V&V of the Hybrid Kinetic-MHD Model

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Outline

Hybrid Kinetic MHD

- computational model
- equations of motion

Giant Sawtooth

- preliminary calculations
- comparison of mode plots
- higher n
- potential plans
- example applications

The Hybrid Kinetic-MHD Equations

C.Z.Cheng, *JGR*, 1991

- $n_h \ll n_0$, $\beta_h \sim \beta_0$, quasi neutrality, MHD momentum equation modified by addition of hot particle pressure tensor:

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla p_b - \nabla\cdot\underline{\mathbf{p}}_h$$

  b, h denote bulk plasma and hot particles
  - $\rho, \mathbf{U}$ for entire plasma, both bulk and hot particle
- steady state equation

$$\mathbf{J}_0\times\mathbf{B}_0 = \nabla p_0 = \nabla p_{b0} + \nabla p_{h0}$$

  - p_{b0} is scaled to accomodate hot particles
  - assumes equilibrium hot particle pressure is isotropic

Linearized Momentum Equation and $\delta \underline{\mathbf{p}}_h$

$$\rho_s \frac{\partial \delta \mathbf{U}}{\partial t} = \mathbf{J}_s \times \delta \mathbf{B} + \delta \mathbf{J} \times \mathbf{B}_s - \nabla \delta p_b - \nabla \cdot \delta \underline{\mathbf{p}}_h$$

- CGL-like $\delta \underline{\mathbf{p}}_h = \begin{pmatrix} \delta p_\perp & 0 & 0 \\ 0 & \delta p_\perp & 0 \\ 0 & 0 & \delta p_\parallel \end{pmatrix}$
- evaluate pressure moment at $\mathbf{x}$

$$\delta \underline{\mathbf{p}}(\mathbf{x}) = \int m \langle \mathbf{v} - \mathbf{V}_h \rangle \langle \mathbf{v} - \mathbf{V}_h \rangle \delta f(\mathbf{x}, \mathbf{v}) d^3 v$$

δf is perturbed phase space density, m mass of particle, and V_h is COM velocity of particles

The Hybrid δf PIC-MHD model

- advance particles and δf using NIMROD fields

$$\mathbf{z}_i^{n+1} = \mathbf{z}_i^n + \dot{\mathbf{z}}(\mathbf{z}_i)\Delta t$$
$$\delta f_i^{n+1} = \delta f_i^n + \dot{\delta f}(\mathbf{z}_i)\Delta t$$

- deposit moment $\delta p(\mathbf{x}) = \sum_{i=1}^{N} \delta f_i m(v_i - V_h)^2 S(\mathbf{x} - \mathbf{x}_i)$ on FE grid
- advance NIMROD field equations with hybrid Kinetic-MHD momentum equation

$$\rho_s \frac{\partial \delta \mathbf{U}}{\partial t} = \mathbf{J}_s \times \delta \mathbf{B} + \delta \mathbf{J} \times \mathbf{B}_s - \nabla \delta p_b - \nabla \cdot \delta \underline{\mathbf{p}}_h$$

- particle dynamics augment fluid dynamics

PIC in FEM - nontrivial

- particles pushed in real space (R, Z) **but** field quantities in logical space (η, ξ)
- particle coordinate (R_i, Z_i) needs to be inverted to logical coordinates

$$R = \sum_j R_j N_j(\eta, \xi), \quad Z = \sum_j Z_j N_j(\eta, \xi),$$

- iterative process
- sorting and parallelization done at same time
- sorting done by a bucket sort
- computationally most demanding - greatest room for performance increase

PIC options

- two equations of motion available
  - drift kinetic equations of motion
  - full Lorentz force equations of motion
- eventually provide capability of direct comparison
- several distribution functions are available
  - slowing down distribution
  - Maxwellian
  - monoenergetic
  - monoenergetic, no $v_{||}$
- several spatial profiles are available
  - proportional to MHD profile
  - uniform
  - peaked gaussian
- room for growth

Slowing Down Distribution for Hot Particles

- for the slowing down distribution function

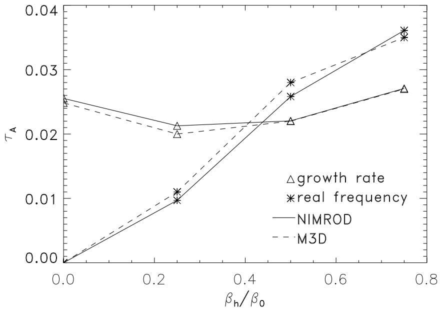
$$\delta\dot{f} = f_{eq}\left\{\frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}\cdot\mathbf{E}\right] + \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_0^{3/2}}\mathbf{v}_D\cdot\mathbf{E}\right\}$$

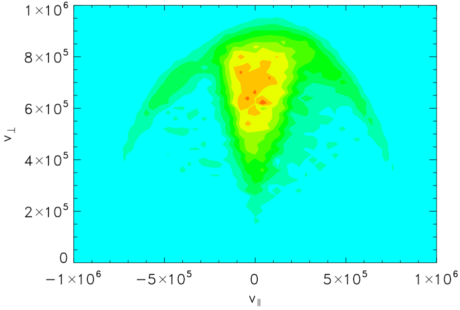
where

$$\mathbf{v}_D = \frac{m}{eB^3}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp$$

$$\delta\mathbf{v} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\cdot\frac{\delta\mathbf{B}}{B}$$

Benchmark with M3D

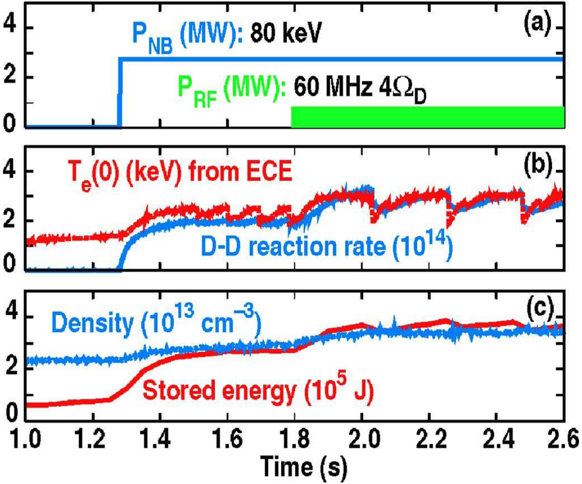




- velocity shows most activity in the most energetic particles
- mostly in trapped region, also in extremes of passing particles

GIANT SAWTOOTH

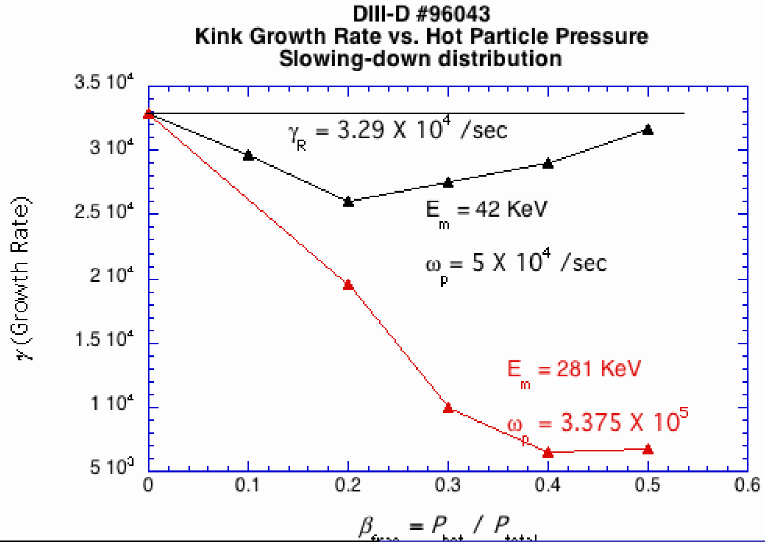
DIII-D shot#096043



V&V of Hybrid-Kinetic MHD Model

- effect of energetic particles on internal kink mode using DIII-D EFIT equilibria
- excitation of fishbones
- role of trapped and passing particles
- role of distribution parameters (ε_c, v_{max})
- identify subdominant modes
- implement and study role of high energy tail
- compare to theory and numeric work (M.Choi PoP2007)
- nonlinear will require all of the aobve

Preliminary Simulations - D. Schnack



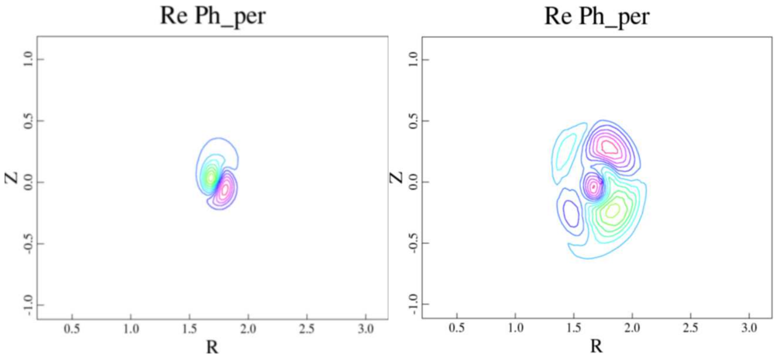
Preliminary Simulations ($t = 1700ms$) - C. C. Kim

$v_{max} \backslash \beta_{frac}$	.25	.5	.75
2×10^6m/s	0.0033/0.012	0.0075/0.017	0.0101/0.021
3×10^6m/s	0.0/0.0	0.0013/0.019	0.0053/0.026
4×10^6m/s	0.0/0.0	0.0/0.0	0.0045/*

Table: Comparison of (growth rate/real frequency $\times \tau_A$) of $(1, 1)$ for DIII-D shot #96043 $t = 1700ms$ shows stabilization with increasing velocity cutoff. Ideal growth rate is $\gamma \tau_A = 0.0054$

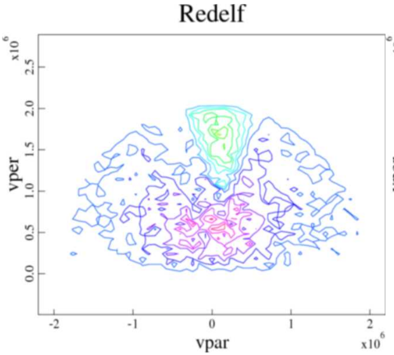
no stabilization seen for $t = 1900, 2020$

eigenmodes



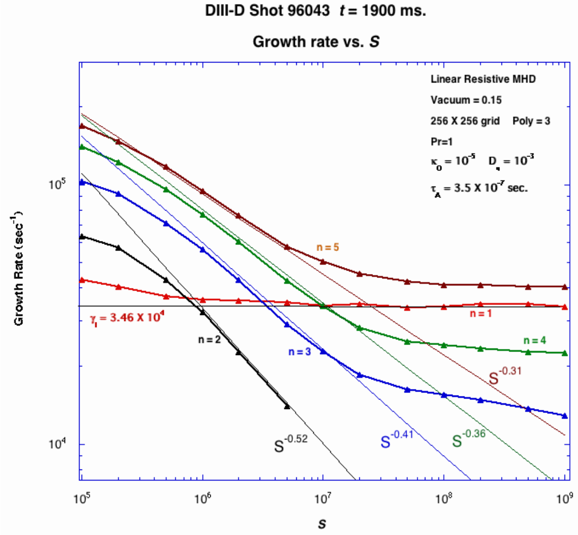
- comparison of case1($\beta_{frac} = .5, v_{max} = 2 \times 10^6$) and case2($\beta_{frac} = .75, v_{max} = 4 \times 10^6$)
- ideal ($\gamma\tau_A = .0054$), case1($\gamma\tau_A = .0075, \omega\tau_A = .017$), case2($\gamma\tau_A = .0045?$)

velocity plots indicates different physics



- case1 shows usual velocity interaction
- case2 shows indication of Landau resonance - subdominant mode?

onto higher *n*, scan in S - D. Schnack



Potential Plans

- development
  - other distribution functions
  - multi-species
  - improve parallelization
  - better shape function
- studies
  - condition equilibria with NIMEQ
  - compare to Maxwellian - for KO
  - passing/trapped runs
  - energetic tail
  - multi-species

δf and the Lorentz Equations

- Lorentz equations of motion

$$\dot{\mathbf{x}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

  - use Boris push
- weight equation is

$$\dot{\delta f} = -\frac{2q}{m}\delta\mathbf{E} \cdot \mathbf{v}\frac{\partial f_0}{\partial v^2}$$

Full orbit recovers drift kinetic result

β_{frac} scan of $(1, 1)$ benchmark kink