Guiding Center Plasma Models in 3D and ELM Simulation Results

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TOPICS

- **Guiding Center plasma models in 3D magnetic fields: update**

  Sherwood poster, Monday morning

  3D geometrical conditions on validity of small gyroradius GC expansions

  Only first order equations exist for general 3D magnetic fields, higher order for 2D symmetry (eg, toroidal axisymmetry)

  Implications

- **Recent progress in M3D ELM NL simulations**

  MPP scaling to several 1000's of cpus

  No toroidal periodicity assumed; want 40 or more toroidal harmonics

  n=1 harmonic in longer time NL phase

- **Other M3D ELM-related work**

  – **RMP and toroidal rotation in MPP**

  – **Better 2D FE grids (TRIANGLE + packed mesh for Alcator C-Mod)**

- **Cray XT4 problems (NCCS jaguarcnl, NERSC franklin)**

Guiding Center plasma models in 3D plasmas

- **Guiding Center (GC) model for single charged particle separates particle motion into fast gyration around magnetic field lines and smoother guiding center motion,**

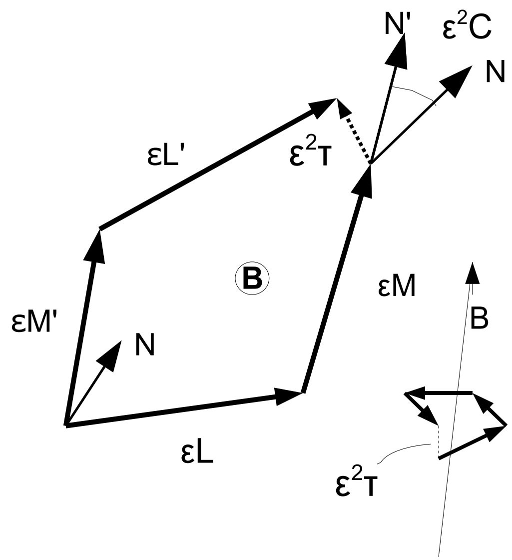
$$\mathrm{x} = \mathrm{X} + \frac{\epsilon v_\perp}{\Omega}\hat{\boldsymbol{\rho}}. \tag{1}$$

Particle velocity $\mathrm{v} = (v_\parallel, \theta, v_\perp)$.

- **2D slab (straight, uniform magnetic field lines) has exact GC expansion in small gyroradius** ρ_i/L **to all orders.**

- **In 3D, for nonzero magnetic field line torsion** $\tau = \hat{\mathrm{b}} \cdot \nabla \times \hat{\mathrm{b}} \neq 0$ **(parallel current** $J_\parallel \neq 0$**). an infinitesimal path around a field line does not close.**

- $(\zeta, v_\perp)$ **defined in local orthogonal, magnetic coordinate systems** $(\hat{\mathrm{e}}_1, \hat{\mathrm{e}}_2, \hat{\mathrm{b}})$ **tied to** B **at guiding center.**



- The GC expansion is nonuniform in velocity phase space starting in first order. Large and small ratios of $v_\parallel/v_\perp$ break the small gyroradius ordering.

  Lowest order in ϵ: $\langle \mathrm{x} \rangle = \mathrm{X}$, $\langle v \rangle = v_\parallel$

  First order:

$$
\begin{aligned}
\dot{\zeta} &= \Omega \left[1 + \frac{\epsilon w}{B} \frac{U_\parallel}{w} \left(\frac{1}{2} \hat{\mathrm{b}} \cdot \nabla \times \hat{\mathrm{b}} + \hat{\mathrm{b}} \cdot \mathrm{R} \right) \right] \\
\langle v_\parallel \rangle &= U_\parallel \left[1 - \frac{\epsilon w}{B} \frac{w}{U_\parallel} \left(\frac{1}{2} \hat{\mathrm{b}} \cdot \nabla \times \hat{\mathrm{b}} - \hat{\mathrm{b}} \cdot \mathrm{R} \right) \right]
\end{aligned} \tag{2}
$$

- Nonuniform gyroangle $\rightarrow$ nonuniform gyroperiod, depending on parallel velocity $U_\parallel$. (Gyroperiod, gyroaverage defined as $\oint d\zeta = 2\pi$.)

  Particle sees longer or shorter gyroperiod depending on whether it moves parallel or anti-parallel to B. Due to torsion, the field lines and ζ rotate by $(1/2)\tau$ when moving along B. Local coordinate axis $\hat{\mathrm{e}}_1$ rotates by $\hat{\mathrm{b}} \cdot \mathrm{R}$, $\mathrm{R} \equiv (\nabla \hat{\mathrm{e}}_1) \cdot \hat{\mathrm{e}}_2$, due to curvature and torsion, from the convective part $\mathrm{v} \cdot \nabla \zeta$ of $d\zeta/dt$.

- The angle coordinate nonuniformity due to torsion is a real physical effect; appears in many areas (Aharonov-Bohm effect, Berry phase, related to Dirac magnetic monopole)

Second order

- **Second order GC equations from Hamiltonian or Lagrangian non-canonical phase-space variable methods, developed to extend the expansion to all orders (Littlejohn 1979-83, Brizard 1989).**

  Goal is to eliminate the 3D geometrical terms from the dynamical equations, keeping only in the gyroangle time derivative $\dot{\zeta}$ and $\langle v_\parallel \rangle$

  Add free functions (gyrogauge) to the Lagrangian and define their gyroaverages to remove $\hat{\mathrm{b}} \cdot \nabla \times \hat{\mathrm{b}}$ and $\hat{\mathrm{b}} \cdot \mathrm{R}$.

- **Derivation is formally correct, but assumes all terms exist.**

- **Effective magnetic vector potential A^* in Lagrangian L allows many terms to be eliminated; corresponds to effective field $\mathrm{B}^* = \nabla \times \mathrm{A}^*$,**

$$\begin{aligned} \mathrm{A}^* &= \mathrm{A} + \epsilon U \hat{\mathrm{b}} - \epsilon^2 \mu \mathrm{R} \\ L &= (1/\epsilon)\mathrm{A}^* \cdot d\mathrm{X} + \epsilon \mu d\zeta - ((1/2)U^2 + \mu B)dt. \end{aligned} \tag{3}$$

- **Problem: R. No connection specified for directions of local coordinate axes at different points. When is the gradient in $\mathrm{R} = \nabla \hat{\mathrm{e}}_1 \cdot \hat{\mathrm{e}}_2$ defined? Also, the curl $\nabla \times \mathrm{A}^*$ in GC space coordinates X needed for the equations of motion.**

- Answer: In general 3D plasmas, equations based on a gyroangle are NOT independent of the choice of the local orthogonal, magnetic coordinate systems $(\hat{\mathrm{e}}_1, \hat{\mathrm{e}}_2, \hat{\mathrm{b}})$ in which the gyroangle is defined.

- Existence of second order expansion requires good magnetic flux surfaces.

  Then can choose $\hat{\mathrm{e}}_1$ to be normal to the surface (not field lines B) and the local magnetic coordinate rotation parameter $\tau_g = \hat{\mathrm{b}} \cdot \mathrm{R} = \hat{\mathrm{b}} \cdot (\boldsymbol{\nabla}\hat{\mathrm{e}}_1) \cdot \hat{\mathrm{e}}_2$ is (negative) of the geodesic torsion of the field line on flux surface.

  Gradient across flux surfaces is the problem; in general, must match field lines across different surfaces. Difficult: for scalar functions, Newcomb solvability. Here, for vector field

- $\tau = 0$ for existence of global planes perpendicular to magnetic field lines.

- $\tau_g = 0$ for existence of triply orthogonal surfaces, one through the field lines.

- In general 3D, $\tau = \tau_g = 0$ required for existence of second order equations.

  2D symmetry, such as toroidal axisymmetry, allows existence for finite τ, τ_g. Do not have to trace field lines on each flux surface to define R.

Time dependence

- Time dependence through Maxwell's equations – the magnetic vector potential term in the electric field in Ohm's law $\mathrm{E} + \mathrm{v} \times \mathrm{B} \simeq 0$, also affects geometrical accuracy in 3D.
  - Ordering the perpendicular component of $-(1/c)\partial \mathrm{A}_\perp/\partial t$ small compared to the electrostatic potential drops the compressional Alfvén wave and makes the geometrical approximation
$\nabla \cdot \left(\hat{\mathrm{b}}\hat{\mathrm{b}} \cdot \nabla\Phi \right) = (\hat{\mathrm{b}} \cdot \nabla)(\hat{\mathrm{b}} \cdot \nabla\Phi) - (1/B)(\hat{\mathrm{b}} \cdot \nabla B)(\hat{\mathrm{b}} \cdot \nabla\Phi) \simeq 0$.
  - Ordering parallel component of $-(1/c)\partial \mathrm{A}_\parallel/\partial t$ small compared to $\nabla_\parallel \Phi$ drops shear Alfvén wave and makes geometrical approximation $\hat{\mathrm{b}} \cdot \nabla\Phi \simeq 0$.
- Both approximations encourage an artificial enhancement of turbulent and zonal poloidal ExB flows with $\Phi \simeq \Phi(r)$.
- Analytic GC and GK models drop compressional wave, keep shear Alfvén.
- Numerical GK particle models are somewhat different from analytic ones — may drop parallel $\partial \mathrm{A}_\parallel/\partial t$ for numerical reasons. Then cannot recover shear Alfvén wave through NL polarization drift.

– May explain why GK (and perhaps gyrofluid) numerical simulations regularly see robust zonal flows $v_{E,\theta} \sim E_r B_\phi / B^2$ in toroidal configurations, while experimental evidence is less certain, suggests weaker drive.

Some Implications

- Twisting of magnetic-field-tied coordinates in 3D is a real physical effect. The velocity space nonuniformities due to τ, τ_g should appear in all guiding center expansion at first order in small gyroradius, including velocity moments. Usually ignored - may help explain differences, inconsistencies between models.

- Since time-varying fields will break axisymmetry, GC models keeping exact 3D geometry are probably at most first order in gyroradius for toroidal plasmas.

- Vector potentials and Lagrangians have interesting properties and existence conditions in three or higher dimensions. GC Lagrangian problem has analogies to the problems encountered by grand unified theories of physics (small scale Lagrangian theory mapped locally to larger scale, higher dimensional space-time).

  - Lagrangian formalism describes strictly local relations; existence is a separate condition.

  - Vector potentials in 3D space always break down on some curve, even though the field corresponding to $\nabla \times \mathrm{A}$ exists there. Breaks down on axis defining

an angle since gradient of angle undefined there ("Dirac curve"). Example

$$\begin{aligned} \mathbf{A} &= \boldsymbol{\psi \nabla \theta} \\ \mathrm{B} &= \boldsymbol{\nabla \psi \times \nabla \theta}. \end{aligned} \tag{4}$$

Gradient $\nabla\theta$ is undefined on axis around which angle is defined. eg, magnetic axis of torus. For GC gyroangle and A^*, this is B at every guiding center point.