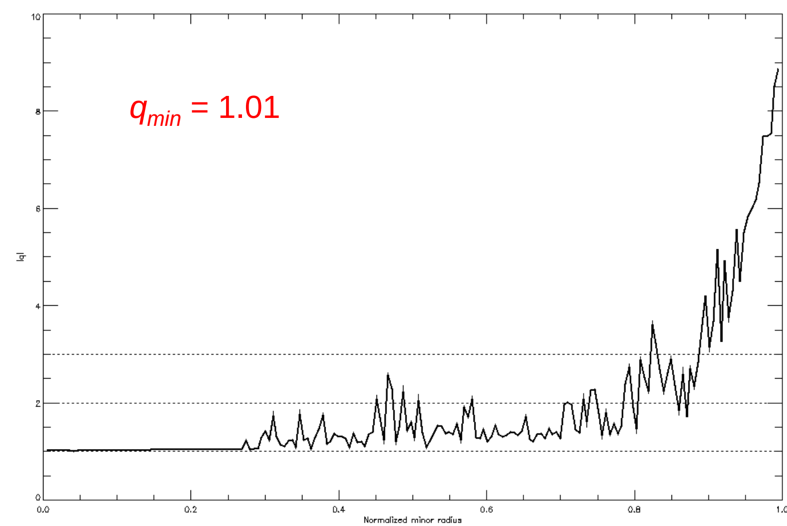


Temperature profile



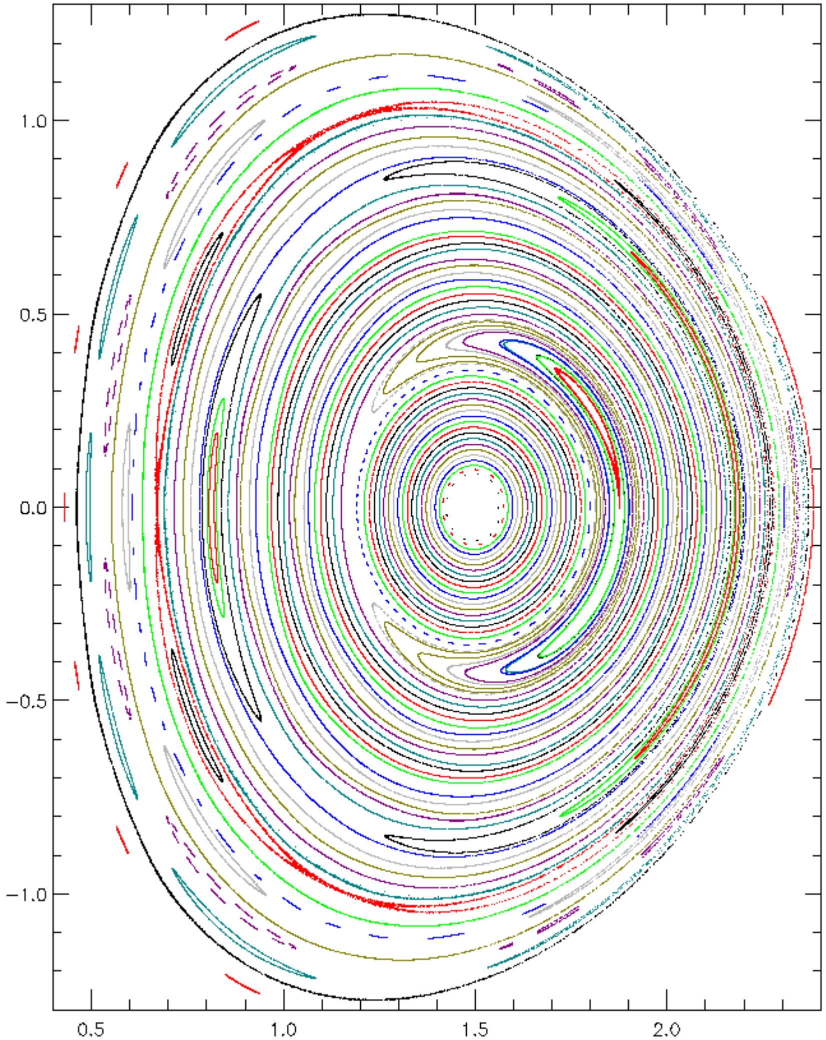


q profile

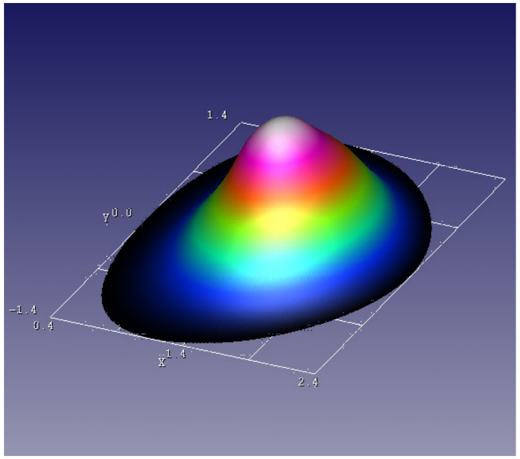


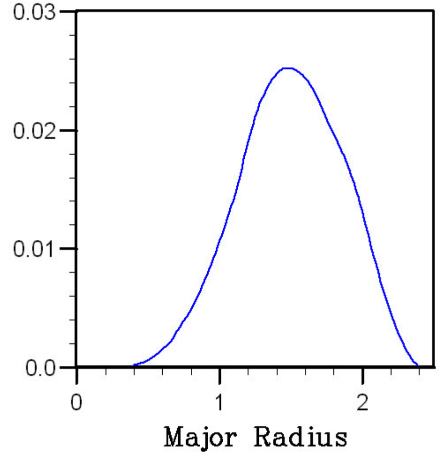
Flux surfaces recovered: t = 2094.08

Poincaré plot

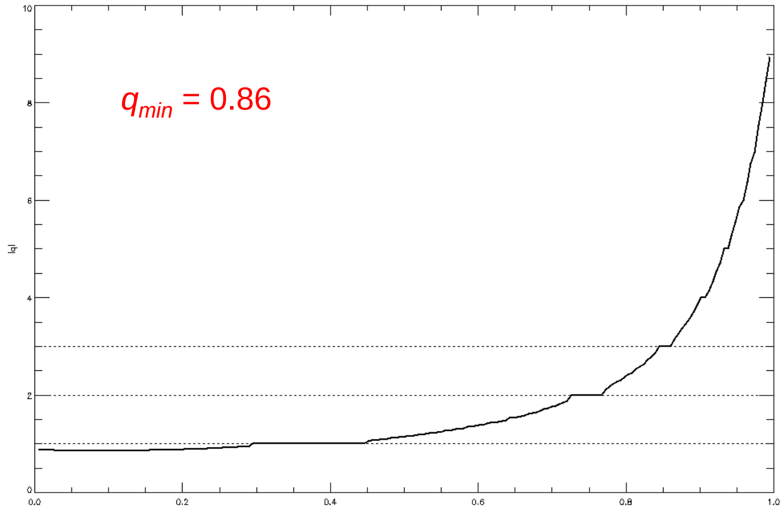


Temperature profile



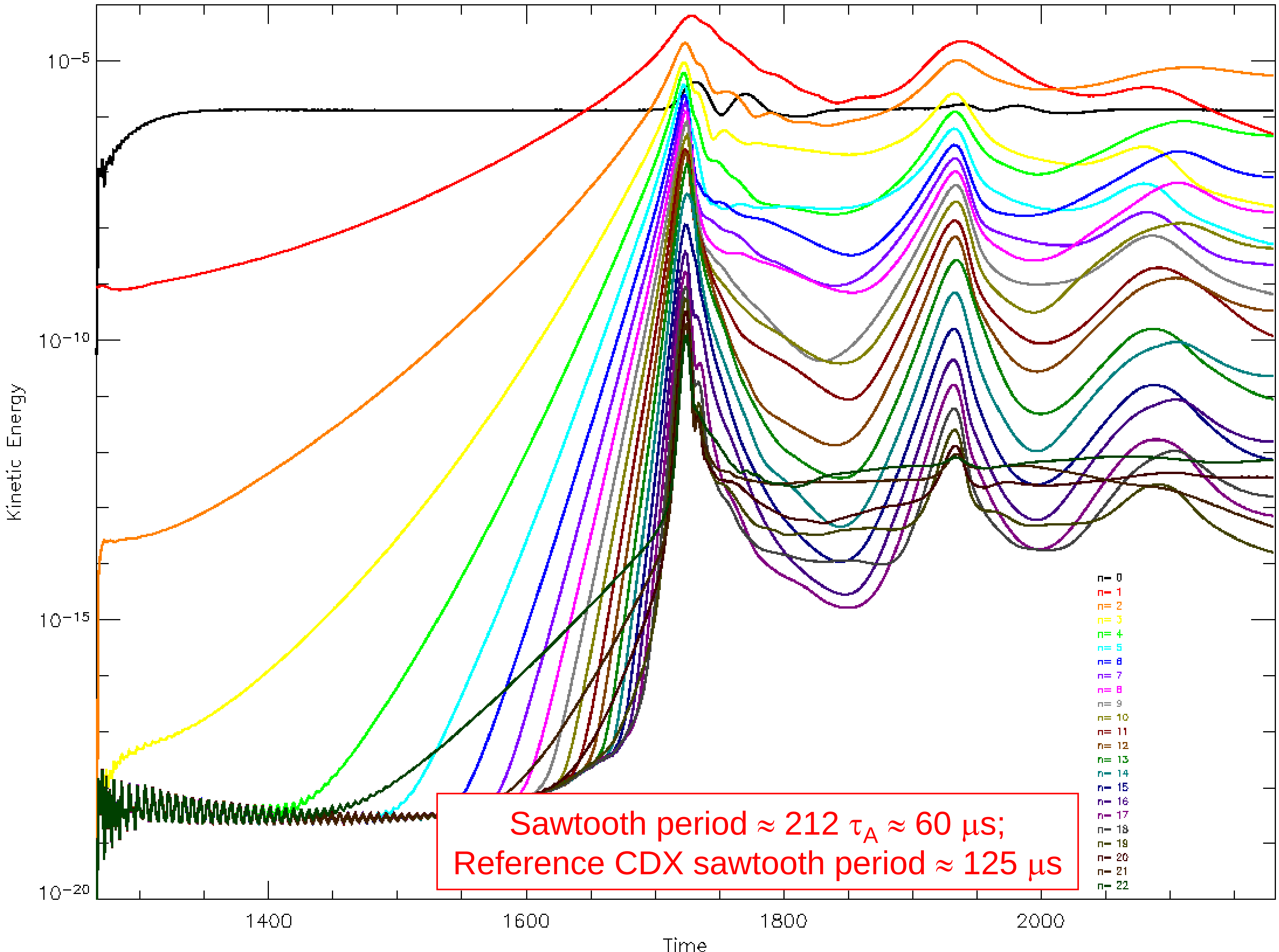


q profile

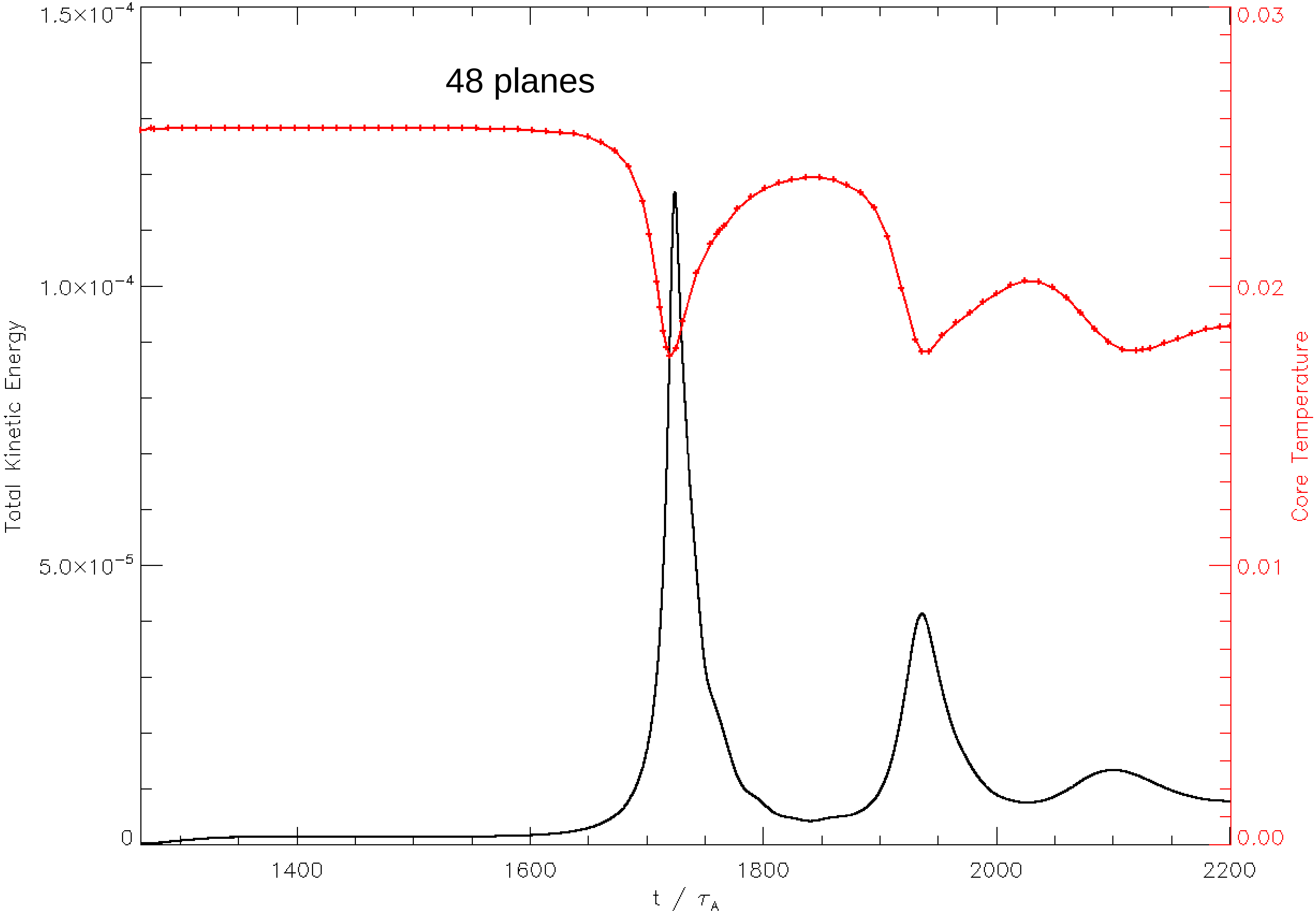


Nonlinear Sawtooth History

22 Modes Retained

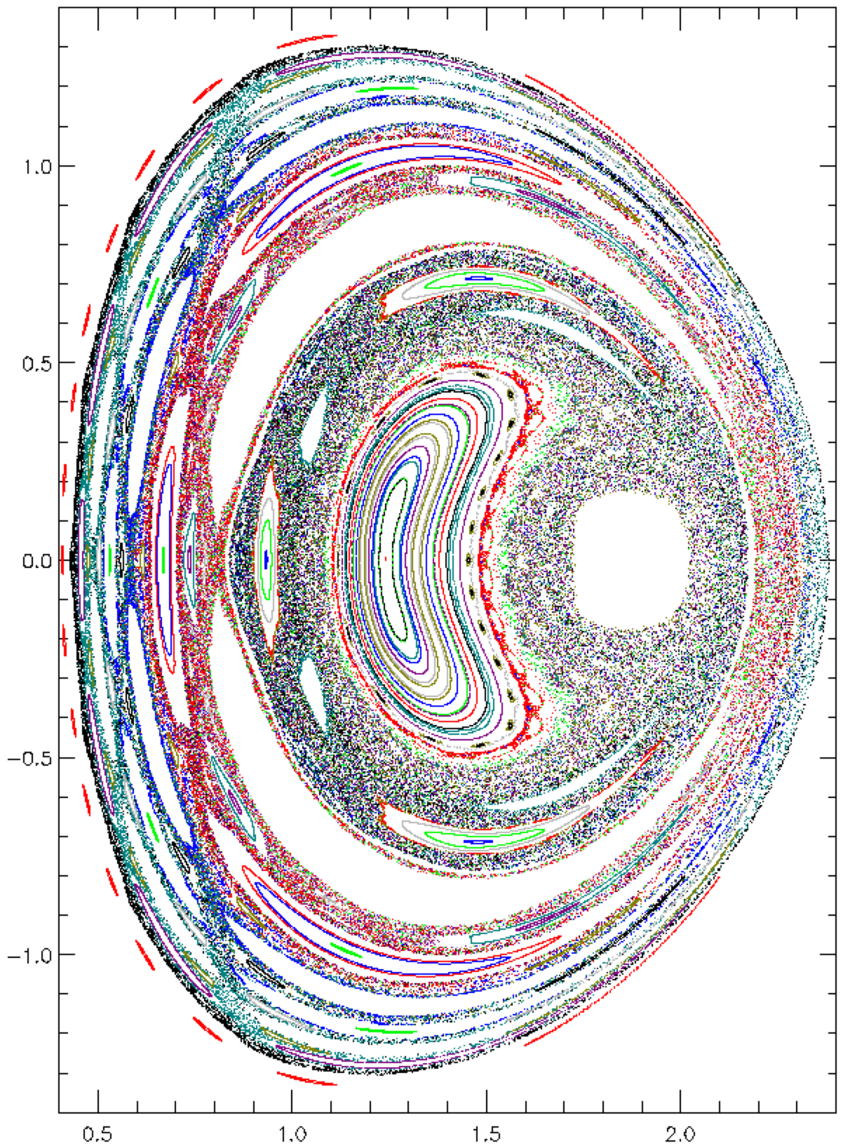


Total Energy and Core Temperature

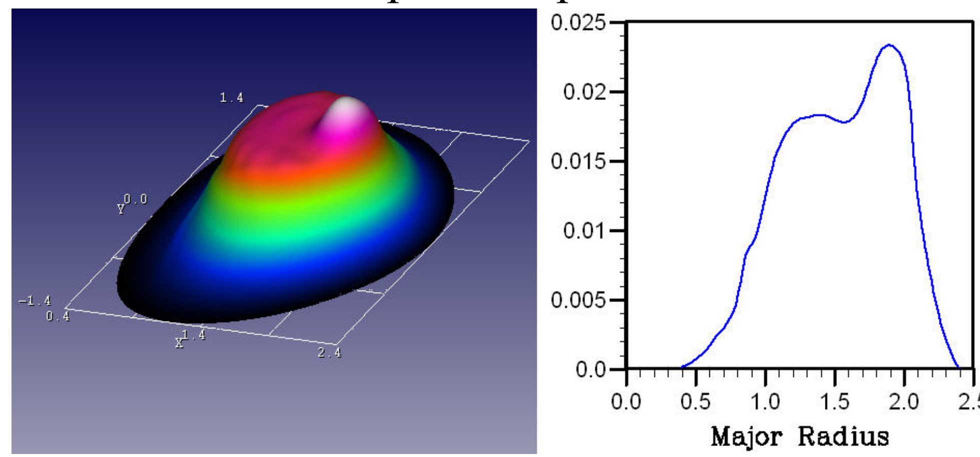


During 1st Crash: *t* = 1717.08

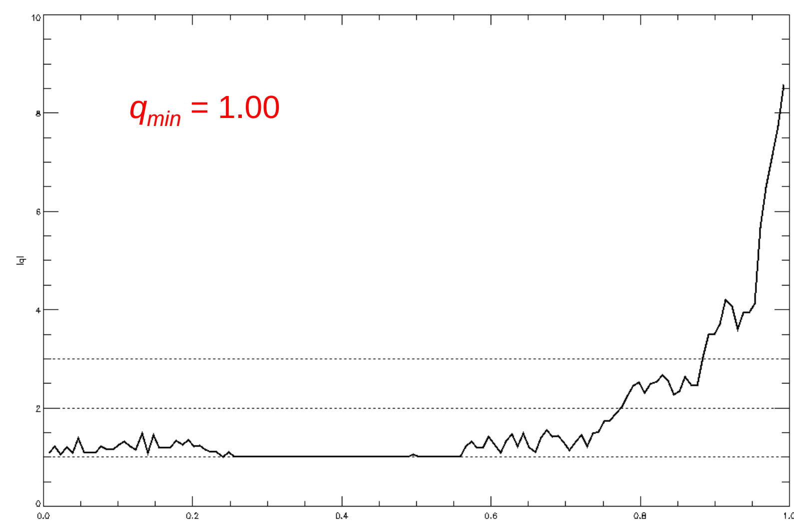
Poincaré plot



Temperature profile

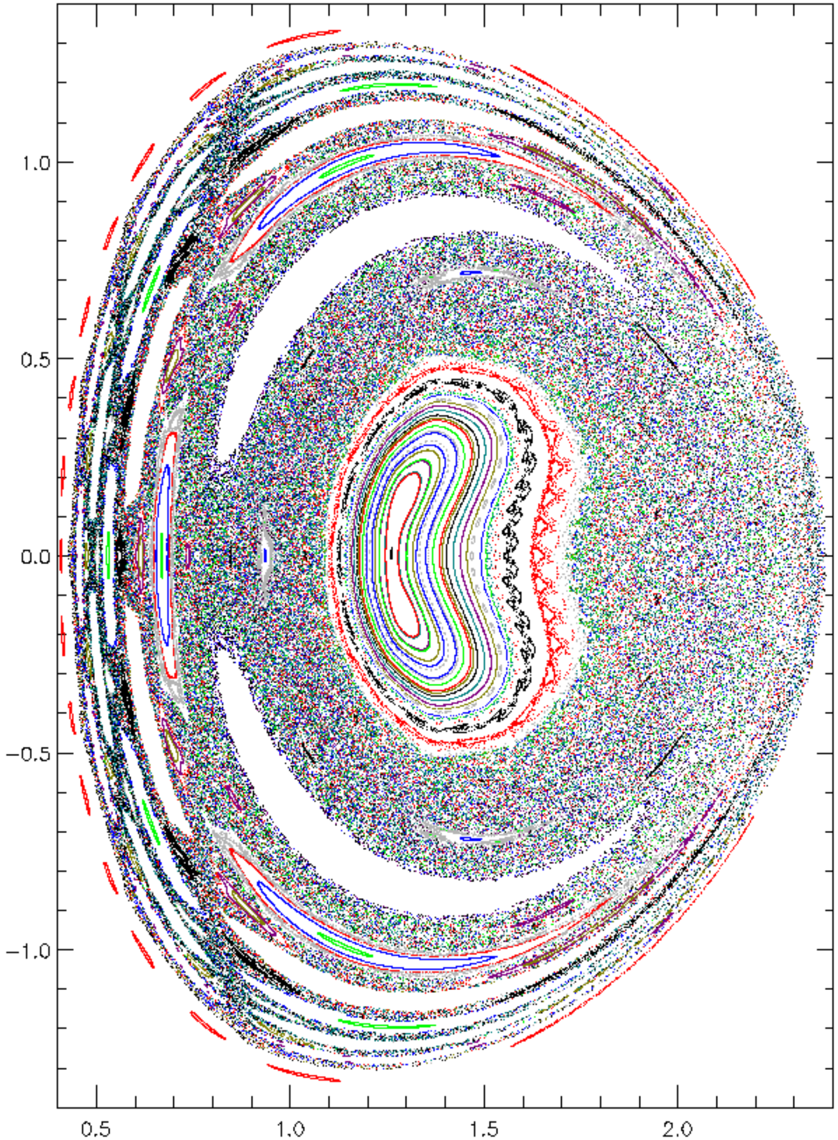


q profile

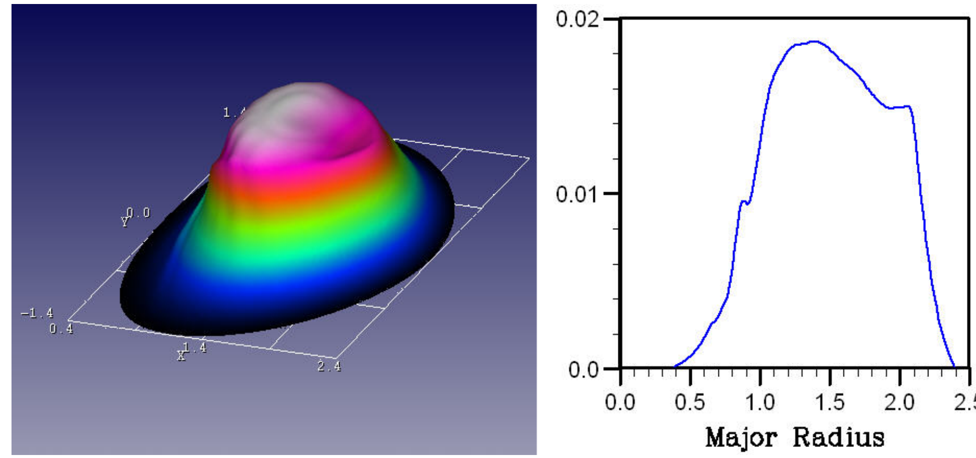


After 1st Crash: t = 1725.34

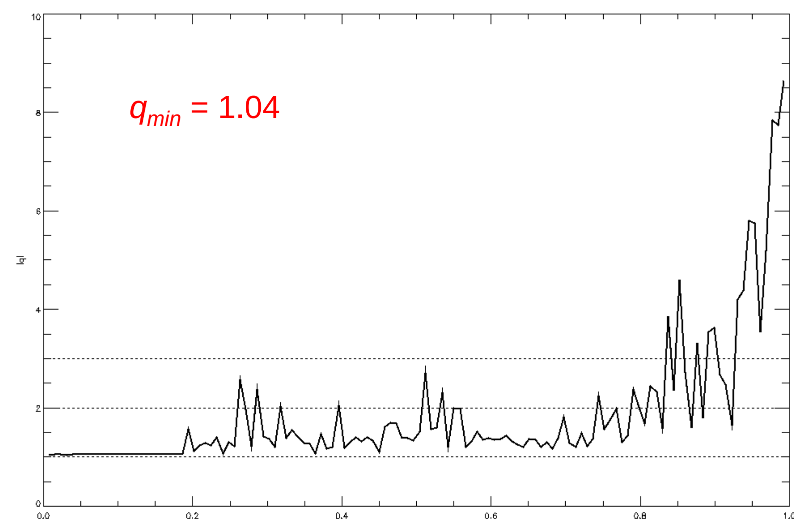
Poincaré plot



Temperature profile

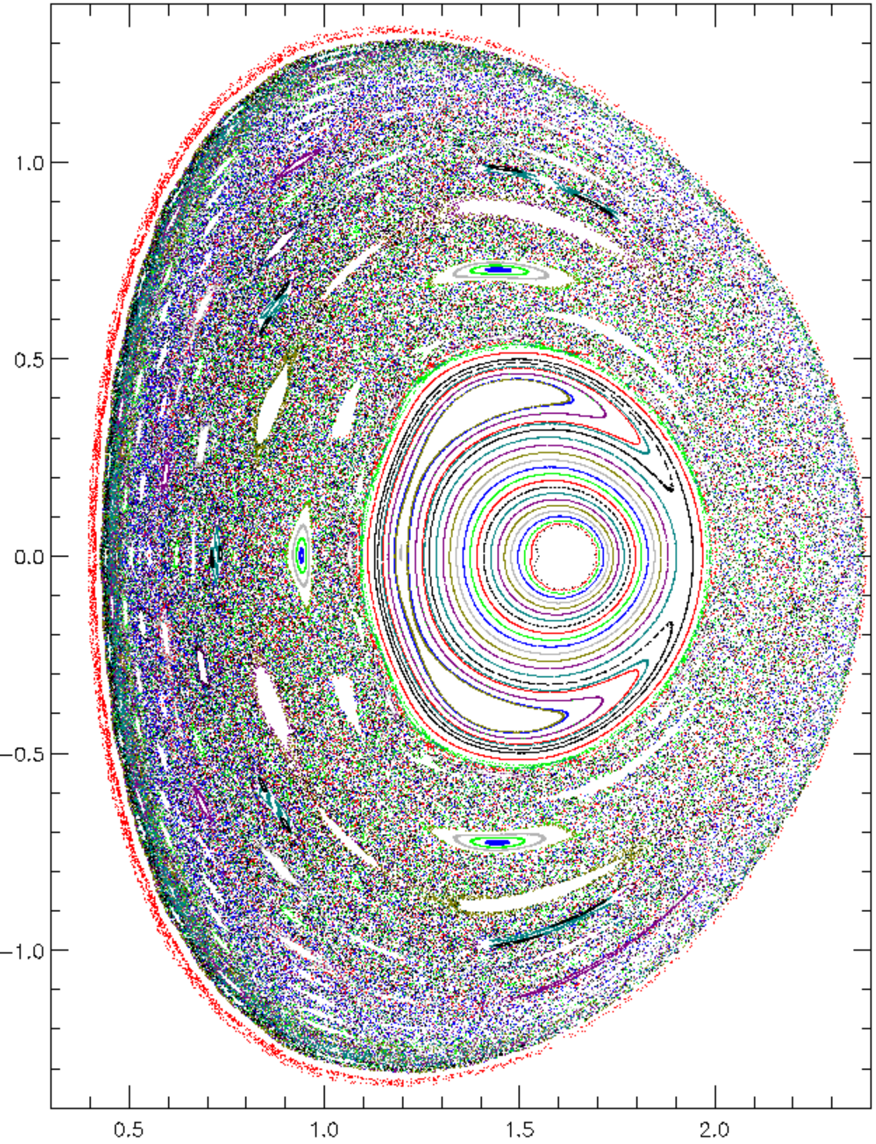


q profile

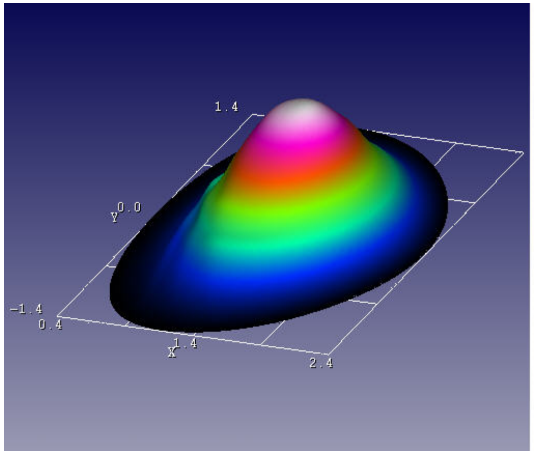


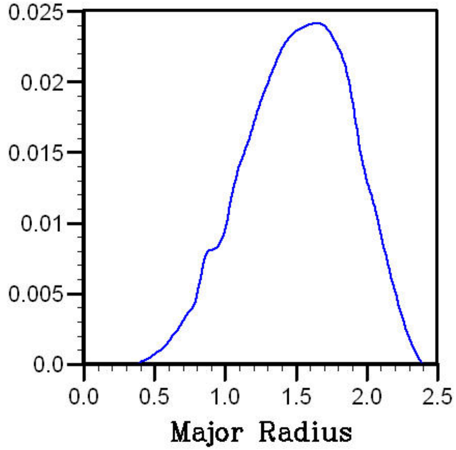
After Incomplete Recovery: t = 1848.30

Poincaré plot



Temperature profile





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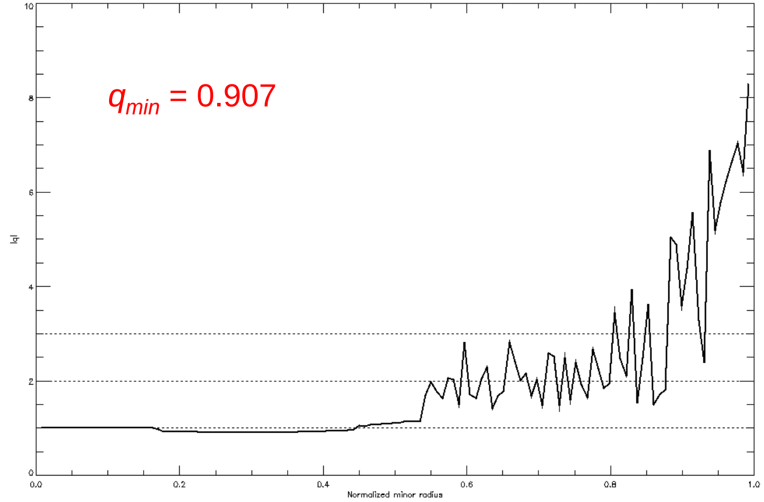
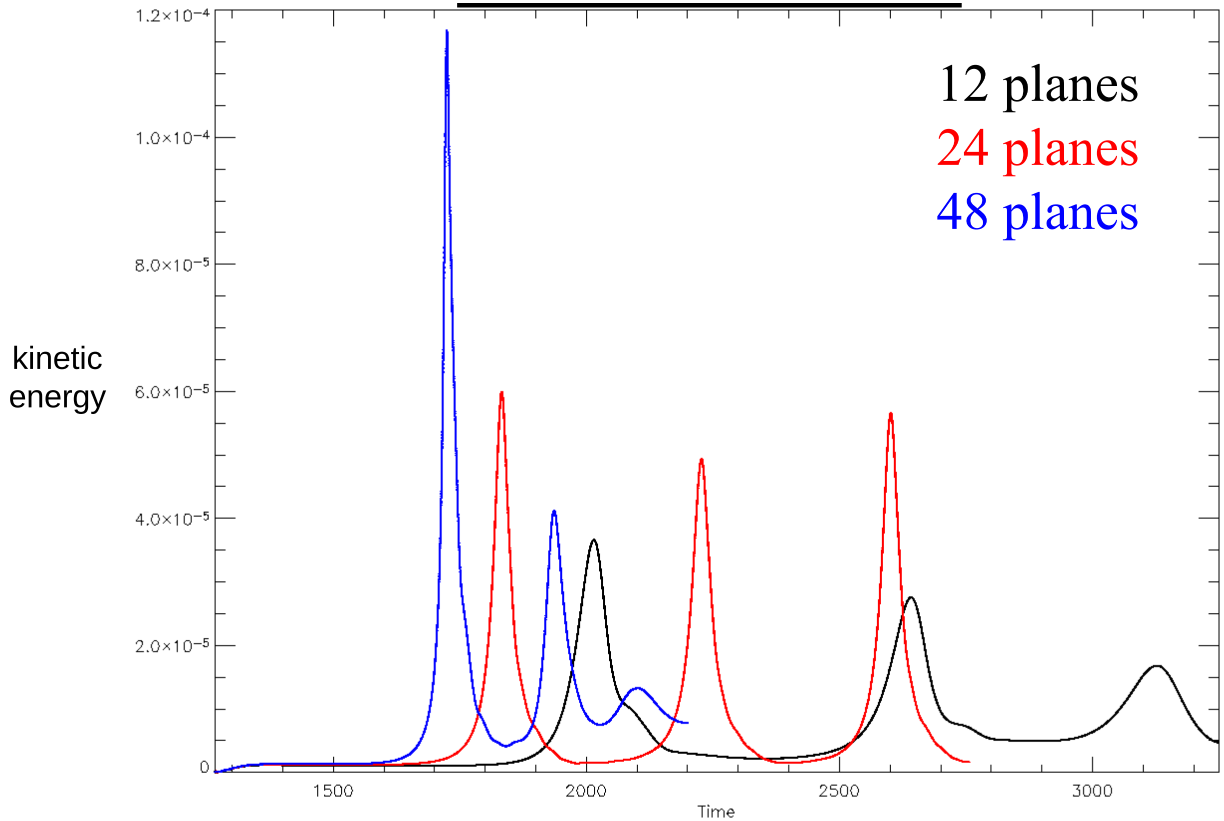


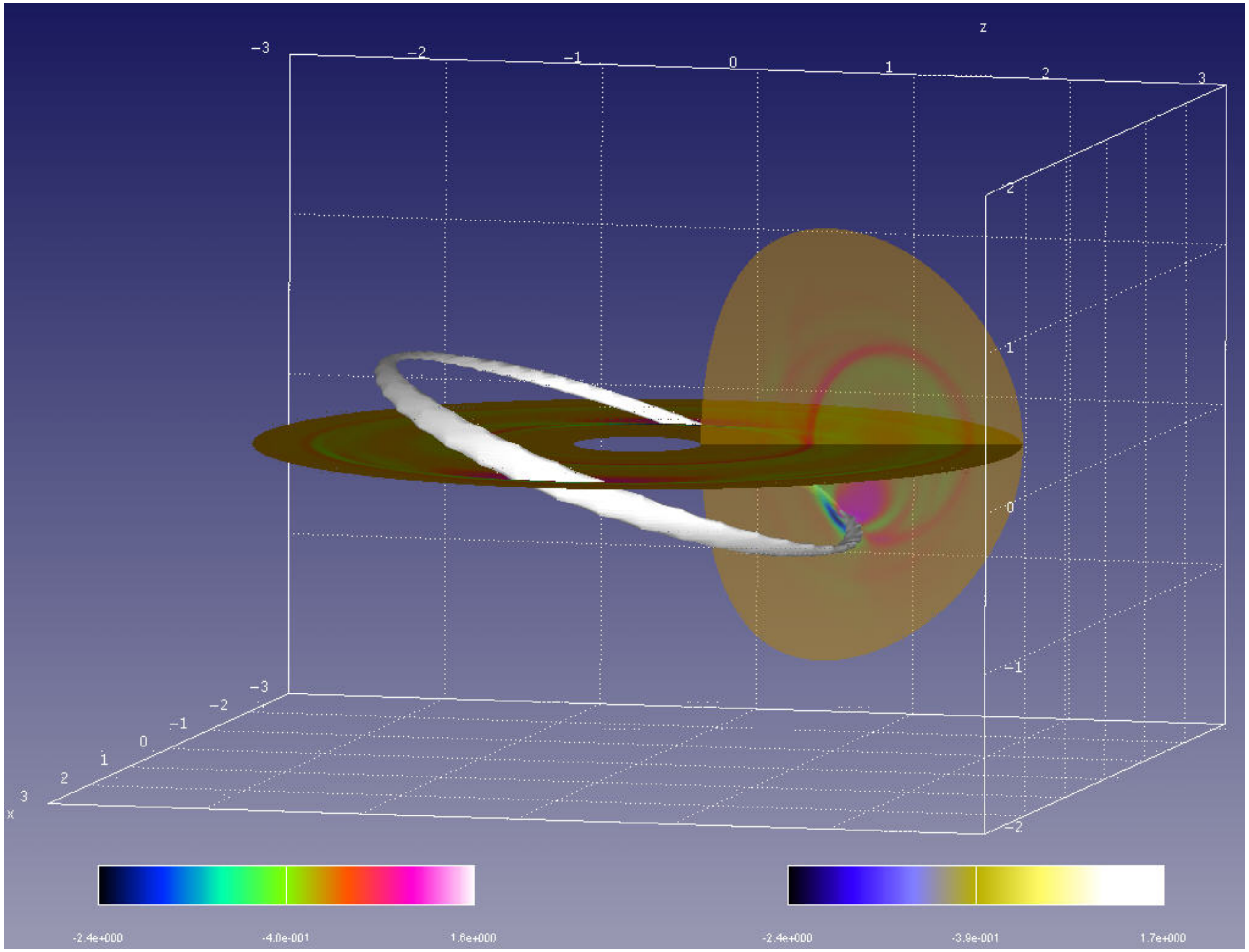
Lack of Convergence in Toroidal Resolution



- Both timing and energy of peaks are different.
- Outer flux surfaces do not heal in highest-resolution case.

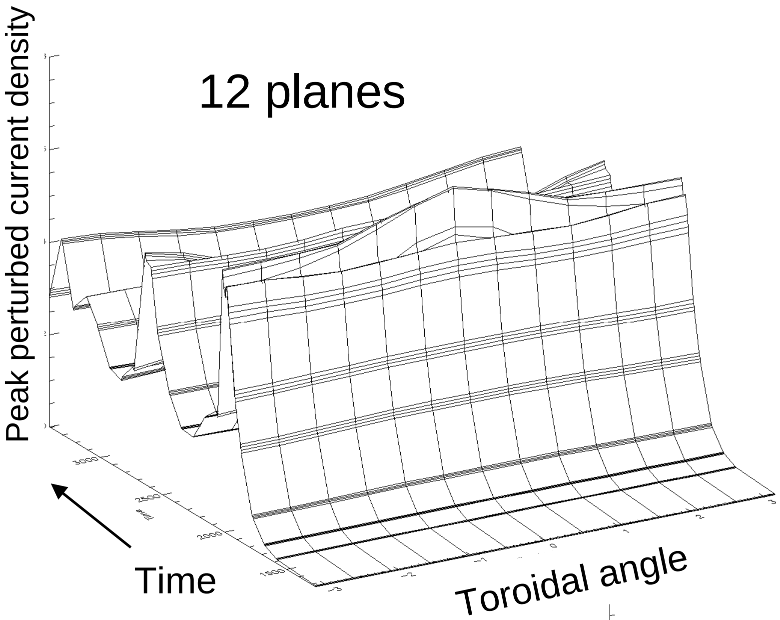
- Energy in higher-*n* modes significantly affects sawtooth evolution.
- Further study is needed to assess convergence on this case.

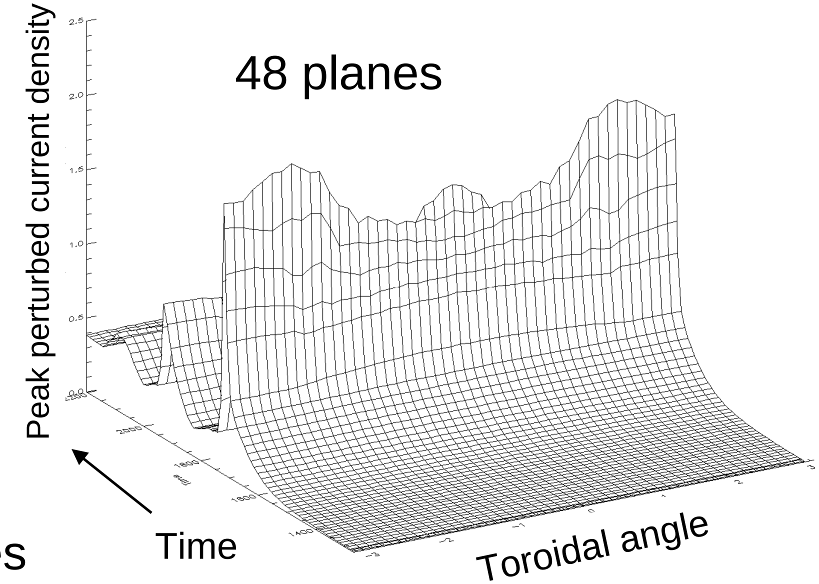
Toroidal Structure of the Sawtooth Current Sheet

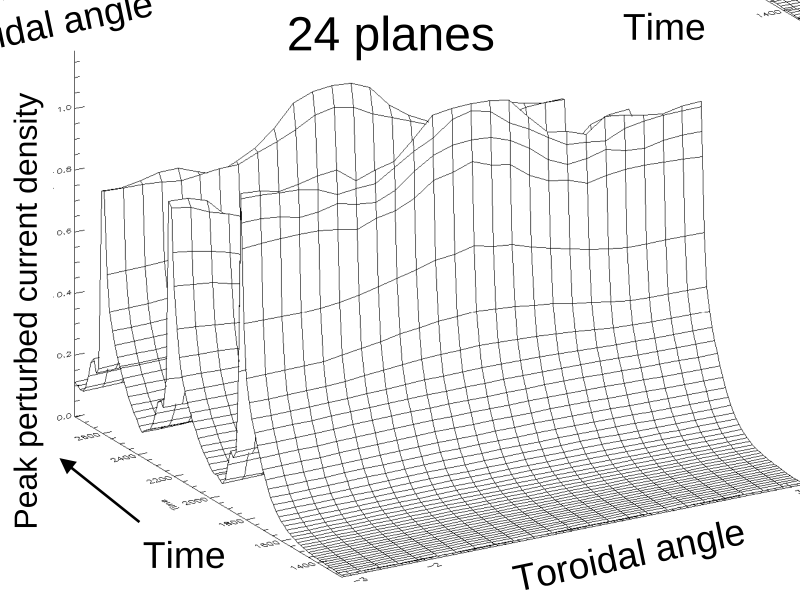


Isosurface of n>0 part of toroidal current density during sawtooth crash shows that the current peak occurs where the 1,1 island is reconnecting. Following this peak around the torus indicates that its variation in ϕ may not be fully resolved in this calculation.

Toroidal Structure of the Sawtooth Current Sheet, continued







Conclusions

- Nonlinear MHD simulation with actual device parameters is capable of tracking evolution through repeated sawtooth reconnection cycles.

- The coupling of reasonable $\kappa_{||}$ with finite $\kappa_{\perp}$ will rapidly smooth out any temperature fluctuations resulting from instabilities strong enough to render the local magnetic field stochastic. This should cause any pressure-gradient-driven modes to saturate.

- The increased energy, faster cycle, and differing toroidal structure in the current sheet indicate that the problem is not fully resolved at 48 planes. More resolution or an extended MHD model may be necessary to achieve convergence.