Continuum approach for hot particles. Benchmark of continuum neoclassical closures in NIMROD with NEO.

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Continuum approach as an alternative to δf PIC.

- Evolve δf for hot, drifting, minority ion species as in C. C. Kim, *Phys. Plasmas* **15**, 072507 (2008):

$$\dot{\delta f} = f_0 \left\{ \frac{mRB_\phi}{e\psi_n B^3} \left[\left(v_{||}^2 + v_\perp^2/2 \right) \delta\mathbf{B}\cdot\nabla B - \mu_0 v_{||} \mathbf{J}\cdot\mathbf{E} \right] + \frac{1}{\psi_n} \left(\mathbf{E}\times\mathbf{B}/B^2 + v_{||}\delta\mathbf{B}/B \right) \cdot \left(\nabla\psi_p - mv_{||}\nabla(RB_\phi)/(qB) \right) + \frac{3}{2} \frac{e\epsilon^{1/2}}{\epsilon^{3/2} + \epsilon^{3/2}} \mathbf{v}_D\cdot\mathbf{E} \right\}$$

- Here the slowing down distribution, $f_0 = P_0 \exp(P_\zeta/\psi_n)/(\epsilon^{3/2} + \epsilon_c^{3/2})$.
- Expand $\delta f = \sum_{l=0}^{nl} \delta f_l(\mathbf{x}, t, s) P_l(v_{||}/v)$, where the coefficients $\delta f_l(\mathbf{x}, t, s)$ are determined on a speed grid, $s = v/v_c$.

Comparison of growth rates.

- Increasing hot particle pressure first stabilizes and then destabilizes mode.
- Continuum approach (red circles) used $l = 0, ..., 15$ for $P_l(v_{||}/v)$,6 speed grid points and quadratic FEs.

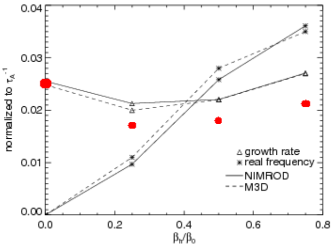


FIG. 2. Comparison of (1,1) hybrid kinetic MHD growth rate and real frequency between NIMROD and M3D shows excellent agreement.

Comparison of anisotropic pressure response for $\beta_{hot} = 0.25\beta_{MHD}$.

- Anisotropic hot particle pressure shifted to outboard side of torus.

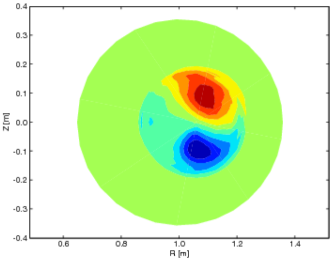
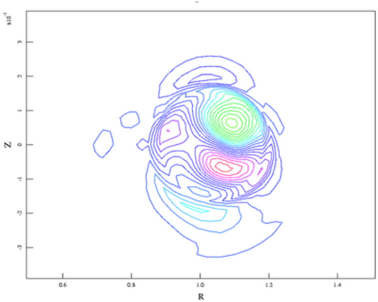


FIG. 4. (Color online) $n=1$ anisotropic pressure contour. The perturbed anisotropic pressure is concentrated on the outboard side and is attributed primarily to the trapped particles.



Comparison of distribution functions.

- δf localization in trapped space.

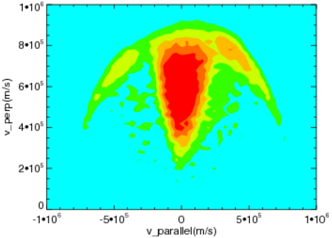
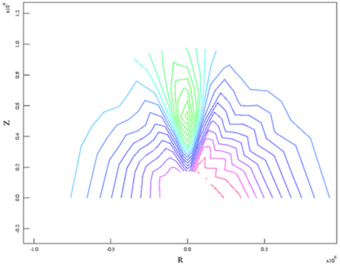


FIG. 5. (Color online) ABS (δf) of the $n=1$ component of $(v_{\|}, v_{\perp})$ space shows activity primarily in the trapped particle region of phase space.



Future work for hot particles with continuum approach.

- Carry out higher resolution runs on parallel machine like ITER at Tech-X.
- Include effects of high-energy tail.
- Include effects of $\Pi_{||}$ from thermal distribution of ions.

Solve Chapman-Enskog-like (CEL) drift kinetic equation for electrons.

- Solve CEL-DKE for a variety of equilibria:

$$\frac{\partial F}{\partial t} + \mathbf{v}_{||} \cdot \nabla F + \frac{q}{m} E_{||} \frac{v_{||}}{v} \frac{\partial F}{\partial v} = \langle C(F + f_M) \rangle - \frac{mv^2}{T} P_2(v_{||}/v) f_M \left(\mathbf{bb} - \frac{\mathbf{I}}{3} \right) : \nabla \mathbf{u} - \frac{2f_m}{3p} L_1^{(3/2)} \left[\nabla \cdot \mathbf{q} + \mathbf{\Pi} : \nabla \mathbf{V} - Q - S_0^{rf} \right] + \mathbf{v}_{||} \cdot \left[\frac{f_m}{p} \left(\nabla \cdot \mathbf{\Pi} - \mathbf{R} - F_0^{rf} \right) + \frac{f_m}{T} L_1^{(5/2)} \nabla T \right]$$

- Expand $F = \sum_{l=0}^{nl} F_l(\mathbf{x}, t, s) P_l(v_{||}/v)$, where the coefficients $F_l(\mathbf{x}, t, s)$ are determined on a grid of *ns* grid points in the normalized speed, $s = v/v_T$.

Include drift drives.

- In PSFC/JA-10-5, Ramos has provided following form for the electron drift drives :

$$\frac{2f_m}{3eB}\left\{2P_0 L_2^{1/2} b\times(\nabla \ln B+\kappa)+ P_2 s^2 \mathbf{b}\times\left[\mathbf{L_1^{3/2}}(\nabla \ln \mathbf{B}-\mathbf{2}\kappa)+\nabla \ln \mathbf{n}\right]\right\}\cdot\nabla\mathbf{T}$$

- NIMROD computes and stores $|B|$ and magnetic curvature $\kappa = \mathbf{b}\cdot\nabla\mathbf{b}$.
- Maxwellian, $f_M = (n/\pi^{3/2} v_T^3)\, e^{-v^2/v_T^2}$, computed and stored at speed grid points for use in volume integrations.

Define flux surface average of $J_{||BS}$.

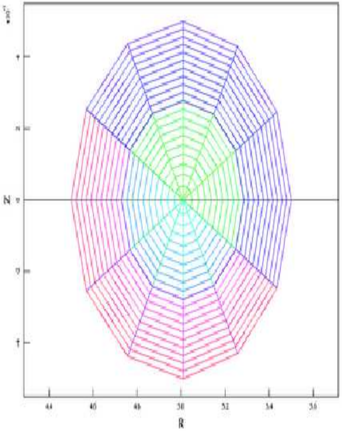
- NEO (Belli and Candy, *PPCF* **51** (2009)) defines:$\langle j_{||} B \rangle = \sum_a z_a e \left\langle B \int d^3v \; v_{||} g_{1a} \right\rangle$.
- NIMROD computes $\pi_{||} = m \int d\mathbf{v} (v_{||}^2 - v_{\perp}^2/2)\, F$ for electrons and ions.
- Bootstrap current is given by

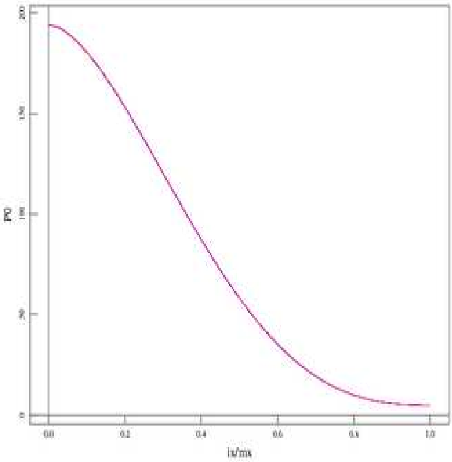
$$J_{||BS} = (\sigma_{||}/ne) \left[\mathbf{b} \cdot \nabla \cdot \pi_{||} \left(\mathbf{bb} - \mathbf{I}/3 \right) \right] = (\sigma_{||}/ne) \left[\frac{2}{3} \mathbf{b} \cdot \nabla \pi_{||} - \pi_{||} \mathbf{b} \cdot \nabla \ln B \right]$$

Flux surface average implemented as $\langle J_{||BS} \rangle = \int_0^{2\pi} d\theta \, J_{||BS} / \mathbf{B} \cdot \nabla\theta / \int_0^{2\pi} d\theta / \mathbf{B} \cdot \nabla\theta.$

Start with high-aspect ratio, collisional case.

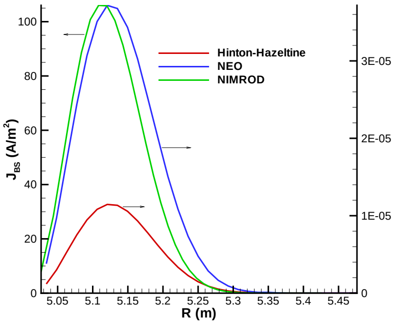
- Circular cross section, $\epsilon = 0.1$, Pfirsch-Schluter regime very low $\beta = 1\%$.

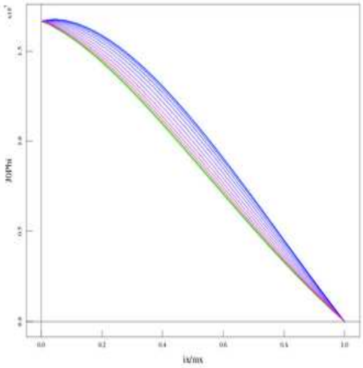




$J_{||BS}$ have similar spatial structure.

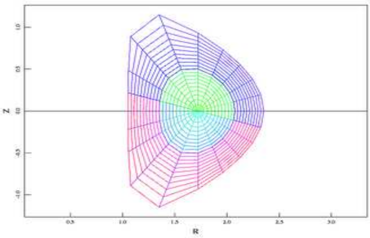
- Theory says $J_{||BS}/J_{||Ohmic} \sim \sqrt{\epsilon}\beta_p \sim .003$. NIMROD has $J_{||BS}/J_{||Ohmic} \sim 100/1.5 \times 10^5 = .006$.

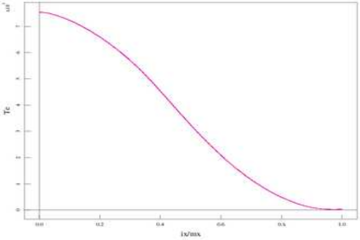




Proceed to shaped, high-β equilibriua.

- Adjust $\nu* = \nu/(v_T/qR)$ by tweaking density and temperature profiles.
- Banana, plateau and PS regimes have $\nu* = 4 \times 10^{-4}$, 0.3, and 10.3, respectively.





Future work on NEO benchmark.

- Test spatial and velocity-space resolution requirements.
- Compare distribution functions between NIMROD and NEO.
- Compare neoclassical electron closures for variety of geometries and collisionality regimes.
- Proceed to continuum computations of neoclassical ion closures with assistance from Ramos.