NIMROD CDX-U Update and Other Developments

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Topics

- CDX-U Update
  - Parameters
  - Axisymmetric computations
  - Linear results
  - 3D computation
- Tearing mode

The new CDX-U equilibrium is constructed to be a steady state for the entire system of equations.

- Equilibrium p, I, and ψ read from JSOLVER output, "fixed129x257."
- Pressure profile is $p_0\left[\alpha\tilde{\psi}+(1-\alpha)\tilde{\psi}^2\right]$, $\tilde{\psi}\equiv(\psi-\psi_{\mathrm{lim}})/(\psi_{\mathrm{axis}}-\psi_{\mathrm{lim}})$
- Number density profile $n_i=n_e=n_0\left[\alpha+(1-\alpha)\tilde{\psi}\right]$, $\alpha=0.1$

  $n_0=1.8626\times10^{19}\ \mathrm{m}^{-19}$ to match $\mu_0 p(0)=7.5\times10^{-4}$ at $T_i=T_e=100$ eV

  from e-mails. Z_{eff} does not affect ion density; $n_i=n_e$.
- Field from JSOLVER: $I(0)=0.0470$ Tm, $R(0)=0.395$ m, $B_\phi(0)=0.119$ T

$$\tau_A=\frac{R(0)\sqrt{\mu_0 m_p n_0}}{B_\phi(0)}=6.58\times10^{-7}\ \mathrm{s}$$

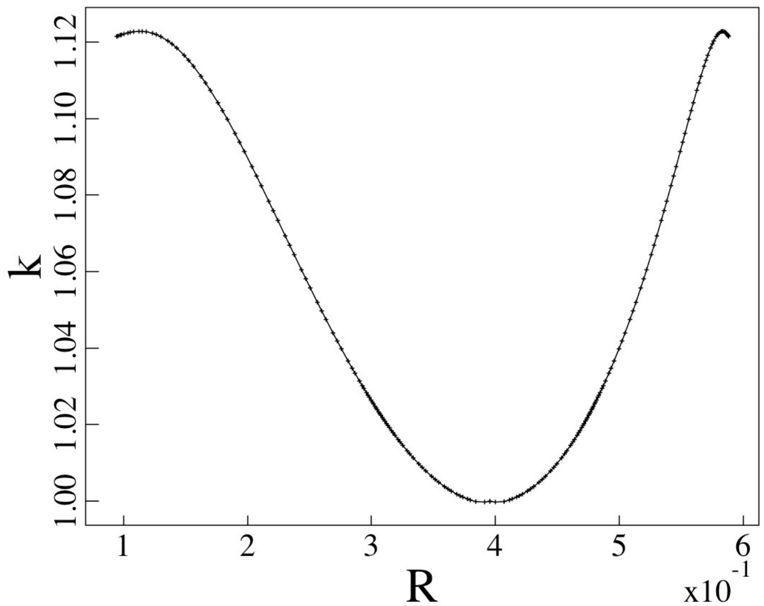
- Perpendicular thermal conductivity to enforce steady state:

$$\kappa_\perp=\frac{1}{T'\left\langle|\nabla\psi|^2\right\rangle}\left[\frac{V_{loop}}{2\pi\mu_0}\left\langle\frac{|\nabla\psi|^2}{R^2}\right\rangle\right]\propto\frac{\left\langle B_{pol}^2\right\rangle}{\left\langle R^2B_{pol}^2\right\rangle}$$

Evaluated in FLUXGRID like n.

More on new CDX-U equilibrium:

κ (not χ) is almost flat.→



- From equilibrium: $J_\phi(0) = 6.01 \times 10^5 \ \text{A/m}^2$
- JSOLVER fort.76 file from SCJ: $\kappa(0)/n(0)$=12.4 m^2/s [No adjustments made.]

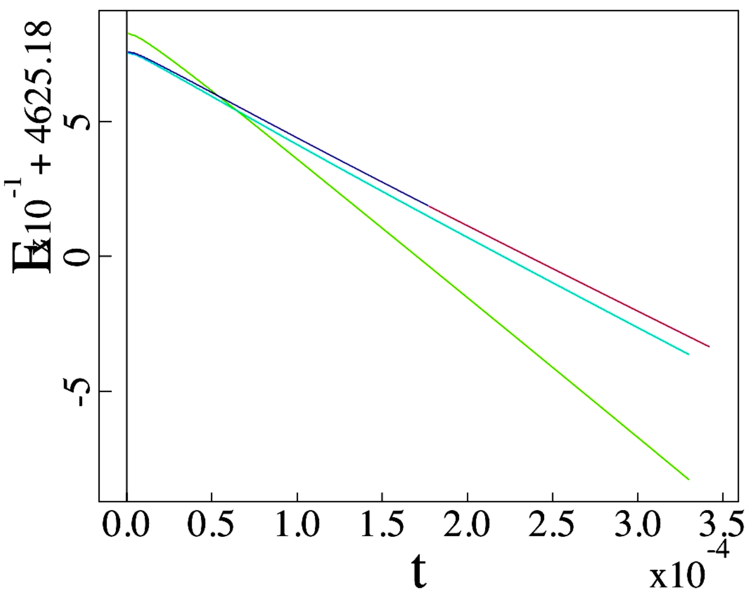
$$V_{loop} = 3.17 \ \text{V} \left[= 2\pi\eta R J_\phi \ \text{for} \ \frac{\eta(0)}{\mu_0} = 1.69 \ \text{m}^2/\text{s} \right]$$

- This resistivity is consistent with Spitzer parallel for Z_{eff}=2, lnΛ=20, T_e=100 eV.

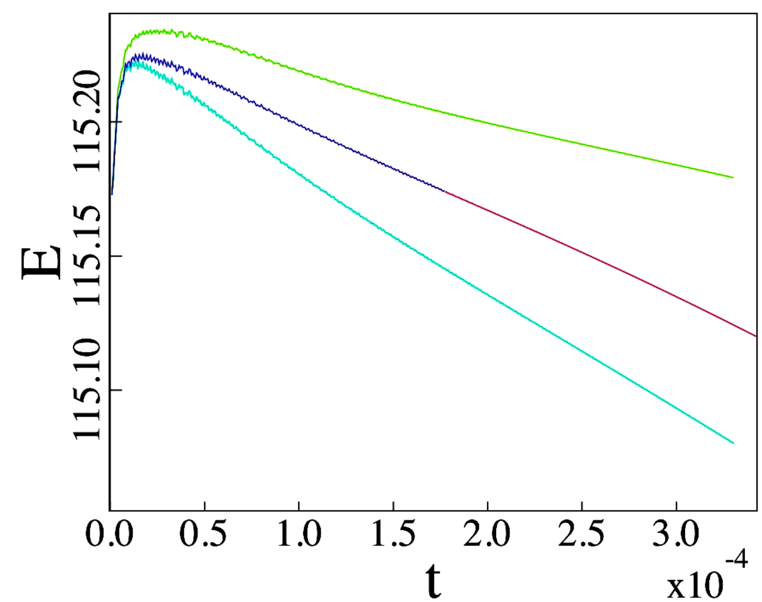
However, $$\tau_r = \frac{\mu_0 a^2}{\eta(0)} = 0.0359 \ \text{s}, \ \text{so S} = 54,500$$

Nonlinear axisymmetric computations check steady state.

- Resistivity $\sim T^{-3/2}$, limited to 100 times axis value; $\chi_{\|} \sim (6 v_A)^2 \tau_e = 2\times 10^7\ \mathrm{m^2/s}$.
- Viscosity is 10 times initial magnetic diffusivity. $D = \dfrac{a^2}{1000\tau_A} = 92.7\ \mathrm{m^2/s}$.
- Ohmic heating, loop-voltage drive from boundary conditions; $I{\cdot}V$ held constant.
- Equilibrium transferred to n=0 component of solution except number density.
- Lower resolution 20×30 bicubic loses 15 of 4626 J over 4400 τ_A.
- Less loss with 32×32 bicubic and biquartic meshes with packing near wall.

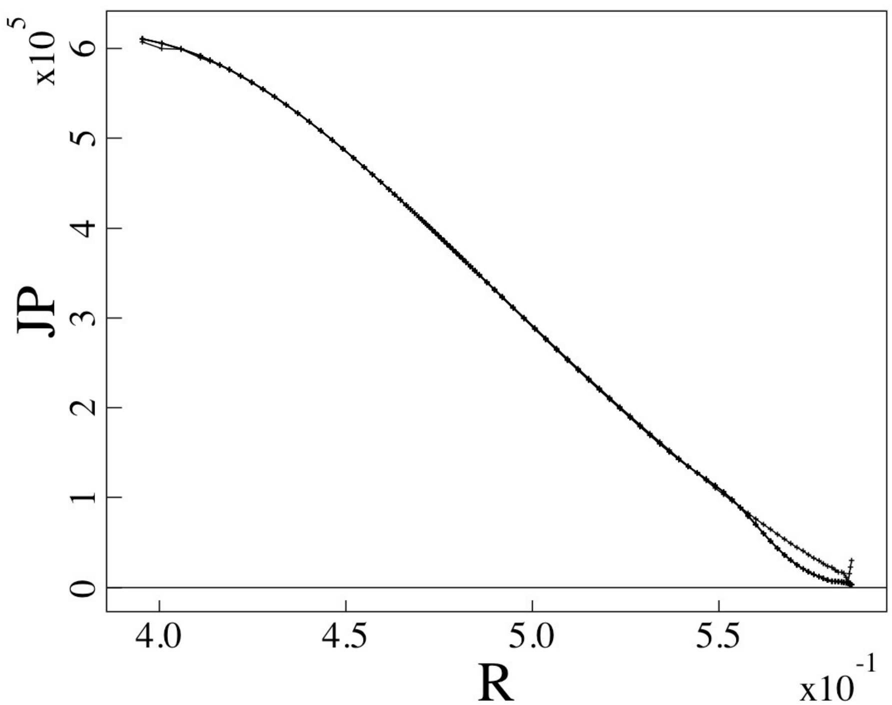


Total energy for 20x30 bicubic and 32x32 bicubic and biquartic computations. (500 τ_A)

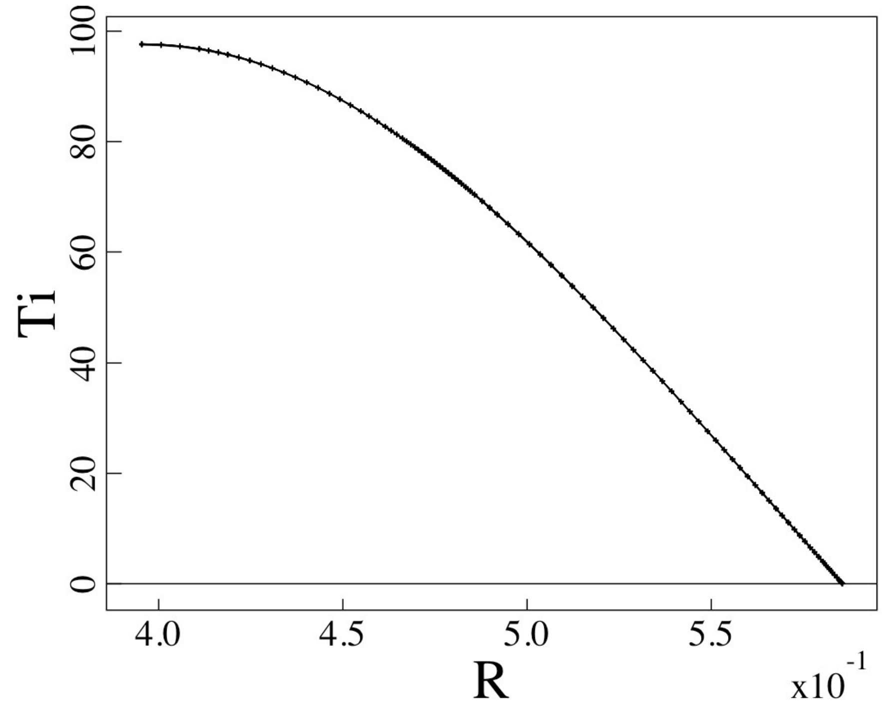


Internal energy for the same computations.

Profile changes are slight except J_ϕ near wall.



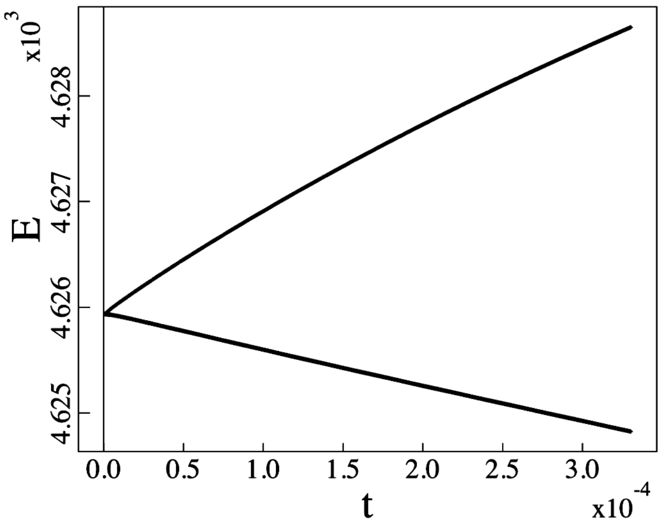
Profiles of J_ϕ for the 32x32 bicubic computation over 500 τ_A.



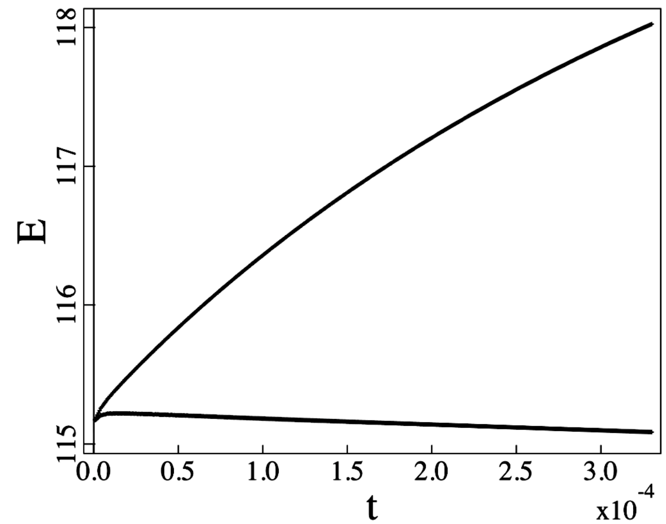
Profiles of T_i for the 32x32 bicubic computation over 500 τ_A.

- Loop voltage dips to a minimum of 3.156 V (from 3.17) at 0.12 ms then slowly starts to increase.

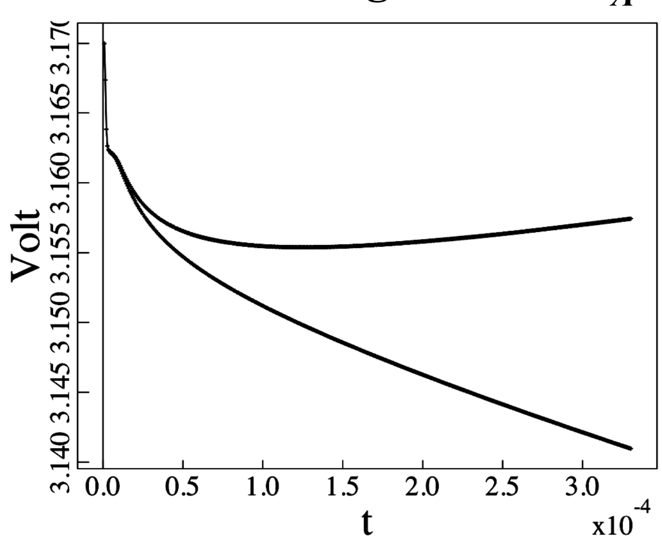
Computation with $\kappa_\perp$ reduced by 10% serves as a sensitivity test.



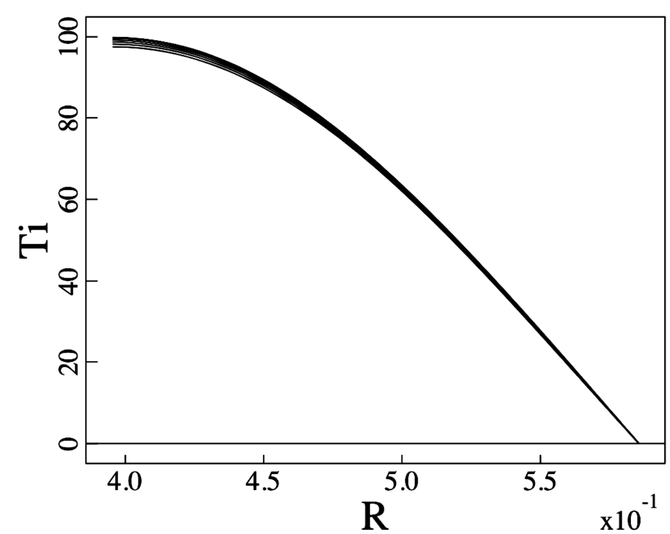
Reducing $\kappa_\perp$ changes total energy evolution to be increasing over 500 τ_A.



Internal energy increases by 3%.

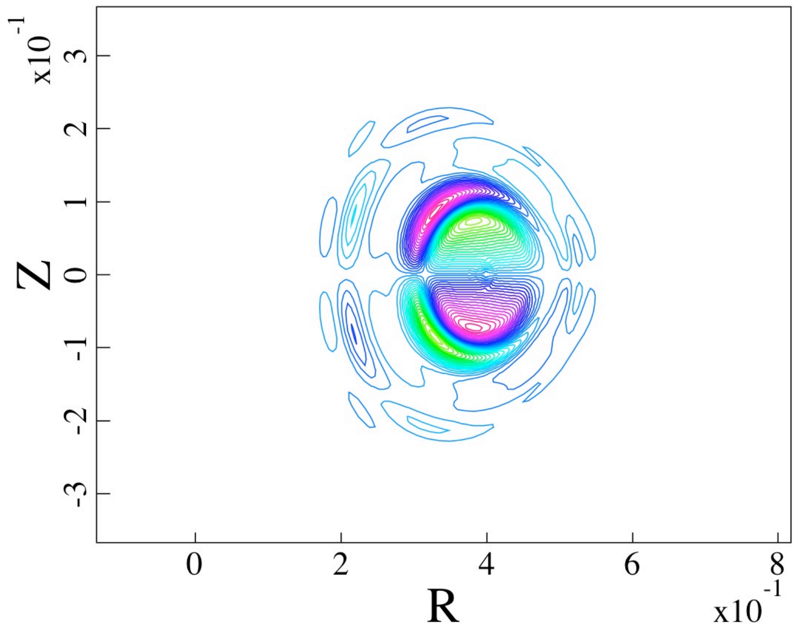


V_{loop} decreases monotonically with reduced $\kappa_\perp$.

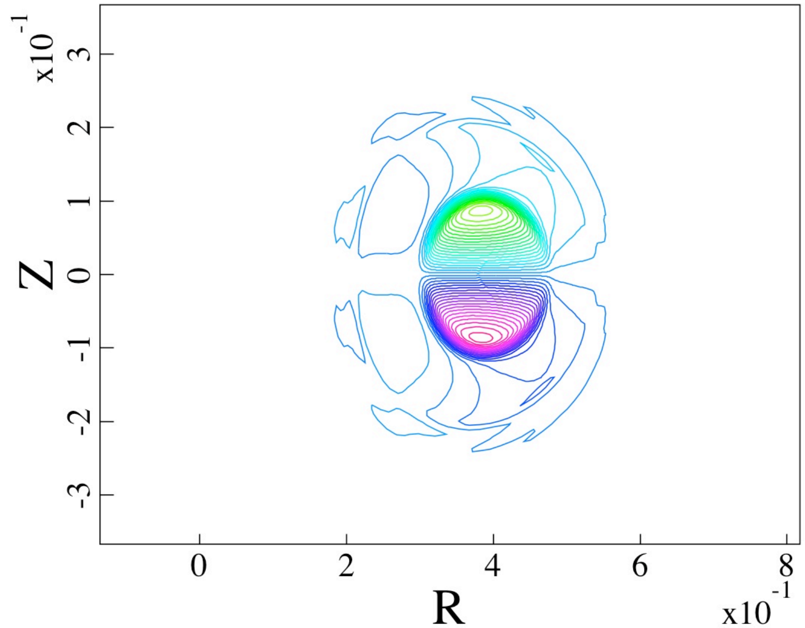


Profiles of T_i over 500 τ_A show on-axis temperature increasing by 2.3 eV.

Linear results (so far) have n=1 but no converged growing n=2.



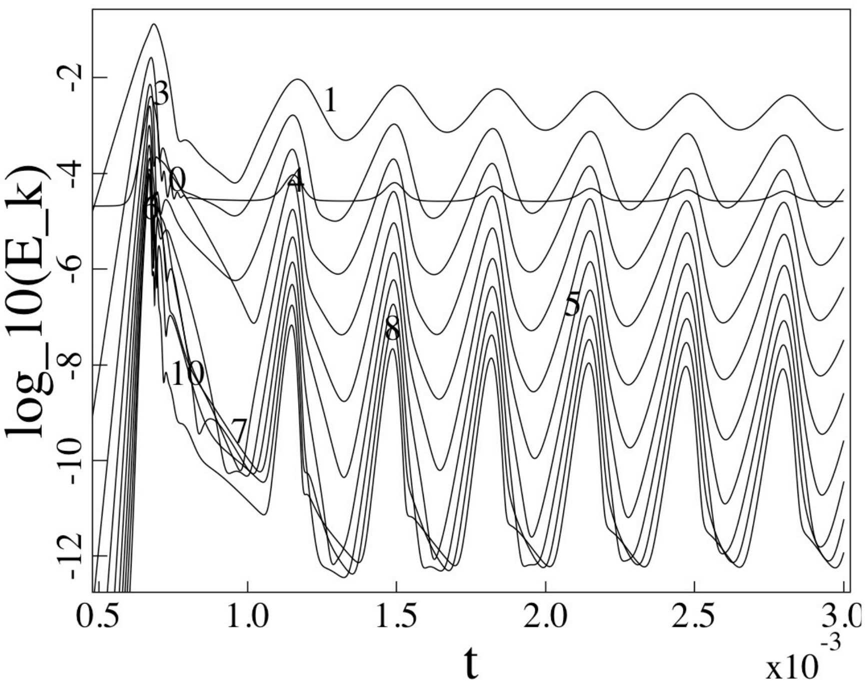
Contours of constant J_ϕ for a 32x32 bicubic n=1 eigenmode.

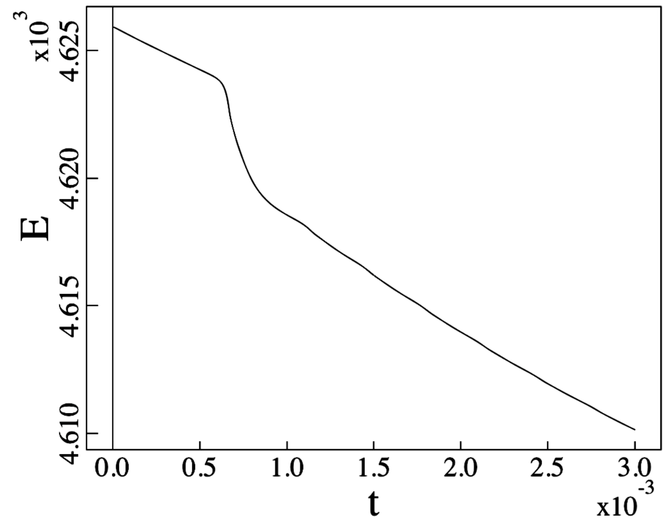


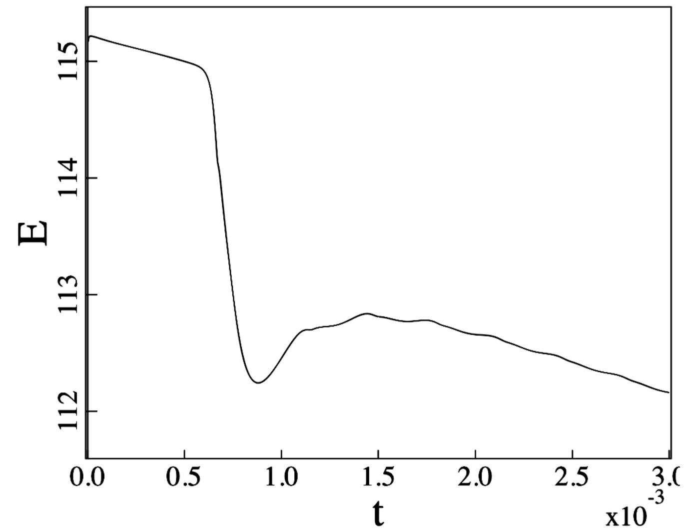
Contours of constant $T_i = T_e$ for a 32x32 bicubic n=1 eigenmode.

- $\Delta t > \tau_A / 3$ produces something that looks like ballooning.
- $\gamma\tau_A = 0.0162$ with anti-rippling $-1.5\eta_0\left(\tilde{T}/T_0\right)\mathbf{J}_0$ term (and close to this in the nonlinear simulation) and $\gamma\tau_A = 0.0114$ without the anti-rippling term.

Nonlinear simulation with *n* up to 10 settles into a sawtooth cycle with period of ~330 *μ*s.





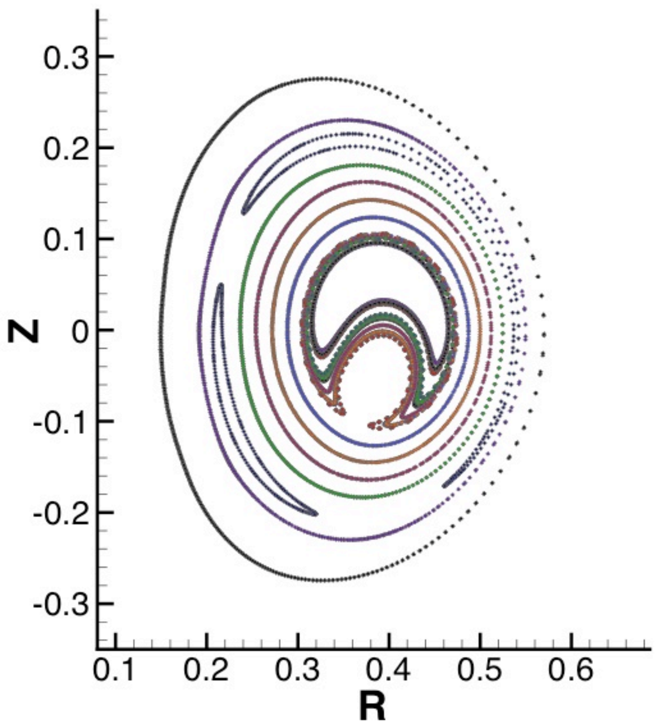


- Computation ran on 44 Franklin processors in 13.6 wall-clock hours.

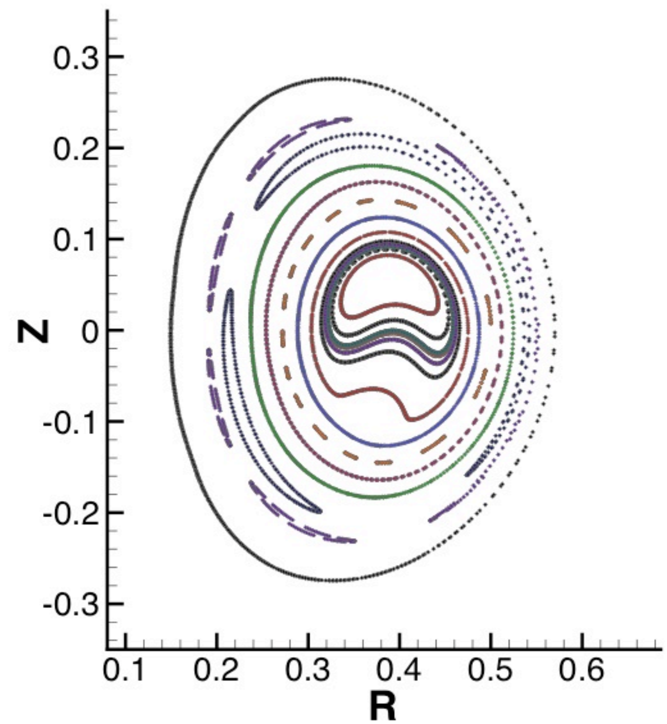
Total and internal energies show a significant drop at the first crash but mostly the same decay as axisym. computation afterward.

There is no full recovery to the original axisymmetric state.

- Temperature remains about 10 eV lower than initial condition after 1st crash.
- The core remains helical with q computed from the mean remaining close to 1.

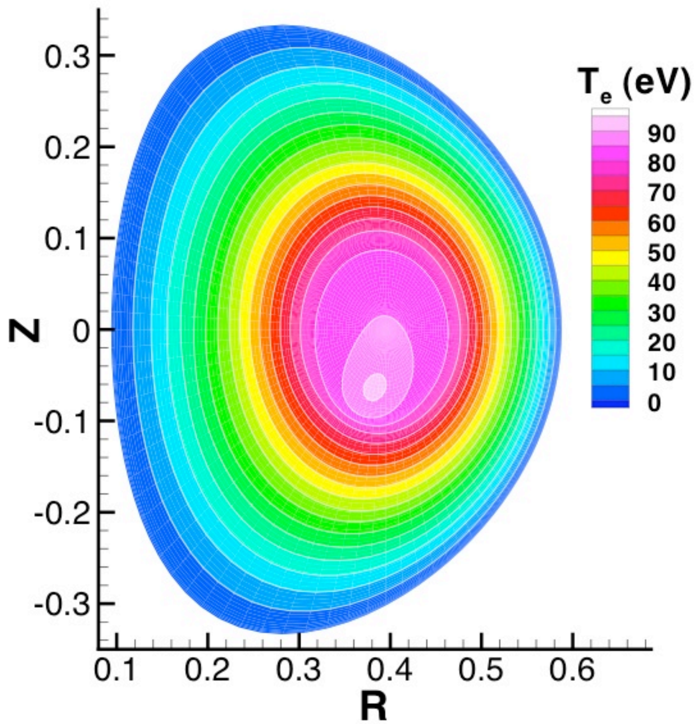


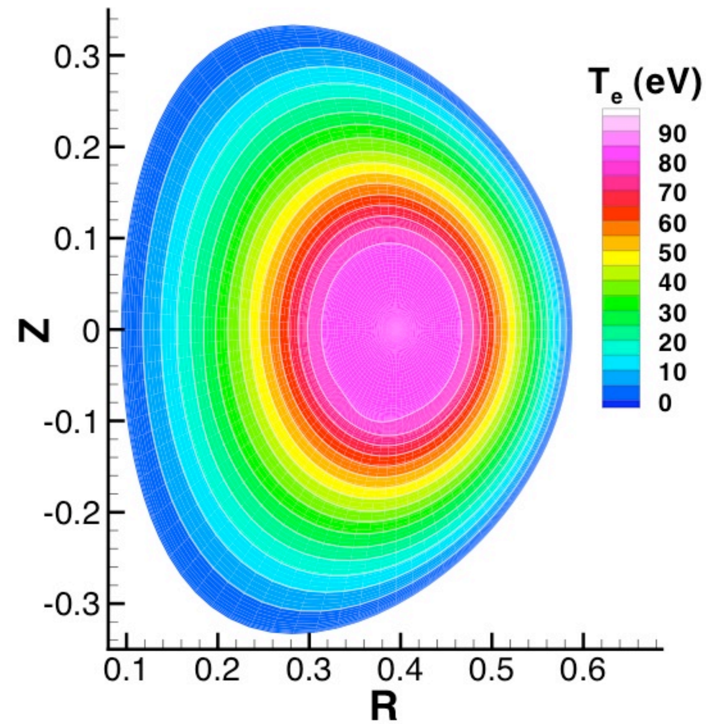
Surface of section just before third peak in *n*=1 energy.



Surface of section 48 μs later (approximately 1.5 ms into simulation).

Peak temperature occurs along a helix, due to enhanced current density and Ohmic heating at the reconnection site, before the loss of the previous magnetic axis.





Temperature just before third peak in *n*=1 energy. T_{max} is 90.7 eV.

Temperature 48 μs later (approximately 1.5 ms into simulation). T_{max} is 84.6 eV.

Tearing-mode Computation

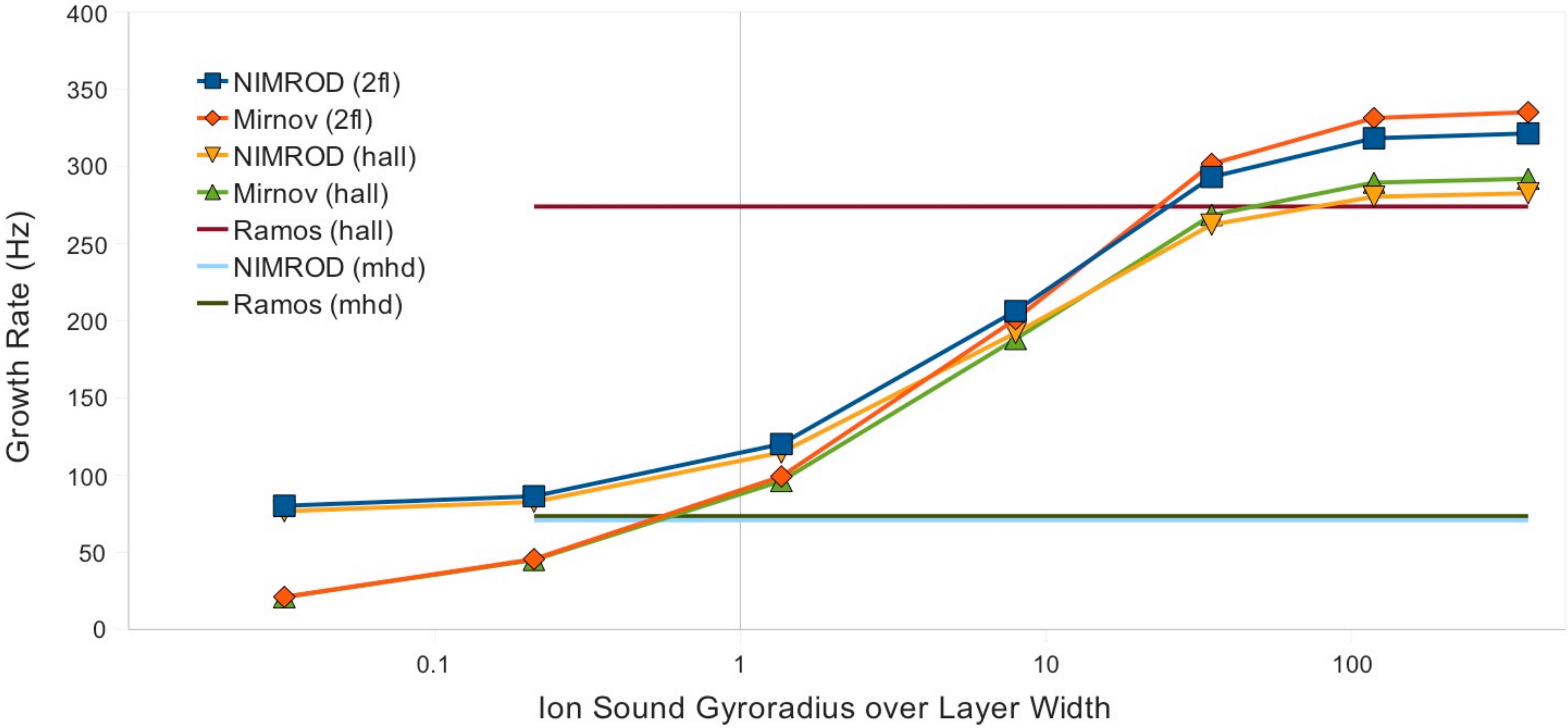
J. King has extended large guide field linear computations for benchmarking with theory by V.V. Mirnov. [GP8.00143]

δ/L	Γ (NIMROD)	Γ (Theory)	Error (%)
0.12	7.48E-004	8.05E-004	7.6
0.09	4.93E-004	5.10E-004	3.44
0.08	3.48E-004	3.61E-004	3.53
0.05	1.61E-004	1.61E-004	0.49

Box Size	1 m
L	0.1667 m
ρ_s	0.1205 m
β	0.1
Δ'	1.66 m^{-1}

- Reaching the asymptotic regime with $\rho_s < L$ and $L \ll$ box requires greater resolution (packed 120×14, biquartic) than previous cases with $L < \rho_s$.
- Table shows small delta-prime, large-beta regime results.

For small δ/L, there is quantitative agreement with Mirnov over a range of beta and with Ramos in the high-beta limit.



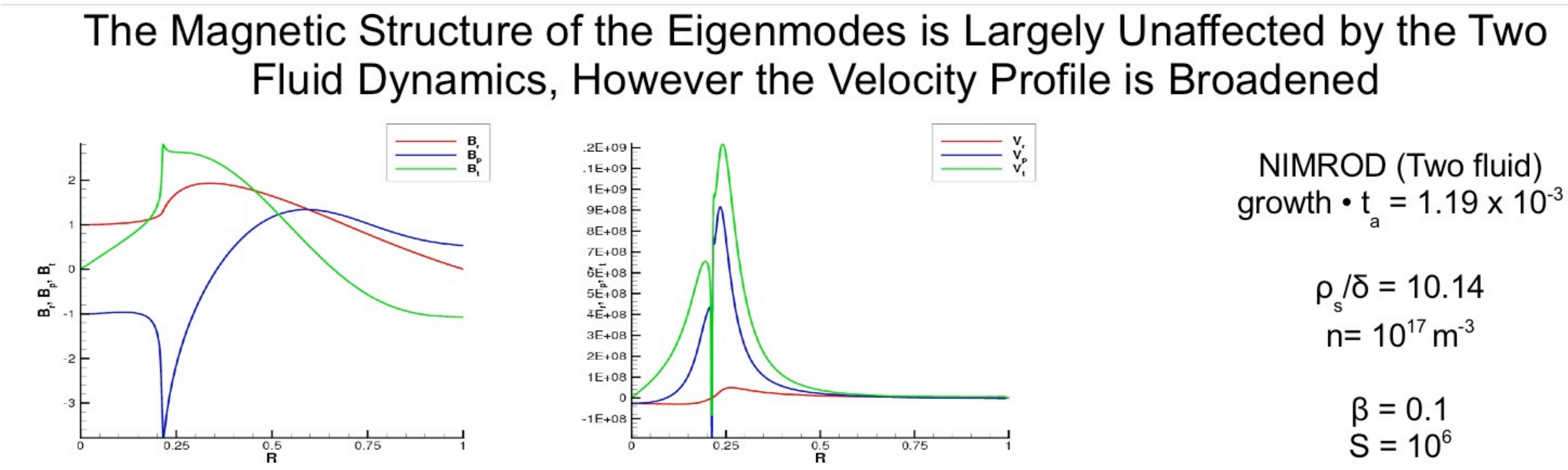
- This scan is done by decreasing the plasma temperature, and thus the plasma beta
- The Ramos theory is independent of beta, and is only valid in the two fluid limit
- The Mirnov theory agrees well in the transition between the two fluid regime and the single fluid regime, however it does not reduce to the single fluid result in that regime

Box Size	1 m
L	0.1667 m
δ	~2×10^{-2} m
Δ'	1.66 m^{-1}
S	6.66×10^{5}

Tearing mode (continued)

• A scan to large Δ' is inconclusive so far. Simulation results approach the MHD limit, and we have a discrepancy in evaluating the analytical prediction.

• Cylindrical benchmarks are being performed for core and edge tearing modes in RFP equilibria.



• We are also investigating stabilization from non-equilibrium rotation.

• Nonlinear computations (now also with $\rho_s < L$) will be extended to consider multiple helicities in slab and cylindrical geometry.

Conclusions

- We are able to run the new CDX-U cases with loop-voltage drive and Ohmic heating.
- Approximate steady state for axisymmetric computations serves as a benchmark with JSOLVER.
- Linear computations are sensitive to perturbed resistivity term.
- Nonlinear 3D computation shows repeated sawteeth, near-constant q after initial crash, and thermal snakes.
- Simulation is roughly over the discharge time. How about transients?
- Menard paper is not a good reference for basic discharge information and sawteeth. What are we using for comparison?
- Slab tearing computations that approach the asymptotic show agreement with Mirnov and recent Ramos results.