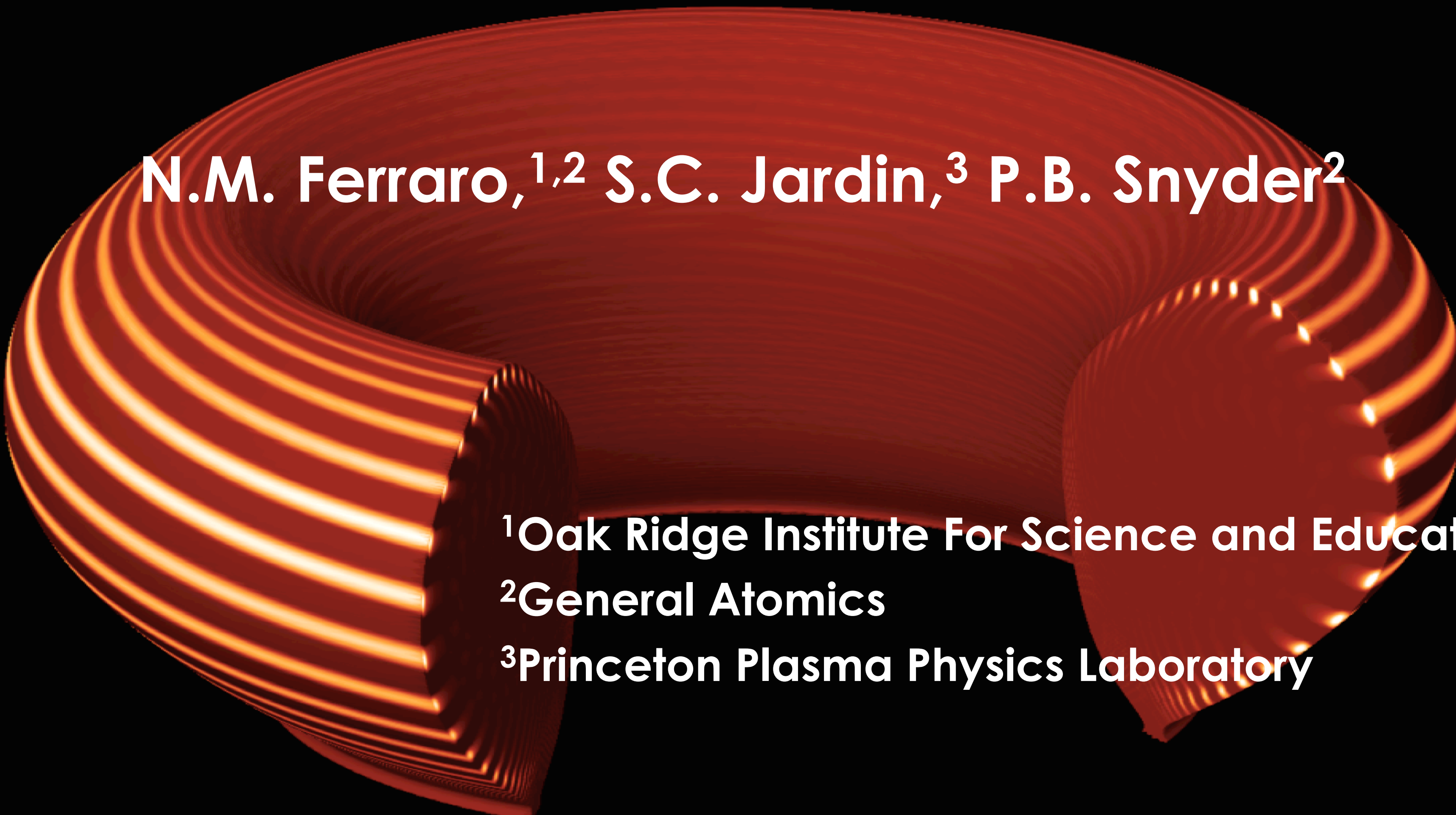
Ideal and Non-Ideal Tokamak Edge Stability Calculations



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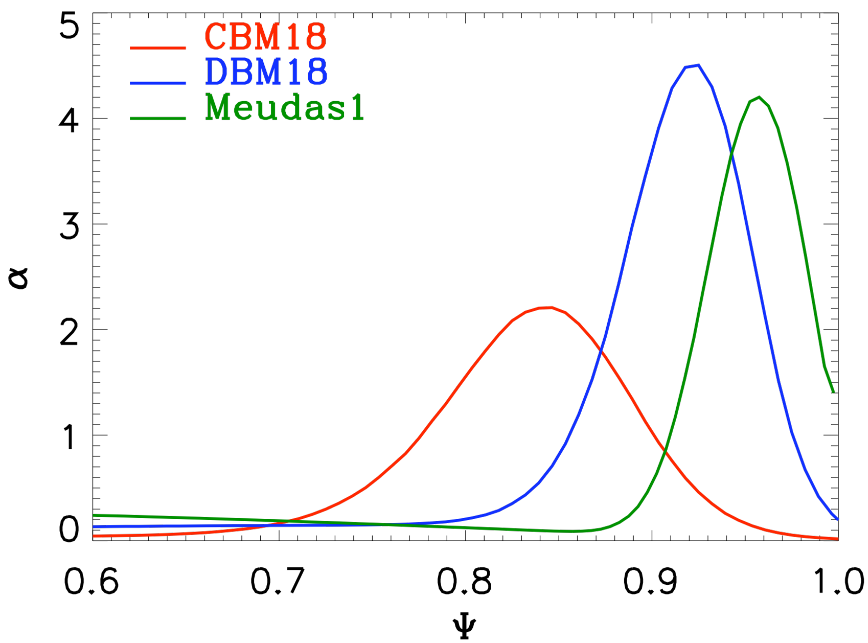
GENERAL ATOMICS

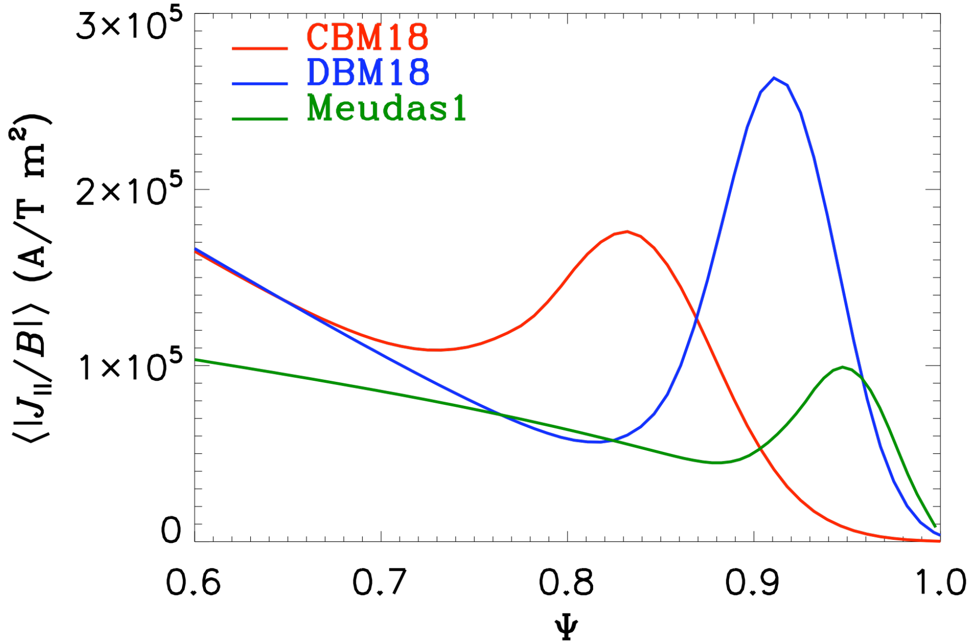
Outline

- Results of ELM benchmark with M3D-C1
- Extension to non-ideal physics
- Numerical Methods
- Conclusions

Three Equilibria Considered

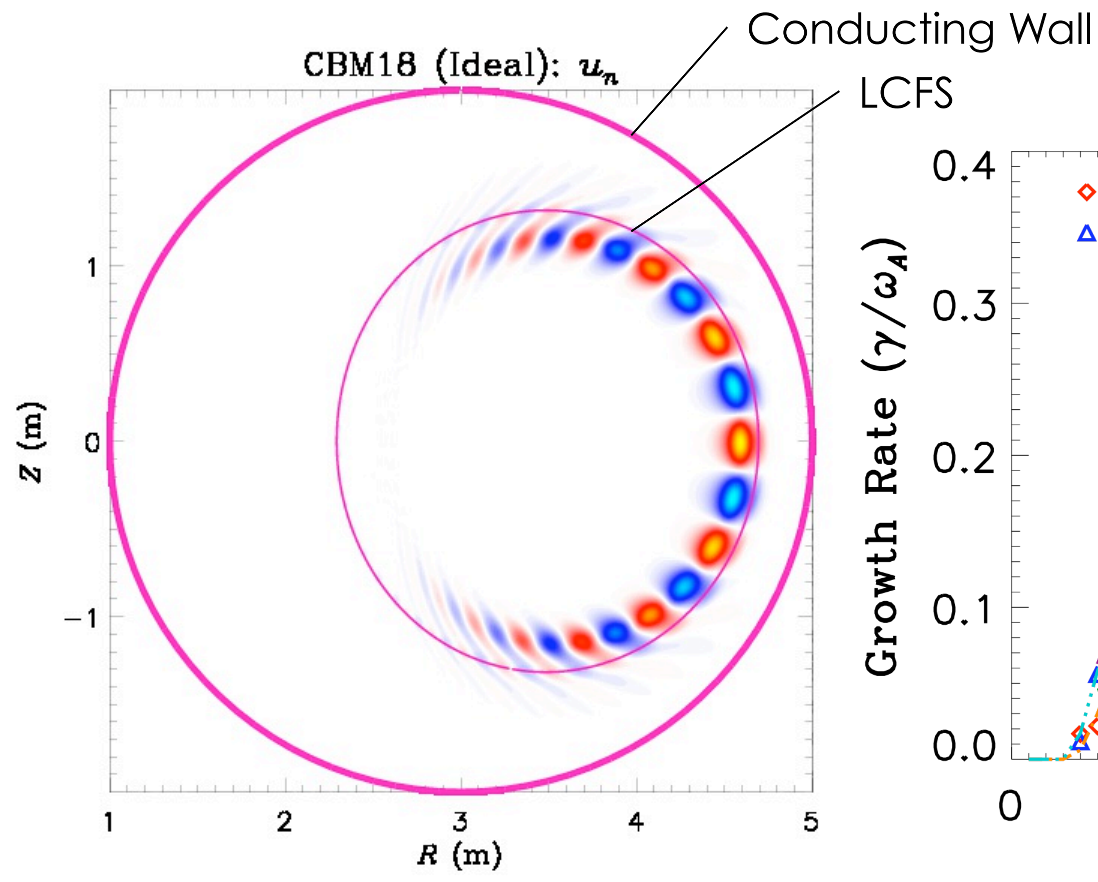
- **CBM18: circular; wide pedestal ($\Delta\Psi \approx .12$)**
- **DBM18: shaped; narrower pedestal ($\Delta\Psi \approx .08$)**
- **Meudas1: diverted; narrowest pedestal ($\Delta\Psi \approx .06$)**

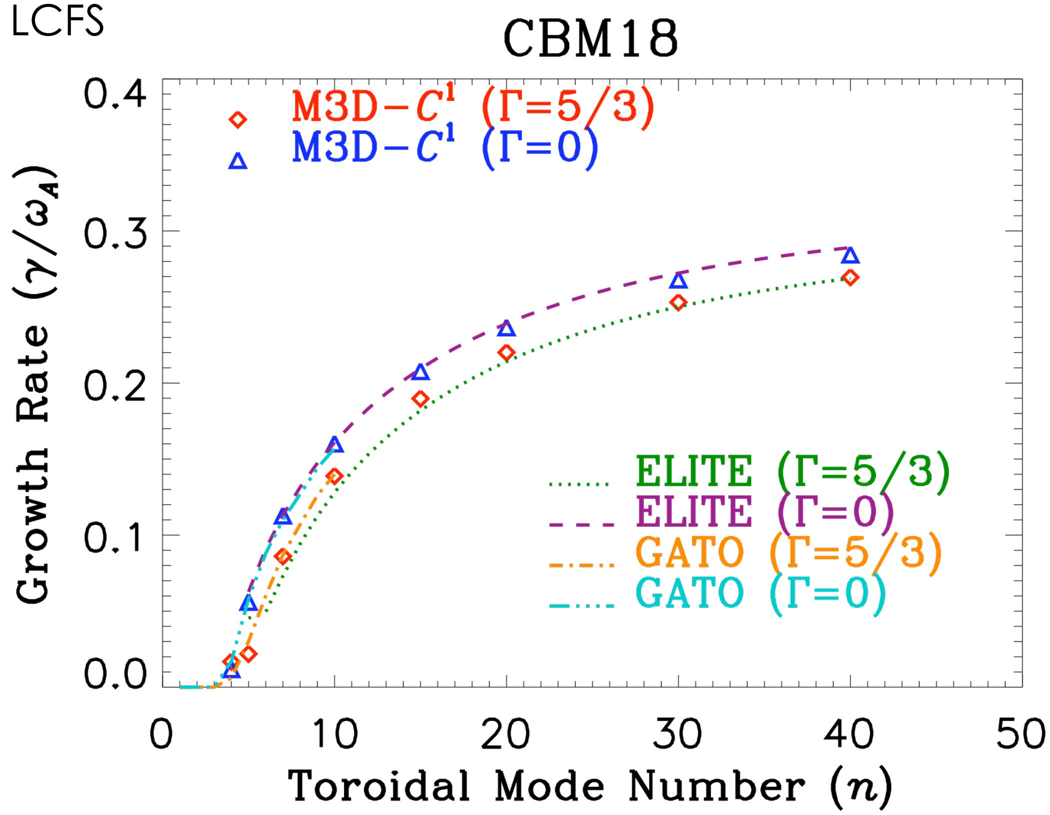




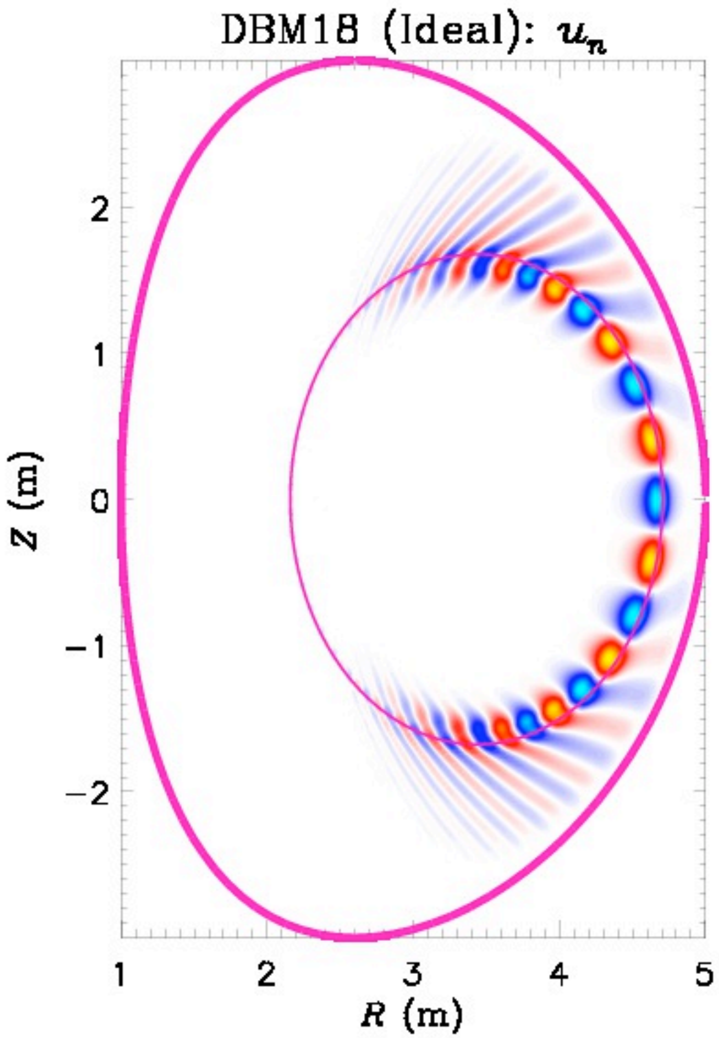
CBM18 Equilibrium: Good Agreement Among Codes

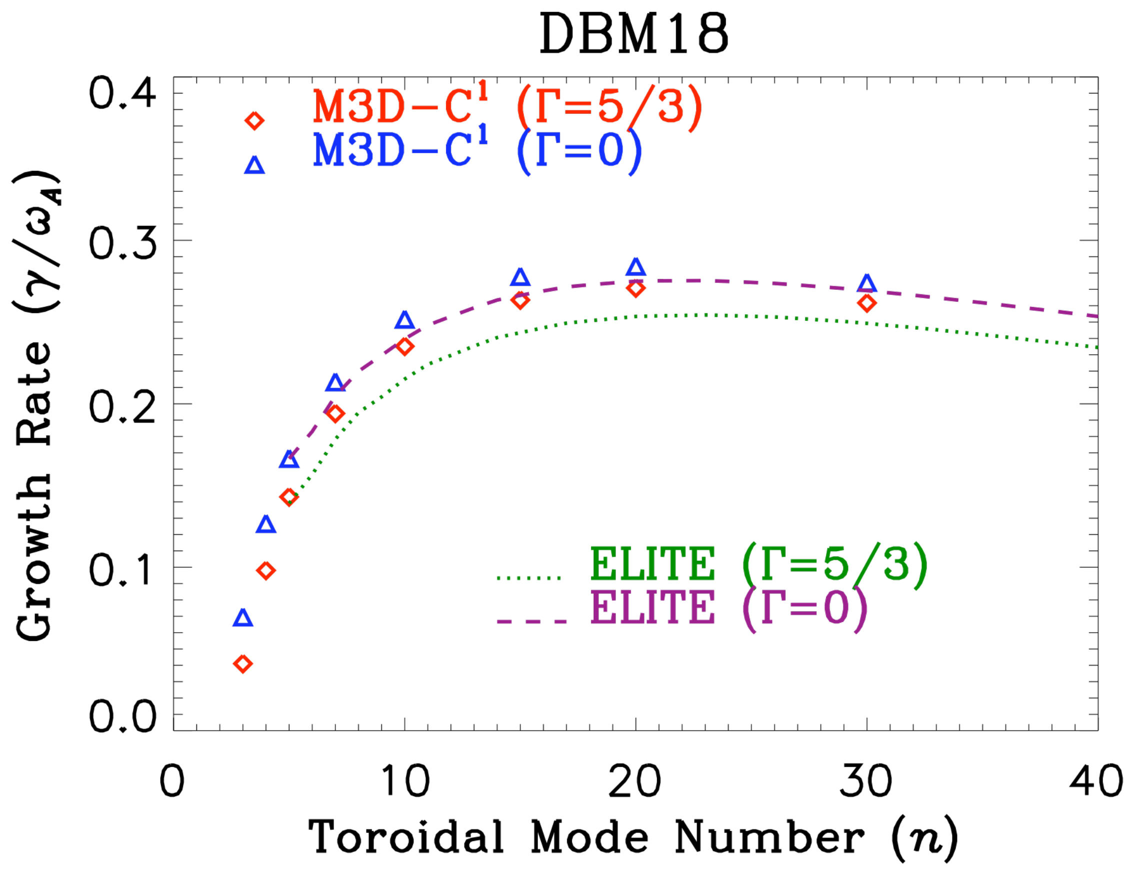
- **Both compressible (Γ =5/3) and compressionless (Γ =0) cases agree well.**



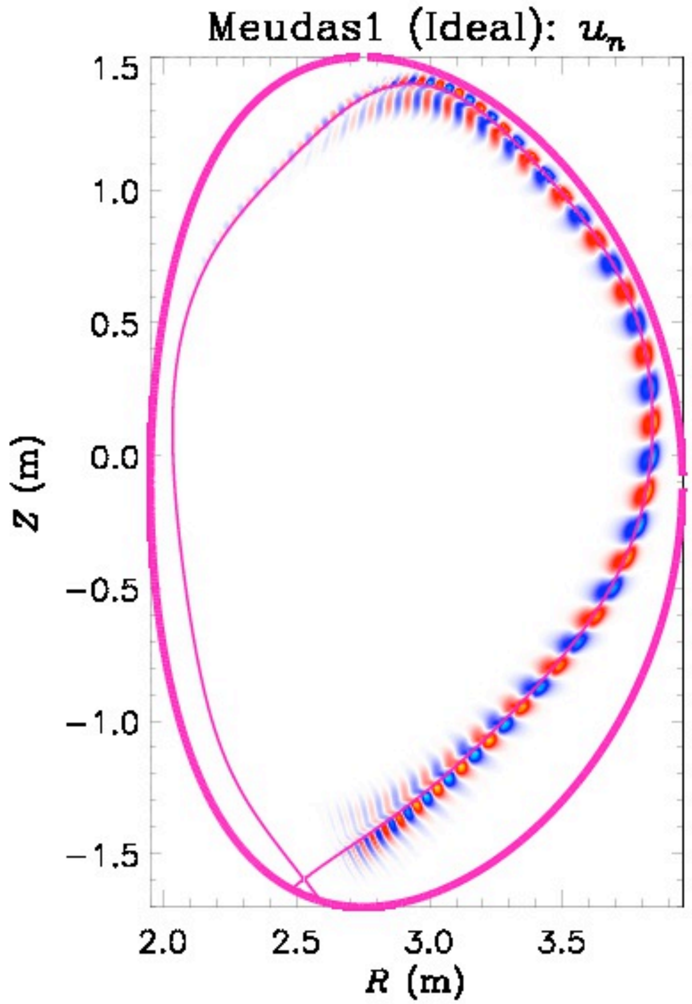


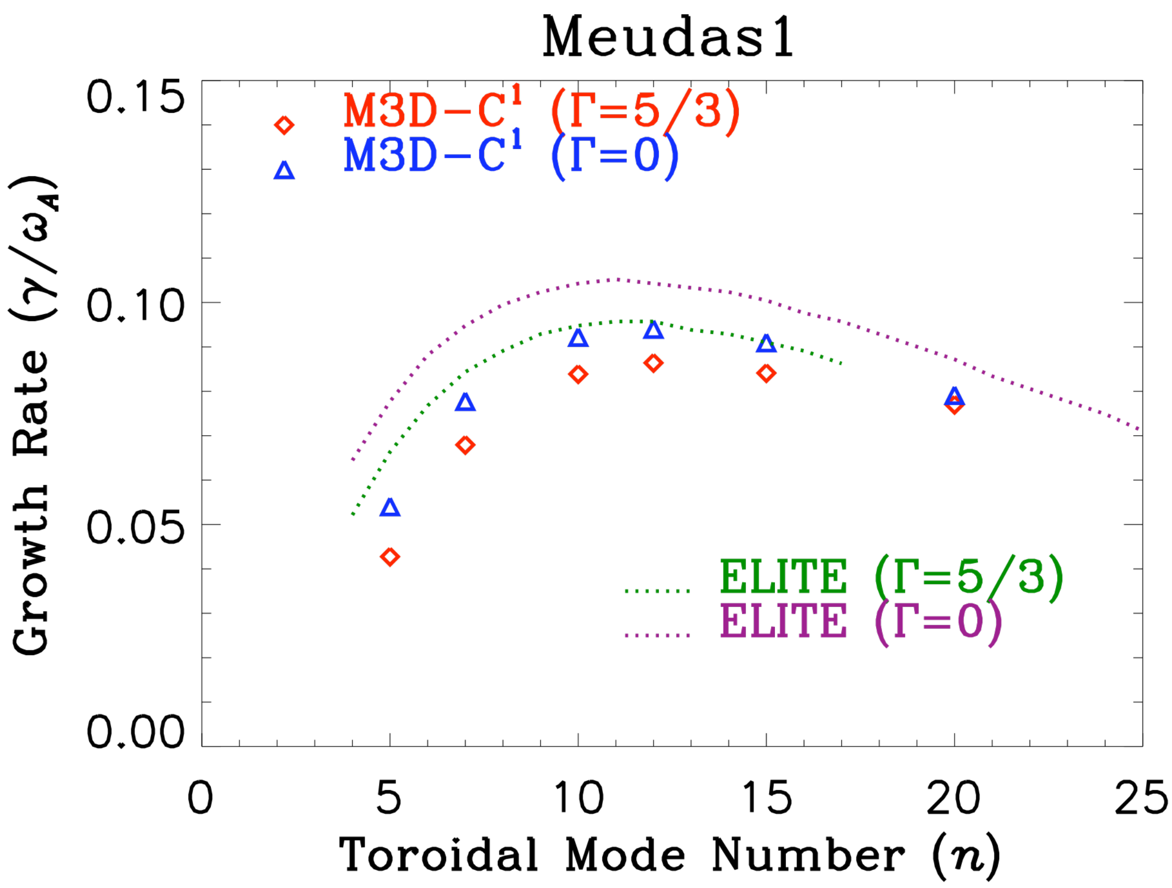
DBM18 Equilibrium: Good Agreement





Meudas1 Equilibrium: Decent Agreement



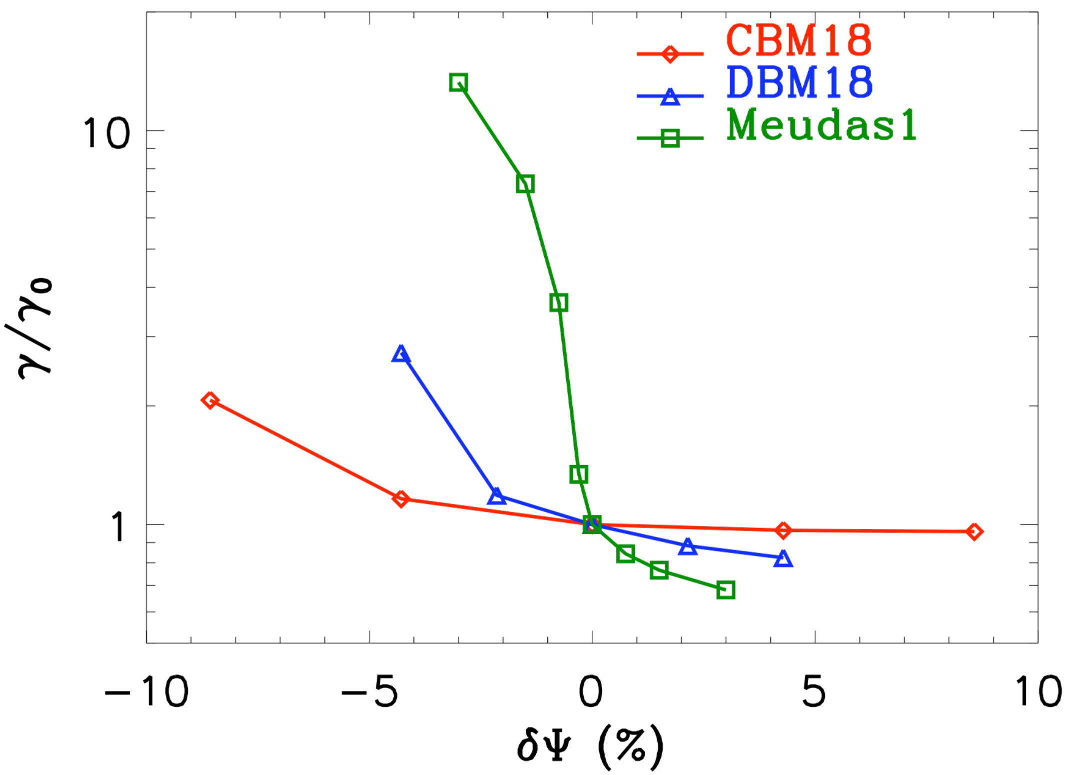


Meudas1 Discrepancy

- **Equilibrium**
  - M3D-C1 does not interpolate equilibrium data, but re-solves equilibrium
  - Burke (2010) showed sensitivity to equilibrium mapping
- **“Vacuum” region is probably too conductive in M3D-C1**
  - Meudas case is highly sensitive to SOL resistivity
- **Ideal Meudas eigenmode is unresolvable**
  - Even ideal codes agree at to ~10%

Growth Rates are Sensitive to Cutoffs Within LCFS

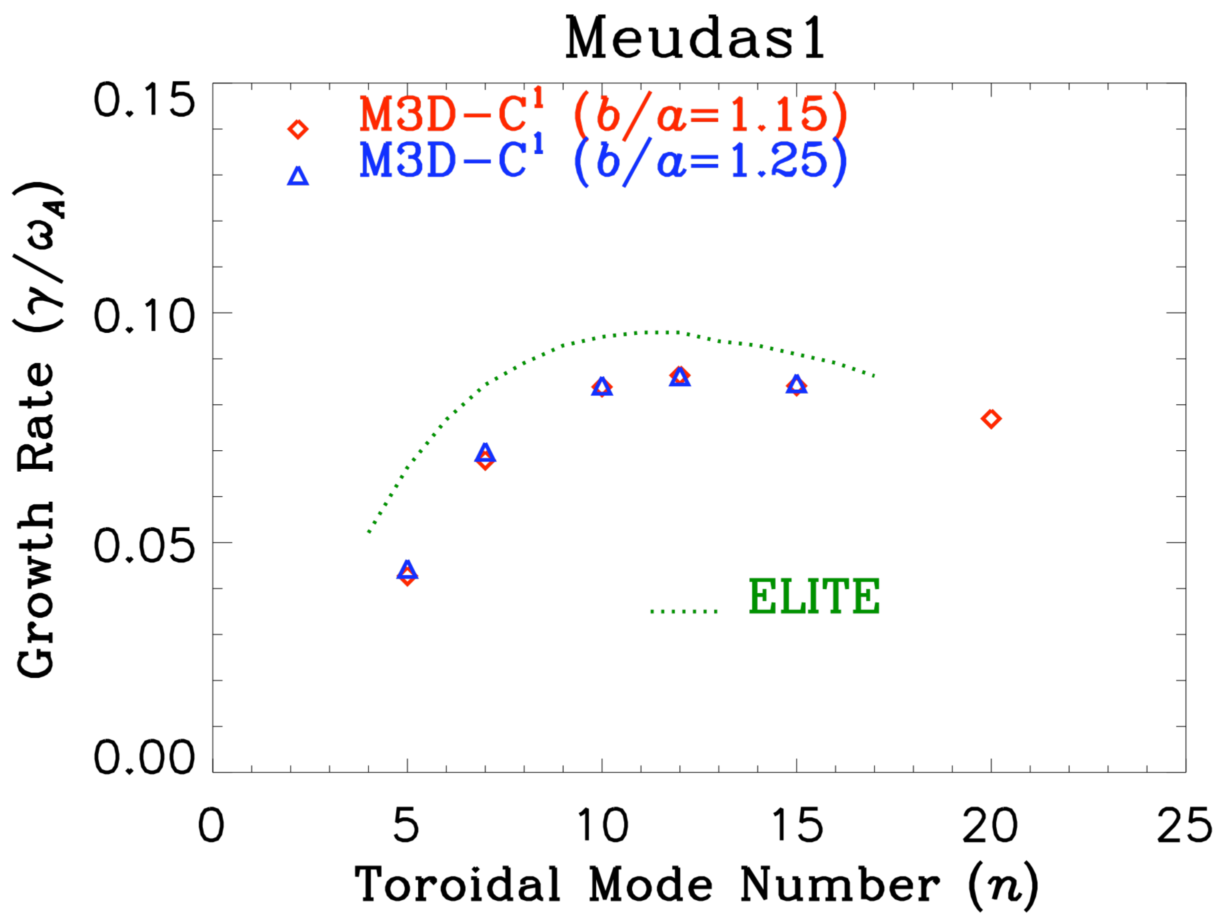
- **Diverted case is extremely sensitive to position of vacuum-plasma interface**
- **Sensitivity is to ρ cutoff, not η**



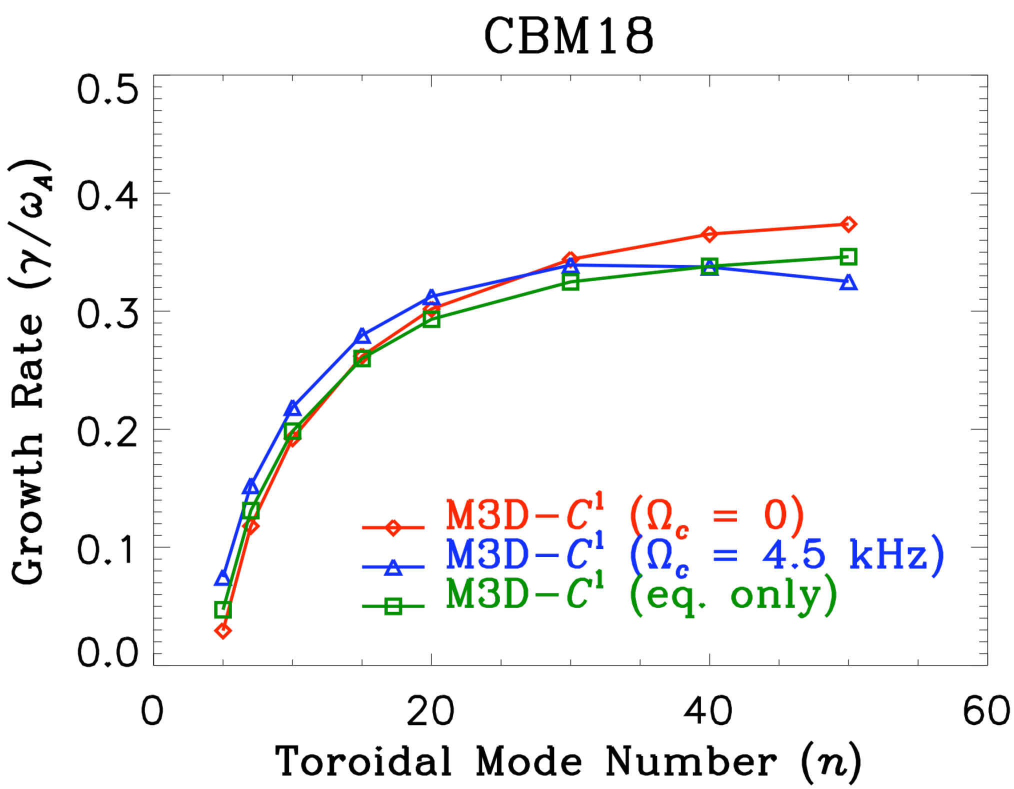
Inward ← | → Outward

Wall Stabilization is Negligible

- **Growth rates are essentially unaffected by the conducting wall, even at *n* = 5**



Toroidal Rotation is Destabilizing and Stabilizing

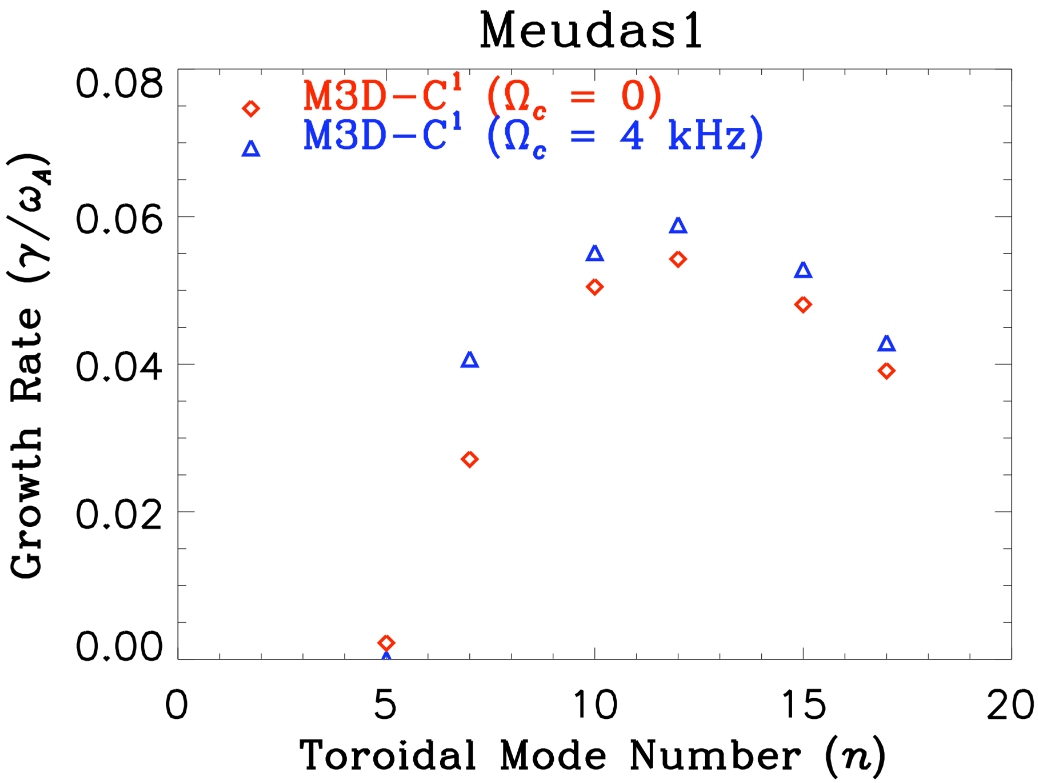


- No Rotation
- Rotation is self-consistently included in equilibrium (Ω ~ p)
- Equilibrium is changed, but Ω=0

- Destabilizing at low-*n*
- Stabilizing at high-*n*
- Some stabilization due to equilibrium change

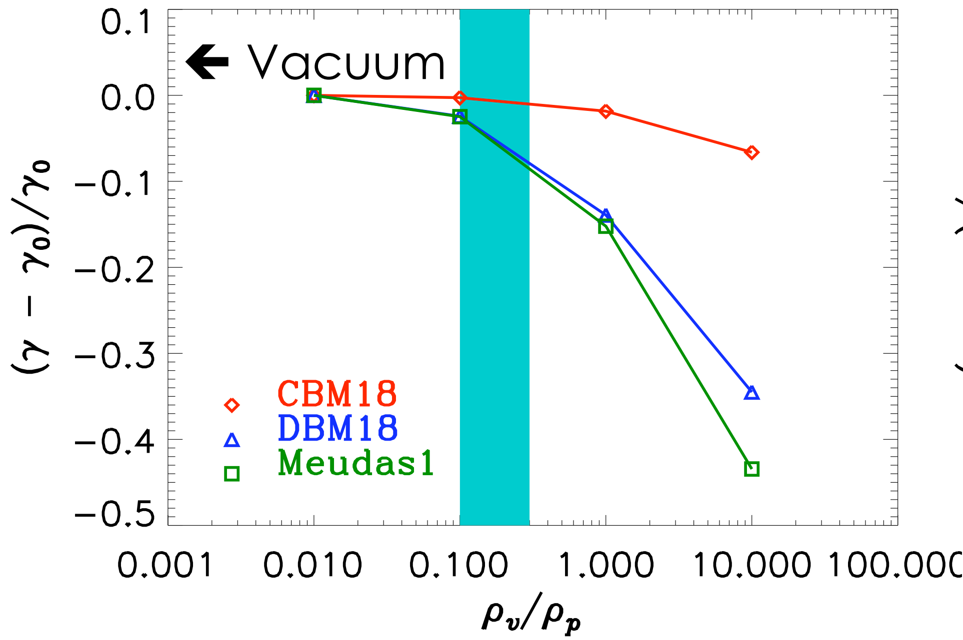
Rotational Destabilization Important for Diverted Case

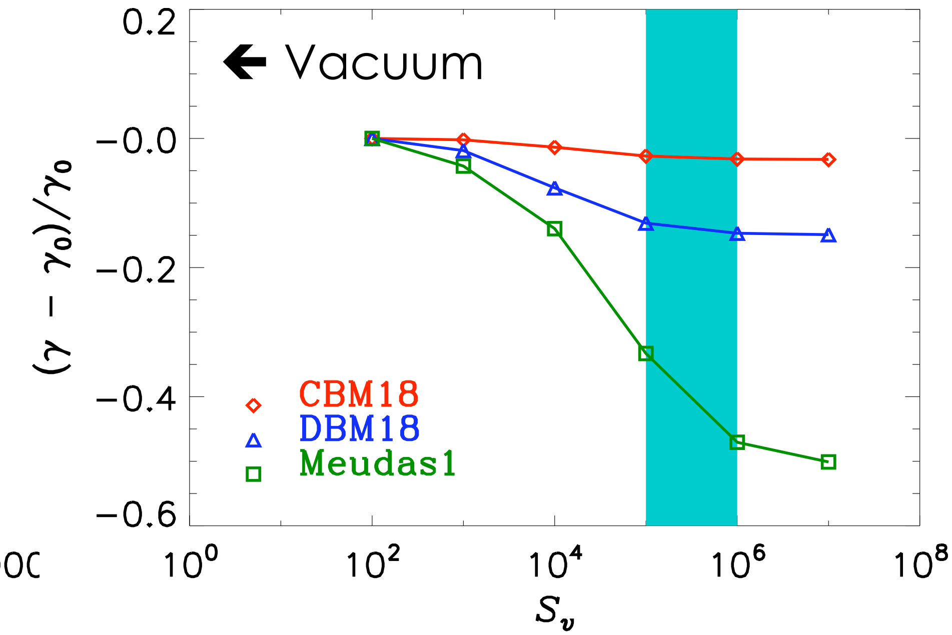
- **Low-*n* modes in the diverted equilibrium are more significantly destabilized**



SOL is not a Vacuum

- How do SOL ρ and η affect growth rates?





- Realistic SOL densities are similar to vacuum model
- Realistic SOL resistivity less similar to vacuum model

Stabilization by Other Non-Ideal Effects

- A simple model for stabilization is:

$$\boxed{1-\frac{\gamma}{\gamma_0}=\left(\frac{D}{D_{crit}}\right)^m}$$

- Here *D* measures size of non-ideal term (e.g. χ). *D* ≥ D_{crit} implies stability.

- γ_0 is the growth rate without the non-ideal effect

- All cases here are run using Spitzer resistivity

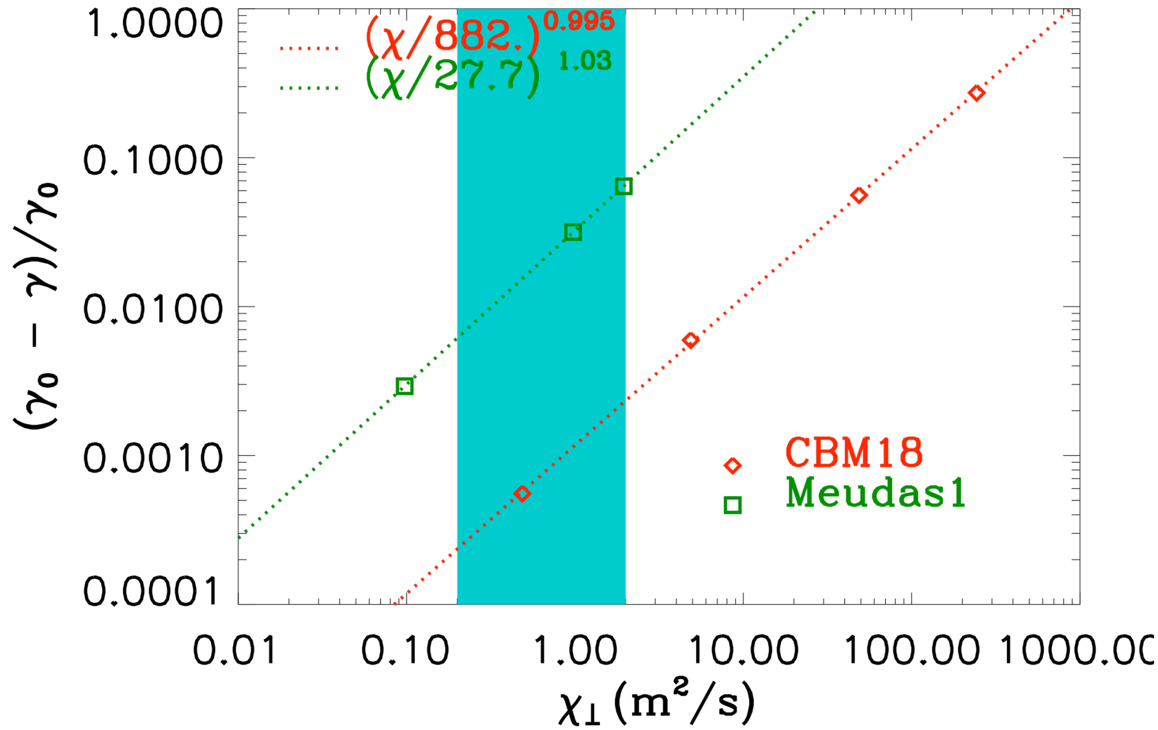
$$\gamma_0=\gamma_{Spitz}$$

Perpendicular Thermal Conductivity

- **Assuming** $\chi_\perp = 1\,\mathrm{m}^2/\mathrm{s}$, **CBM and Meudas cases are stabilized at**

$$n_{crit}^{CBM} \approx 800$$

$$n_{crit}^{Meudas} \approx 40$$



$$n = 10$$

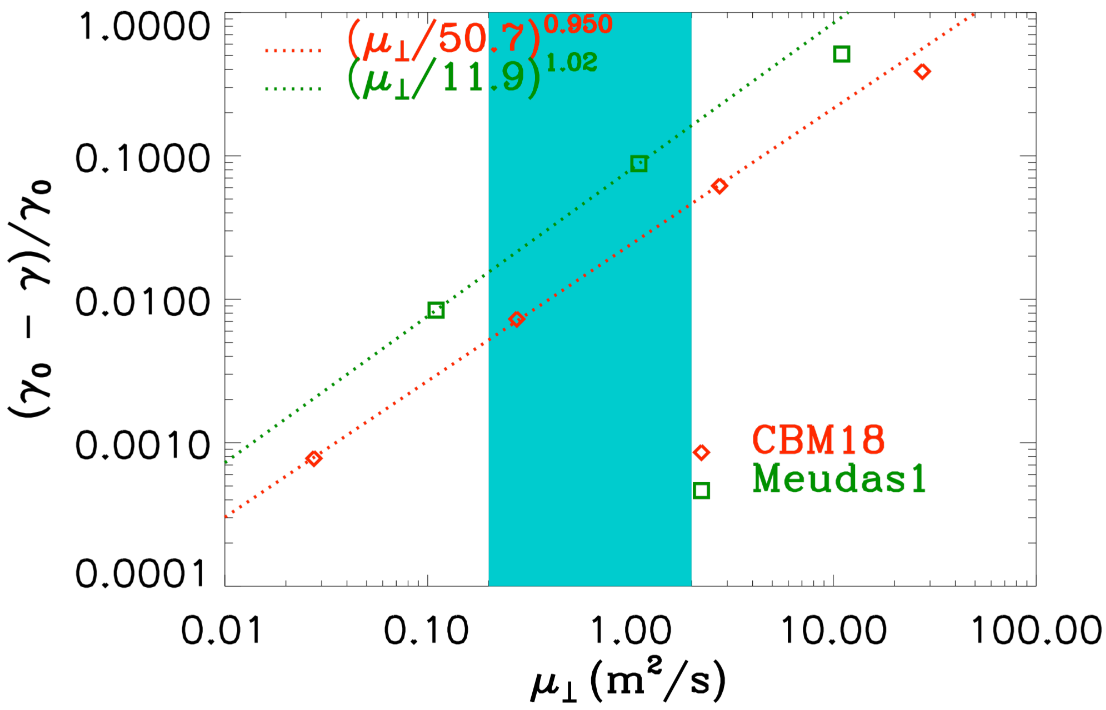
Perpendicular Viscosity is Stabilizing

- **Assuming $\mu_\perp = 1\ \mathrm{m^2/s}$, CBM and Meudas cases are stabilized at**

$$n_{crit}^{CBM} \approx 240$$

$$n_{crit}^{Meudas} \approx 40$$

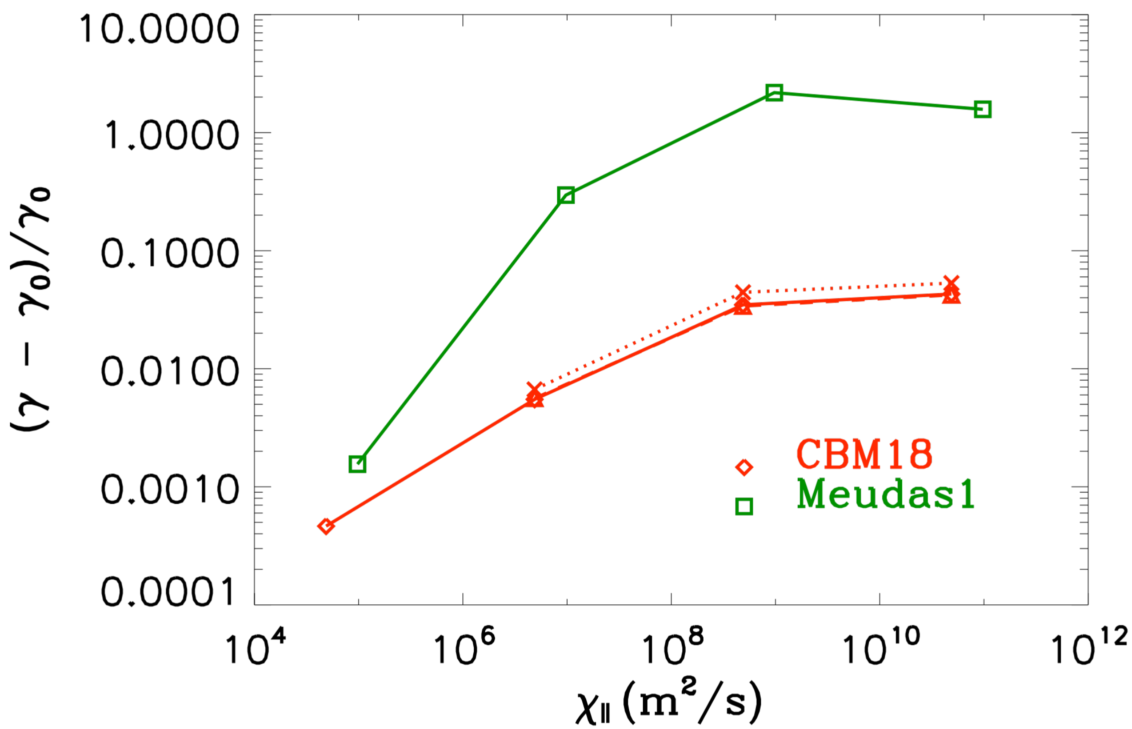
- **Viscosity does not yield a clean "cutoff"**



$$n = 10$$

Parallel Thermal Conductivity is Destabilizing

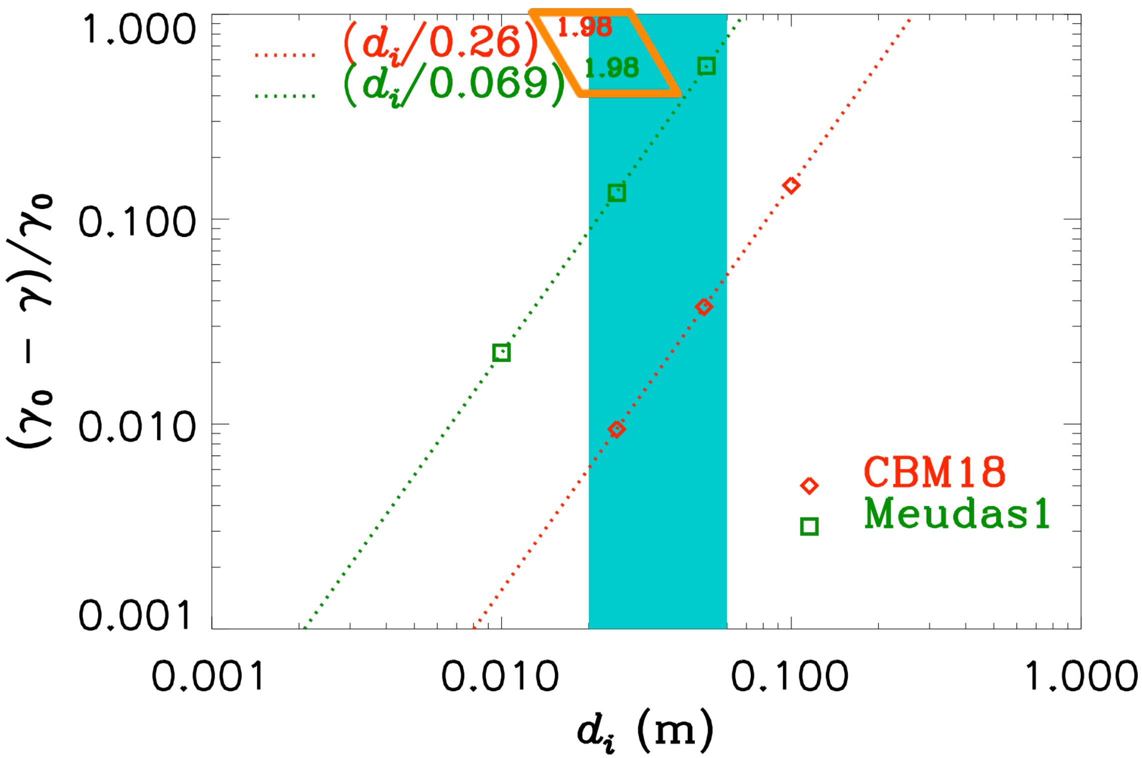
- **Growth rate increases, saturates as $\chi_{\parallel}$ increases**



$n = 10$

- **Lower-resolution runs give same result**
- **Could have some relation to MTI**

Stabilization by Gyroviscosity: Results



$n = 10$

- **CBM and Meudas cases are stabilized by gyroviscosity at**

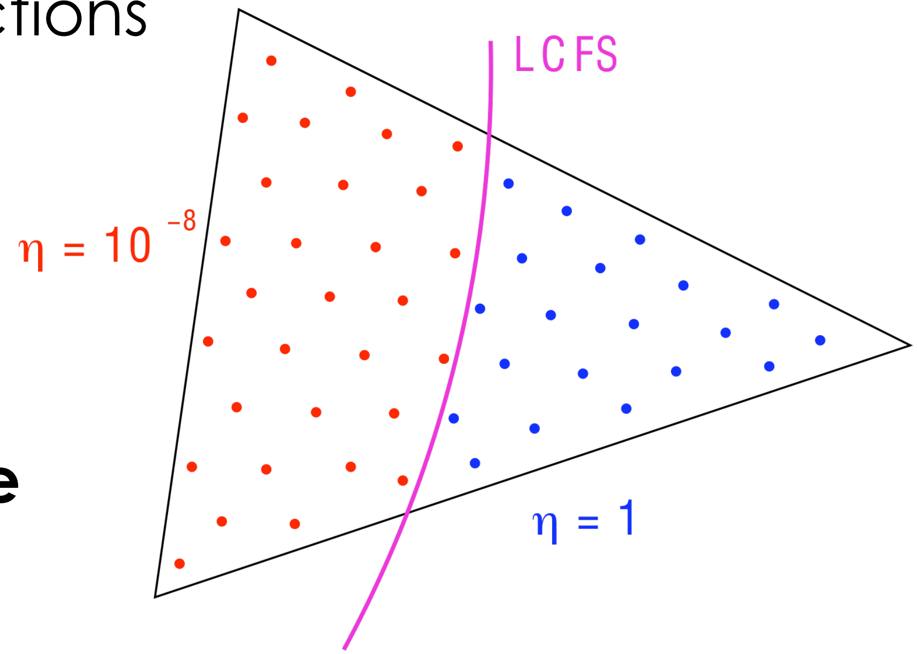
$$n_{crit}^{CBM} \approx 120$$

$$n_{crit}^{Meudas} \approx 14$$

- **In reality, Meudas cases is stabilized at** $n_{crit}^{Meudas} \approx 20$

Ideal Benchmark: Plasma-Vacuum Interface

- **M3D-C1 Solution to plasma-vacuum interface:**
  - Don't represent η, ρ on finite element basis
  - Instead, calculate $\eta(\psi)$, $\rho(\psi)$
  - ψ is a smooth function
  - η, ρ are true step functions



- **If density is dynamical, ρ must be represented on finite element basis**

- **No reason for η ever to be represented on finite element basis**

M3D-C1: Time Step Methods

- **M3D-C1 uses a split time step**

$$(1-\theta^2\delta t^2 L)u^{n+1} = (1-\alpha\delta t^2 L)u^n + \delta t\, F(B^n)$$

$$B^{m+1} - \theta\,\delta t\, G(u^{n+1}) = B^m + (1-\theta)\delta t\, G(u^n)$$

  - Split Crank-Nicholson:

$$\alpha = \theta(\theta - 1)$$

$$m = n$$

  - "Implicit Leapfrog":

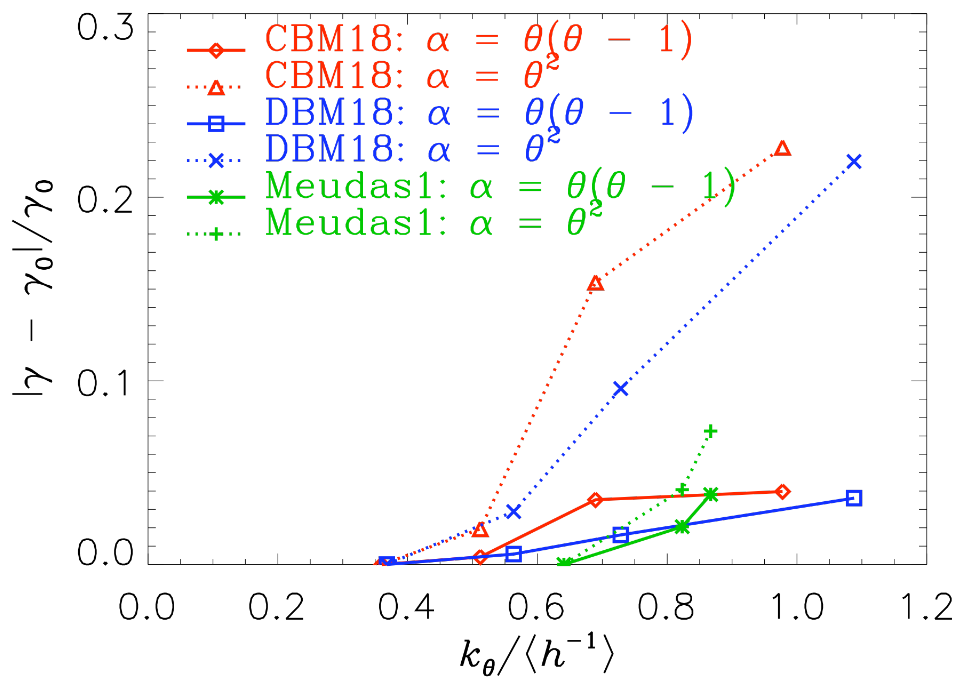
$$\alpha = \theta^2$$

$$m = n + 1/2$$

M3D-C1: Time Step Methods

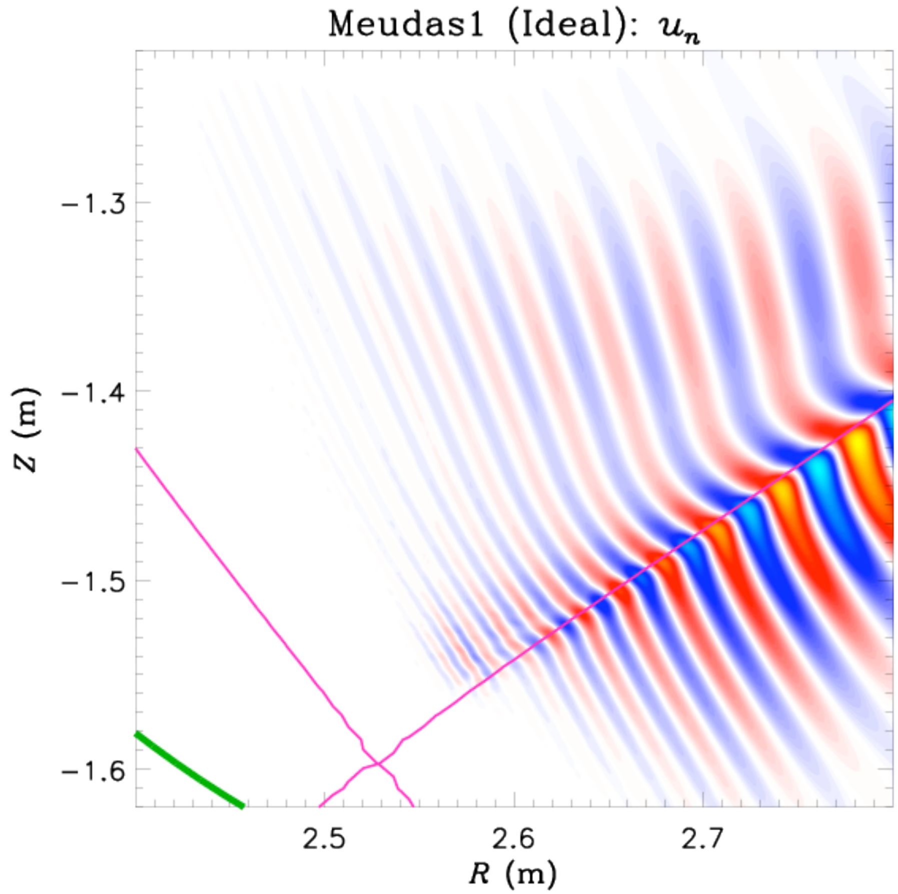
- **Implicit Leapfrog introduces less numerical dissipation than split CN.**

- **For some problems, IL converges faster with δt**
  - Ferraro, Jardin, *JCP* **228**(20):7742 (2009)



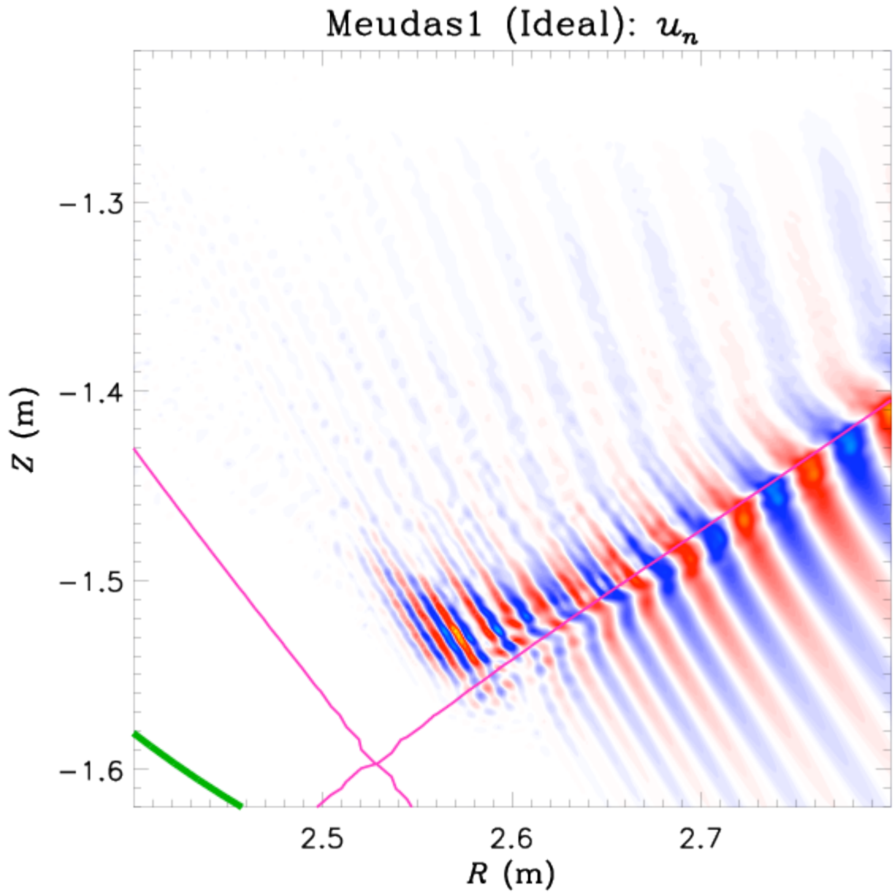
- **In linear peeling-ballooning calculations, split CN converges faster with δx than IL**
  - IL does not damp grid-scale oscillations arising from unresolved spatial structures
  - We were not able to overcome problem with explicit damping terms

Split CN is Smoother Than Implicit Leapfrog



Split CN

$$\alpha = \theta(\theta - 1)$$



Implicit Leapfrog

$$\alpha = \theta^2$$

Conclusions

- **Successfully reproduced ideal results with M3D-C1**
- **Resistive SOL is more accurately modeled as force-free plasma than as a vacuum (factor of 2 difference in diverted case)**
  - Within plasma, Spitzer resistivity same as "ideal."
- **Growth rates are more sensitive to moving cutoff inward from separatrix than outward**
  - Good news: sensitivity is to ρ, not η
- **Wall stabilization is insignificant except (possibly) at very low-*n* ($n \leq 3$).**
- **Crank-Nicholson converges better without explicit diffusive term than Implicit Leapfrog for this application**
  - Likely due to spatial unresolvability