Two-fluid Extended MHD Calculations of Collisionless Reconnection in Magnetized Plasmas with a Strong Guide Field

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Project Sherwood Meeting

Annapolis, MD

April 23-25 2007

abstract

We have developed a new, high-order, implicit method for solving the time-dependent extended magneto-hydrodynamic (X-MHD) equations in two dimensions [1], [2], [4] and have applied this to the problem of collisionless magnetic reconnection in the presence of a strong guide field. The reconnection calculations extend the GEM [5], [4] reconnection problem to include a strong background (guide) magnetic field as is present in a fusion plasma. We find that the background field significantly delays the onset of the fast reconnection phase and reduces the maximum reconnection rate and the amplitude of the velocities that develop in the reconnection region. The flow structure changes to be nearly incompressible and its localization presents severe resolution requirements. These calculations are performed with a new simulation code, M3D-*C1,* which uses a fully unstructured triangular mesh that is packed into regions with high gradients, but the time-step is limited only by resolution requirements. Each finite element contains a complete 4th degree polynomial with additional terms to provide *C1* continuity across element boundaries as required to efficiently solve the stream function/ potential form of the equations including the high-order derivatives that appear in the electron and ion viscosities.

[1] S. C. Jardin, J. Comp. Phys, 200, 133 (2004)
[2] S. C. Jardin and J. A. Breslau, Phys. Plasma, **12**, 056101 (2005)
[3] N Ferraro and S. C. Jardin, Phys. Plasma **13**, 092101 (2006)
[4] S. C Jardin, J. Breslau, and N. Ferraro, PPPL-4209, (2007)
[5] Birn, et al, J. Geophys Res 106, 3715 (2001)

The Extended MHD equations for a magnetized (fusion) plasma are a high-order system of 8 scalar variables that are characterized by a wide range of space and timescales.

The M3D-C^1 approach is as follows:

Multiple space scales	→	unstructured adaptive elements
Multiple time scales	→	implicit time differencing
High order derivatives	→	C^1 continuity elements (up to 4^{th} order)
8 scalar variables	→	split implicit time advance & compact rep.
Strong magnetic field	→	stream function/potential representation

2-Fluid MHD Equations:

Resistive MHD

2-fluid Extended MHD terms

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\vec{V}) = 0$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \qquad \vec{J} = \nabla \times \vec{B}$$

$$\vec{V} = \nabla U \times \hat{z} + \nabla_{\perp}\chi + V_z$$

$$\vec{B} = \nabla \psi \times \hat{z} + I\hat{z}$$

$$nM_i\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}\right) + \nabla p = \vec{J} \times \vec{B} - \nabla \bullet \Pi_{GV} + \mu \nabla^2 \vec{V}$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{J} + \frac{1}{ne}\left(\vec{J} \times \vec{B} - \nabla p_e\right) - \lambda_H (\Delta x)^2 \nabla^2 \vec{J}$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \nabla \bullet \left(\frac{3}{2} p_e \vec{V}\right) = -p_e \nabla \bullet \vec{V} + \eta J^2 + \frac{\vec{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n\right] - \nabla \bullet \vec{q}_e + Q_{\Delta}$$

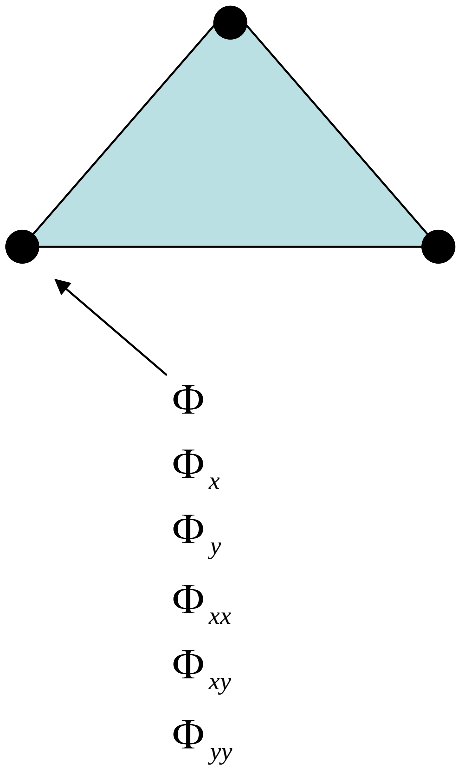
$$\frac{3}{2}\frac{\partial p_i}{\partial t} + \nabla \bullet \left(\frac{3}{2} p_i \vec{V}\right) = -p_i \nabla \bullet \vec{V} + \mu \left|\nabla V\right|^2 - \nabla \bullet \vec{q}_i - Q_{\Delta}$$

8 scalar variables: $\psi, I, U, \chi, V_z, n, p_e, p_i$

Δx is typical zone (element) size

Scalar data is represented using 18 degree of freedom quintic triangular finite elements Q_{18}

- All data is at nodes: function + first 5 derivatives (6 DOF)
- Complete quintic polynomial has 21 coefficients
    - 18 values come from the 3 nodes (3 x 6)
    - 3 values come from requirement that the normal derivative along each edge be only a (univariate) cubic….leads to C^1 continuity
- Contains a complete Taylor series through 4th order…error ~ h^5
- Compact representation … only 3 DOF/triangle
- C^1 continuity allows up to 4th derivatives in space without introducing auxiliary variables
- Unstructured triangular mesh allows adaptive zoning



Implicit velocity time-advance substitutes in from field equations to contain all Ideal MHD wave phenomena

$$\rho\dot{\vec{V}} = \left(\vec{J} + \theta\delta t\dot{\vec{J}}\right)\times\left(\vec{B} + \theta\delta t\dot{\vec{B}}\right) - \nabla\left(P + \theta\delta t\dot{P}\right) + \cdots$$

$$\dot{\vec{J}} = \nabla\times\nabla\times\left[\left(\vec{V} + \theta\delta t\dot{\vec{V}}\right)\times B\right] + \cdots$$

$$\dot{\vec{B}} = \nabla\times\left[\left(\vec{V} + \theta\delta t\dot{\vec{V}}\right)\times B\right] + \cdots$$

$$\dot{P} = -\left(\vec{V} + \theta\delta t\dot{\vec{V}}\right)\bullet\nabla P - \frac{5}{3}P\nabla\bullet\left(\vec{V} + \theta\delta t\dot{\vec{V}}\right)$$

let $\dot{\vec{V}} = \dfrac{\vec{V}^{n+1} - \vec{V}^{n}}{\delta t}$, move all $\vec{V}^{n+1}$ terms to left side of equation

$$L_1\left\{V^{n+1}\right\} = L_2\left\{V^{n}\right\} + \cdots$$

Unconditionally Stable for $\theta \geq 0.5$

Use SuperLU_dist to invert this linear operator to get from time n to (n+1)

A similar technique is used on the magnetic field equations. Fully implicit Extended MHD (2-fluid) equations-- time step determined by accuracy only:

$$\begin{bmatrix} S_{11}^v & S_{12}^v & S_{13}^v \\ S_{21}^v & S_{22}^v & S_{23}^v \\ S_{31}^v & S_{32}^v & S_{33}^v \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n+1} = \begin{bmatrix} D_{11}^v & D_{12}^v & D_{13}^v \\ D_{21}^v & D_{22}^v & D_{23}^v \\ D_{31}^v & D_{32}^v & D_{33}^v \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n} + \begin{bmatrix} R_{11}^v & R_{12}^v & R_{13}^v \\ R_{21}^v & R_{22}^v & R_{23}^v \\ R_{31}^v & R_{32}^v & R_{33}^v \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ P_e \end{bmatrix}^{n}$$

$$\vec{V} = \nabla U \times \hat{z} + \nabla_{\perp}\chi + V_z$$
$$\vec{B} = \nabla \psi \times \hat{z} + I\hat{z}$$

Alfven Wave physics

$$S_{11}^n \bullet N^{n+1} = D_{11}^n \bullet N^n + \begin{bmatrix} R_{11}^n & R_{12}^n & R_{13}^n \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^n & Q_{12}^n & Q_{13}^n \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n} + Q_{14}^n$$

density

$$S_{11}^p \bullet P^{n+1} = D_{11}^p \bullet P^n + \begin{bmatrix} R_{11}^p & R_{12}^p & R_{13}^p \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^p & Q_{12}^p & Q_{13}^p \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n} + Q_{14}^p$$

pressure

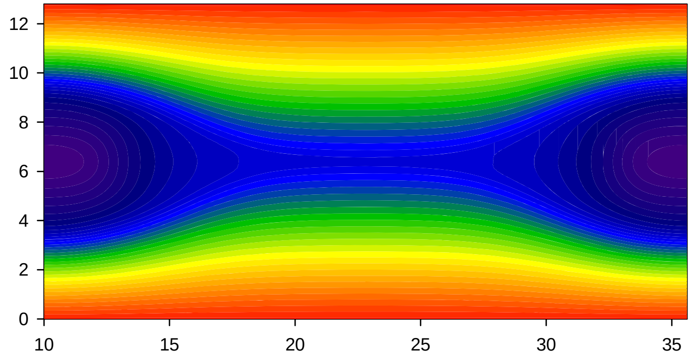
$$\begin{bmatrix} S_{11}^p & S_{12}^p & S_{13}^p \\ S_{21}^p & S_{22}^p & S_{23}^p \\ S_{31}^p & S_{32}^p & S_{33}^p \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ P_e \end{bmatrix}^{n+1} = \begin{bmatrix} D_{11}^p & D_{12}^p & D_{13}^p \\ D_{21}^p & D_{22}^p & D_{23}^p \\ D_{31}^p & D_{32}^p & D_{33}^p \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ P_e \end{bmatrix}^{n} + \begin{bmatrix} R_{11}^p & R_{12}^p & R_{13}^p \\ R_{21}^p & R_{22}^p & R_{23}^p \\ R_{31}^p & R_{32}^p & R_{33}^p \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^p & Q_{12}^p & Q_{13}^p \\ Q_{21}^p & Q_{22}^p & Q_{23}^p \\ Q_{31}^p & Q_{32}^p & Q_{33}^p \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n}$$

Whistler, KAW, field diffusion physics

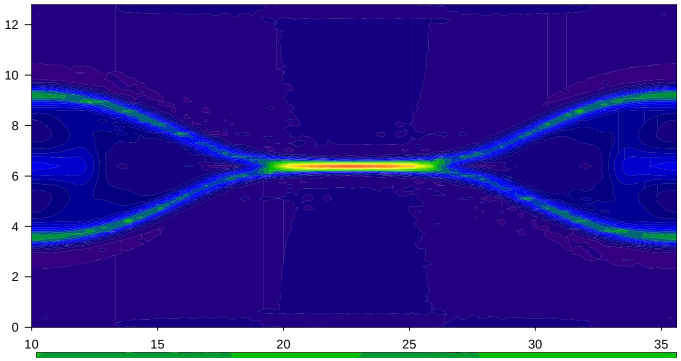
- 4 sequential matrix solves per time step
- 3 non-trivial subsets with 6,4,2 variables

GEM Nonlinear Benchmark

Poloidal Flux Ψ



Current Density J



GEM Reconnection Problem

$$\psi^0(x,y) = \frac{1}{2}\ln(\cosh 2y)$$

$$P^0(x,y) = \left[\operatorname{sec}h^2 2y + 0.2\right]$$

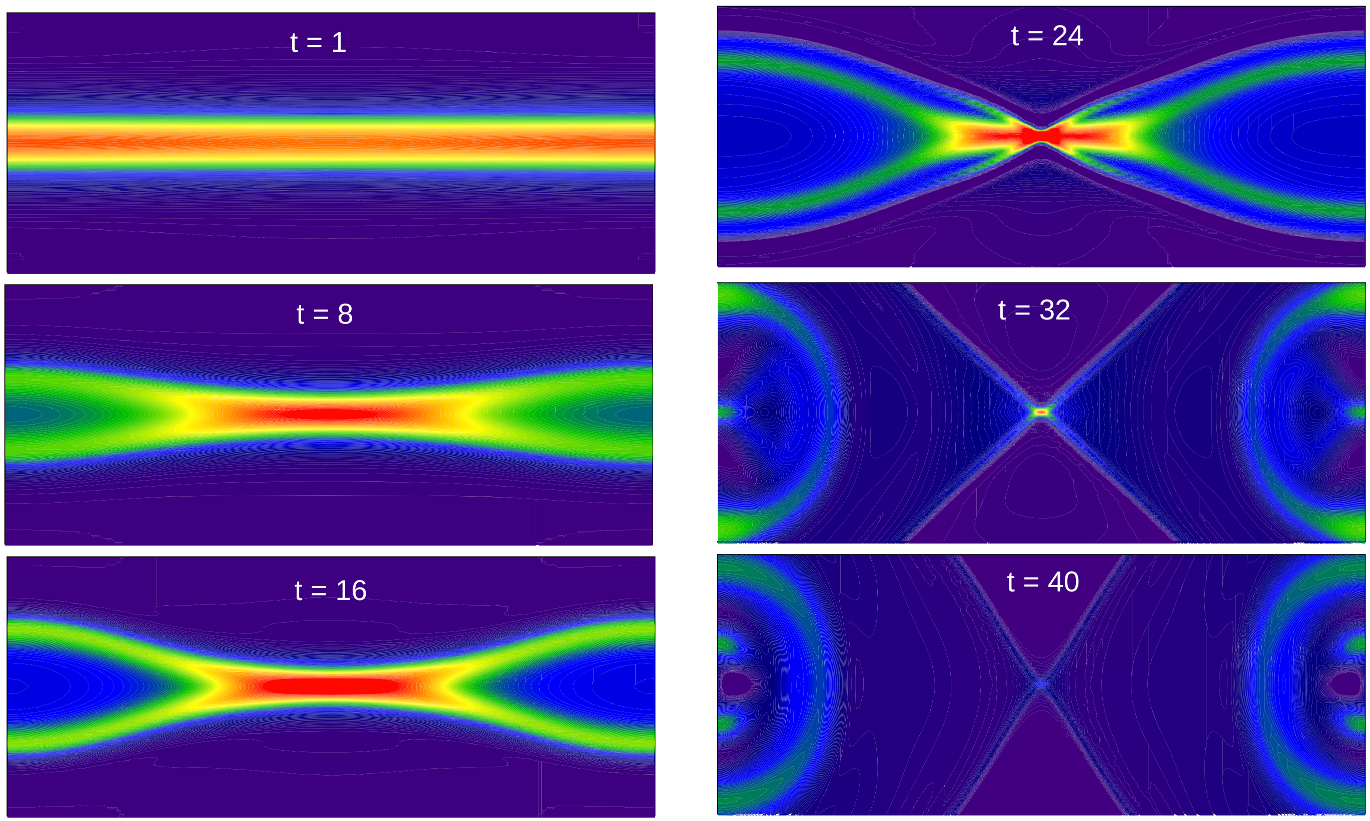
$$\tilde{\psi}(x,y) = \varepsilon \cos k_x x \cos k_y y$$

1. Resistive MHD
   High and Low Viscosity
   (μ = 10 η, μ = 0.1 η)
2. Two-Fluid

- Provides a non-trivial, convenient test problem for code verification and validation and cross-code comparison
- Also, extending this by adding an equilibrium magnetic field into the plane (guide field)

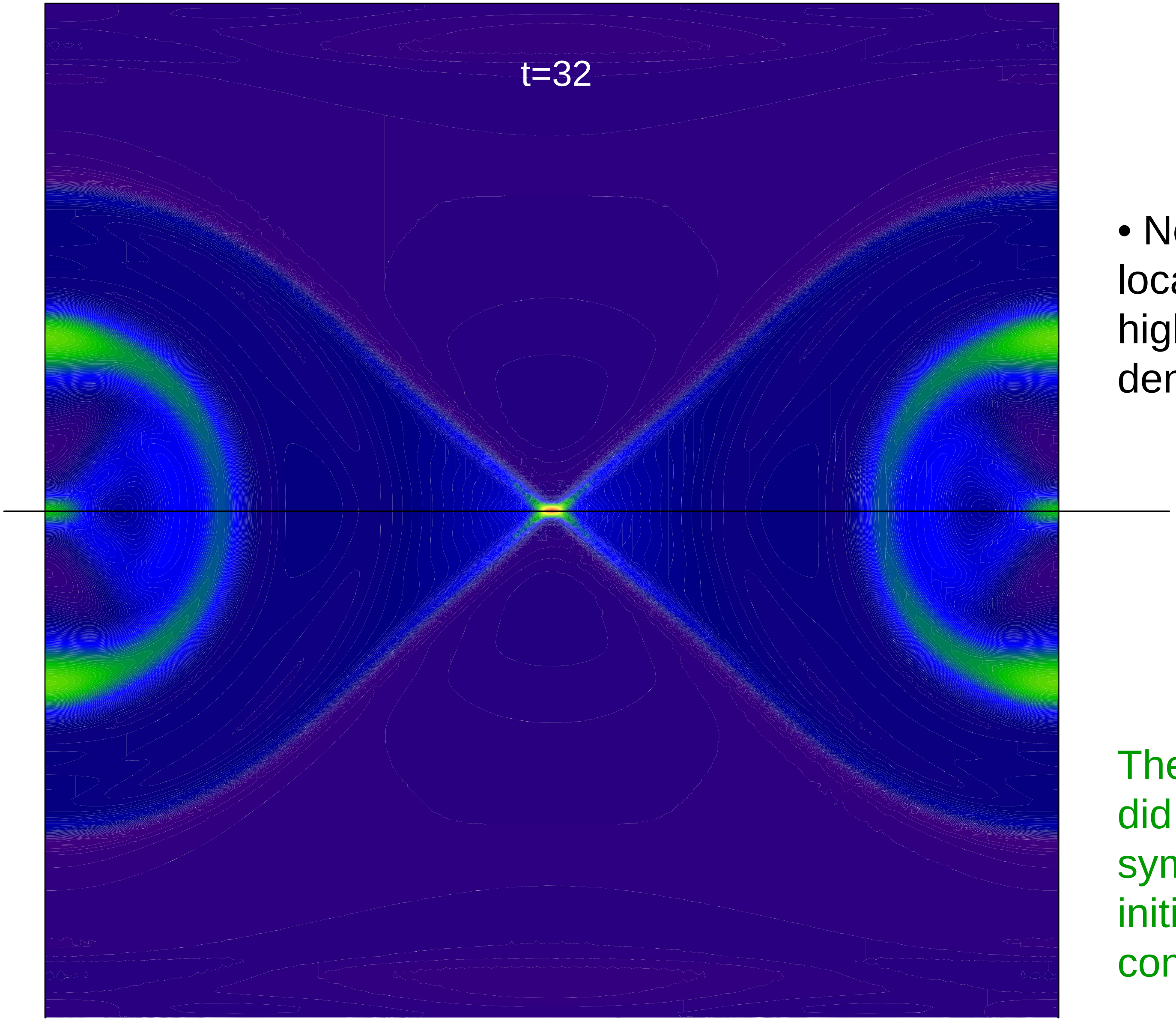
J.Birn, J.F.Drake, M.A. Shay *et al.*,J. Geophys. Res. **106** 3715 (2001)

Current Density contours for 2-fluid MHD



- Starts like resistive MHD
- Dramatic change in configuration for t > 20

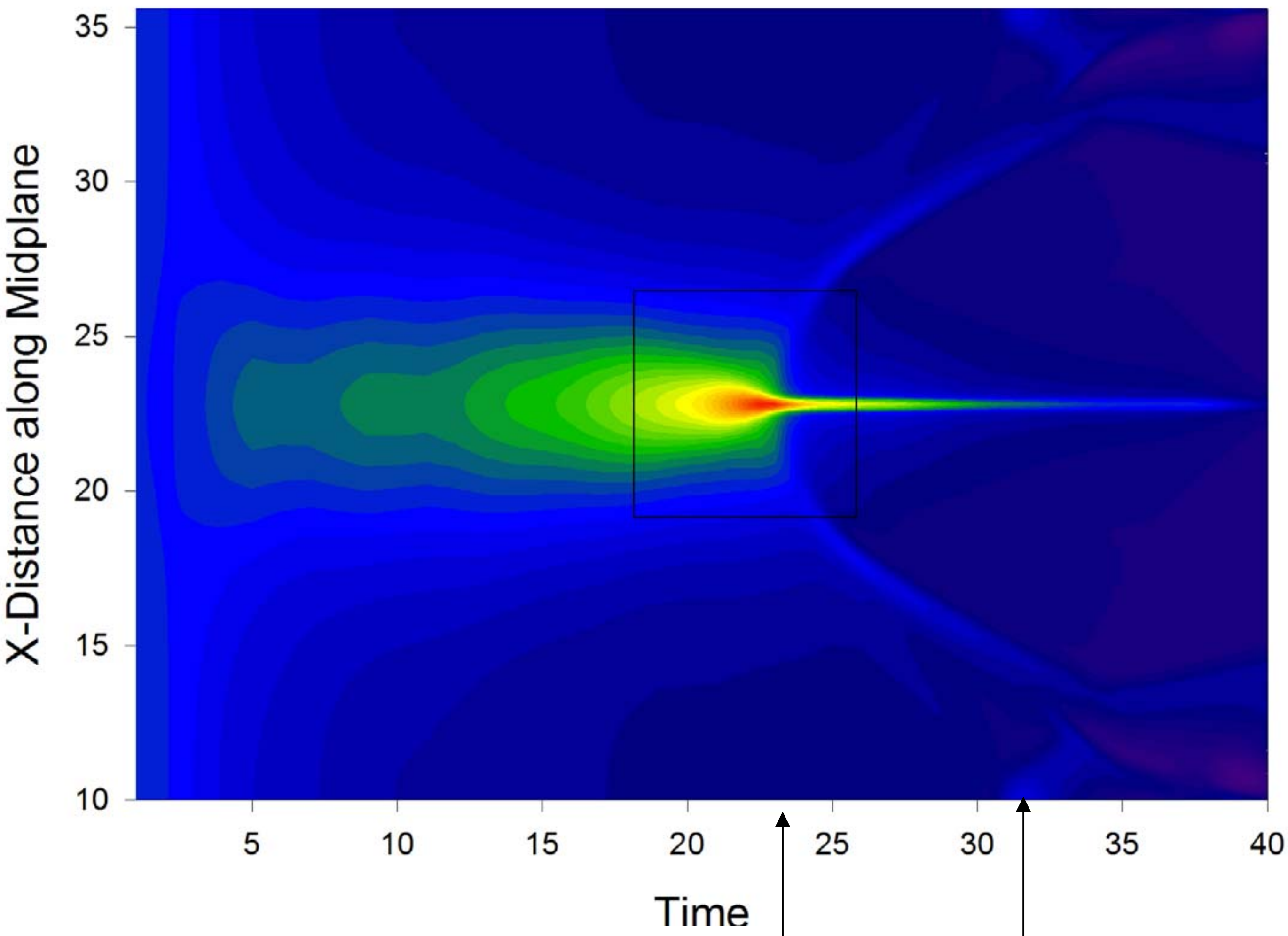
Close-up of 2-fluid current density at t=32



- Note very localized region of high current density in center

These calculations did not assume any symmetry, except for initial and boundary conditions

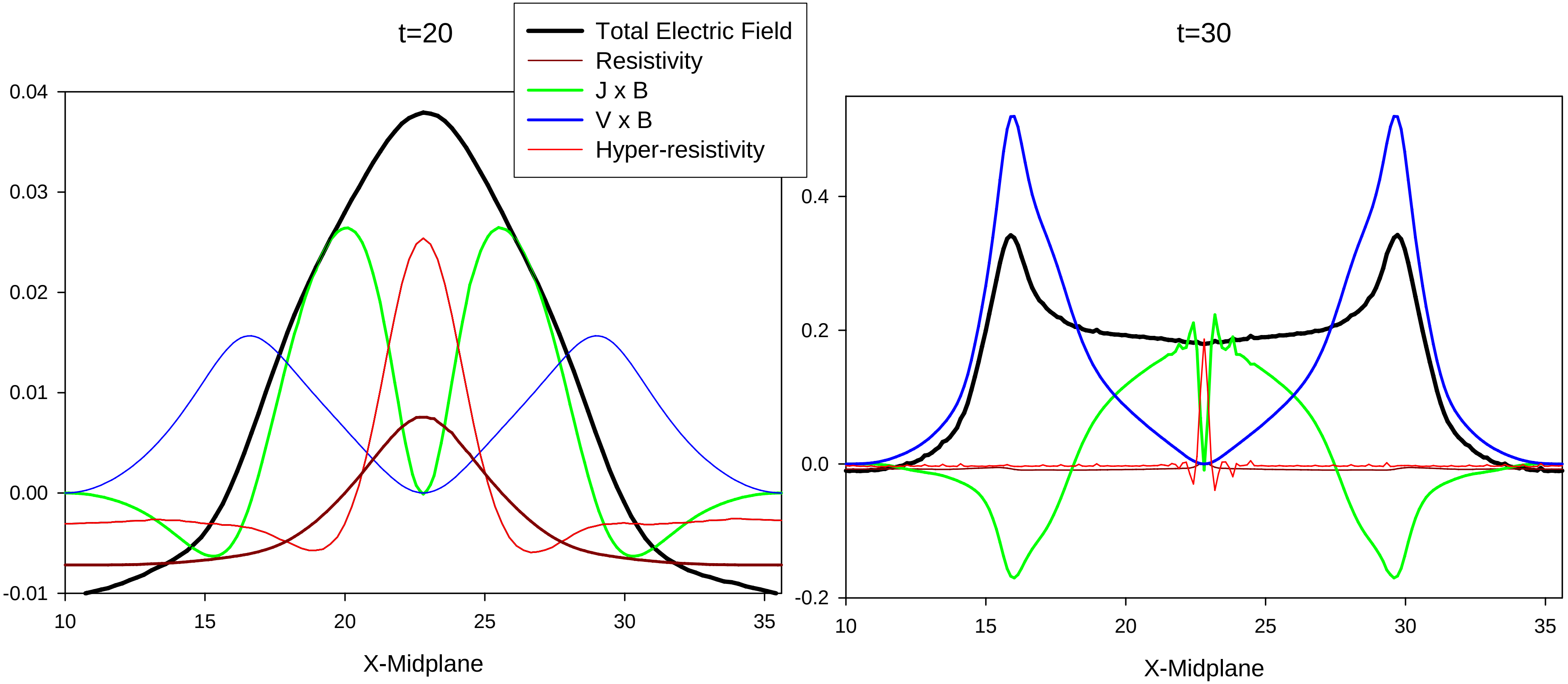
Midplane Current density collapses to the width of 1-3 triangular elements



t=32 time of previous contour plot

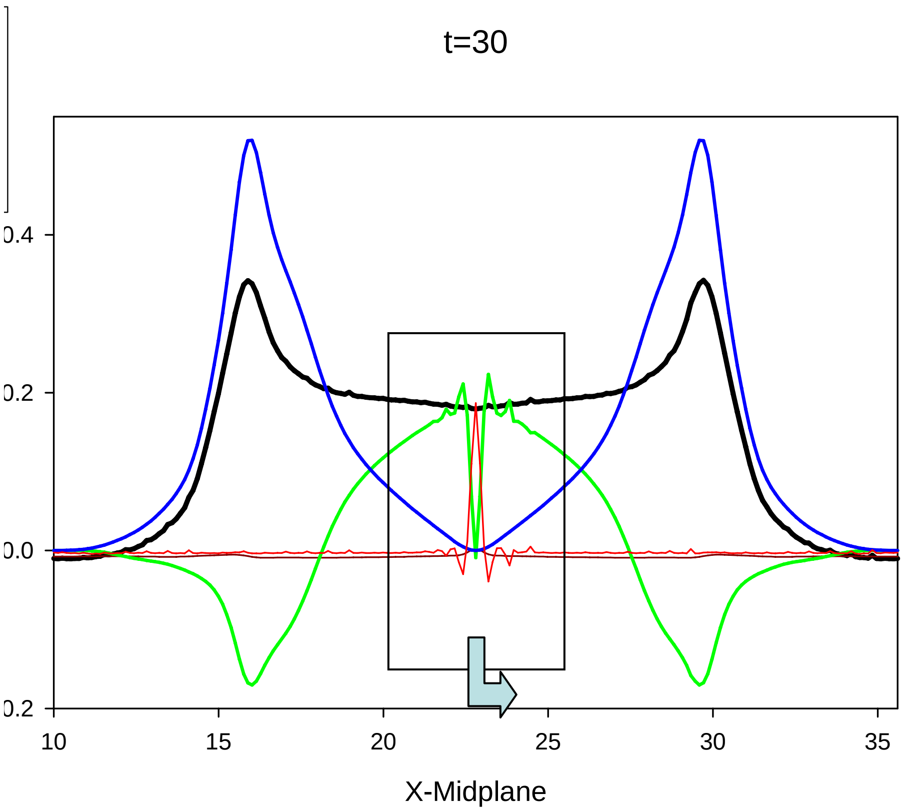
(note sudden collapse at t=23+)

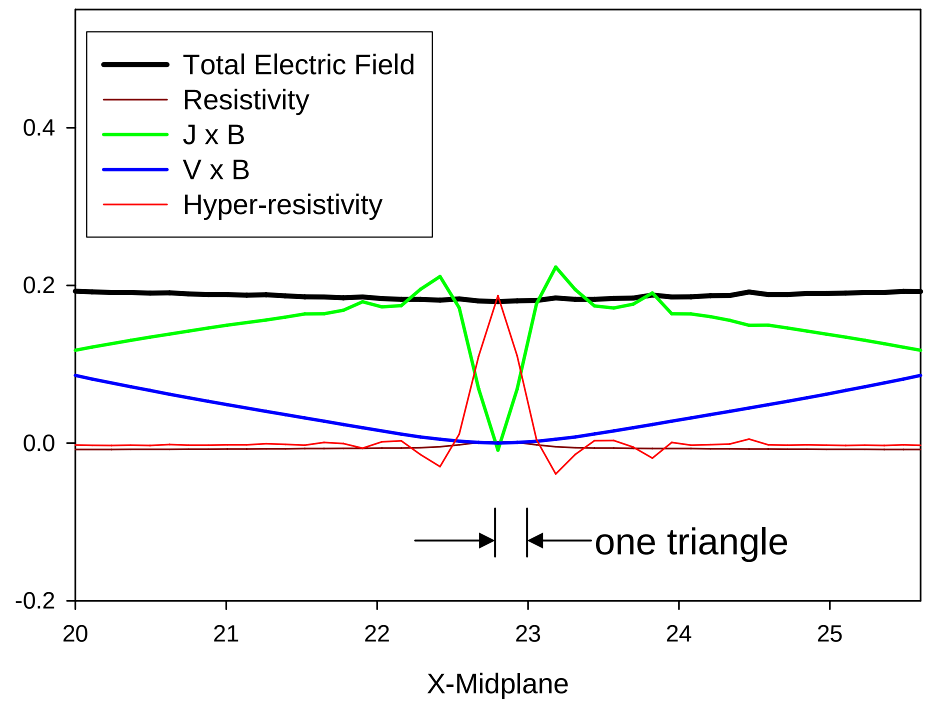
Midplane electric field before and after transition



Reconnection rate: $\hat{z}\bullet\left[\vec{E} = -\vec{V}\times\vec{B} + \eta\vec{J} + \frac{1}{ne}\left(\vec{J}\times\vec{B} - \nabla p_e\right) - \lambda_H (\Delta x)^2 \nabla^2 \vec{J}\right]$

Blowup showing electric field after transition

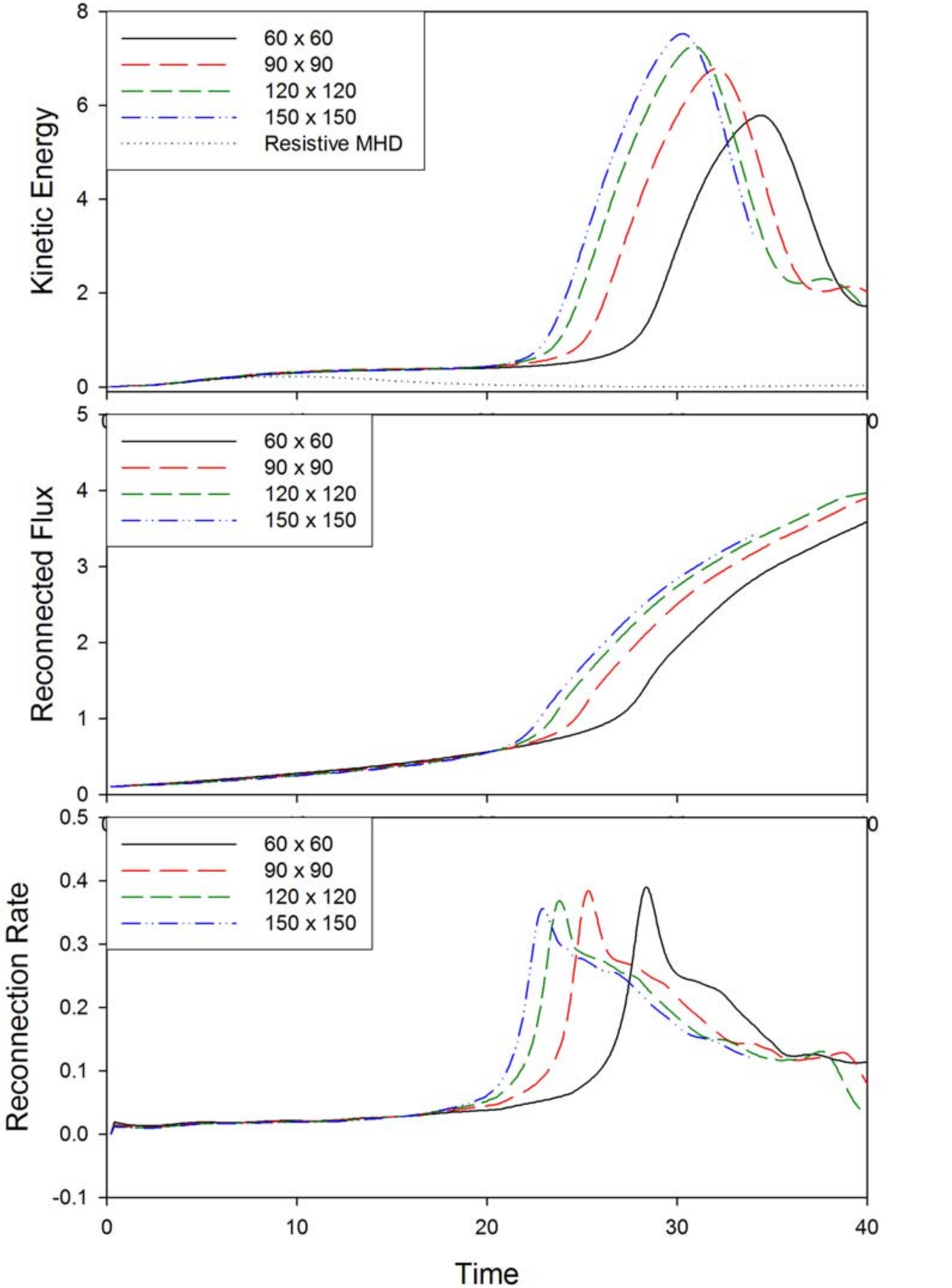




$$\hat{z}\bullet\left[\vec{E} = -\vec{V}\times\vec{B} + \eta\vec{J} + \frac{1}{ne}\left(\vec{J}\times\vec{B} - \nabla p_e\right) - \lambda_H(\Delta x)^2\nabla^2\vec{J}\right]$$

Hyper-resistivity coefficient must be large enough that current density collapse is limited to 1-2 triangles: *reason for factor (Δx)^2*

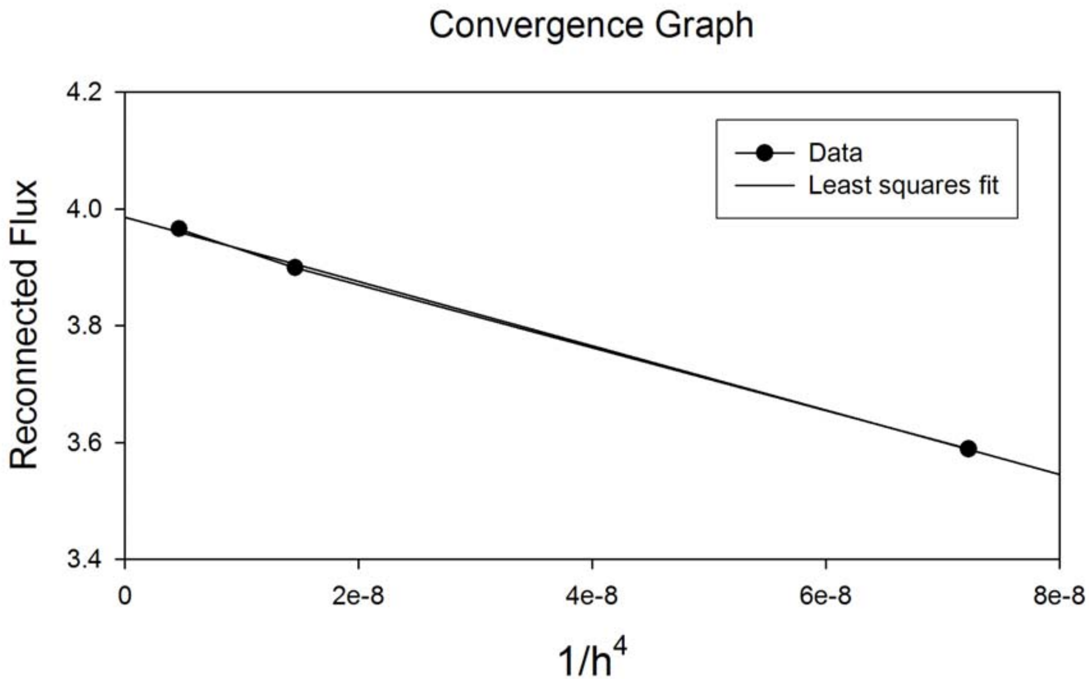
2-fluid reconnection requires high resolution for convergent results



- Note sudden transition where velocity abruptly increases

- These calculations used a hyperviscosity term in Ohm's law proportional to $(\Delta x)^2 \cdots$ required for a stable calculation

Results are converging



Now working on 180 x 180 and higher: Details for bassi.nersc.gov:

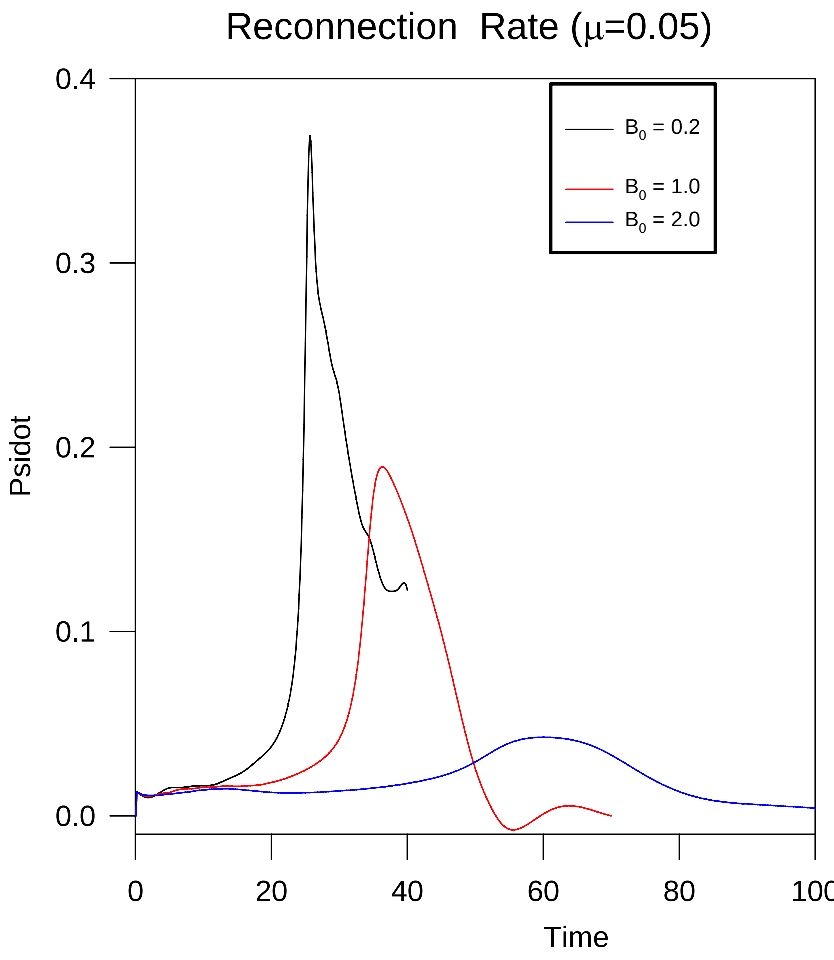
Mesh points	180 x 180
Matrix Rank	5.9×10^5
# Non-zeros	9.5×10^7
# NZ in L/U	8.8×10^8

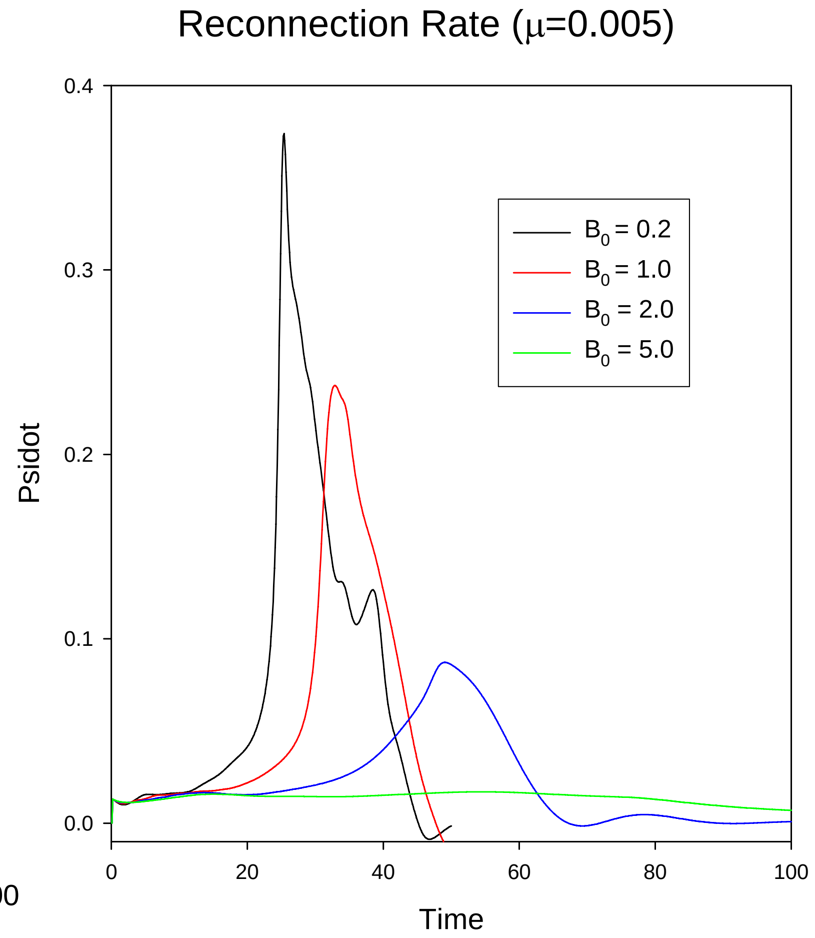
# processors	8	32	128
Factor (s)	69.5	38.1	16.9
Gflop/s	27.2	50.1	112.8

Total problem time (8 processors) = 208 s x 400 cycles x 8p = 185 p-hrs

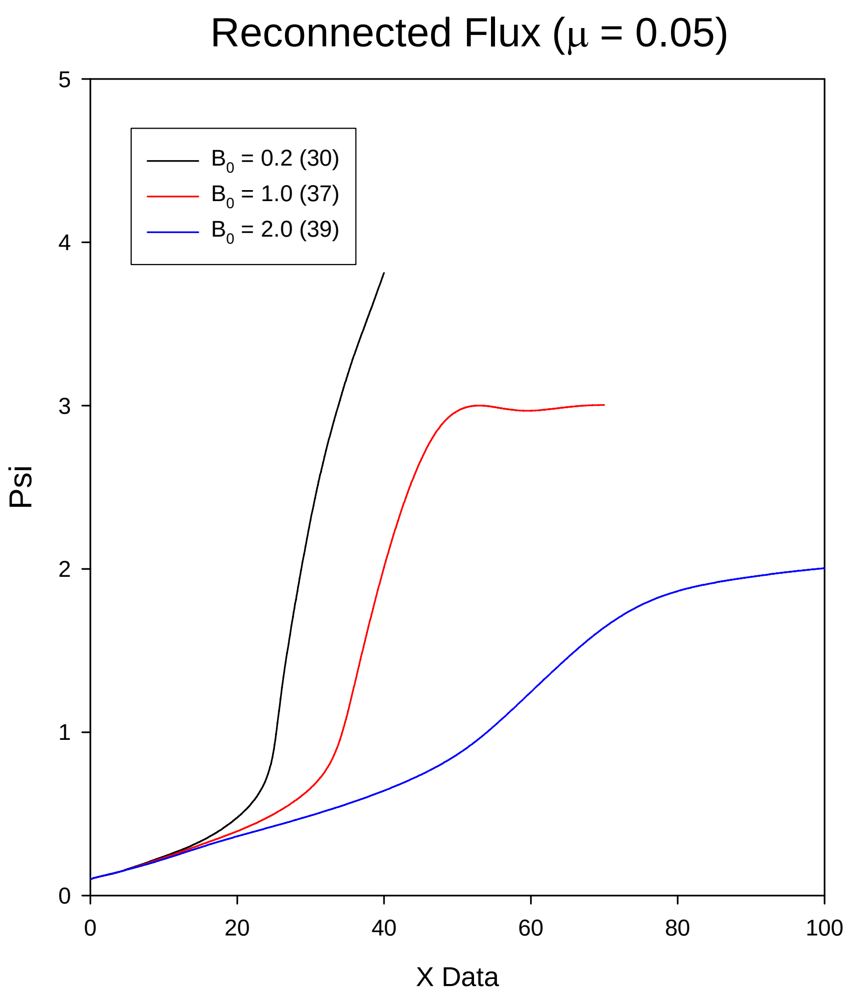
NOTE: mesh adaptation (almost implemented) should bring this down sharply

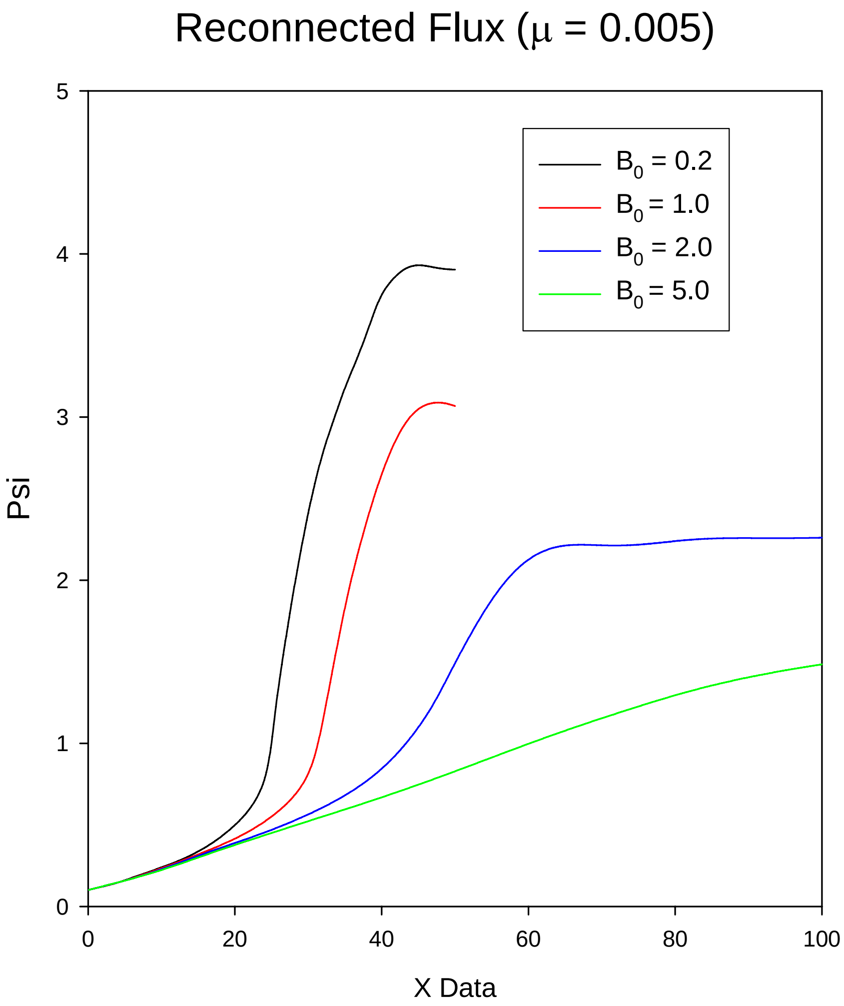
Adding a guide field B_0 to the GEM reconnection problem causes the reconnection rate to decrease significantly. Reducing the viscosity offsets this some



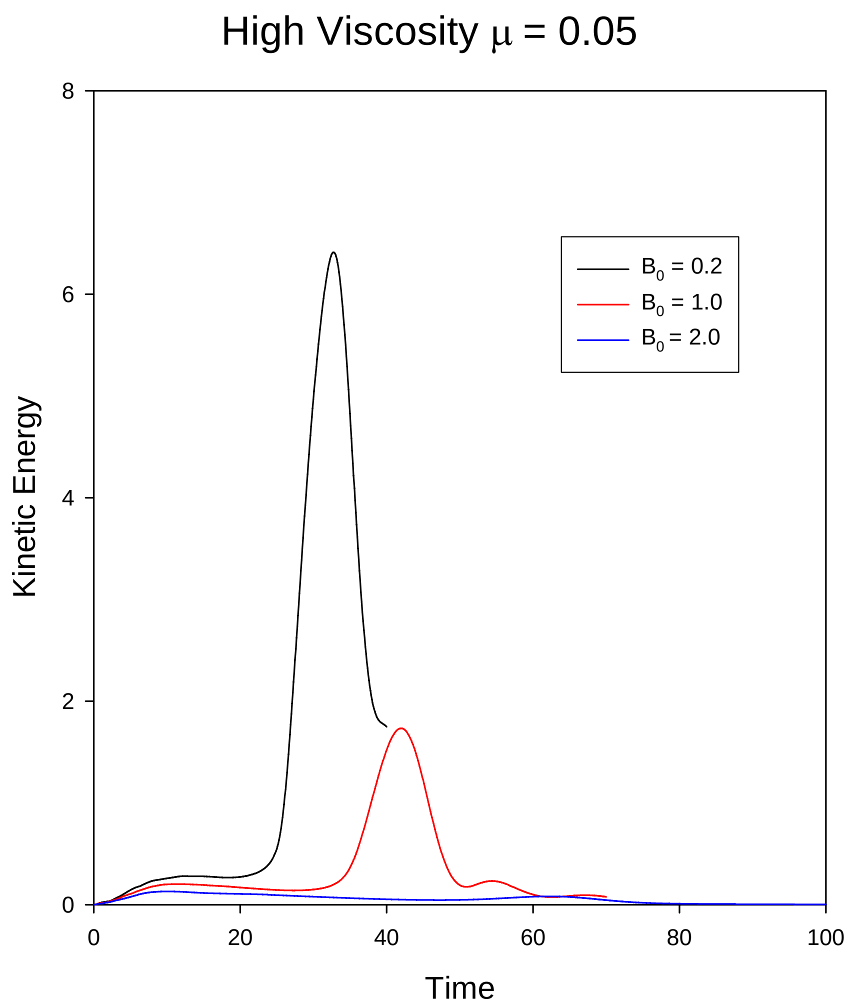


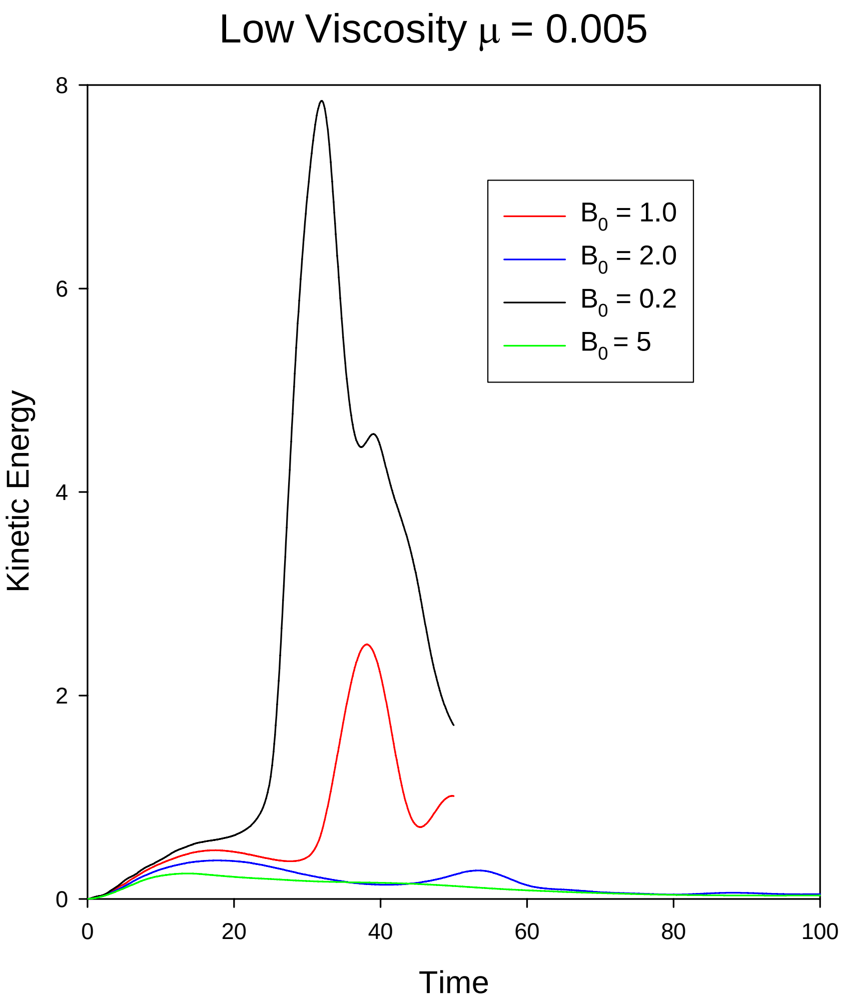
Adding a guide field B_0 to the GEM reconnection problem causes the reconnected flux to saturate at a lower value. Reducing the viscosity causes the saturated value to be obtained slightly earlier.



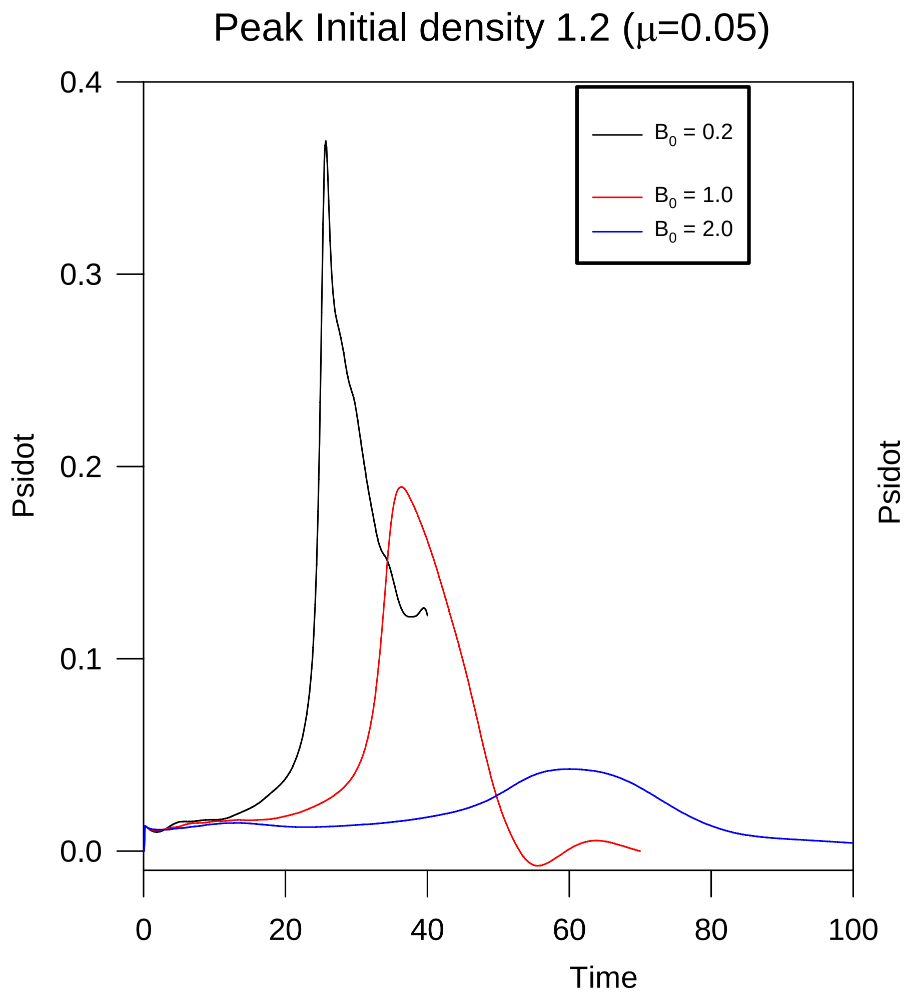


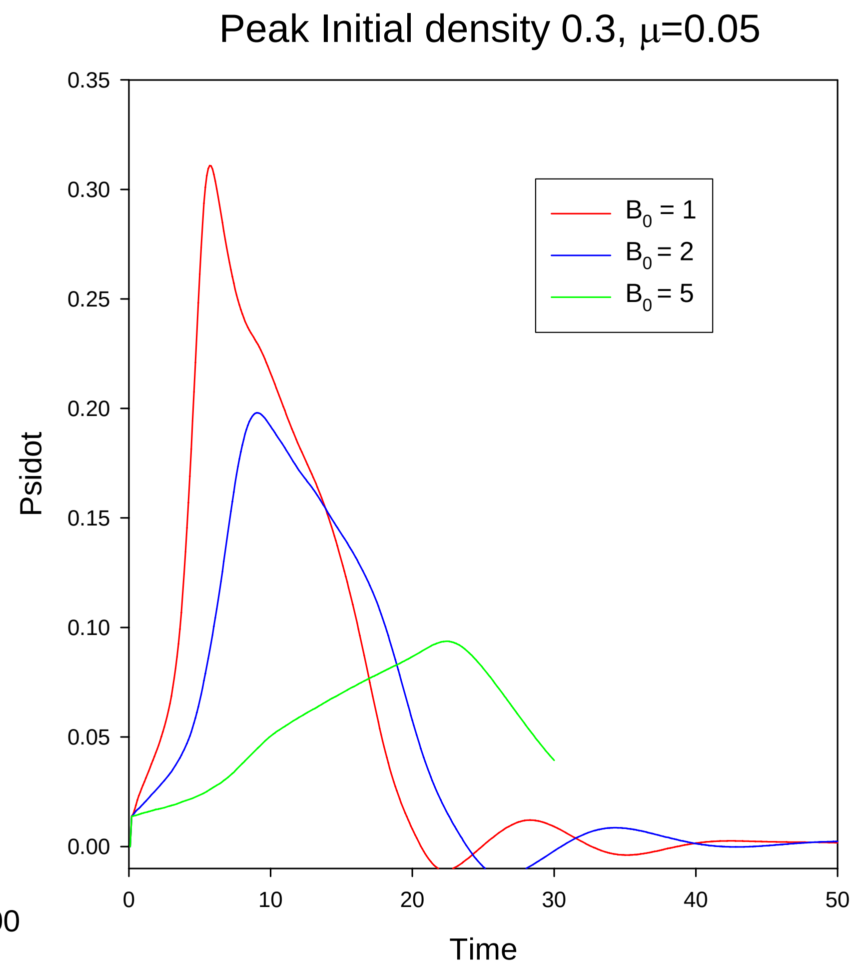
Adding a guide field B_0 to the GEM reconnection problem causes the kinetic energy to decrease significantly. Reducing the viscosity offsets this some



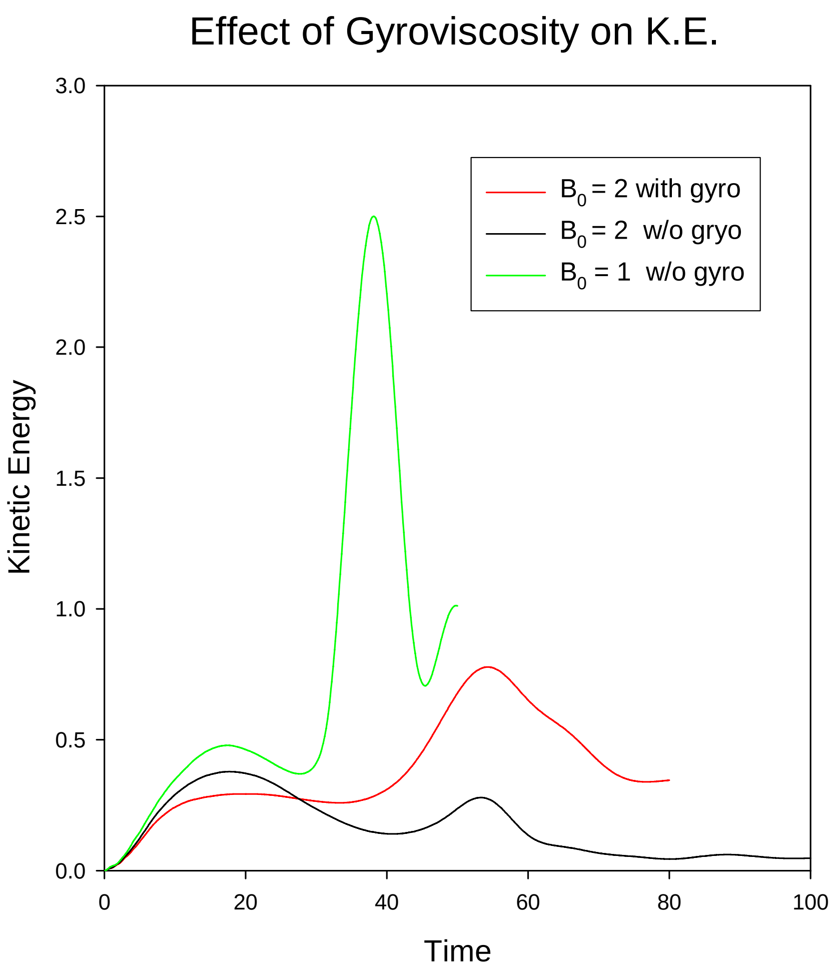


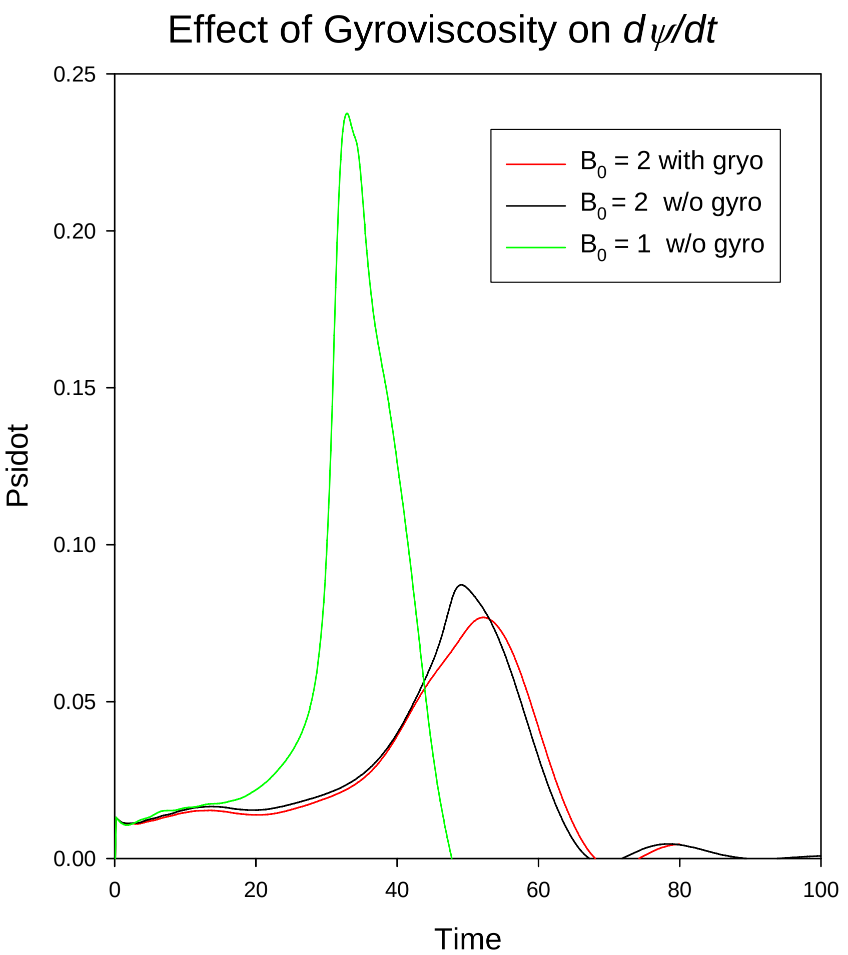
Lowering the peak density from 1.2 to 0.3 significantly increases the reconnection rates for the higher guide field cases



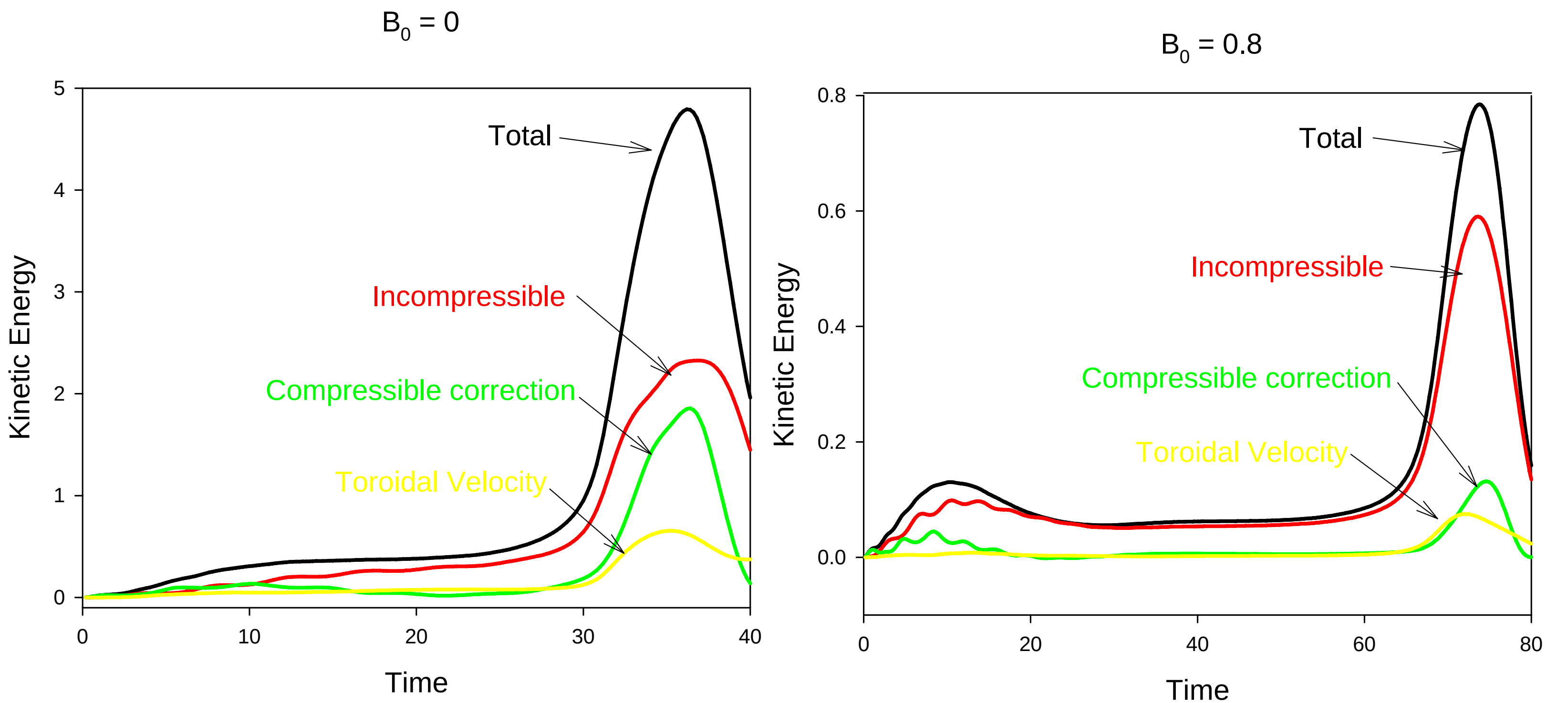


Gyroviscosity leads to somewhat larger kinetic energy, but does not change the reconnection rate much



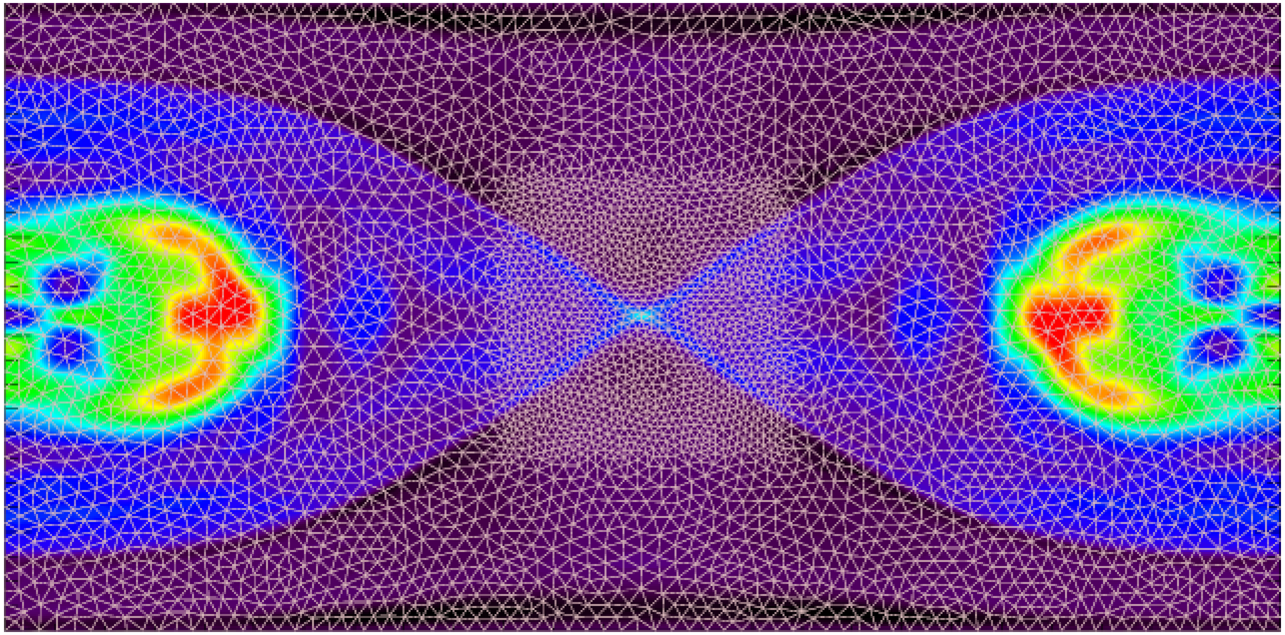


Change in velocity field with toroidal field strength



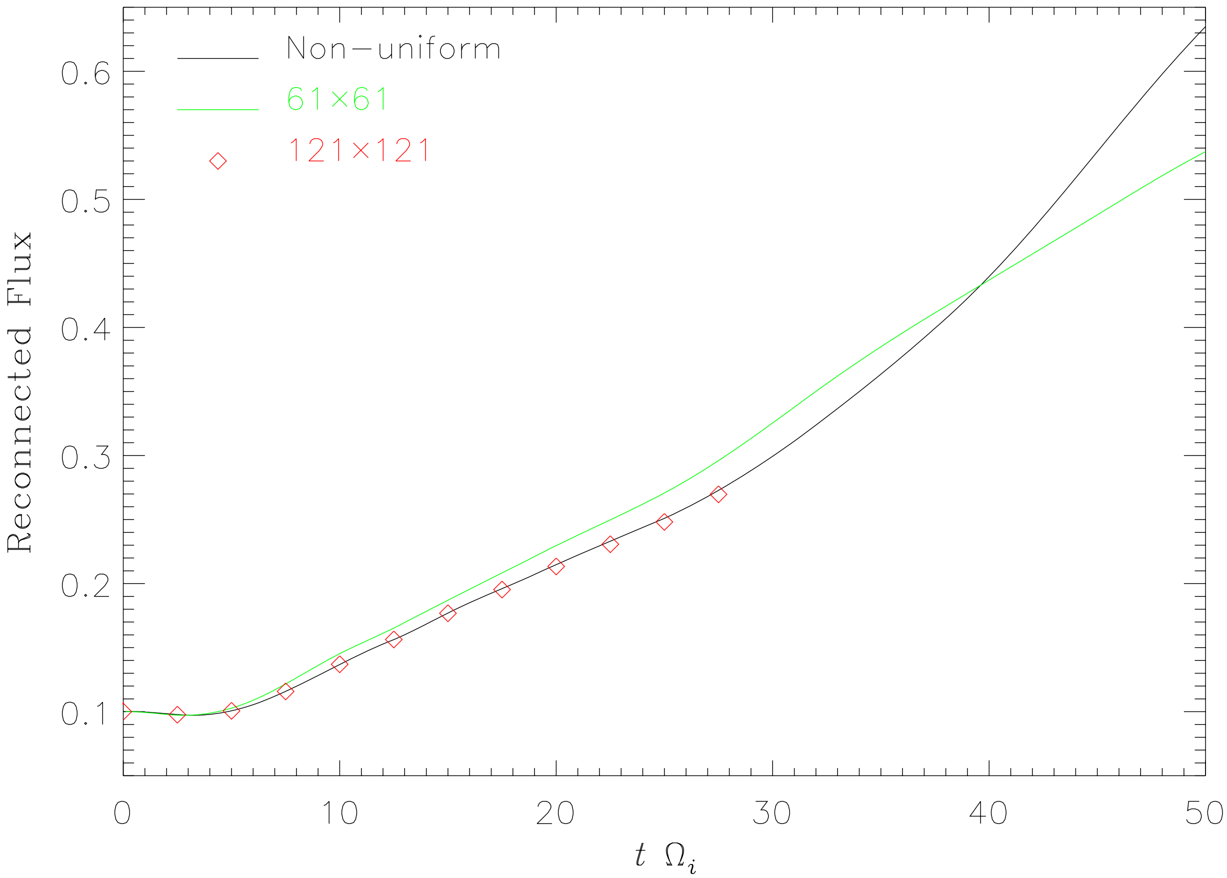
- Velocity field becomes more like incompressible flow as toroidal field strength increases

Adaptive Meshing



Andy Bauer (RPI) has implemented an arbitrary unstructured mesh in the M3D-C^1 code and is exploring different adaptive strategies. This greatly improves the efficiency of the 2-fluid reconnection problem.

Nonuniform mesh with resolution of 121x121 structured mesh near center gives same results as 121x121 calculation in 1/3 the time.



(4-field calculation)

Summary and Conclusions

- Full 8-field 2F-MHD equations solved in 2D slab geometry with stream function/potential form
- Guide field significantly reduces “fast reconnection” phase in slab geometry
- Gyroviscosity has little effect on reconnection rate when guide field is present
- 2-fluid reconnection problems require localized regions with high resolution…natural for adaptive refinement
- Now generalized to toroidal geometry (Ferraro)
- 3D extensions underway