On FSP and Extended MHD Models

J. D. Callen presented by C. C. Hegna
University of Wisconsin
Madison, WI

Basic philosophy is articulated in recent publications:

- J. D. Callen et al "Toroidal rotation in tokamak plasmas," Nucl Fusion 49, 085021 (2009).
- J. D. Callen et al, "Toroidal flow and particle flux in tokamak plasmas," PoP 16, 082504 (2009).
- J. D. Callen et al, "Transport equations in tokamak plasmas," PoP 17, 056113 (2010).

2010 CEMM Meeting
Seattle, WA
April 17

The advocated approach to FSP is consistent with basic CEMM philosophy

- One set of equations --- consistent with the basic extended MHD philosophy
  - Capability to access long transport times scales while retaining the ability to accurately portray faster timescale effects

- Outline
  - Desired physics capability that is ultimately required
  - What equations?
    - Why not gyokinetics for the entire plasma description?
  - Proposed hybrid kinetic/fluid model
  - What should extended MHD codes be doing now to proceed

What physics need to be included ultimately?

- Full magnetic geometry from hot core, through separatrix, out to divertor/walls and coils
- MHD waves and their constraints (in 3D) -- compressional, shear Alfven and sound
- Collisional equilibration to Maxwellian and poloidal flows, neo Ohm's law in tokamak plasmas
- Second order in gyroradius transport fluxes due to collisional and microturbulence processes
- Transport (mainly particle) fluxes caused by first order 3D departures from axisymmetry
- Relevant sources and sinks from heating, non-inductive CD, neutrals, nuclear reactions, etc.
- Extra "energetic particle" effects due to NBI, ICRF and AEs, to extent they are separable, additive

What physics need to be included ultimately? (continued)

- "Radial" motion and diffusion of poloidal and toroidal magnetic fluxes on magnetic diffusion time scale -- but faster for localized non-inductive current drives
- Third order plasma toroidal rotation equation (since V_t ~ first order) -- from toroidal torques --- more later --- use radial force balance and neoclassical poloidal flow damping to write $V_t \cong \frac{E_r}{B_p} - \frac{1}{n_i q_i}\frac{dp_i}{dr} + \frac{B_t}{B_p}V_p \cong \frac{E_r}{B_p} - \frac{1}{n_i q_i}\frac{dp_i}{dr} + \frac{1.17}{q_i B_p}\frac{dT_i}{dr}$
- Allow (in long run) for changes in topology to magnetic islands and locally stochastic fields
  - but only allow gyroradius small 3D perturbations (except fast MHD processes: ELMs, disruptions)
- Long-term issue -- what to do when lowest order distribution is not Maxwellian (e.g., due to ICRH)

IIC. Toroidal Rotation Eqn. Includes Many Different Effects

- Equation for the toroidal angular momentum density $L_t \equiv m_i n_{i0} \langle R^2 \Omega_t \rangle$ is:

$$\boxed{\underbrace{\frac{1}{V'}\frac{\partial}{\partial t}\bigg|_{\psi_p}(V' L_t)}_{\text{inertia}} \simeq -\underbrace{\langle \vec{e}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{i\parallel}^{\text{NA}} \rangle}_{\text{NTV from } \tilde{B}_\parallel} - \underbrace{\langle \vec{e}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{i\perp} \rangle}_{\text{cl, neo, paleo}} - \underbrace{\frac{1}{V'}\frac{\partial}{\partial \rho}(V' \Pi_{i\rho\zeta})}_{\text{Reynolds stress}} + \underbrace{\langle \vec{e}_\zeta \cdot \overline{\tilde{\vec{J}} \times \tilde{\vec{B}}} \rangle}_{\text{res. FE, Max}} - \underbrace{\dot{\rho}_{\psi_p} \frac{\partial L_t}{\partial \rho}}_{\psi_p \text{ motion}} + \underbrace{\langle \vec{e}_\zeta \cdot \textstyle\sum_s \tilde{\vec{S}}_{sm} \rangle}_{\text{sources}}.}$$

- Neoclassical toroidal viscous (NTV) damping[9] by 3D $\tilde{\vec{B}}$ fields of Ω_t to Ω_*:

$$-\langle \vec{e}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{i\parallel}^{\text{NA}} \rangle \simeq -m_i n_{i0} \mu_{i\,t} \left(\frac{\tilde{B}_{\text{eff}}}{B_0}\right)^2 (\langle R^2 \Omega_t \rangle - \langle R^2 \Omega_* \rangle), \quad \Omega_* \simeq \frac{c_p + c_t}{q_i} \frac{dT_i}{d\psi_p}, \text{ offset velocity.}$$

Damping frequency $\mu_{it} \sim 1/\omega_E^2$ in low ν regime yields max NTV torque where $\vec{E} \times \vec{B}_0 \to 0$.[9]

- Collisional $\perp$ viscous stresses are dominated by paleoclassical processes:

$$-\langle \vec{e}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{i\perp} \rangle \simeq -\frac{1}{V'}\frac{\partial}{\partial \rho}\left[V'\left(\bar{D}_\eta \frac{\partial L_t}{\partial \rho} + L_t V_{\text{pc}}\right)\right], \qquad \text{only significant for } T_e \lesssim 1 \text{ keV.}$$

- Microtubulence-induced ion Reynolds stresses cause radial transport of L_t:[9]

$$\Pi_{i\rho\zeta} \equiv m_i n_{i0} \langle \overline{(\vec{\nabla}\rho \cdot \tilde{\vec{V}}_i)(\tilde{\vec{V}}_i \cdot \vec{e}_\zeta)} \rangle + \langle \vec{\nabla}\rho \cdot \overline{\overleftrightarrow{\pi}_{i\wedge}} \cdot \vec{e}_\zeta \rangle \sim \underbrace{-\chi_t \frac{\partial L_t}{\partial \rho}}_{\text{diffusion}} + \underbrace{L_t V_{\text{pinch}}}_{\text{pinch}} + \underbrace{\Pi_{i\rho\zeta}^{\text{RS}}}_{\text{Residual Stress}} .$$

[9] See experimental studies of these effects see next invited talk NI3.002: W.M. Solomon, "Generation and Sustainment of Rotation in Tokamaks."

What equations?

- Advocating fluid moment approach with kinetically-determined closures --- this leads naturally to fluid equations ala extended MHD- ----- pre-Maxwell equations
- Why not gyrokinetics for entire plasma description?
  - Maxwell equations only density and current moments
  - Present gyrokineitic incarnations use and/or are limited to:
    - Axisymmetric reduced MHD model for B-field geometry (no 3-D effects or evolution)
    - First order in gyro-radius
      - Second order has singularities and may not be rigorously definable (Sugiyama)
      - Implementing all second order terms would be heroic if not impractical

What equations?

- Why not gyrokinetics for entire plasma description? (continued)
  - Current gyrokinetic issues illustrate needs for long time scales required for realistic simulations
    - Combinations of electron and ion parallel streaming time scales
    - Electromagnetic plus electrostatic effects (fluid moments are being used to develop algorithms)
    - Collisional equilibration with full F-P operator to obtain poloidal flows, neo Ohm's law
- A hybrid kinetic/fluid approach should be possible and best because
  - Only first order gyrokinetics is needed to obtain Maxwell and Reynolds stresses, heat fluxes
  - Fluid model can supply needed E_r (or V_t) from 3rd order fluid equation
  - Also, gyrokinetics does not want (or need) to deal with all the transport time scale effects --- heat and particle sources and sinks, small 3D fields, poloidal flux motion and diffusion,etc.

Proposed Hybrid Kinetic/Fluid Model

- Assume axisymmetric magnetic field to lowest order in gyroradius
- Expand non-axisymmetries (due to fluctuations, field errors, etc.) in Fourier expansion
  - Assume Fourier spectrum is approximately separable (this may be questionable; check later) --- handle n = 0 and n up to say 20 fluid-like responses with extended MHD codes (M3D, NIMROD) --- handle n > 20 with gyrokinetics (presumably in Chapman-Enskog form for self-consistency)
- Couple the fluid/MHD and gyrokinetic descriptions/simulations as follows:
  - Solve n = 0, 2D multi-collisionality extended MHD model for axisymmetric flux surfaces --- for "equilibrium" plasma description use available plasma profiles and Omega_t, E_r
  - use analytic parallel stress closures (UW-CPTC 09-6R), later solve kinetics (Held, Ramos)
  - evolve n = 0 magnetic field on transport time scale, determine poloidal flux surfaces of it
  - Evolve n = 1 to 20 modes in extended MHD simulation (proceed only if MHD effects are "modest")
    - calculate Reynolds, Maxwell stresses induced by n = 1 to 20 modes

Proposed Hybrid Kinetic/Fluid Model

- Run gyrokinetics using latest available plasma profiles, including Omega_t, E_r profiles
  - determine microturbulence and the Reynolds, Maxwell stresses and heat fluxes it induces
- Combine preceding two sets of results in extended MHD transport time scale evolution simulation
- solve for all $n = 0$ plasma profiles, poloidal flux surfaces and Omega_t, E_r
- Cycle through these steps, perhaps only doing gyrokinetics every 10-100 extended MHD time steps
- note: first $n = 0$ step only needed in first iteration or for collision time scale changes

- Possible complication: how to treat magnetic island effects due to tearing-type modes:
  - Treat regions outside island as above with island just producing $n > 0$ magnetic perturbations?
  - But really need full 3D magnetic topology (or 2D helical for thin island) inside island
  - Not obvious how to connect these two descriptions across (stochastic?) island separatrix.

What should extended MHD codes be doing now to proceed in this direction?

- Continue development of "fast MHD" hybrid-type models (SWIM) where only gyroviscosity is needed
- Facilitate "slow MHD" models by adding and including collisional parallel stress effects:
  - Analytic multi-collisionality parallel stress --- JDC CPTC Seminar Dec. '09
  - in long run one could obtain these effects from Held, Ramos kinetic approaches
- Begin to develop n = 0 2-D extended MHD model on transport time scale
- Seek to develop "extended MHD transport model" like that in UW-CPTC 09-11R inside separatrix
  - Decide on and test out (verify) procedure on some model transport problems (e.g. OH equil.)
  - Decide how to develop a reasonable 2D plasma/boundary model outside the separatrix

Some CEMM Issues to Discuss

- Is this hybrid fluid/kinetic model the right approach for FSP?
  - what might be its limitations?
- What next steps should CEMM do to proceed in this direction?
  - as indicated above? or ?
- Other issues?