A mixed finite-element method for parallel heat conduction and a continuum solution to CEL-DKE in NIMROD

with NIMROD Team

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Mixed finite element method for parallel heat conduction in NIMROD.

- NIMROD successfully uses higher-order elements to capture effects of anisotropic heat conduction.
- However, spatial convergence for confinement calculations with stochastic fields requires many Fourier modes (> 11) and high polynomial order (> 4).
- Such resolution is prohibitive when computing integral heat flow closure. Also, anisotropic conduction is needed for semi-implicit stabilization.
- Goal: accurately capture effects of anisotropic heat conduction with fewer Fourier modes and lower polynomial degree.

Consider temperature evolution due to anisotropic conduction only.

- Simplified temperature equation is:

$$\frac{3}{2} n \frac{\partial T}{\partial t} = -\vec{\nabla} \cdot \vec{q}$$

$$= \vec{\nabla} \cdot \left[\kappa_{\perp} \vec{\nabla} T + \left(\kappa_{\parallel} - \kappa_{\perp} \right) \hat{b} \hat{b} \cdot \vec{\nabla} T \right].$$

- NIMROD expands T using C^0 finite-element basis functions, which are appropriate when solving weak form of problem.

Mixed finite-element method (MFEM) treats $\mathbf{q}_{||}$ as fundamental variable.

- Define parallel conduction type term in T evolution:

$$\frac{3}{2} n \frac{\partial T}{\partial t} = \vec{\nabla} \cdot \left[\kappa_{\perp} \vec{\nabla} T - q_{||} \hat{b} \right],$$

$$q_{||} = -\left(\kappa_{||} - \kappa_{\perp} \right) \hat{b} \cdot \vec{\nabla} T$$

- Expand $q_{||} = \Sigma_i q_{||i} \alpha_i$ (like T) and solve expanded system for T and $q_{||}$ simultaneously.
- Leads to non-Hermitian matrices which require non-symmetric solvers.

Integrate conduction terms in T equation by parts but leave $q_{||}$ equation in strong form.

- Ignoring surface terms yields coupled system:

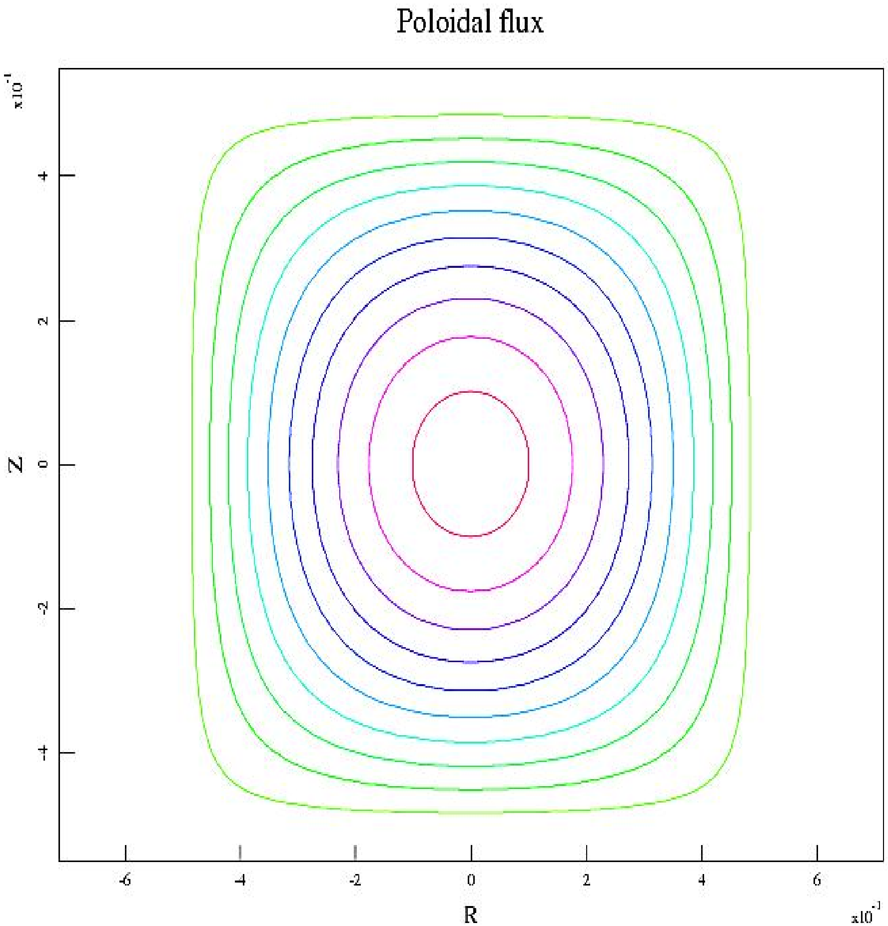
$$\int dV\left[\alpha_j \frac{3}{2} n \frac{\partial T}{\partial t} + \vec{\nabla}\alpha_j \cdot \left(\kappa_\perp \vec{\nabla} T - n_0 \kappa_0^{1/4} q_{||}\hat{b}\right)\right] = 0,$$

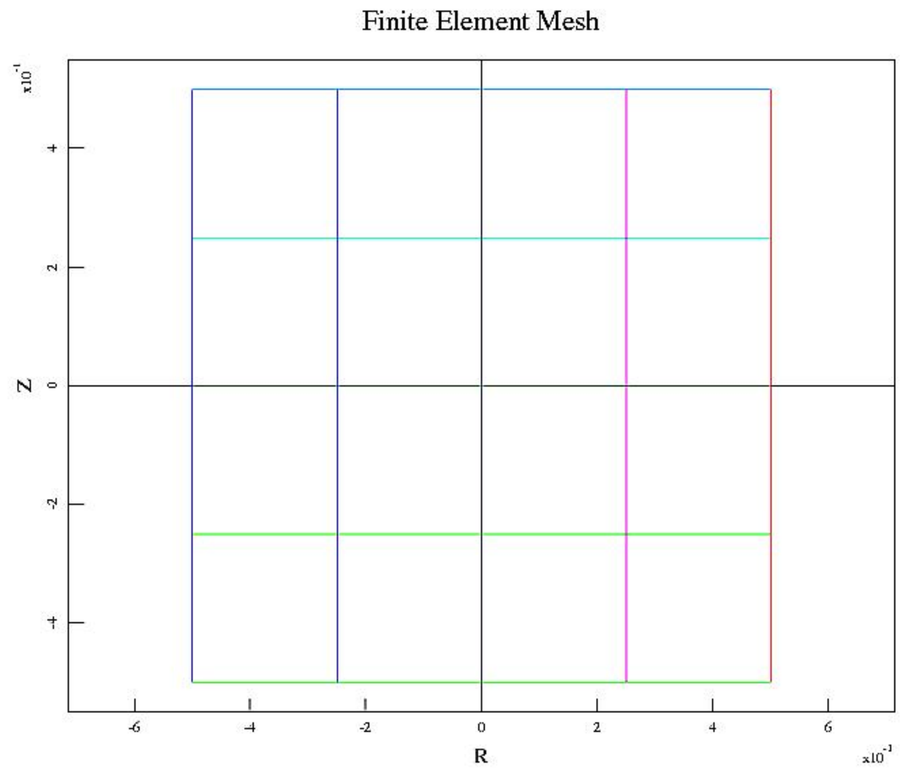
$$\int dV \alpha_j \left[\left\{\frac{n_0^2}{\kappa_0^{1/2}\left\langle \kappa_{||} - \kappa_\perp \right\rangle} q_{||} + n_0 \kappa_0^{1/4} \hat{b} \cdot \vec{\nabla} T\right\}\right] = 0,$$

- Note $q_{||}$ is undifferentiated

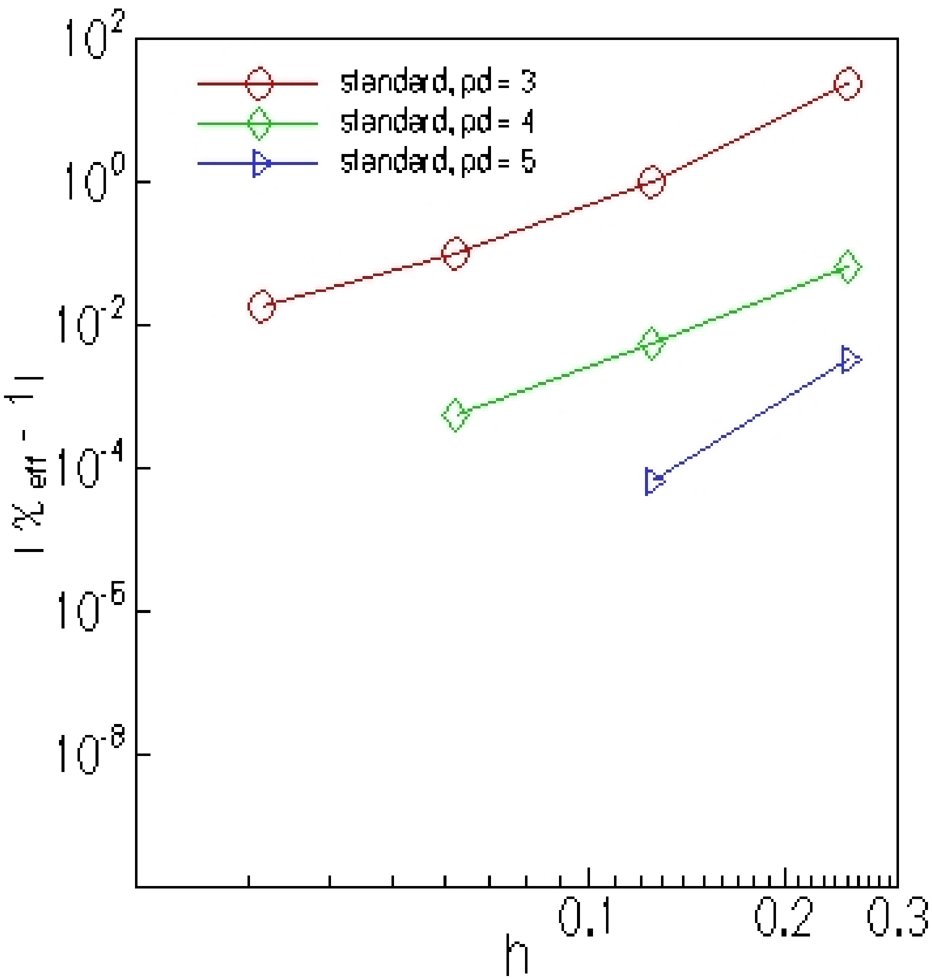
Apply to JCP anisotropic conduction test problem.

- Magnetic field and grid not aligned.
- Accuracy tested with problem that has flux-function heat source in rectangular geometry.

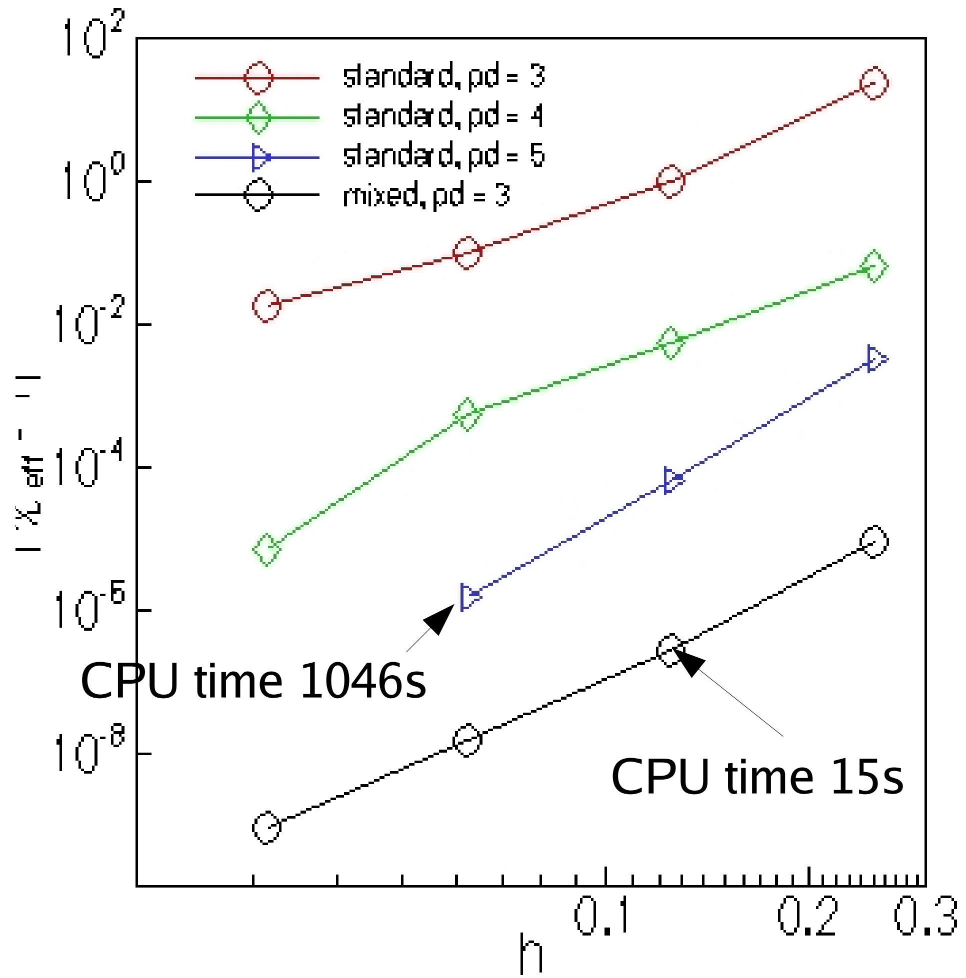




Error reduced considerably with MFEM method (bottom curve on right plot).



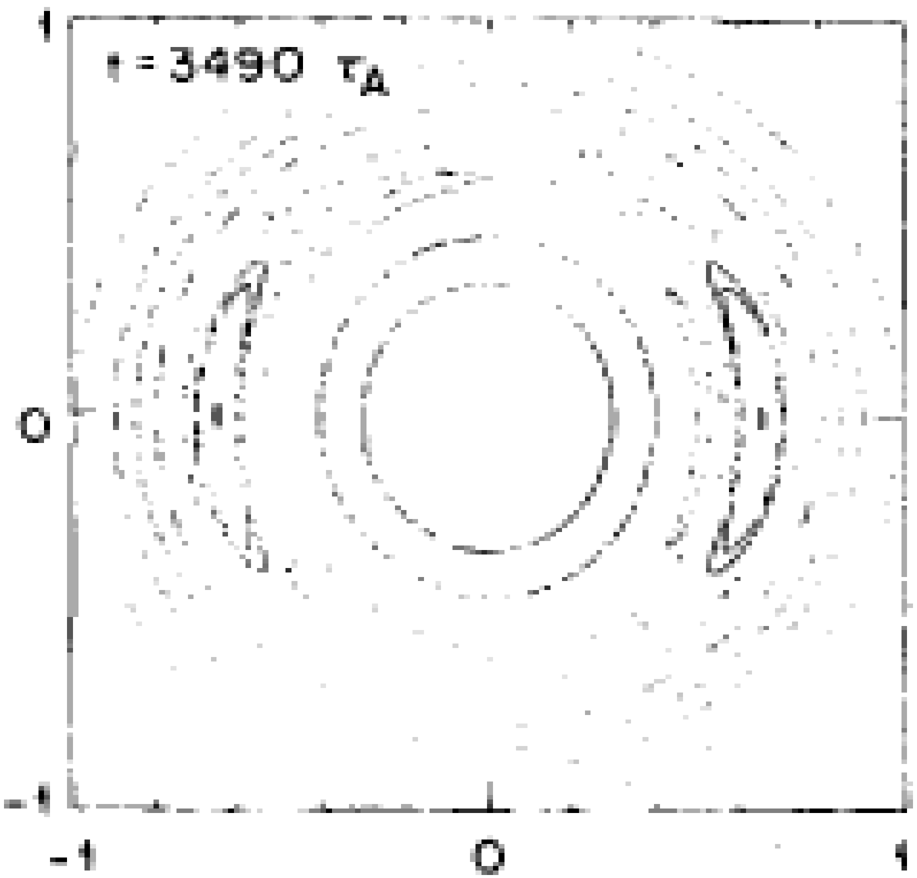
previous result



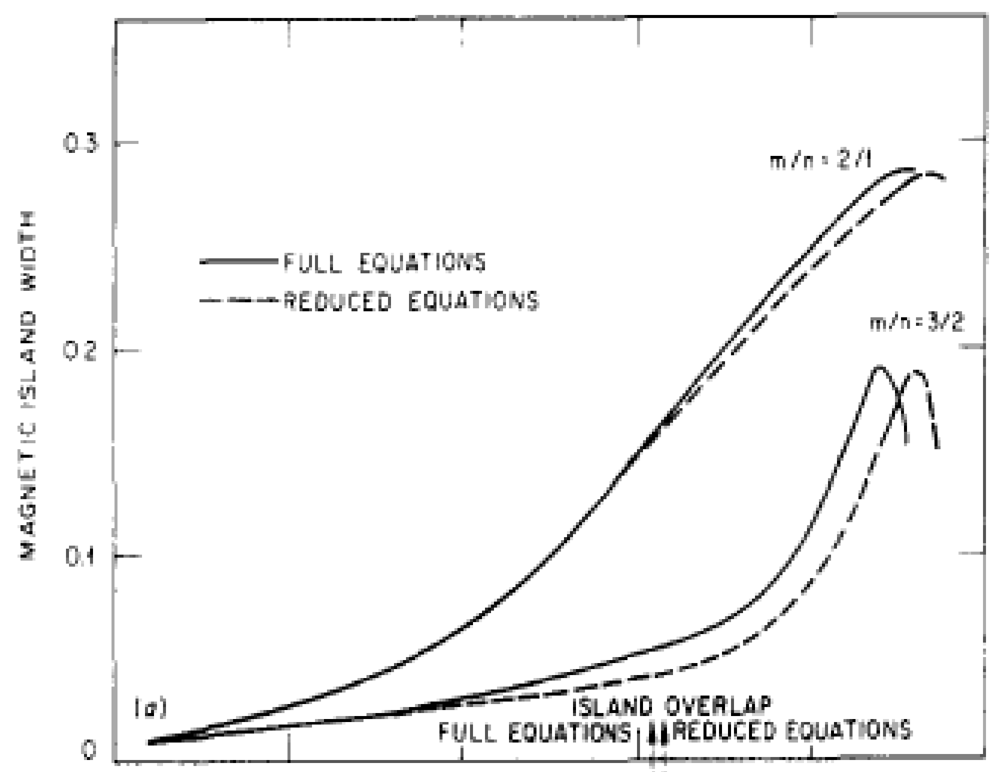
MFEM result (bottom curve)

Apply MFEM to cylindrical tearing mode case as additional test.

- Coupled 2/1 and 3/2 islands interact leading to stochasticity (Holmes, *et al*. 1983 Phys Fluids).



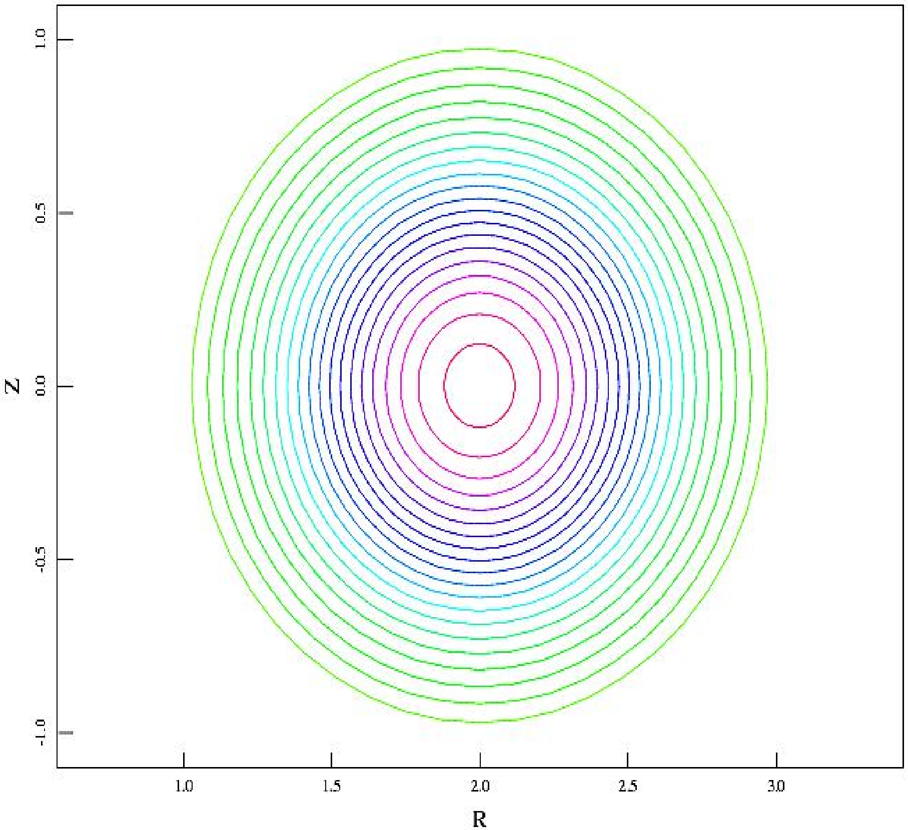
Poincare plot from Holmes' case prior to “disruption”



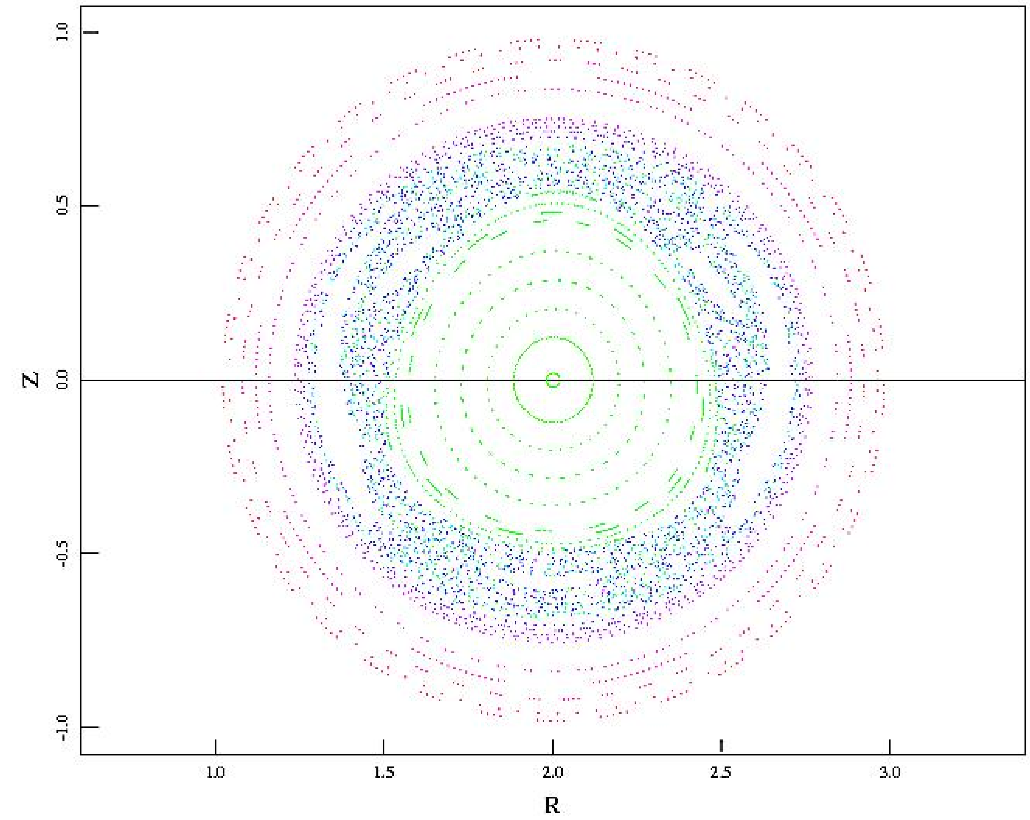
2/1 and 3/2 islands widths versus time

Compare parallel conduction models on this case as part of CEMM work.

- Freeze **B** at different time slices, heat with symmetric source $Q \sim \exp(-r^2)$ and evolve T only to steady-state.



heat source contours, $Q \sim \exp(-r^2)$



juxtaposition of NIMROD linear eigenfunctions

Future work on MFEM applied to anisotropic heat conduction.

- Develop effective preconditioning in Fourier direction.
  - compute diagonal in finite-element basis index matrices and apply in preconditioning step.
- Revisit SSPX heat confinement calculations using MFEM approach.
- Employ MFEM anisotropic, semi-implicit conduction operator for stabilizing integral heat flow closure in SSPX heat confinement studies.

Continuum solution to Chapman-Enskog-like drift kinetic equation in NIMROD.

- For initial $q_{||}$ calculation, solve equations of form:

$$\frac{\partial f_i}{\partial t}+\frac{\nu_L}{2} f_i+\gamma_i v \hat{b}\cdot\vec{\nabla} f_i = W_{i1}^{-1} L_1^{3/2} v(\hat{b}\cdot\vec{\nabla}\ln T) f_{Max},$$

$$where\ kinetic\ distortion\quad F_1=\sum_i W_{1i} f_i,$$

$$q_{||}=-T\int d\vec{v}\, v_{||} L_1^{3/2} P_1\left(\frac{v_{||}}{v}\right) F_1$$

- Solve equations on separate groups of processors.
- Use v grid that makes Gauss-Laguerre quadrature exact.

Stagger *F* and *T*.

- First solve for ΔF (actually $\Delta f_i = f_i^{k+1} - f_i^{k}$) using $T^{k+1/2}$:

$$\left[1+\theta\,\Delta t\left(\frac{\nu_L}{2}+\gamma_i v\,\hat{b}\cdot\vec{\nabla}\right)\right]\Delta f_i = \Delta t\left(\frac{\nu_L}{2}+\gamma_i v\,\hat{b}\cdot\vec{\nabla}\right) f_i^k$$

$$+\Delta t\, W_{i1}^{-1} L_1^{3/2} v\,(\hat{b}\cdot\vec{\nabla}\ln T^{k+1/2})\, f_{Max}^{k+1/2}$$

- Then solve for $\Delta T = T^{k+3/2} - T^{k+1/2}$ using centered f^{k+1}:

$$\left[1+\theta\,\Delta t\,\vec{\nabla}\cdot\kappa_\perp\vec{\nabla}\right]\Delta T = \theta\,\Delta t\,\vec{\nabla}\cdot\kappa_\perp\vec{\nabla}\,T^{k+1/2}$$

$$-\Delta t\,\vec{\nabla}\cdot q_{\|}^{k+1}\,,$$

where $q_{\|}^{k+1} = -T\int d\vec{v}\, v_{\|}\, L_1^{3/2} P_1 \sum_i W_{1i} f_i^{k+1}\,,$

Acceleration term brings in differentiation with respect to speed, *v*.

- Can include nonlinear parallel electric acceleration:

$$\left[1+\theta\Delta t\left(\frac{\nu_L}{2}+\gamma_i\left[v\hat{b}\cdot\vec{\nabla}+\frac{qE_{\|}}{m}\frac{\partial}{\partial v}\right]\right)\right]\Delta f_i=\ldots$$

  at the expense of coupling solutions on *v* grid.

- At present, can compute full matrix that arises from spatial coupling and solve for $f_i(v_j,\boldsymbol{x},t)$ using SuperLU.

- Either develop 3-D preconditioning for non-symmetric systems or combine equations for f_i's to make system symmetric.

Future work.

- Apply 3-D iterative solves in continuum solution of CEL-DKE and/or higher order moment equations.