

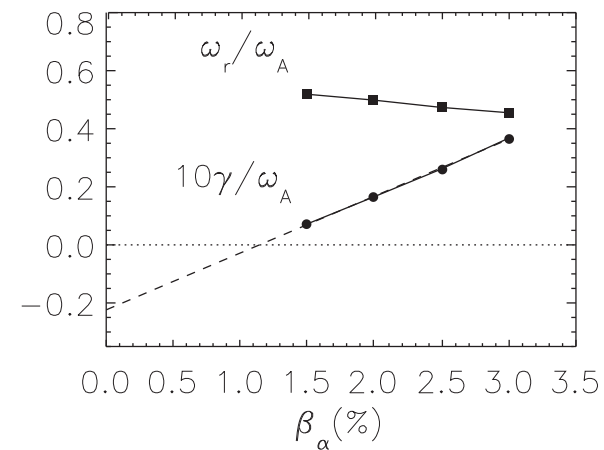
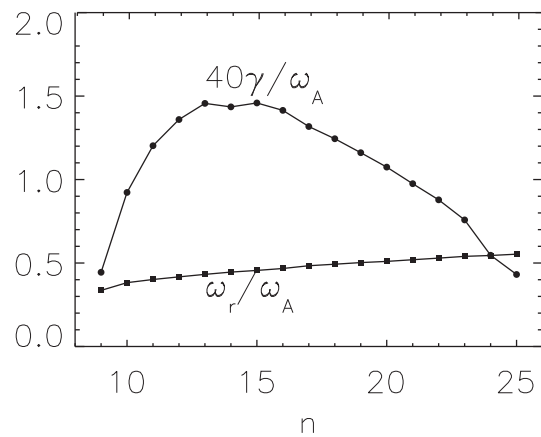
# Progress and plans for particle based kinetic MHD closures

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## TAE calculation of ITER hot alpha critical beta using GEM code

Gyrokinetic core ions and hot alphas  
Drift-fluid electrons  
Shaped equilibrium

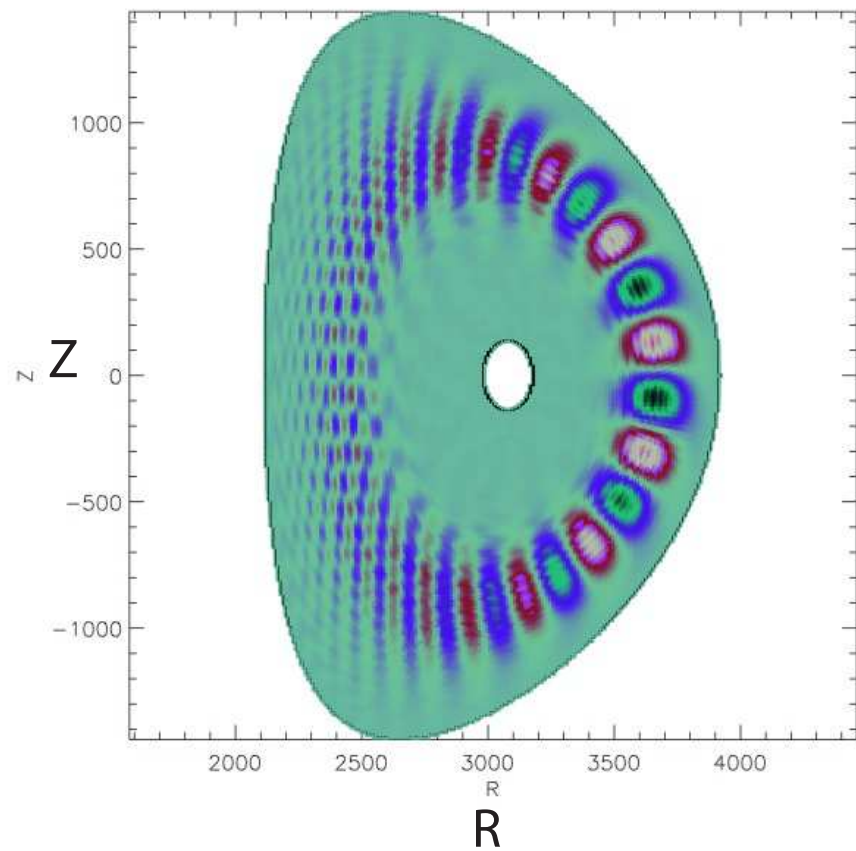


$\omega$  –  $n$  scan for  $\beta_\alpha(0) = 3\%$ .

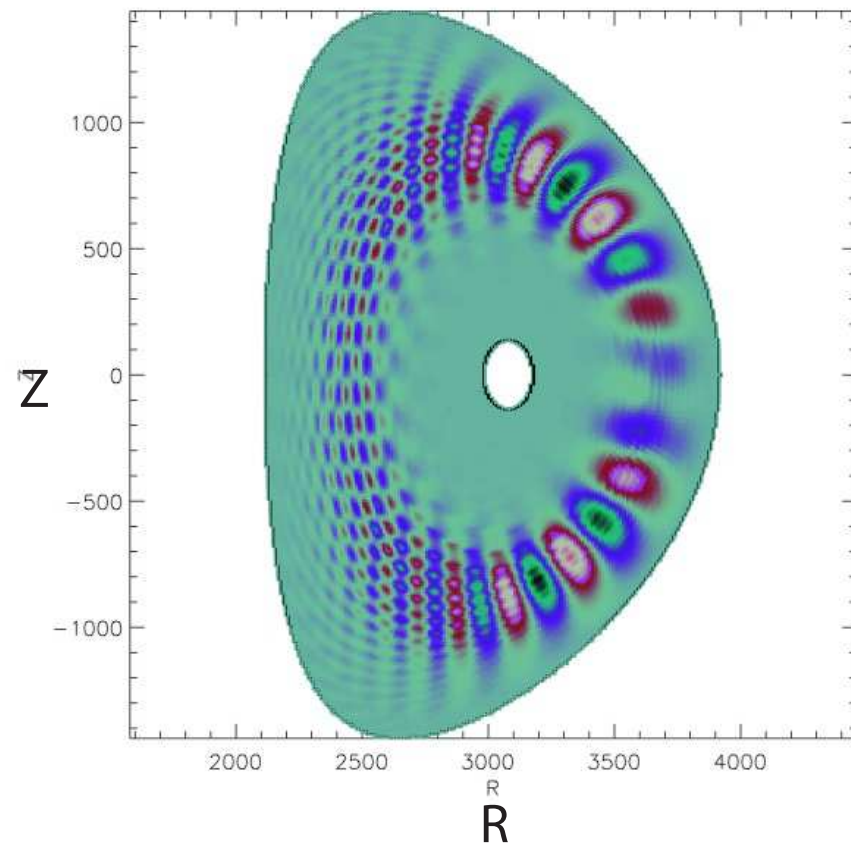
$\gamma$  peaks at  $n = 15$ .

$n = 15$  instability threshold is about  $\beta_{\alpha c} = 1.1\%$ .

electrostatic potential



$A_{||}(R,Z)$



$n=10$

## Recent CEMM publications

*"Low-noise particle algorithms for extended MHD closure,"*

D. Barnes, J. Cheng, S. Parker, *Phys. Plasmas* **15** 055702 (2008)

*"Particle-in-cell simulation with Vlasov ions and drift kinetic electrons,"*

Y. Chen, S. Parker, *Phys. Plasmas* **16** 052305 (2009)

*"Gyrokinetic delta-f particle simulation of the TAE,"*

J. Lang, Y. Chen, S. Parker, G. Fu, *Phys. Plasmas* **16** 052305 (2009)

Expect current/future work (Jianhua Cheng's Ph.D. thesis) will result in two additional publications. One on the second order implicit algorithm and one on reconnection with fully kinetic ions.

# CEMM Proposed work

1. Develop drift kinetic electron model for closure of hybrid model.
  - Test model on slab tearing mode problem.
2. Implement kinetic MHD model in NIMROD code.
3. Simulate classical tearing mode in toroidal geometry with GEM and compare with M3D two-fluid (drift ordering).  
Verification of M3D two-fluid model. In collaboration with L. Sugiama.
4. Initialize NIMROD with  $\delta E$  and  $\delta B$  from GEM electromagnetic turbulence simulations to see if tearing parity perturbations zonal fields can seed NTMs. In collaboration with S. Kruger.

# Motivations

- In certain problems, such as Tokamak edge ETG and weak guide field (or no guide field) magnetic reconnection, the gyro-kinetic orderings are not valid. Therefore the current gyro-kinetic model should be extended.
- Also, for GEM turbulence code on small devices like NSTX, the timestep constraint is  $\Omega_i \Delta t < 0.2$ .
- If electrons and ions are treated as fluid and full-kinetic particles respectively, this simple hybrid model could include the kinetic ion physics and capture MHD physics in a natural way. Meanwhile, realistic electron ion mass ratio could be preserved. It will also serve as a good check for the validity of gyro-kinetic model in the edge.
- We are using the GEM code as a test bed for the model and algorithm. To include kinetic electron effects, drift-kinetic and gyro-kinetic electrons could be added.

# Lorentz ion and fluid electron model

- Lorentz force ions:

$$\frac{d\mathbf{v}_i}{dt} = \frac{q}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B})$$
$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

- Isothermal fluid electrons as a simple test:

$$\delta P_e = \gamma \delta n_e T_e$$

I am working on drift-kinetic and will add gyrokinetic electrons.

- Ampere's law:

$$\nabla \times \mathbf{B} = \mu_0 e (n_i \mathbf{u}_i - n_e \mathbf{u}_e)$$

- Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

## Generalized Ohm's law

- Starting from the electron momentum equation:

$$en_e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) = en_e\eta \mathbf{j} - \nabla \cdot \mathbf{\Pi}_e - m_e \frac{\partial(n_e \mathbf{u}_e)}{\partial t}$$

where  $\mathbf{\Pi}_e = \int f_e m_e \mathbf{v} \mathbf{v} d\mathbf{v}$ .

- Substitute in Ampere's law  $\mathbf{j} = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e) = \frac{1}{\mu_0} \nabla \times \mathbf{B}$ , we could rewrite the above equation as

$$en_e \mathbf{E} = -\mathbf{j}_i \times \mathbf{B} + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{en_e}{\mu_0} \eta (\nabla \times \mathbf{B}) - \nabla \cdot \mathbf{\Pi}_e - m_e \frac{\partial(n_e \mathbf{u}_e)}{\partial t}$$

where  $\mathbf{j}_i = en_i \mathbf{u}_i$



- Taking time derivative of Ampere's law

$$\mu_0 e \left( \frac{\partial n_i \mathbf{u}_i}{\partial t} - \frac{\partial n_e \mathbf{u}_e}{\partial t} \right) = \nabla \times \frac{\partial \mathbf{B}}{\partial t} = - \nabla \times \nabla \times \mathbf{E}$$

The first term on the left hand side comes from ion momentum equation

$$m_i \frac{\partial n_i \mathbf{u}_i}{\partial t} = e n_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla \cdot \mathbf{\Pi}_i$$

the electron inertial term could be written as

$$m_e \frac{\partial (n_e \mathbf{u}_e)}{\partial t} = \frac{m_e}{m_i} (e n_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla \cdot \mathbf{\Pi}_i) + \frac{m_e}{\mu_0 e} \nabla \times (\nabla \times \mathbf{E})$$

- Dropping terms with  $m_e/m_i$ , we arrive at the generalized Ohm's law

$$\mathbf{E} + \frac{c^2}{\omega_{pe}^2} \nabla \times (\nabla \times \mathbf{E}) = - \frac{1}{e n_e} \mathbf{j}_i \times \mathbf{B} + \frac{1}{\mu_0 e n_e} (\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{\eta}{\mu_0} \nabla \times \mathbf{B} - \frac{\nabla \cdot \mathbf{\Pi}_e}{e n_e}$$

## Implicit $\delta f$ algorithm

- Given an ion distribution function  $f_i = f_{i0} + \delta f_i$ , utilizing the usual  $\delta f$  method, the weight equation is

$$\frac{d}{dt}w_i = -\frac{d \ln f_{i0}}{dt}$$

- For  $\rho_i$  scale instabilities  $k_{\perp}\rho_i \sim 1, \beta \sim 0.01$ , the compressional wave frequency  $\frac{\omega}{\Omega_i} \geq 10$ , therefore  $\Omega_i\Delta t \ll 0.01$  is needed. To get rid of the fast frequency, we employ an implicit scheme.
- A first-order scheme has been developed. Here we come up with a second-order scheme with an adjustable centering parameter and improved the field solver.

## Second order implicit scheme

- Particle push

$$\begin{aligned}\frac{\mathbf{x}^{n+1} - \mathbf{x}^n}{\Delta t} &= (1 - \theta) \mathbf{v}^n + \theta \mathbf{v}^{n+1} \\ \frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t} &= \frac{q}{m} \left( (1 - \theta) (\mathbf{E}^n + \mathbf{v}^n \times \mathbf{B}^n) + \theta (\mathbf{E}^{n+1} + \mathbf{v}^{n+1} \times \mathbf{B}^{n+1}) \right) \\ \frac{w^{n+1} - w^n}{\Delta t} &= \frac{q}{T_{i0}} \left( (1 - \theta) (\mathbf{E}^n \cdot \mathbf{v}^n) + \theta (\mathbf{E}^{n+1} \cdot \mathbf{v}^{n+1}) \right)\end{aligned}$$

- Faraday's law

$$\frac{\delta \mathbf{B}^{n+1} - \delta \mathbf{B}^n}{\Delta t} = -[(1 - \theta) \nabla \times \mathbf{E}^n + \theta \nabla \times \mathbf{E}^{n+1}]$$

- Ohm's law:

$$\begin{aligned}(n_{e0} + \delta n_e) \mathbf{E}^{n+1} + \frac{m_e}{\mu_0 e} \nabla \times (\nabla \times \mathbf{E}^{n+1}) &= -\delta \mathbf{j}_i^{n+1} \times \mathbf{B}_0 - \delta \mathbf{j}_i^{n+1} \times \delta \mathbf{B}^{n+1} + \frac{1}{\mu_0} (\nabla \times \delta \mathbf{B}^{n+1}) \times \mathbf{B}_0 \\ &+ \frac{1}{\mu_0} (\nabla \times \mathbf{B}_0) \times \delta \mathbf{B}^{n+1} + \frac{1}{\mu_0} (\nabla \times \delta \mathbf{B}^{n+1}) \times \delta \mathbf{B}^{n+1} \\ &+ \frac{\eta e n_{e0}}{\mu_0} \nabla \times \delta \mathbf{B}^{n+1} + \frac{\eta e \delta n_e}{\mu_0} \nabla \times \delta \mathbf{B}^{n+1} - \gamma T_e \nabla \delta n_i^{n+1}\end{aligned}$$

## Ion current

- First half push cycle

$$\begin{aligned}\mathbf{v}^* &= \mathbf{v}^n + (1 - \theta)\Delta t \frac{q}{m} (\mathbf{E}^n + \mathbf{v}^n \times \mathbf{B}^n) \\ \mathbf{x}^* &= \mathbf{x}^n + (1 - \theta)\Delta t \mathbf{v}^n \\ w^* &= w^n + (1 - \theta)\Delta t \frac{q}{T_{i0}} (\mathbf{E}^n \cdot \mathbf{v}^n)\end{aligned}$$

- Dependence of  $\delta \mathbf{j}_i^{n+1}$  on  $\mathbf{E}^{n+1}$

$$\begin{aligned}\delta \mathbf{j}_i^{n+1} &= q \sum_j w_j^{n+1} \mathbf{v}_j^{n+1} \\ &= \delta \mathbf{j}_i^* + \theta \Delta t \frac{V}{N} \sum_j \frac{1}{\Delta V} \frac{q}{T_i} \mathbf{v}_j \mathbf{E}^{n+1}(\mathbf{x}_j^{n+1}) \cdot \mathbf{v}_j S(\mathbf{x} - \mathbf{x}_j^{n+1}) \\ &\simeq \delta \mathbf{j}_i^* + \theta \Delta t \frac{q^2}{m} \mathbf{E}^{n+1} \equiv \mathbf{J}'_i\end{aligned}$$

where this equation follows as the marker distribution is Maxwellian.

- For accuracy issues, we iterate on the differences between  $\delta \mathbf{j}_i^{n+1}$  and  $\mathbf{J}'_i$  while solving Ohm's law to obtain  $\mathbf{E}^{n+1}$ .

- Once we have  $\mathbf{E}^{n+1}$ ,  $\delta\mathbf{B}^{n+1}$  is advanced according to the Faraday's law.

$$\frac{\delta\mathbf{B}^{n+1} - \delta\mathbf{B}^n}{\Delta t} = -[(1 - \theta) \nabla \times \mathbf{E}^n + \theta \nabla \times \mathbf{E}^{n+1}]$$

- With  $\mathbf{E}^{n+1}$  and  $\delta\mathbf{B}^{n+1}$ , we could proceed to complete the second half push cycle

$$\begin{aligned} \mathbf{v}^{n+1} &= \mathbf{v}^* + \theta\Delta t \frac{q}{m} (\mathbf{E}^{n+1} + \mathbf{v}^{n+1} \times \mathbf{B}^{n+1}) \\ \mathbf{x}^{n+1} &= \mathbf{x}^* + \theta\Delta t \mathbf{v}^{n+1} \\ w^{n+1} &= w^* + \theta\Delta t \frac{q}{T_{i0}} (\mathbf{E}^{n+1} \cdot \mathbf{v}^{n+1}) \end{aligned}$$

## Field solver

- Zero-order  $B$  field

$$\mathbf{B}_0 = \mathbf{e}_y B_{0y} + \mathbf{e}_z B_{0z}.$$

- In the Ohm's law,

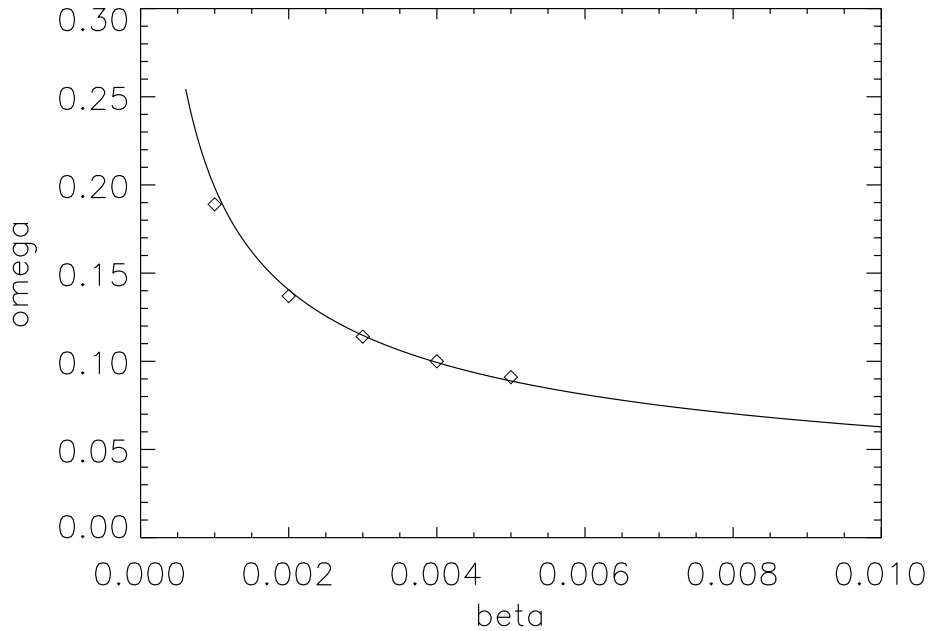
$$\begin{aligned} n_e \mathbf{E}^{n+1} + \frac{n_e}{\beta_e} \left( \frac{m_e}{m_i} + \theta \Delta t \eta \right) \nabla \times (\nabla \times \mathbf{E}^{n+1}) + \theta \frac{\Delta t}{\beta_e} (\nabla \times \nabla \times \mathbf{E}^{n+1}) \times \mathbf{B}_0 + \dots \\ = -\delta \mathbf{j}_i^{n+1} \times \mathbf{B}_0 + \dots \end{aligned}$$

the third term on the left hand side involves the cross product of  $\mathbf{E}^{n+1}$  and  $\mathbf{B}_0$ ,

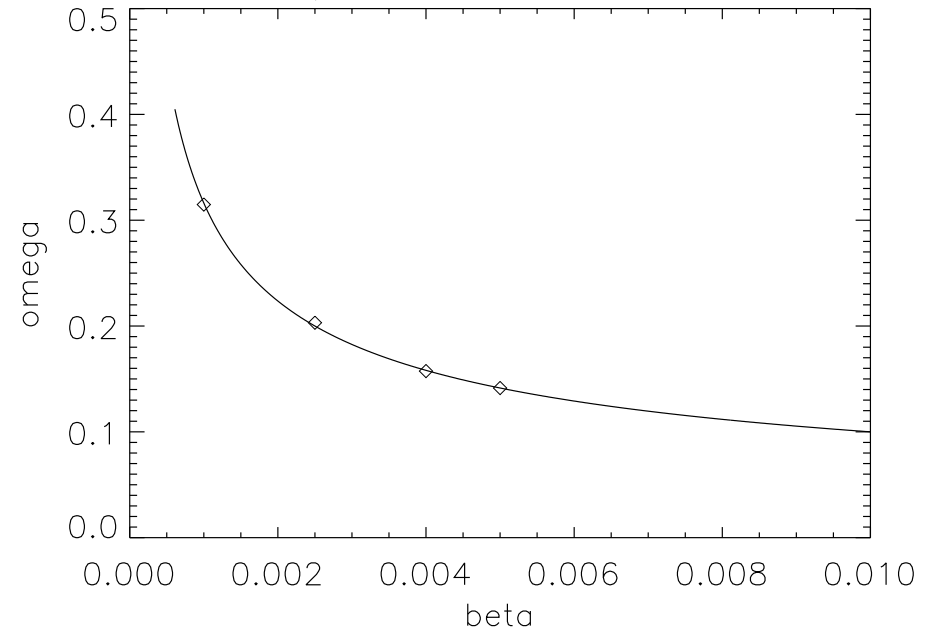
- If  $\mathbf{B}_0$  is space-dependent, we could not obtain a clean single mode equation through Fourier transformation. As in the Harris sheet equilibrium,  $\mathbf{B}_0$  only depends on  $x$ , we could Fourier transform  $\mathbf{E}^{n+1}(x, y, z)$  to  $\mathbf{E}^{n+1}(x, k_y, k_z)$  and solve the latter by direct matrix inversion for every  $k_y, k_z$  mode.

## 3-D Shearless Slab Alfvén waves

shear Alfvén wave



compressional Alfvén wave



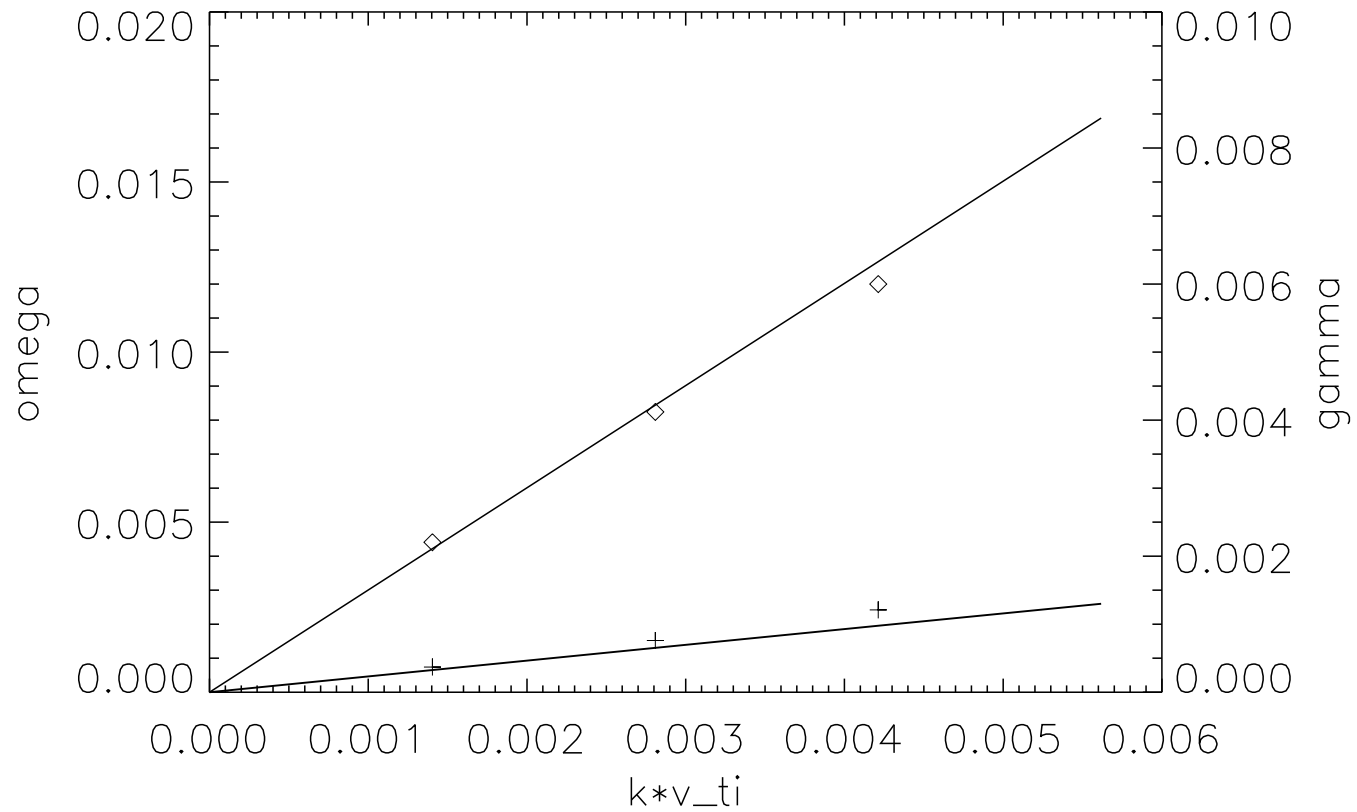
$2 \times 32 \times 32$  grids, 131072 particles.

For shear Alfvén wave,  $k_{\perp} = 0$ ,  $k_{\parallel} \rho_i = 0.00628$ , initialize with  $\delta \mathbf{B}_{\perp}$ .

For compressional Alfvén wave,  $k_{\parallel} = 0$ ,  $k_{\perp} \rho_i = 0.01$ , initialize with  $\delta \mathbf{B}_{\parallel}$ .

These simulations are done in a tilted  $B_0$  field.

# Ion acoustic wave



$2 \times 32 \times 32$  grids, 131072 particles.  $k_{\perp} = 0$ .



## Whistler wave

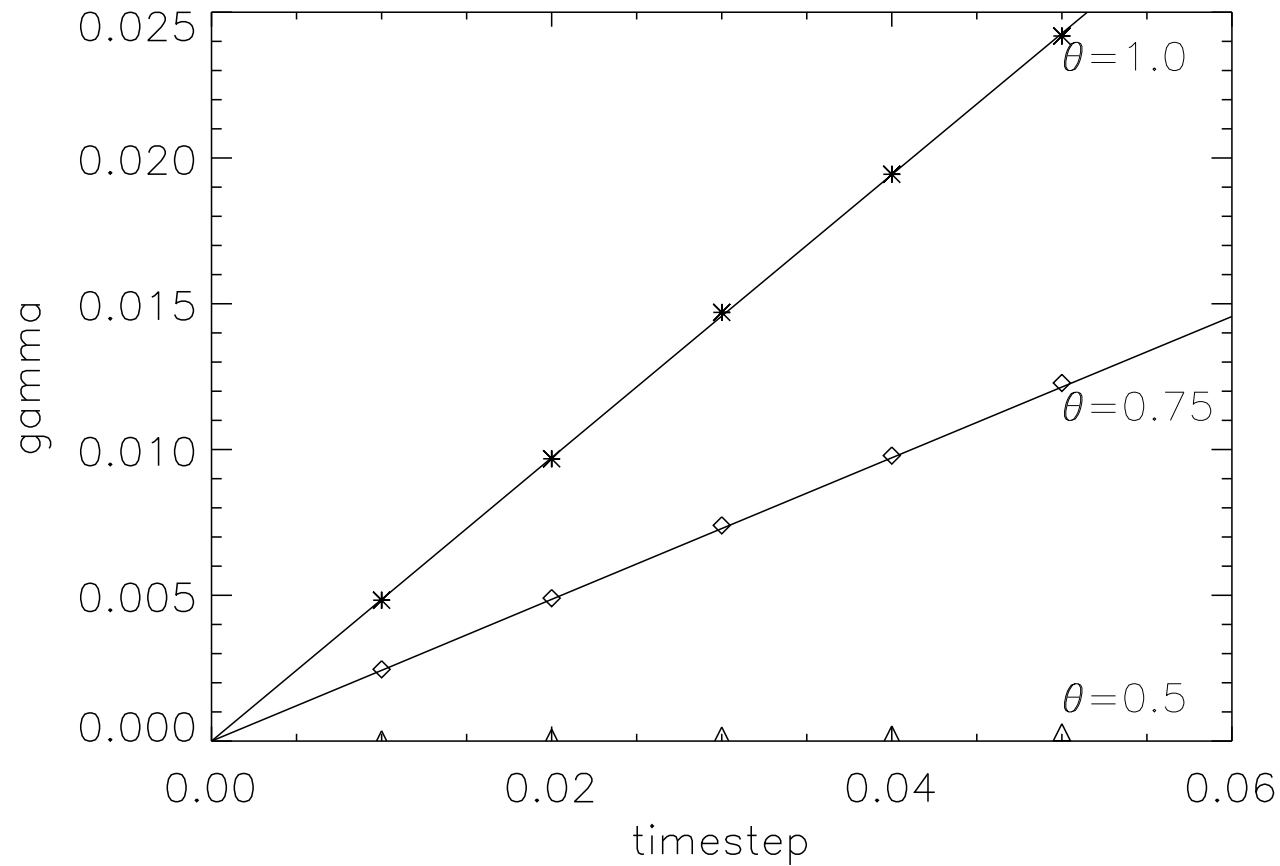
- By neglecting ion current and electron inertia, the Ohm's law yields

$$\mathbf{E} = \frac{1}{\beta_e} (\nabla \times \delta \mathbf{B}) \times \mathbf{B}_0.$$

- The numerical dispersion relation from a Von Neumann stability analysis

$$\tan(\omega_r \Delta t) = \frac{\frac{k^2}{\beta} \Delta t}{1 - \left(\frac{k^2}{\beta} \Delta t\right)^2 \theta(1 - \theta)}$$
$$\omega_i \Delta t = -\frac{1}{2} \ln \left( \frac{\left(1 - \left(\frac{k^2}{\beta} \Delta t\right)^2 \theta(1 - \theta)\right)^2 + \left(\frac{k^2}{\beta} \Delta t\right)^2}{\left(1 + \left(\frac{k^2}{\beta} \Delta t\right)^2 (1 - \theta)^2\right)^2} \right)$$

# Numerical dispersion relation



$16 \times 16 \times 32$  grids, 131072 particles,  $k_{\perp} = 0$ ,  $k_{\parallel} \rho_i = 0.0628$ ,  $\beta = 0.004$ .

## Harris sheet equilibrium

- Zero-order  $\mathbf{B}$

$$\mathbf{B}_0(\mathbf{x}) = B_{y0} \tanh\left(\frac{x}{L}\right) \hat{\mathbf{y}} + B_G \hat{\mathbf{z}}$$

- The equilibrium distribution function is

$$f_{0s} = n_h \operatorname{sech}^2\left(\frac{x}{L}\right) \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left[-\frac{m(v_x^2 + v_y^2 + (v_z - v_{ds})^2)}{2T_s}\right] \\ + n_b \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left(-\frac{mv^2}{2T_s}\right)$$

- Load particles as Maxwellian

$$g_s = n_0 \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left(-\frac{m_s \mathbf{v}^2}{2T_s}\right)$$

- Weight equation

$$\frac{dw_i}{dt} = \frac{q_s}{T_s} \left( \mathbf{E} \cdot \mathbf{v} \left( \frac{f_h}{g_s} + \frac{n_b}{n_0} \right) - \mathbf{v}_d \cdot (\mathbf{E} + \mathbf{v} \times \delta \mathbf{B}) \frac{f_h}{g_s} \right) \\ \frac{f_h}{g_s} = \frac{n_h}{n_0} \operatorname{sech}^2\left(\frac{x}{L}\right) \exp\left(\frac{m_s}{2T_s} (2\mathbf{v}_d \cdot \mathbf{v} - v_d^2)\right).$$

## Boundary conditions

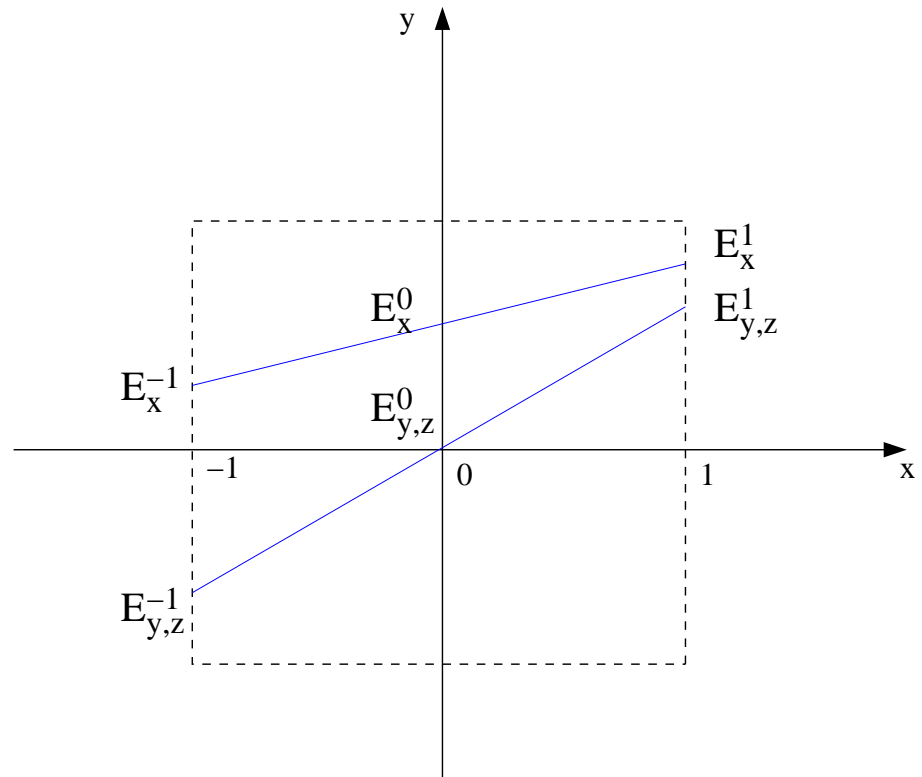
- Perfect conducting wall boundary condition is employed in  $x$  while periodic boundary conditions in  $y$  and  $z$  direction.
- Boundary condition for  $\delta\mathbf{B}$  is assumed in Faraday's equation.

$$\begin{aligned} \mathbf{E}_{y,z}|_{x=\pm l_x/2} &= 0 \\ \delta\mathbf{B}_x|_{x=\pm l_x/2} &= 0 \end{aligned}$$

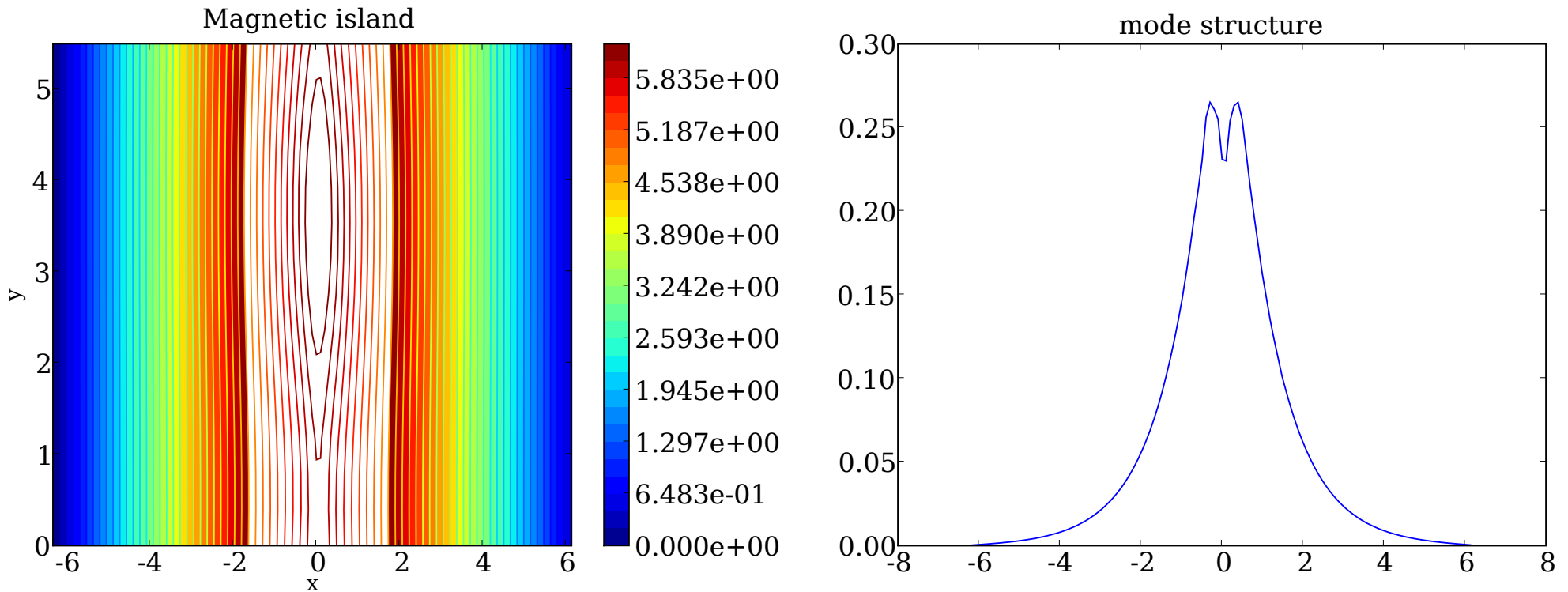
- Numerically, the boundary condition for  $\mathbf{E}$  can be treated as

$$\begin{aligned} \frac{\mathbf{E}_{y,z}^{-1} + \mathbf{E}_{y,z}^1}{2} &= 0 \\ \frac{\mathbf{E}_x^{-1} + \mathbf{E}_x^1}{2} &= \mathbf{E}_x^0 \end{aligned}$$

at  $x = \pm l_x/2$ .



# Resistive Tearing mode



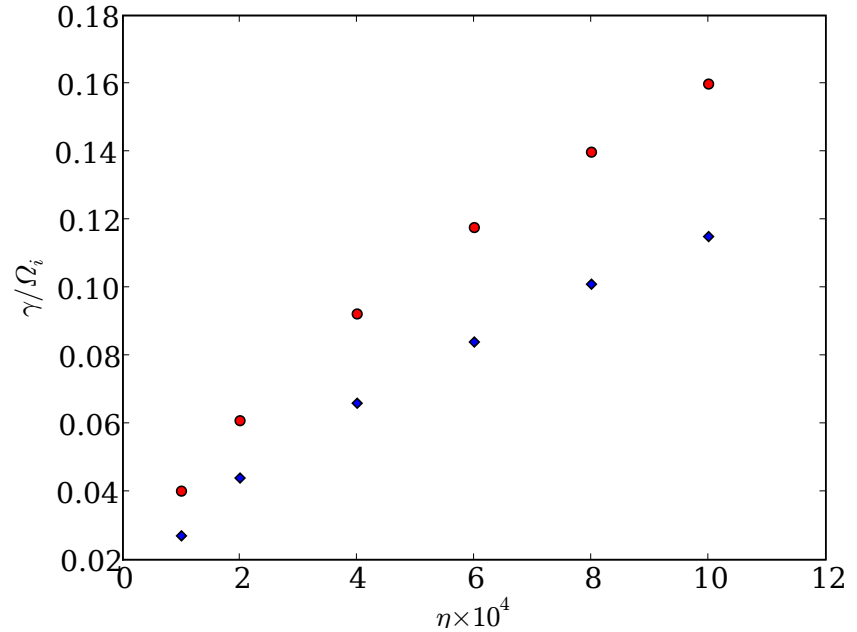
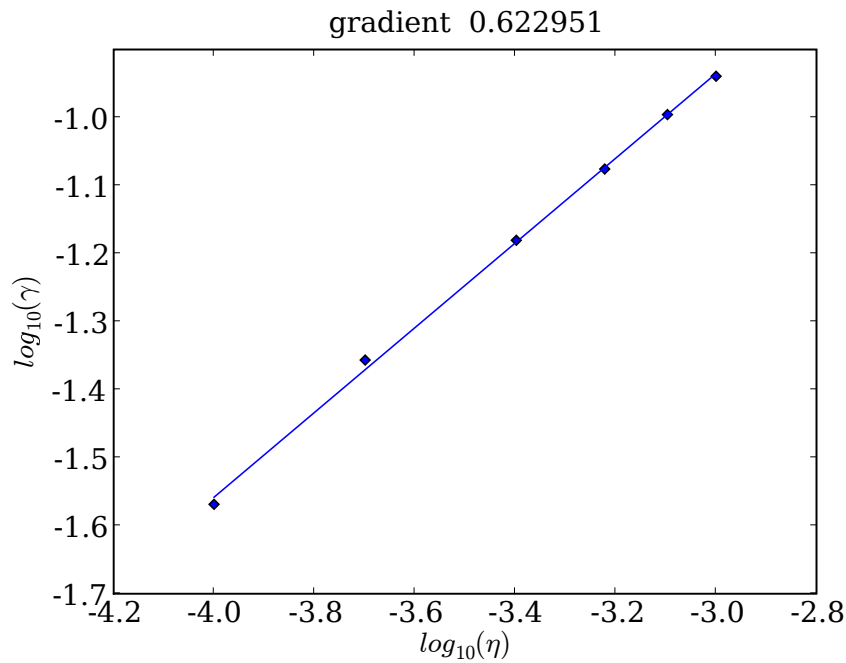
$128 \times 16 \times 4$  grids, 131072 particles.  $k_y \rho_i = 1$ ,  $\frac{L}{\rho_i} = 0.25$ ,  $\beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.2$ ,

$$\eta \frac{en_0}{B_0} = 0.0008, \frac{B_G}{B_0} = 2, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.56, \frac{l_y}{\rho_i} = 6.28$$

# Tearing mode growth rate

- Linear Tearing mode theory shows that the growth rate is (scaled)

$$\gamma = 0.55 \left(\frac{\Delta'}{\beta}\right)^{4/5} \eta^{3/5} (k B'_{y0})^{2/5}.$$



$256 \times 16 \times 4$  grids, 131072 particles.  $k_y \rho_i = 0.5$ ,  $\frac{L}{\rho_i} = 0.4$ ,  $\beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.2$ ,

$$\frac{B_G}{B_0} = 1, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \frac{l_y}{\rho_i} = 12.56$$

# The Lorentz ion/Drift kinetic electron model

Lorentz ions:

$$\frac{d\mathbf{v}_i}{dt} = \frac{q}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \quad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

Drift kinetic electrons:  $\varepsilon = \frac{1}{2}m_e v^2$

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{v}_G \equiv v_{\parallel} \left( \mathbf{b} + \frac{\delta\mathbf{B}_{\perp}}{B_0} \right) + \mathbf{v}_D + \mathbf{v}_E \\ \frac{d\varepsilon}{dt} &= -e\mathbf{v}_G \cdot \mathbf{E} + \mu \frac{\partial B}{\partial t}, \quad \frac{d\mu}{dt} = 0 \end{aligned}$$

Ampere's equation

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J}_i - en_e(\mathbf{V}_{e\perp} + u_{\parallel e}\mathbf{b}))$$

$$\mathbf{V}_{e\perp} = \frac{1}{B}\mathbf{E} \times \mathbf{b} - \frac{1}{enB}\mathbf{b} \times \nabla P_{\perp e}$$

$$\mathbf{J}_i = \int f_i \mathbf{v} d\mathbf{v}, \quad u_{\parallel e} = \int f_e v_{\parallel} d\mathbf{v}, \quad P_{\perp e} = \int f_e \frac{1}{2}m_e v^2 d\mathbf{v}$$

Faraday's equation,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

- The Harris sheet equilibrium distribution function is a drifted Maxwellian with a non-uniform density profile in  $x$ , which doesn't meet the drift-kinetic assumptions. If a strong guide field in  $z$  exists, the nonuniformity could be treated as a perturbation, then the drift-kinetic equations is applicable in this case.
- With drift-kinetic electrons, we could study the roles of kinetic electrons in reconnection problem by making direct comparison with our fluid electron model.
- Currently I am adding the drift-kinetic electrons into the code.



## Including gyrokinetic electrons

- Gyrokinetic equations are usually derived in terms of  $\mathbf{A}$  and  $\phi$ , to make explicit the ordering

$$\frac{\partial \mathbf{A}}{\partial t} \sim \epsilon_\delta \nabla_\perp \phi$$

- The Frieman-Chen gyrokinetic equation, assuming isotropy ( $\partial F_0 / \partial \mu = 0$ ),

$$\hat{L}_g \delta H_0 \equiv \left( \frac{\partial}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla + \mathbf{v}_D \cdot \nabla \right) \delta H_0 = -\frac{q}{m} (S_L + \langle R_{\text{NL}} \rangle),$$

where  $\delta H_0$  is related to the perturbed distribution  $\delta F$  through  $\delta F = \frac{q}{m} \phi \frac{\partial F_0}{\partial \epsilon} + \delta H_0$

$$S_L = \frac{\partial}{\partial t} \langle \phi - \mathbf{v} \cdot \mathbf{A} \rangle \frac{\partial F_0}{\partial \epsilon} - \nabla \langle \phi - \mathbf{v} \cdot \mathbf{A} \rangle \times \frac{\mathbf{b}}{\Omega} \cdot \nabla F_0,$$

$$\langle R_{\text{NL}} \rangle = -\nabla \langle \phi - \mathbf{v} \cdot \mathbf{A} \rangle \times \frac{\mathbf{b}}{\Omega} \cdot \nabla \delta H_0.$$

- Define  $\delta f = \frac{q}{m} \langle \phi \rangle \frac{\partial F_0}{\partial \epsilon} + \delta H_0$ . The gyrokinetic equation for  $\delta f$  is, written in terms of  $\mathbf{E}_1$  and  $\mathbf{B}_1$

$$\frac{D}{Dt} \delta f = - \left( \frac{1}{B_0} \langle \mathbf{E}_1 \rangle \times \mathbf{b} + v_\parallel \frac{\langle \mathbf{B}_{1\perp} \rangle}{B_0} \right) \cdot \nabla F_0 + \frac{1}{m} \dot{\epsilon} \frac{\partial F_0}{\partial \epsilon}$$

$$\frac{D}{Dt} = \hat{L}_g + \left( \frac{1}{B_0} \langle \mathbf{E}_1 \rangle \times \mathbf{b} + v_\parallel \frac{\langle \mathbf{B}_{1\perp} \rangle}{B_0} \right) \cdot \nabla, \quad \dot{\epsilon} = q \left( v_\parallel \mathbf{b} + \mathbf{v}_D + v_\parallel \frac{\langle \mathbf{B}_{1\perp} \rangle}{B_0} \right) \cdot \langle \mathbf{E}_1 \rangle + q \langle \mathbf{v}_\perp \cdot \mathbf{E}_{1\perp} \rangle$$

- The perturbed electron diamagnetic flow comes from  $\delta f$ ,

$$n_0 \mathbf{V}_D(\mathbf{x}) = \int (v_{\parallel} \mathbf{b} + \mathbf{v}_{\perp}(\mathbf{R}', \epsilon, \mu, \alpha)) \delta f(\mathbf{R}', \epsilon, \mu) \delta(\mathbf{x} - \mathbf{R}' - \boldsymbol{\rho}) J d\mathbf{R}' d\epsilon d\mu d\gamma$$

$n_0 \mathbf{V}_D$  is computed by depositing the particle current along the gyro-ring. In the drift-kinetic limit  $\mathbf{V}_D$  reduces to the electron diamagnetic flow.

- The electron  $\mathbf{E} \times \mathbf{B}$  flow comes from the first term in  $\delta F$ ,

$$n_0 \mathbf{V}_E(\mathbf{x}) = \frac{\mathbf{q}}{\mathbf{m}} \int \mathbf{v} (\phi(\mathbf{x}) - \langle \phi \rangle(\mathbf{x} - \boldsymbol{\rho}, \epsilon, \mu)) \frac{\partial \mathbf{F}_0}{\partial \epsilon} \mathbf{J} d\epsilon d\mu d\gamma$$

in eikonal form,

$$n_0 \mathbf{V}_E = n_0 \frac{h}{B_0} \delta \mathbf{E}_k \times \mathbf{b}$$

with  $b = k_{\perp}^2 v_T^2 / \Omega^2$  and

$$h(b) = -\frac{1}{b^2} \int_0^{\infty} e^{-x^2/2b} J_0(b) J_0'(b) x^2 dx$$

In the limit of small  $k\rho \ll 1$  the factor  $h(b)$  become unity, so that  $n_0 \mathbf{V}_E$  become the total guiding center  $\mathbf{E} \times \mathbf{b}$  flow.

# Summary

1. We implemented an implicit algorithm with Lorentz force ions and isothermal fluid electrons which is
  - Quasi-neutral and fully electromagnetic.
  - Suitable for MHD scale plasmas.
2. Second order implicit scheme allows bigger time step,  $\Omega_i \Delta t \gtrsim 0.1$ .
  - Compared the first order and second order scheme with Whistler waves
3. Demonstrated 3-D slab simulation for compressional and shear Alfvén waves, Whistler wave, and the ion acoustic wave.
4. With conducting wall boundary condition on  $x$  and periodic boundary condition on  $y$  and  $z$ , the resistive tearing mode instability is investigated with Harris sheet equilibrium.
5. Working on nonlinear tearing mode saturation and drift-kinetic electrons.